1a

Here,
approximate value of π (x_a) = 3.1428571
true value " " (x_t) = 3.1415926

absolute error = $|x_t - x_a|$
= |3.1415926 - 3.1428571|
= 0.0012645

relative error = $\frac{x_t - x_a}{x_t}$

= $\frac{-0.0012645}{3.1415926}$

= -0.0004025

C ~~erro~~
relative error percentage = 0.0004025 × 100%
= 0.0402502 %
(percentage cann't be -ve

7

1b

given,

$f(x) = 3x + \sin x - e^x$ using false position method.

x	$f(x)$
0	-1
1	1.12318

Here, $x_0 = 0$, $f_0 = -1$

$x_1 = 1$, $f_1 = 1.12318$

Since, $f_0 \cdot f_1 < 1$, root lies between 0 & 1.

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0}$$

$$= \frac{0 \times 1.12318 - 1 \times (-1)}{1.12318 + 1}$$

$$= 0.47098$$

$$f(x_2) = 3 \times 0.47098 + \sin(0.47098) - e^{0.47098}$$

$$= 0.26516$$

1b

iterative table:

itno	x_0	f_0	x_1	f_1	x_2	$f(x_2)$
1	0	-1	1	1.12318	0.47099	0.26516
2	0	-1	0.47099	0.26516	0.37228	0.02953
3	0	-1	0.37228	0.02953	0.36160	0.00295
4	0	-1	0.36160	0.00295	0.36054	0.00029
5	0	-1	0.36054	0.00029	0.36044	0.00003
6	0	-1	0.36044	0.00003	0.36043	0.00002

$\therefore$ root of $f(x)$ upto four decimal point $= 0.3604$.

10

2a

given,

$f(x) = x^3 + x - 1$ using secant method.

x	$f(x)$
0	-1
1	1

Here, $x_0 = 0, \ f_0 = -1$

$x_1 = 1, \ f_1 = 1$

$$x_2 = \frac{x_0 \times f_1 - x_1 f_0}{f_1 - f_0}$$

$$= \frac{0 \times (1) - 1 \times (-1)}{1 + 1}$$

$$= 0.5$$

$$f(x_2) = 0.5^3 + 0.5 - 1$$

$$= -0.37500$$

x^3+x-1

2a

iterative table:

itno	x_0	f_0	x_1	f_1	x_2	$f(x_2)$
1	0	-1	1	1	0.5	-0.37500
2	1	1	0.5	-0.37500	0.63636	-0.10594
3	0.5	-0.37500	0.63636	-0.10594	0.69005	0.01863
4	0.63636	-0.10594	0.69005	0.01863	0.68202	-0.00074
5	0.69005	0.01863	0.68202	-0.00074	0.68233	0.00000

Since, $f(x_2)=0$ which has error less than 10^{-4}, So root of $f(x)=0.6823$

9

Q1.

Algorithm for bisection method

1) Start
2) define $f(x)$
3) choose initial guess x_0 and x_1 such that $f_0 f_1 < 0$.
4) ~~find~~ define tolerable error (e)

5) calculate, $x_2 = \frac{x_1 + x_0}{2}$

6) calculate $f(x_2)$
i) if $f(x_2) = 0$, then root $= x_2$ and goto stop.

7) i) if $f_0 \cdot f(x_2) < 0$ set,
$x_0 = x_0$, $x_1 = x_2$
$f_0 = f_0$, $f_1 = f(x_2)$ and goto step 8

ii) if $f_0 \cdot f(x_2) > 0$ set,
$x_0 = x_2$, $x_1 = x_1$
$f_0 = f(x_2)$, $f_1 = f_1$ and goto step 8

iii) if $f(x_2) = 0$, then root $= x_2$ and goto stop.

8. if $|f(x_2)| < e$, goto stop
else goto step 5.

9. Stop.

3a

given,

$f(x) = \log x - \cos x$

$f'(x) = \frac{1}{x} + \sin x$

x	$f(x)$
2	1.20929
3	2.08860

using, newton's method:

let, initial guess $(x_0) = 2.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2.5 - \frac{1.71743}{0.99847}$

$= 0.77994$

$f(x_1) = \log(0.77994) - \cos(0.77994)$

$= -0.95949$

$f'(x_1) = 1.98539$

Q.a.

Iterative table

itno.	x_0	$f(x_0)$	$f'(x_0)$	x_1	$f(x_1)$
1	2.5	1.71743	0.99847	0.77597	-0.35949
2	0.77597	-0.35949	1.98539	1.26322	-0.06909
3	1.26322	-0.06909	1.74470	1.30282	-0.00025
4	1.30282	-0.00025	1.73188	1.30294	-0.00001

since, $|f(x_1)| = 0$ which has less error than tolerable error.

$\therefore$ root of $f(x) = 1.3029$

9

Q.a

given system of equations:

$$2x - y - 7z = 3$$
$$5x - 2y + 3z = -1$$
$$-3x + 9y + z = 2$$

given system in matrix form:

$$\begin{bmatrix} 2 & -1 & -7 \\ 5 & -2 & 3 \\ -3 & 9 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$AX = B \quad -① $$

where,

$$A = \begin{bmatrix} 2 & -1 & -7 \\ 5 & -2 & 3 \\ -3 & 9 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

4 a

again using LU decomposition method:

$A = LU$

using crout's factorization method:

$$\begin{bmatrix} 2 & -1 & -7 \\ 5 & -2 & 3 \\ -3 & 9 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

Comparing:

$L_{11} = 2$, $L_{21} = 5$, $L_{31} = -3$

$L_{11}U_{12} = -1 \Rightarrow U_{12} = -1/2$

$L_{11}U_{13} = -7 \Rightarrow U_{13} = -7/2$

$L_{21}U_{12} + L_{22} = -2$

$5 \times (-1/2) + L_{22} = -2 \Rightarrow L_{22} = 1/2$

$L_{21}U_{13} + L_{22}U_{23} = 3$

$5 \times (-7/2) + 1/2 \times U_{23} = 3 \Rightarrow U_{23} = \cancel{40} + 4$

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4a

again using LU decomposition method:

$A = LU$

using crout's factorization method:

$$\begin{bmatrix} 2 & -1 & -7 \\ 5 & -2 & 3 \\ -3 & 9 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} L_{11} & L_{11}U_{12} & L_{11}U_{13} \\ L_{21} & L_{21}U_{12}+L_{22} & L_{21}U_{13}+L_{22}U_{23} \\ L_{31} & L_{31}U_{12}+L_{32} & L_{31}U_{13}+L_{32}U_{23}+L_{33} \end{bmatrix}$$

comparing:

$L_{11} = 2, \quad L_{21} = 5, \quad L_{31} = -3$

$L_{11}U_{12} = -1 \Rightarrow U_{12} = -1/2$

$L_{11}U_{13} = -7 \Rightarrow U_{13} = -7/2$

$L_{21}U_{12} + L_{22} = -2$

$5 \times (-1/2) + L_{22} = -2 \Rightarrow L_{22} = 1/2$

$L_{21}U_{13} + L_{22}U_{23} = 3$

$5 \times (-7/2) + 1/2 \times U_{23} = 3 \Rightarrow U_{23} = \cancel{40} \; 41$

-essing ?

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process multiple once.

→ Disadvantages

• more memor

tion

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4a

$L_{31}U_{12} + L_{32} = 9$

$-3 \times (-1/2) + L_{32} = 9$

$\Rightarrow L_{32} = 15/2$

$L_{31}U_{13} + L_{32}U_{23} + L_{33} = 1$

$-3 \times (-7/2) + 15/2 \times \cancel{76}\ 41 + L_{33} = 1$

$L_{33} = -317$

$$\therefore L = \begin{bmatrix} 2 & 0 & 0 \\ 5 & 1/2 & 0 \\ -3 & 15/2 & -317 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1/2 & -7/2 \\ 0 & 1 & 41 \\ 0 & 0 & 1 \end{bmatrix}$$

eqn ① can be written as:

$L(UX) = B$

again, $UX = Z$ — (ii)

$LZ = B$ — (iii)

Solving (iii)

$$\begin{bmatrix} 2 & 0 & 0 \\ 5 & 1/2 & 0 \\ -3 & 15/2 & -317 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2z_1 \\ 5z_1 + 1/2 z_2 \\ -3z_1 + 15/2 z_2 - 317 z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$z_1 = 3/2$

$5 \times 3/2 + \frac{1}{2} z_2 = -1$

$z_2 = -17$

$-3 \times 3/2 + 15/2 \times (-17) - 317 z_3 = 2$

$z_3 = -\frac{134}{137}$

solving: (11)

$$UX = Z$$

$$\begin{bmatrix} 1 & -1/2 & -7/2 \\ 0 & 1 & 41 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/2 \\ -17 \\ -134/137 \end{bmatrix}$$

$$\begin{bmatrix} x - 1/2\,y - 7/2\,z \\ y + 41z \\ z \end{bmatrix} = \begin{bmatrix} 3/2 \\ -17 \\ -134/137 \end{bmatrix}$$

$$\therefore \boxed{z = \frac{-134}{137}}$$

$$y + 41 \times \left(\frac{-134}{137}\right) = -17$$

$$\therefore \boxed{y = \frac{3165}{137}}$$

$$x - \frac{1}{2} \times \frac{3165}{137} - \frac{7}{2} \times z = \frac{3}{2}$$

16

$$\therefore \boxed{x = \frac{2747}{137}}$$

$$\therefore \boxed{\begin{aligned} x &= 20.05 \\ y &= 23.10 \\ z &= -0.97 \end{aligned}}$$

3b

Given system of equations:

$$10x_1 + 3x_2 + 4x_3 = 15 \quad \text{(i)}$$
$$2x_1 - 10x_2 + 3x_3 = 37 \quad \text{(ii)}$$
$$3x_1 + 2x_2 - 10x_3 = -10 \quad \text{(iii)}$$

Solving (i), (ii) & (iii) for x_1, x_2, x_3 respectively

$$x_1 = \frac{1}{10}[15 - 3x_2 - 4x_3] \quad (1)$$

$$x_2 = -\frac{1}{10}[37 - 2x_1 - 3x_3] \quad (2)$$

$$x_3 = -\frac{1}{10}[-10 - 3x_1 - 2x_2] \quad (3)$$

put, $x_2 = x_3 = 0$ for initial
using gauss seidel method:

$$x_1 = \frac{1}{10}[15] = \frac{3}{2} = 1.50$$

$$x_2 = -\frac{1}{10}[37 - 2 \times 1.50 - 0]$$
$$= -3.40$$

$$x_3 = -\frac{1}{10}[-10 - 3 \times 1.50 - 2 \times (-3.40)]$$
$$= 0.77$$

4b

Iterative table :

it.no	x_1 from eqⁿ ①	x_2 from eqⁿ ②	x_3 from eqⁿ ③
1	1.56	-3.40	0.77 1.05
2	2.21	~~1.89~~ -3.02	~~2.64~~ ~~3.02~~
~~3~~	~~0.11~~	~~1.64~~	~~1.36~~
~~4~~	~~0.46~~	~~-3.26~~	~~0.79~~
3	1.98	-2.98	0.99
4	~~2.00~~ 2.00 ~~1.99~~	-3.00	0.99
5	2.00	-3.00	0.99

$\therefore$

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= -3 \\ x_3 &= 0.99 \end{aligned}}$$

6

5a.

Given systems:

$$5x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 + 7x_2 + x_3 + x_4 = 12$$
$$x_1 + x_2 + 6x_3 + x_4 = -5$$
$$x_1 + x_2 + x_3 + 4x_4 = -6$$

augmented coefficient matrix:

$$A = \left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & 6 \end{array}\right]$$

Using gauss elimination partial pivoting method:

$R_2 \to 5R_2 - R_1$, $R_3 \to 5R_3 - R_1$, $R_4 \to 5R_4 -$

$$\left[\begin{array}{cccc|c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 19 & 56 \\ 0 & 4 & 29 & 4 & -29 \\ 0 & 4 & 4 & 19 & 26 \end{array}\right]$$

5a

$R_3 \to \frac{17}{2} R_3 - R_2, \quad R_4 \to \frac{17}{2} R_4 - R_2$

$$\left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 0 & 34 & 4 & 19 & 56 \\ 0 & 0 & 485/2 & 15 & -605/2 \\ 0 & 0 & 0 & 285/2 & 165 \end{array}\right]$$

$\because \quad \frac{285}{2} x_4 = 165 \quad -①$

$$x_4 = \frac{22}{19} = 1.15 \#$$

$\because \frac{485}{2} x_3 + 15x_4 = -605/2$

$$\frac{485}{2} x_3 + 15 \times \frac{22}{19} = -\frac{605}{2}$$

6

$$x_3 = -\frac{2431}{1843} = -1.31 \#$$

$$34x_2 + 4x_3 + 19x_4 = 56$$
$$x_2 = \frac{2129}{1843} = 1.15 \#$$

$$5x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 = \frac{1108}{1843} = 0.60 \#$$

5b.

Given system:

$$2x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 2x_2 + 3x_3 = 6$$
$$x_1 - x_2 + x_3 = 0$$

Augmented matrix:

$$A = \left[\begin{array}{ccc:c} 2 & 2 & 1 & 6 \\ 4 & 2 & 3 & 6 \\ 1 & -1 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to 2R_3 - R_1$

$$\left[\begin{array}{ccc:c} 2 & 2 & 1 & 6 \\ 0 & -2 & 1 & -6 \\ 0 & -4 & 1 & -6 \end{array}\right]$$

$R_1 \to R_1 + R_2$, $R_3 \to R_3 \pm 2R_2$

$$\left[\begin{array}{ccc:c} 2 & 0 & 2 & 0 \\ 0 & -2 & 1 & -6 \\ 0 & 0 & -1 & 6 \end{array}\right]$$

$R_1 \to R_1 + 2R_3$, $R_2 \to R_2 + R_3$

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Sol.

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 12 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 6 \end{array}\right]$$

$R_1 \to R_1/2$, $R_2 \to R_2/-2$, $R_3 \to R_3/-1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$\therefore$
$$\boxed{\begin{aligned} x_1 &= 6 \\ x_2 &= 0 \\ x_3 &= -6 \end{aligned}}$$

7

6a.

Sol. x be the cube root of 11.

i.e. $x = 11^{1/3}$

$f(x) = x^3 - 11$

$f'(x) = 3x^2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{16}{27} = 2.407$

x	$f(x)$
0	-11

Sol. initial guess $(x_0) = 3$

$f(x_0) = 3^3 - 11 = 16$

$f'(x_0) = 3 \times 3^2 = 27$

iterative table

x_0	$f(x_0)$	$f'(x_0)$	x_1	$f(x_1)$
3	16	27	2.40741	2.95240
2.40741	2.95240	26.15000	2.29451	1.08004
2.29451	1.08004	15.79493	2.22613	0.03191
2.22613	0.03191	14.86696	2.22398	0.00000

Since, $f(x_2) = 0$

$\therefore$ root of $f(x) = 2.22398$ #

neural networks.

6b

Error: Error is the difference between actual value and calculated value.

i.e. $\boxed{error = X_a - X_c}$

There are different types of error arises during calculations like trucation error, inherent errors etc.

a) Round off errors

Round off errors are arised due to roundoff decimal point.

Let, we have, $x = \frac{100}{3} = 33.3333...$

if we write, $x = 33.33$ it is rounded.

$x = 33.333$ is more accurate than $x = 33.33$

Solution: write more number after decimal point.

To get more precise value we must use as more as possible the bit after decimal point.

6b

b) Truncation Errors.

Truncation Errors are arised due to replacing infinite terms by a finite one.

Example: we have a series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

let, put $x = 2$

$$e^2 = 1 + 2 + \frac{2^2}{2!} = 5 \quad -(i)$$

$$e^2 = 1 + 2 + \frac{2^2}{2} + \frac{2^3}{3!} = 6.33 \quad -(ii)$$

eqn (ii) is more accurate than eqn (i).

Solution: writing more term reduce the truncation error. The more you need accuracy, the more you write the terms.

7

$x^2(x+1) = 1$
$x = 1$
$x = 5$

3b

$f(x) = x^3 + x^2 - 1$

x	$f(x)$
0	-1
1	1

$\therefore$ initial guess $= 0.5$

Using, fixed point iteration method:

$$x = g(x) = (1 - x^2)^{1/3}$$

$$g'(x) = \frac{1}{3}(1 - x^2)^{2/3}(-2x)$$

$$= -\frac{2x}{3}(1 - x^2)^{2/3}$$

$$g'(0.5) = -0.20637 < 1$$

$$x_1 = g(0.5) = (1 - 0.5^2)^{1/3} = 0.90856$$

$$x_2 = (1 - 0.90856^2)^{1/3} = 0.55883$$

$$x_3 = (1 - 0.55883^2)^{1/3} = 0.88268$$

Similarly,

$$x_4 = 0.60448$$

$$x_5 = 0.85935$$

36

$x_6 = (1 - 0.85935^2)^{1/3}$

$= 0.63949$

$x_7 = (1 - 0.63949^2)^{1/3}$

$= 0.83922$

$x_8 = 0.66623$

5

and so on ...