Path Planning Method Design and Dynamic Model Simplification of Free-Flying Space Robot

Zhuoyi XING, Yang ZHAO*, Shunjie Zhu

Abstract—**Space robot is indispensable for complex operations such as in-orbit maintenance of spacecraft, space debris defense, etc.. In this paper, a path planning method based on joint parameters is proposed to resolve the problem of dynamic singularity while planning the trajectory for free-flying space robot. Parameters-interpolating is used in this path planning method to convert the equation of path planning motion based on generalized Jacobian matrix to non-linear equations with the solution of numerical calculation. In this way, the path planning process of free-flying space robot merely takes forward kinematics problem into consideration. Therefore, the dynamic singularity problem is resolved. Meanwhile, a nonlinear analytical method of dynamic model is also proposed in this paper to access to the engineering application of space robot. This method is used to reduce the computational complexity of in-orbit operation process while keep the accuracy of the planning result. Methods proposed in this paper have broad prospect of applications for space robot.**

***Index Terms*—Free-Flying space robot, dynamic model simplification, path planning, dynamic singularity.**

I. Introduction

THE free-flying robot plays important role in space exploration in the coming future. To replace the astronaut with free-flying robot will become the new trend for set-up space station, satellite in-orbit maintenances and recovery. Therefore, to study the free-flying control technology is one of the hot topics on space exploration. However, without fixed base, the movement of free-flying robot will generate interference force and torque on its base, i.e. the satellite platform and to change the attitude and position of satellite. Whereas the attitude stability of satellite is crucial for the safety of power supply and reliable communication, it is of great importance on disturbance determination and to deal with the problem of dynamic decoupling of the satellite platform and the free-flying robot. In this way, the influence on satellite attitude caused by robot can be controlled.

Papadopoubs E.and Dubowsky S. have proposed the dynamic modelling method based on Lagrange formulation. The model they setup can directly reflect the dynamic characteristics of system [1]. However, this method cost vast time on computation. Vafa Z. and Dubowsky S. have proposed the method of virtual manipulator and it is used on calculating the working space of robot and inverse kinematics problem [2]. Bin Liang, et al have proposed modelling method of dynamically equivalent manipulator [3].

On the path planning problem, Dubowsky and Torres used enhanced disturbance map (EDM) to plan path for the movement of space manipulator, therefore to decrease the disturbance on the attitude of base [4]. However, to collect EDM is of much more difficulties. Yoshida k, et al. have proposed the idea on zero reaction maneuver flight control method to plan the path for manipulator without disturbing the base [5]. However, the path planned is limited to the manipulator of non-dynamic redundancy. Besides, Yoshida k, et al. have also proposed arm coordination method to balance the disturbance on base with one-side manipulator [6]. But it cost too much on configuring multiple manipulator to stabilize the attitude of satellite. Umetani Y and Yoshida K. combined the characteristics of linear momentum conservation and angular momentum conservation to resolve the motion rate control problems with generalized Jacobin matrix, which saved the computational time [7].

In this paper, we propose a path planning method based on joint parameters to resolve the problem of dynamic singularity while planning the trajectory for free-flying space robot . Parameters-interpolating is used in this path planning method to convert the equation of path planning motion based on generalized Jacobian matrix to non-linear equations with the solution of numerical calculation. Therefore, the dynamic singularity problem is resolved. Meanwhile, we setup a nonlinear analytical method of dynamic model to access to the engineering application of space robot.

II. Dynamic Modelling and Model Simplification of the Free-Flying Space Robot

A. Basic Configuration and Dynamic Modelling of Free-Flying Robot

The basic configuration of free-flying space robot is body (flight base) and manipulator, as shown in Fig.1.

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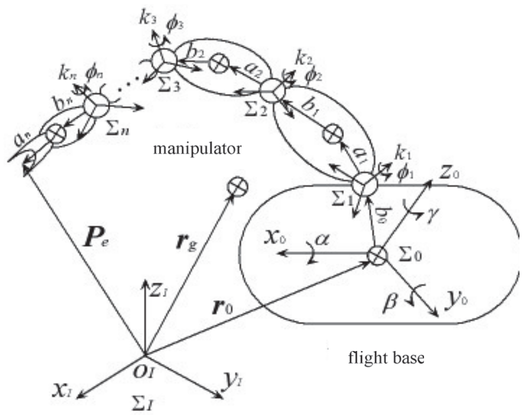

Fig.1. Basic configuration of free-flying robot

Mark the base with number 0, the connecting rods with number 1 to n. Define the ith joint as the joint between the ith connecting rod and the (i-1)th connecting rod and the angle of joint is i. The free-flying robot can be defined as TABLE I. shows:

TABLE I
PARAMETER LIST

Parameters	Definition
$r_i \in R^3$	Vector of the mass center of the ith connecting rod
$r_e \in R^3$	Vector of the manipulator end position
$\varphi_e \in R^3$	Attitude angle of the manipulator end
m_i	Mass of the ith connecting rod
$I_i \in R^{3\times3}$	Moment of inertia of the ith connecting rod
$\omega_i \in R^3$	Angular rate of the ith joint
$r_0 \in R^3$	Vector of the mass center of the satellite
m_0	Mass of the free-flying robot
$I_0 \in R^{3\times3}$	Moment of inertia of the satellite
$\omega_0 \in R^3$	Angular rate of the satellite
$r_g \in R^3$	Vector of the mass center of the whole robot system
M	Mass of the whole robot system
$P_i \in R^3$	Vector of the position of the ith joint of the manipulator
$P_e \in R^3$	Vector of the position of the manipulator end
$\dot{P}_e \in R^3$	Vector of the velocity of the manipulator end
$\omega_e \in R^3$	Angular rate of the manipulator end
$\varphi_m \in R^6$	Angular vector of the manipulator end
$\varphi_0 \in R^3$	Attitude angles of the satellite
$v \in R^6$	Linear velocity and angular rate of the manipulator end
K_i	Unit vector of the ith joint of the manipulator at rotating coordinates

The dynamic model of free-flying robot at free-flying status is built as:

$$H^*(\phi_m)\ddot{\phi}_m + C(\phi_m,\dot{\phi}_m)\dot{\phi}_m = \tau \tag{1}$$

where

$$C(\phi_m,\dot{\phi}_m)\dot{\phi}_m = H^*(\phi_m)\dot{\phi}_m - \frac{\partial}{\partial\phi_m}\left\{\frac{1}{2}\dot{\phi}_m^T H^*\dot{\phi}_m\right\}$$

$$H^*(\phi) = H_\phi - \begin{bmatrix} J_{T\omega}^T & H_{\omega\phi}^T \end{bmatrix}\begin{bmatrix} ME & M\tilde{r}_{g0}^T \\ M\tilde{r}_{g0} & H_\omega \end{bmatrix}\begin{bmatrix} J_{T\omega} \\ H_{\omega\phi} \end{bmatrix}$$

$$\boldsymbol{J}_{T\omega} = \sum_{i=1}^{n} m_i \boldsymbol{J}_{Ti}$$

$$H_\omega = I_0 + \sum_{i=1}^{n}\left(I_i + m_i\tilde{r}_{0i}^T\tilde{r}_{0i}\right)$$

$$H_{\omega\phi} = \sum_{i=1}^{n}\left(I_i J_{Ri} + m_i\tilde{r}_{0i}J_{Ti}\right)$$

$$H_\phi = \sum_{i=1}^{n}\left(J_{Ri}^T I_i J_{Ri} + m_i J_{Ti}^T J_{Ti}\right)$$

$$J_{Ti} = \left[{}^I K_1 \times ({}^I r_i - {}^I P_1) \quad \cdots \quad {}^I K_n \times ({}^I r_i - {}^I P_n)\right]$$

$$\boldsymbol{r}_{0g} = \boldsymbol{r}_g - \boldsymbol{r}_0\ ;\quad \boldsymbol{r}_{0i} = \boldsymbol{r}_i - \boldsymbol{r}_0$$

$$\boldsymbol{J}_{Ri} = \left[\boldsymbol{K}_1 \quad \boldsymbol{K}_2 \quad \ldots \quad \boldsymbol{K}_n\right]$$

B. Simplification and Simulation of Dynamic Model

$C(\varphi_m,\dot{\varphi}_m)\dot{\varphi}_m$ is a non-linear item and is difficult to describe with analytical expressions. Therefore, this item should be simplified by nonlinear equivalence for less cost on computation and increase the efficiency of simulation, verification and control strategy design of free-flying space robot. So we define:

$$C(\phi_m,\dot{\phi}_m)\dot{\phi}_m = \begin{bmatrix} c_0 \\ c_m \end{bmatrix} \in R^{(n+6)x1}$$

where $c_0 \in R^{6\times1}$ and $c_m \in R^{n\times1}$ represent the correlated nonlinear parameters of the robot base and manipulator respectively.

The generalized force is defined as:

$$\tau = \begin{bmatrix} F_0 \\ \tau_m \end{bmatrix} + \begin{bmatrix} J_0^T \\ J_m^T \end{bmatrix} F_e \tag{2}$$

where F_0 and F_e are external force or torque of the base and manipulator end respectively, τ_m is external force of each joint of the manipulator.

Define a inertial matrix:

$$H^* = \begin{bmatrix} ME & M\tilde{r}_{0g}^T & J_{T\omega} \\ M\tilde{r}_{0g} & H_\omega & H_{\omega\phi} \\ J_{T\omega}^T & H_{\omega\phi}^T & H_\phi \end{bmatrix}$$

Therefore we get:

$$H^*\begin{bmatrix}\ddot{x}_0\\ \ddot{\phi}_m\end{bmatrix}+\begin{bmatrix}c_0\\ c_m\end{bmatrix}=\begin{bmatrix}F_0\\ \tau_m\end{bmatrix}+\begin{bmatrix}J_0^T\\ J_m^T\end{bmatrix}F_e \quad (3)$$

where the acceleration of bas is:

$$\ddot{x}_0=\begin{bmatrix}\ddot{r}_0\\ \dot{\omega}_0\end{bmatrix}$$

Then obviously, we can simplify the nonlinear item $C(\phi_m,\dot{\phi}_m)\dot{\phi}_m$ by setting the value of parameters $\ddot{x}_0$, $\ddot{\phi}_m$, F_b and F_e as zero. Therefore the inertial force of the connecting rod n to base can be calculated out according to inverse dynamic. The calculation results are described as c_0 and c_m. Where the detailed calculation steps are as follows:

a) Determine the joint force torque τ_m and the force and torque on the base according to the control law of manipulator;

b) At the time of *t*, make recursive calculation on the position and velocity from the base to connecting rod *n*;

c) Calculate the inertial matrix *H;*

d) Set the value of parameters $\ddot{x}_0$, $\ddot{\phi}_m$, F_b and F_e as zero and calculate the inertial force according to inverse dynamic. The results are c_0 and c_m;

e) Calculate the acceleration value:

$$\begin{bmatrix}\ddot{x}_b\\ \ddot{\phi}_m\end{bmatrix}=H^{-1}\left\{\begin{bmatrix}F_b\\ \tau_m\end{bmatrix}+\begin{bmatrix}J_b^T\\ J_m^T\end{bmatrix}F_e-\begin{bmatrix}c_b\\ c_m\end{bmatrix}\right\} \quad (4)$$

And calculate the velocity value by integration with acceleration and calculate the position value by integration with velocity;

f) Repeat steps of a)~e) at the beginning of period of simulation, until the end of simulation.

III. KINEMATICS MODELLING AND PATH PLANNING OF THE FREE-FLYING ROBOT

A. *Kinematics Modelling of the Free-Flying Robot*

According to the movement relationship between the free-flying robot and connecting rods, the linear velocity and angular rate of manipulator end can be described as:

$$v=\begin{bmatrix}\dot{P}_e\\ \omega_e\end{bmatrix}=\begin{bmatrix}E & -(\widetilde{P}_{0e})\\ 0 & E\end{bmatrix}\begin{bmatrix}\dot{r}_0\\ \omega_0\end{bmatrix}+\begin{bmatrix}J_{Ti}\\ J_{Ri}\end{bmatrix}\cdot\dot{\phi}_m \quad (5)$$

Because of the disturbance of manipulator movement and following the law of conservation of momentum, we can get the description of momentum P and angular momentum L as:

$$\begin{bmatrix}P\\ L\end{bmatrix}=\begin{bmatrix}ME & -M\widetilde{r}_{0g}\\ M\widetilde{r}_g & I_\omega\end{bmatrix}\begin{bmatrix}\dot{r}_0\\ \omega_0\end{bmatrix}+\begin{bmatrix}J_{T\omega}\\ H_{\omega\phi}\end{bmatrix}\cdot\dot{\phi}_m \quad (6)$$

Suppose the momentum and angular momentum value at initial time as zero, by solving the equation(6), we get:

$$\begin{aligned}&I_s\omega_0+I_m\dot{\phi}_m=0\\ &\dot{r}_0=-\left(J_{T\omega}/M+\widetilde{r}_{0g}I_s^{-1}I_m\right)\dot{\phi}_m\equiv J_v\dot{\phi}_m\end{aligned} \quad (7)$$

Where $I_s=M\widetilde{r}_g\widetilde{r}_{0g}+I_0+\sum_{i=1}^{n}(I_i-m_i\widetilde{r}_i\widetilde{r}_{0i})$ and $I_m=I_\phi-\widetilde{r}_gJ_{T\omega}$ are the generalized inertial matrix of the free-flying robot and its manipulator. If we combine equation(7) and (6), we can get:

$$v=\begin{bmatrix}\dot{P}_e\\ \omega_e\end{bmatrix}=\begin{bmatrix}-(\widetilde{P}_{0e})\\ E\end{bmatrix}\omega_0+\begin{bmatrix}J_v+J_{mp}\\ J_{m\omega}\end{bmatrix}\cdot\dot{\phi}_m\equiv J_s\omega_0+J_m\dot{\phi}_m \quad (8)$$

and $\omega_0=-I_s^{-1}I_m\dot{\phi}_m$.

Therefore, from equation (8), we can get the kinematic model of free-flying robot based on generalized Jacob matrix as:

$$v=\left(J_m-J_sI_s^{-1}\dot{\phi}_m\right)\equiv J^*\dot{\phi}_m \quad (9)$$

where *J** is the generalized Jacob matrix of the velocity of manipulator end of the robot, which reveals the relationship between the manipulator end velocity and the manipulator joint angular rate at free-flying status.

B. *Path Planning of the Free-Flying Robot*

According to our study result, the position and attitude of manipulator end in the space is decided by the movement of each joint, which can be described by vector form as:

$$X_e=[r_e,\ \varphi_e]^{\mathrm{T}}=f(\phi_0,\phi_m) \quad (10)$$

Suppose the transformation matrix of Euler angular rate and absolute angular rate of manipulator end as N_e, we can describe the kinematic equation of free-flying robot based on generalized Jacob matrix from equation (9) and (10) as:

$$\dot{X}_e=[\dot{r}_e,\ \dot{\varphi}_e]^{\mathrm{T}}=\begin{bmatrix}I_3 & O_3\\ O_3 & N_e^{-1}\end{bmatrix}J^*\dot{\phi}_m \quad (11)$$

where I_3 and O_3 are unit matrix and zero matrix with the dimensions of 3×3. So we can get the integration result of equation(11) as:

$$X_e=\int_0^{tf}\left(\begin{bmatrix}I_3 & O_3\\ O_3 & N_e^{-1}\end{bmatrix}J^*\dot{\phi}_m\right)dt \quad (12)$$

Obviously, from equation (12), we cannot directly get analytical solution of it. So we refer to the method proposed by Yoji Umetani [8] to plan the expected angular rate value of the manipulator end and calculate inverse result of generalized Jacob matrix to get the velocity of joint. But we faced to the problem of dynamic singularities.

Therefore, we propose the method of planning the path of robot based on the joint parameters, which will convert the planning problem as solving a nonlinear equation or the problem of parameter identification. In this way, the dynamic singularity problem can be resolved.

So we deal with the angular function of joints with this equation:

$$\phi_i=a_{i5}t^5+a_{i4}t^4+a_{i3}t^3+a_{i2}t^2+a_{i1}t+a_{i0} \quad (13)$$

where $i=1,\cdots6,\ \ 0\le t\le t_f$.

Suppose the velocity and acceleration of each beginning and end point of the manipulator as zero, the angle of joints at initial position is described as ϕ_{i0}, then the velocity and acceleration of the i[th] joint can be described as:

$$\begin{aligned}
\phi_i &= a_{i5}\left(t^5 - \frac{5}{2}t_f t^4 + \frac{5}{3}t_f^2 t^3\right) + \phi_{i0} \\
\dot{\phi}_i &= a_{i5}\left(5t^4 - 10t_f t^3 + 5t_f^2 t^2\right) \\
\ddot{\phi}_i &= a_{i5}\left(20t^3 - 30t_f t^2 + 10t_f^2 t\right)
\end{aligned} \qquad (14)$$

where we find out that only one parameter of a_{i5} exists in the function of angle, angular rate and angular acceleration for each joint. So we define:

$$F(a) = X_{ed} - \int_0^{t_f}\left(\begin{bmatrix} I_3 & O_3 \\ O_3 & N_e^{-1} \end{bmatrix} J^* \dot{\phi}_m\right) dt \qquad (15)$$

where $a = [a_{15}, a_{25}, \cdots a_{65}]$ and X_{ed} is the end position of manipulator.

Therefore, the path planning problem can be equivalent as solving a nonlinear equation, i.e. to calculate the value of a (Newton Iteration is suggested), we can get the result of path planning and the accuracy of manipulator movement can be satisfied, i.e. $F(a) < \varepsilon$.

IV. Simulation and Verification

Suppose the mass of a satellite as 500kg, the mass of each manipulator joint is [10,5,10,5,10,5] kg, respectively. The manipulator has redundant design of double-joint and each joint is rotary. The parameters of the manipulator are shown in TABLE II.

TABLE II
PARAMETER LIST

Connecting Rods No.	θ_i	α_i	a_i (mm)	d_i (mm)
1	90°	-90°	0	0
2	0°	0°	1000	0
3	90°	90°	0	0
4	0°	-90°	0	-800
5	0°	90°	0	0
6	0°	0°	0	400

Suppose the backward movement path of the manipulator as a circle and it stretch out with the forward movement of arc. If we use the path planning method in this paper, the simulation result of manipulator end movement is shown in Fig.2.

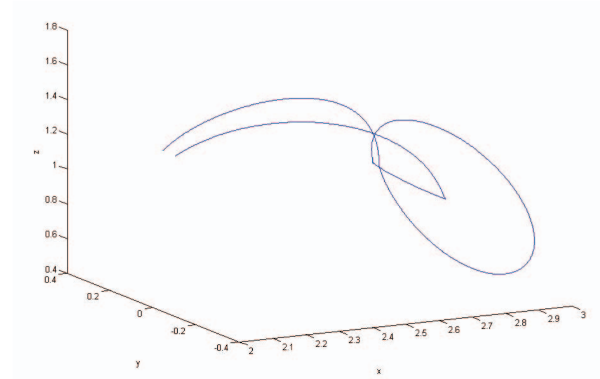
Fig.2. Movement path of the manipulator end

According to the simulation results, the disturbance force torque caused by the movement of manipulator in three directions of x,y,z (at orbital coordinates) is shown in Fig.3. This is controllable and can be described by planning the path of the manipulator movements.

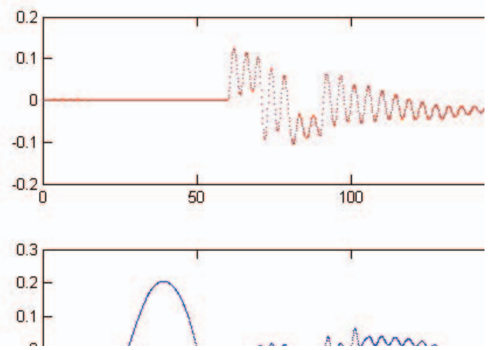

Fig.3. Disturbance force torque of the free-flying robot manipulator movement onto the satellite platform

V. Conclusion

In this paper we resolve the problem of dynamic and kinematic modelling and simulation for free-flying space robot under the occasion of practical application. We proposed the dynamic model of free-flying space robot on the base of in-orbit satellite. Therefore, to deal with the problem of robot path planning, we proposed the modelling simplification method and verification method. Meanwhile, we conducted the kinematic model based on generalized Jacob matrix and put forward the path planning method based on simplifying the parameters of manipulator joints of the robot. Therefore, in this way, we resolved the problem of dynamic singularities in path planning process.

According to our simulation results, the methods of both dynamic simplification and path planning we proposed in this paper can save the time of computation and the result of robot movement also satisfy the requirement of the free-flying mission.

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