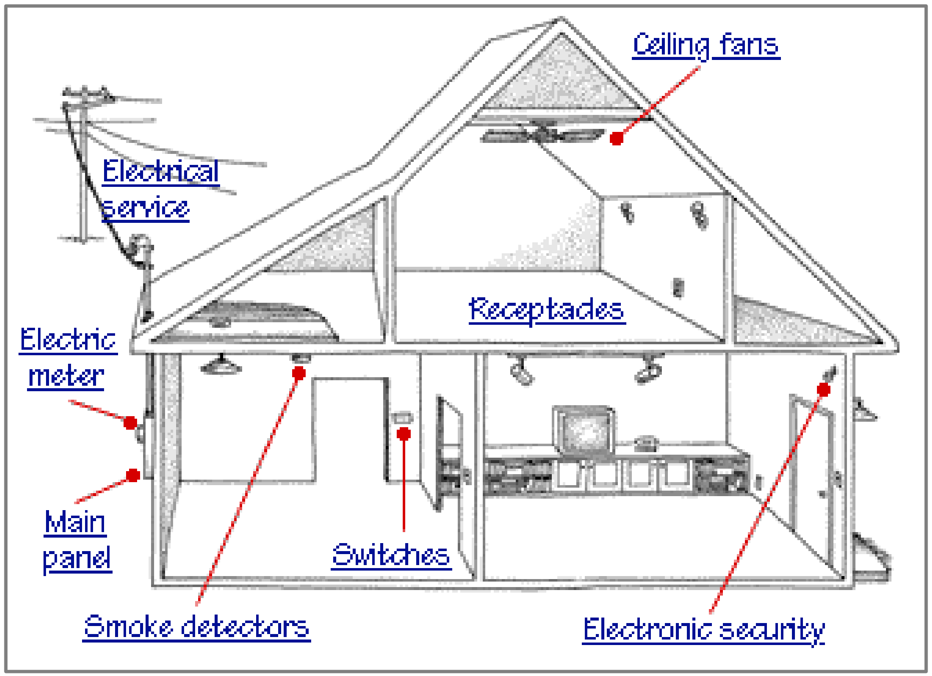
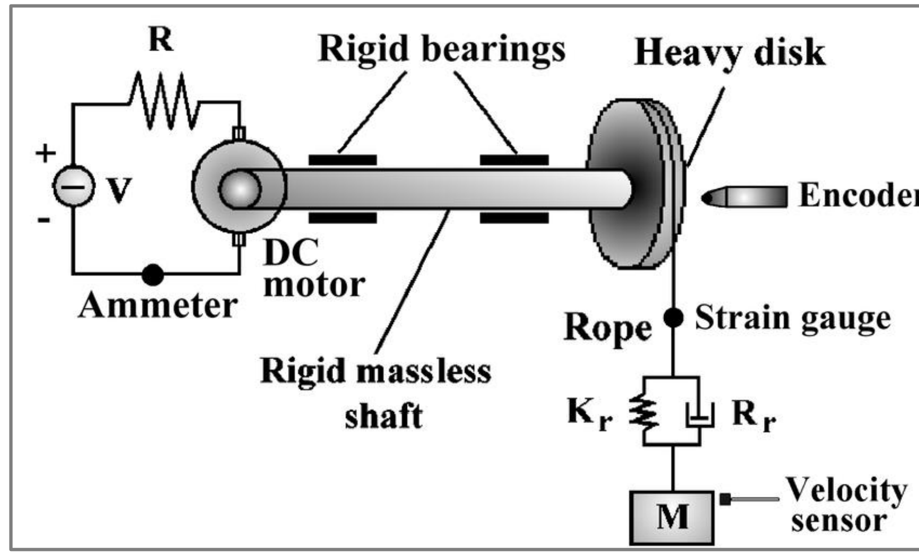
Faculty of Engineering

ENGR 311: System Dynamics

Fall 2020



https://www.hometips.com/how-it-works/electrical-panel-circuit-breakers.html



https://www.researchgate.net/figure/An-electromechanical-system-with-its-instrumentation_fig36_257351591

Chapter 7

30/11/2020

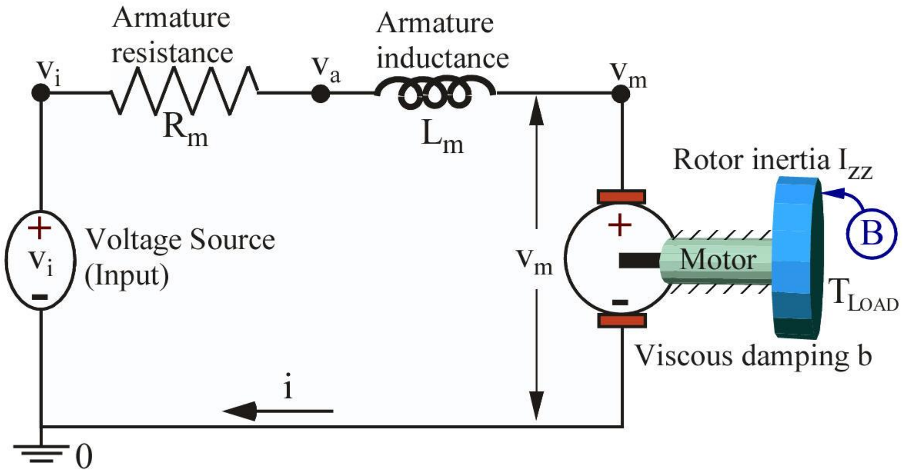
Electrical and Electromechanical Systems

Outline

- Introduction
- Fundamental of Electrical Circuits
- Mathematical Modeling of Electrical Systems
- ~~Analogous Systems~~
- ~~Mathematical Modeling of Electromechanical Systems~~

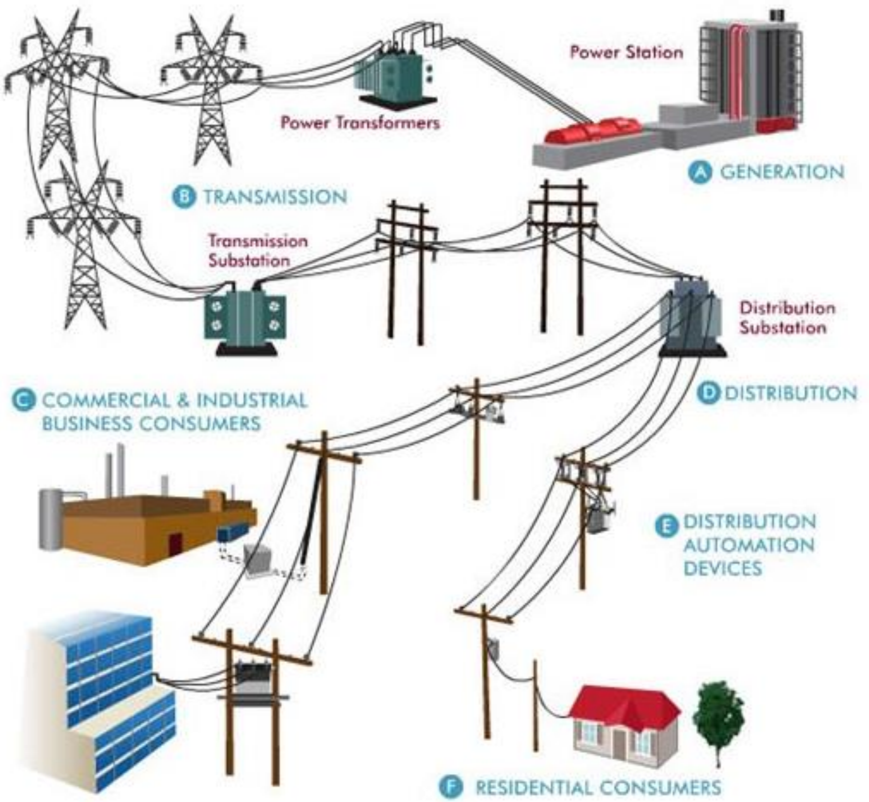
Introduction

- The majority of engineering systems now have at least one electrical subsystem.
- This may be a power supply, sensor, motor, controller, or an acoustic device such as a speaker. So an understanding of electrical systems is essential to understanding the behavior of many systems.
- Electrical systems permit us to easily transmit energy from a source of supply to a point of application.



http://www.motiongenesis.com/MGWebSite/MGGetStarted/MGExampleDCMotorWithRLCircuit/MGExampleMotorWithRLCircuit.html

Electromechanical: Simple DC motor



https://slideplayer.com/slide/5872983/

Basic of electrical system

Introduction

Electrical Elements: Energy Sources

- An active two-terminal element that supplies energy to a circuit is a source of energy.

- A *voltage source* is a two-terminal device that causes a specified voltage to exist between two points in a circuit. The voltage may be time varying or time invariant (for a sufficiently long time). Examples include batteries and generators.

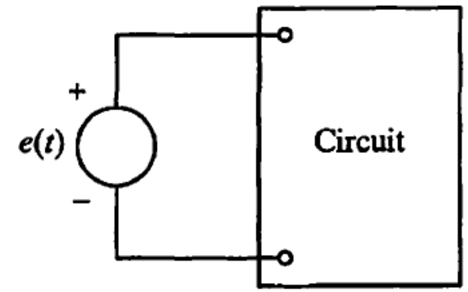
- An *ideal voltage source* maintains a fixed voltage drop across its terminals, independent of the load resistance or the output current.

- A *current source* causes a specified current to flow through a wire containing this source. Example is transistor current source.

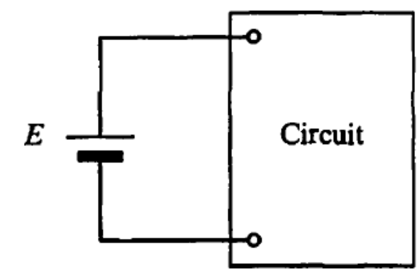
- An *ideal current source* is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

http://carunderstanding.com/how-to-charge-rv-battery-with-generator/

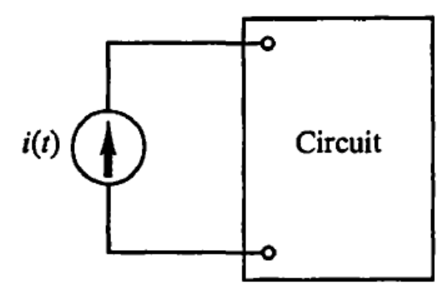
Generator and Battery



Voltage Source



Time Invariant Voltage source

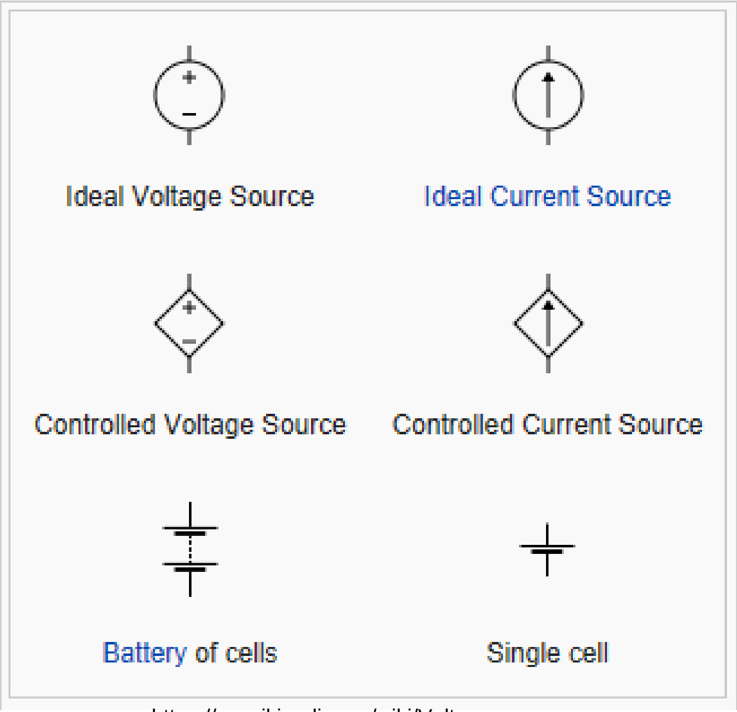


Current Source

Introduction

Electrical Elements: Energy Sources

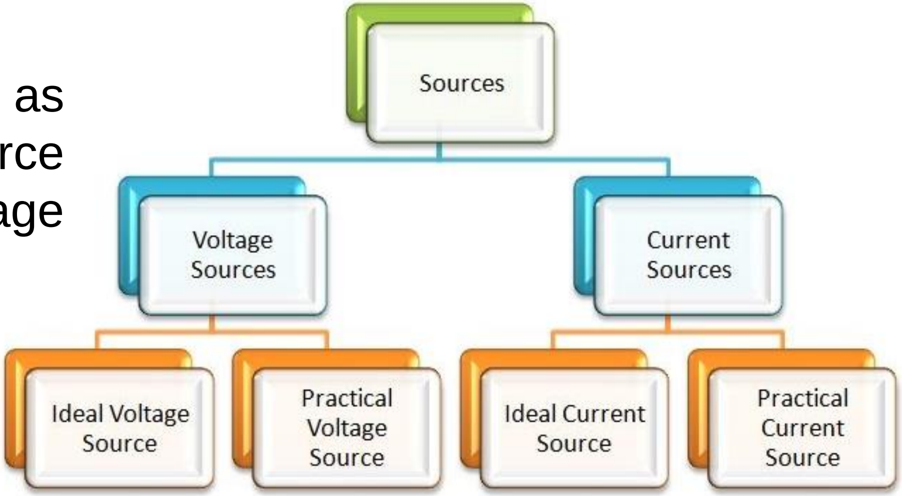
- These circuit elements do not exist as practical devices. A real-world voltage source cannot supply unlimited current. A voltage source is the dual of a current source.

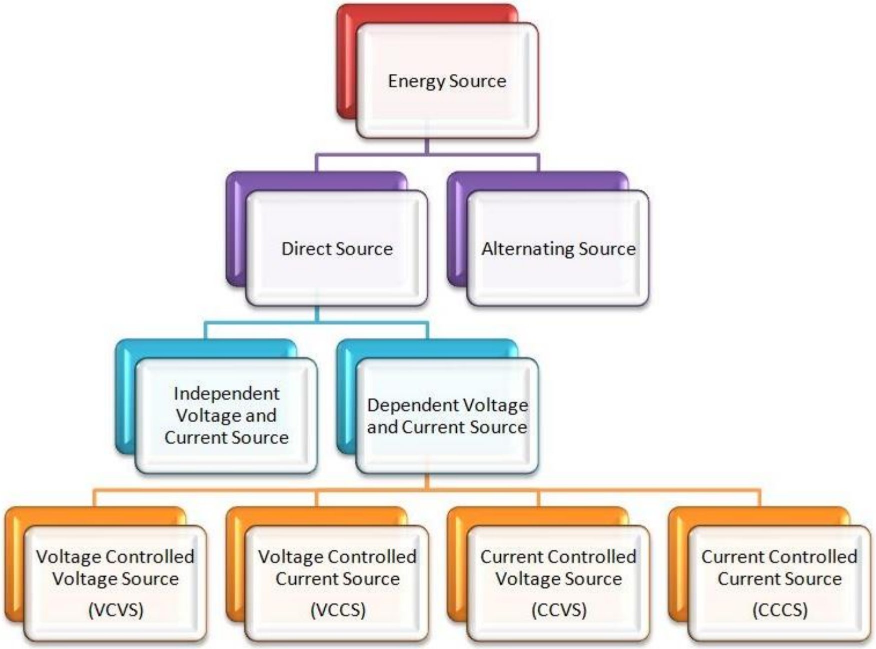


https://en.wikipedia.org/wiki/Voltage_source

Symbols used for voltage and current sources

- *Voltage* and *current* are the primary variables used to describe a circuit's behavior.





https://circuitglobe.com/independent-dependent-voltage-and-current-source.html

Circuit Energy Sources

Introduction

Electrical Elements: Current and Voltage

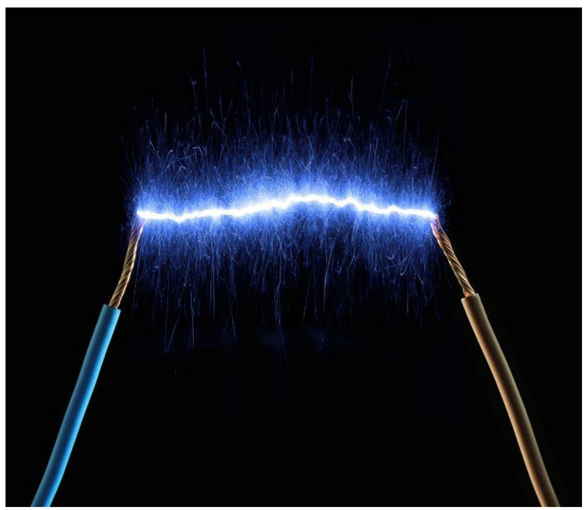
- Current is the flow of electrons. It is the time rate of change of electrons passing through a defined area, such as the cross section of a wire. Because electrons are negatively charged, the positive direction of current flow is opposite to that of the electron flow.

- The mathematical description of the relation between the number of electrons (called charge Q) and current i is

$$\boxed{i = \frac{dQ}{dt}}$$

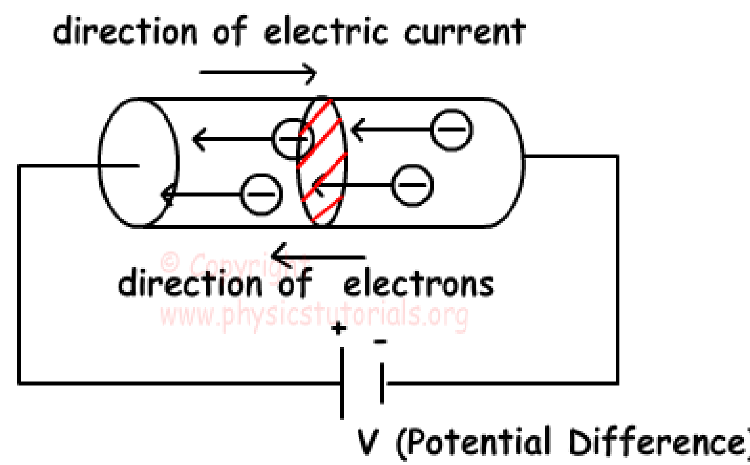
or

$$\boxed{Q(t) = \int i dt}$$



https://www.thoughtco.com/electrical-current-2698954

Electrical current



http://www.physicstutorials.org/home/electric-current/electric-current-and-flow-of-charge

Electrical current and flow of charge

- The unit of charge is the *coulomb* (C), and the unit of current is the *ampere* (A), which is one coulomb per second. These units, and the others we will use for electrical systems, are the same in both the SI and FPS systems.

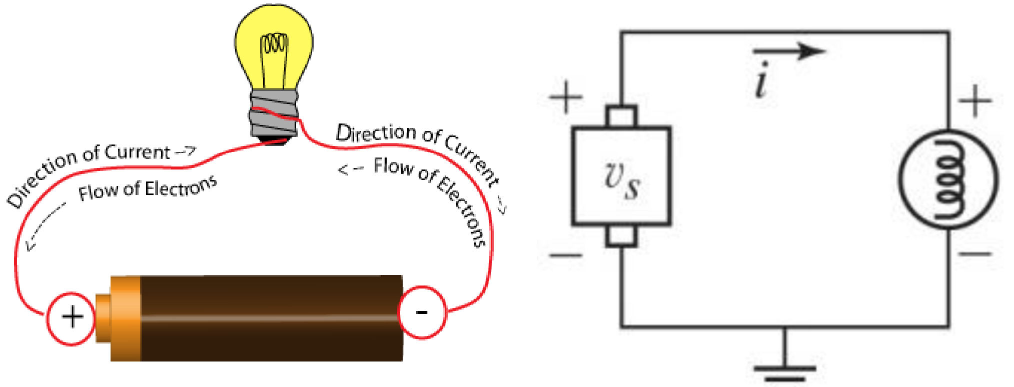
Introduction

Electrical Elements: Current and Voltage

- Energy is required to move a charge between two points in a circuit. The work per unit charge required to do this is called *voltage*. The unit of voltage is the *volt* (V), which is defined to be one joule per coulomb.

- The sign of voltage difference is important. The Figure shows a battery connected to a lightbulb. The electrons in the wire are attracted to the battery's positive terminal; thus the positive direction of current is clockwise, as indicated by the arrow.

- Because the battery supplies energy to move electrons and the lightbulb dissipates energy (through light and heat), the sign of voltage difference across the battery is the opposite of the sign of the voltage difference across the lightbulb.



A battery-lightbulb circuit

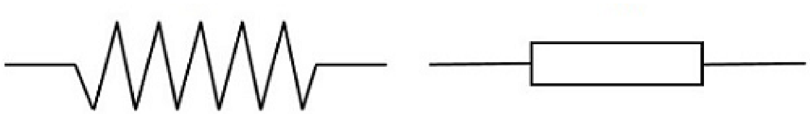
Introduction

Electrical Elements: Resistor

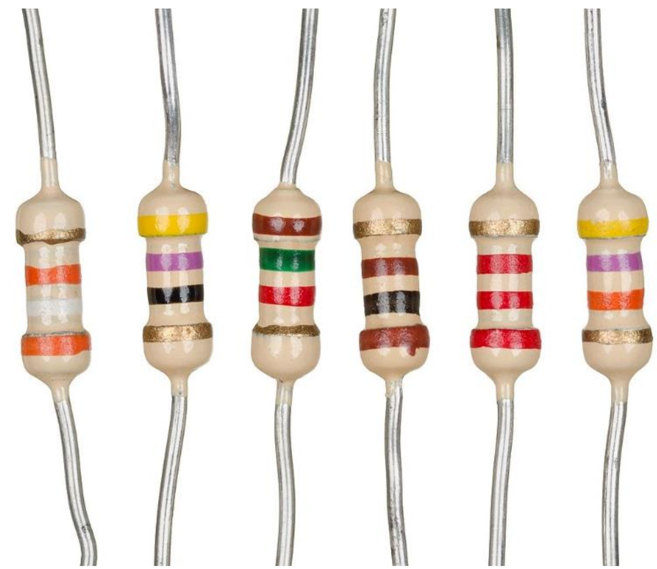
- The bulb current depends on the voltage difference and the bulb's material properties, which resist the current and cause the energy loss.
- When a current flows through wire or other circuit elements, it encounters *resistance*. Sometimes this resistance is desirable and intentionally introduced; sometimes not.
- A *resistor* is an element designed to provide resistance. Most resistors are designed to have a linear relation between the current passing through them and the voltage difference across them. This linear relation is Ohm's law. It states that

$$\boxed{v = iR} \quad \text{or} \quad \boxed{R = \frac{v}{i}}$$

where i is the current, v is the voltage difference, and R is the *resistance*. The unit of resistance is the *ohm* (Ω), which is one volt per ampere.



Symbol for a fixed resistor

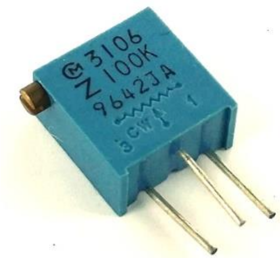


https://en.wikipedia.org/wiki/Resistor

An array of axial-lead resistor

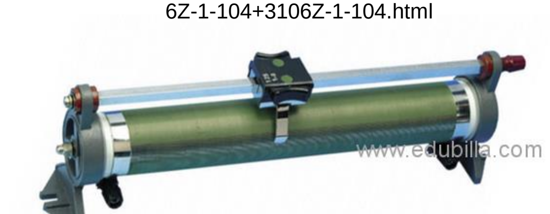
Introduction

Electrical Elements: Resistor



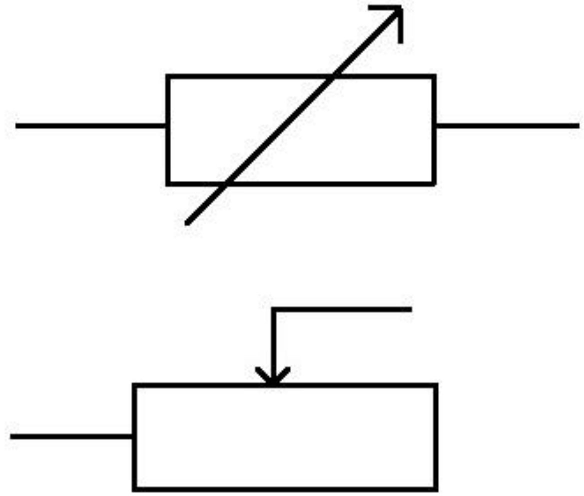
https://www.westfloridacomponents.com/P178PD/100K+ohm+Variable+Resistor+Trimpot++POT3106Z-1-104+3106Z-1-104.html

https://www.lelong.com.my/potentiometer-variable-resistor-50k-ohm-realove-F745807-2007-01-Sale-I.htm



http://www.edubilla.com/invention/variable-resistor/

Variable Resistors: Potentiometer/Rheostat

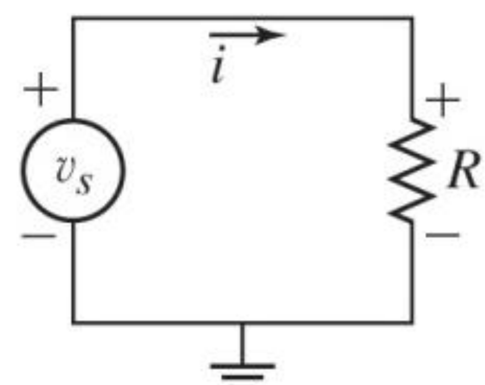


Symbol for a variable resistor

- Example, because of conservation of energy, the voltage increase v_s supplied by the source must equal the voltage drop iR across the resistor. Thus the model of this circuit is

$$v_s = iR$$

- Resistors do not store electric energy in any form, but instead dissipate it as heat. Note that real resistors may not be linear and may also exhibit some capacitance and inductance effects.

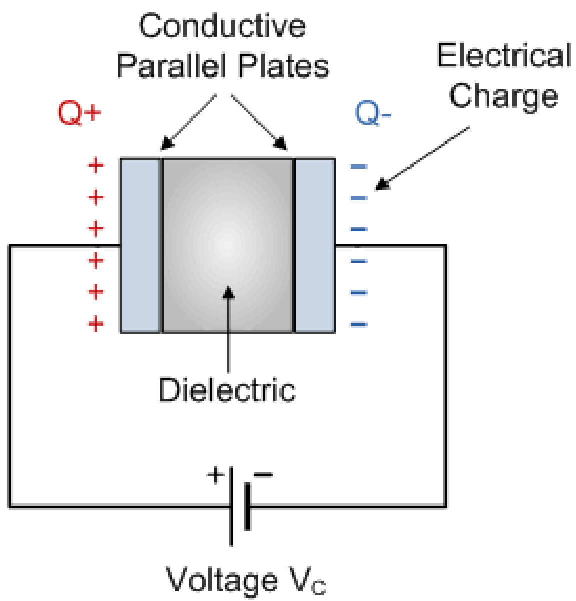


Circuit diagram representation of the battery-lightbulb circuit

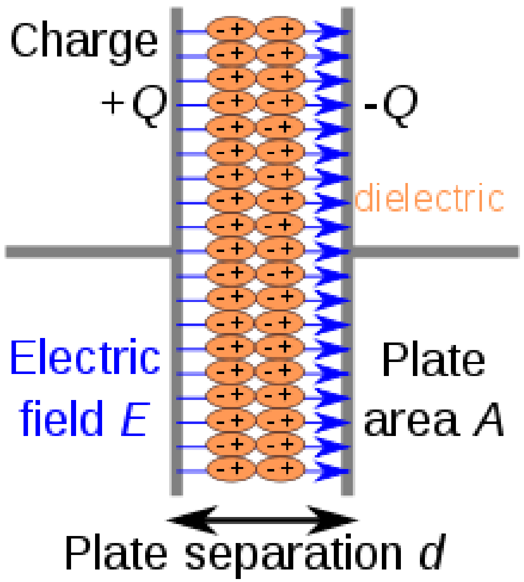
Introduction

Electrical Elements: Capacitor

- Two conductors separated by a nonconducting medium form a capacitor, so two metallic plates separated by a very thin dielectric material (glass, air, paper, plastic, ceramic, etc.) form a capacitor.

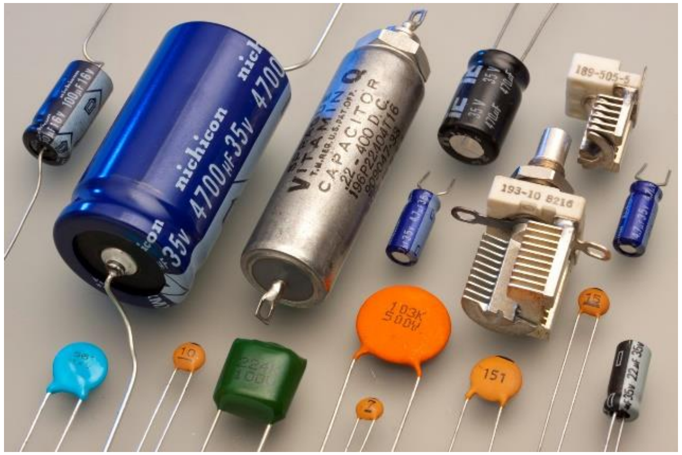


https://www.electronics-tutorials.ws/capacitor/cap_1.html



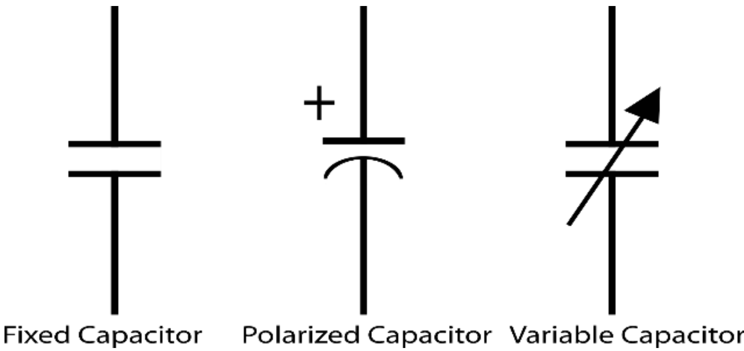
https://en.wikipedia.org/wiki/Capacitor

Charge separation in a parallel-plate capacitor causes an internal electric field. A dielectric (orange) reduces the field and increases the capacitance



https://en.wikipedia.org/wiki/Capacitor

Capacity



Symbol for a capacity

Introduction

Electrical Elements: Capacitor

- The *capacitance* is a measure of the quantity of charge that can be stored for a given voltage across the plates.
- An ideal (pure) capacitor is characterized by a constant capacitance C, in *farads* (F) in the SI system of units, defined as the ratio of the positive or negative charge Q on each conductor to the voltage V between them.

$$\boxed{v = \frac{Q}{C}} \quad \text{and} \quad \boxed{i = \frac{dQ}{dt}} \quad \rightarrow \quad \boxed{i = C\frac{dv}{dt}}$$

- Thus,

  - The unit of capacitance is the farad (F).

$$\boxed{v(t) = \frac{1}{C}\int i dt = \frac{1}{C}\int_0^t i dt + \frac{Q_o}{C}} \quad \text{or} \quad \boxed{v(t) = \frac{1}{C}\int_0^t i dt + v(0)}$$

where Q_o is the initial charge on the capacitor at time $t = 0$.

- Although a pure capacitor stores energy and can release all of it, real capacitors exhibit various losses. These energy losses are indicated by a *power factor*, which is the ratio of the energy lost per cycle of ac voltage to the energy stored per cycle. Thus, a small-valued power factor is desirable.

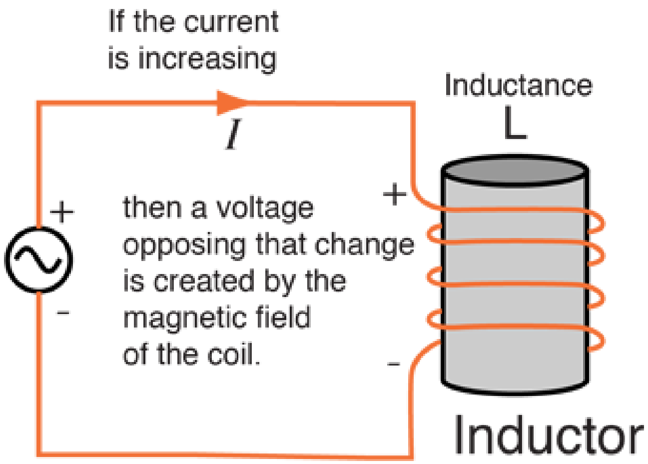
Introduction

Electrical Elements: Inductor

- A magnetic field (a flux) surrounds a moving charge or current. If the conductor of the current is coiled, the flux created by the current in one loop affects the adjacent loops.
- If a circuit lies in a time-varying magnetic field, an electromotive force is induced in the circuit. The inductive effects can be classified as *self-inductance* and *mutual inductance*.
- If the circuit does not contain ferromagnetic elements (such as an iron core), the rate of change of flux is proportional to di/dt.

$$\boxed{\emptyset = Li}$$

where L is the inductance and $\emptyset$ is the flux across the inductor.



http://hyperphysics.phy-astr.gsu.edu/hbase/electric/induct.html

Inductor

https://www.miniphysics.com/uy1-self-inductance-inductors.html

Coil Inductance

Introduction

Electrical Elements: Inductor

- And the flux is proportional to the time integral of the applied voltage.

$$\boxed{\emptyset = \int v dt}$$

- For self-inductance, or simply inductance, we have

$$\boxed{i = \frac{1}{L}\int v dt} \quad \text{or} \quad \boxed{v = L\frac{di}{dt}}$$

- Thus,
  - The unit of inductance is the <u>henry</u> (H).

$$\boxed{i(t) = \frac{1}{L}\int_0^t v dt + i(0)} \quad \text{or} \quad \boxed{i(t) = \frac{1}{L}\int_0^t v dt + \frac{\emptyset_o}{L}}$$

where $\emptyset_o$ is the initial flux across the inductor at time $t = 0$.

- Because most inductors are coils of wire, they have considerable resistance. The energy loss due to the presence of resistance is indicated by the *<u>quality factor</u>* Q, which denotes the ratio of stored to dissipated energy. A high value of Q generally means that the inductor contains small resistance.

Introduction

Electrical Elements: Active and Passive

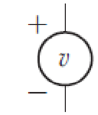
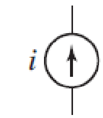
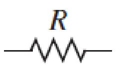
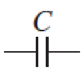
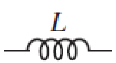
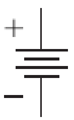
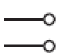
- Circuit elements may be classified as *active* or *passive*.

- *Passive elements* such as resistors, capacitors, and inductors are not sources of energy, although the latter two can store it temporarily.

- Elements that provide energy are *sources*, and elements that dissipate energy are *loads*.

- The *active elements* are energy sources that drive the system. There are several types available; for example, chemical (batteries), mechanical (generators), thermal (thermocouples), and optical (solar cells).

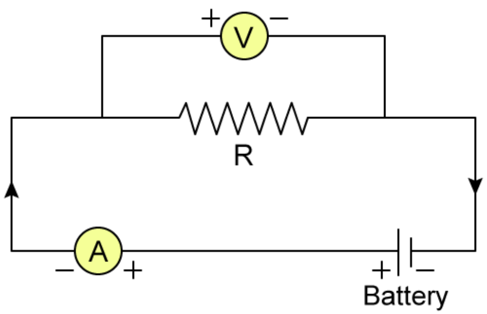
Electrical quantities

Quantity	Units	Circuit symbol
Voltage	volt (V)	Voltage Source 
Charge	coulomb (C) = N · m/V	
Current	ampere (A) = C/s	Current Source 
Resistance	ohm (Ω) = V/A	
Capacitance	farad (F) = C/V	
Inductance	henry (H) = V · s/A	
Battery	—	
Ground	—	
Terminals (input or output)	—	

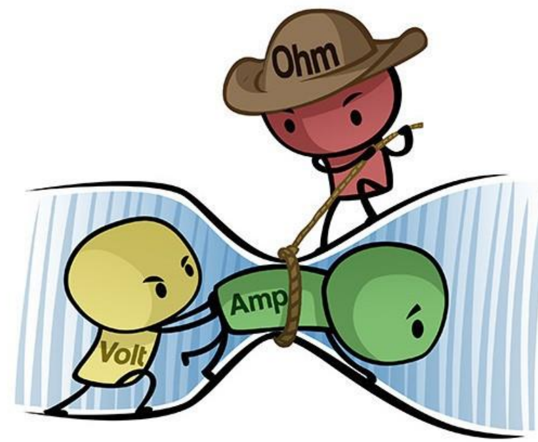
- Active elements are modeled as either ideal voltage sources or ideal current sources.

Fundamentals of Electrical Circuits

Ohm's Law

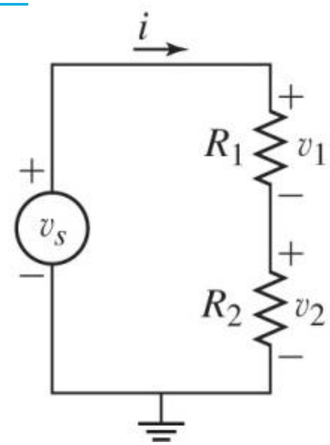


$$\boxed{v = iR}$$



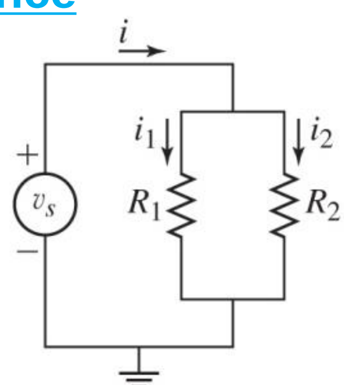
https://www.build-electronic-circuits.com/ohms-law/

Series Resistance



$$\boxed{R_T = R_e = R = R_1 + R_2}$$

Parallel Resistance



$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

or

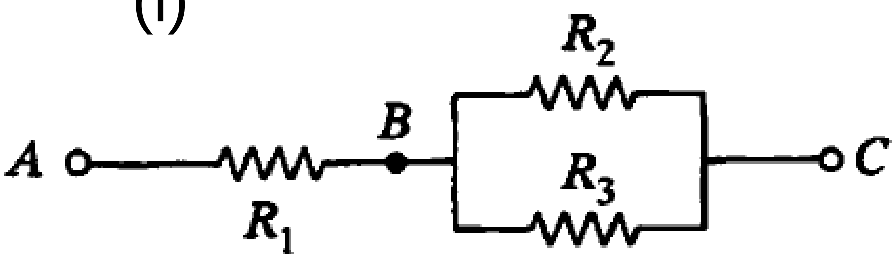
$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Fundamentals of Electrical Circuits

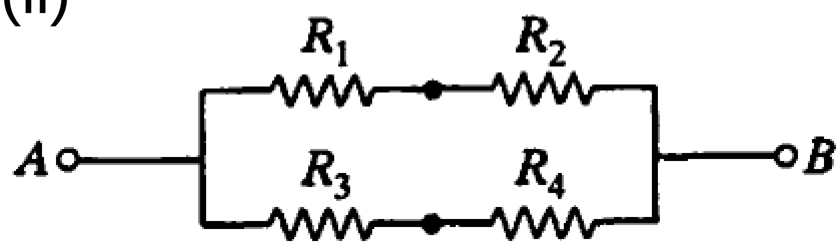
Example 1

(a) Determine the combined resistance R.

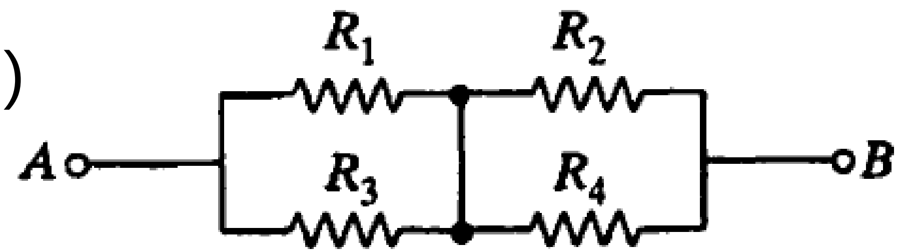
(i)



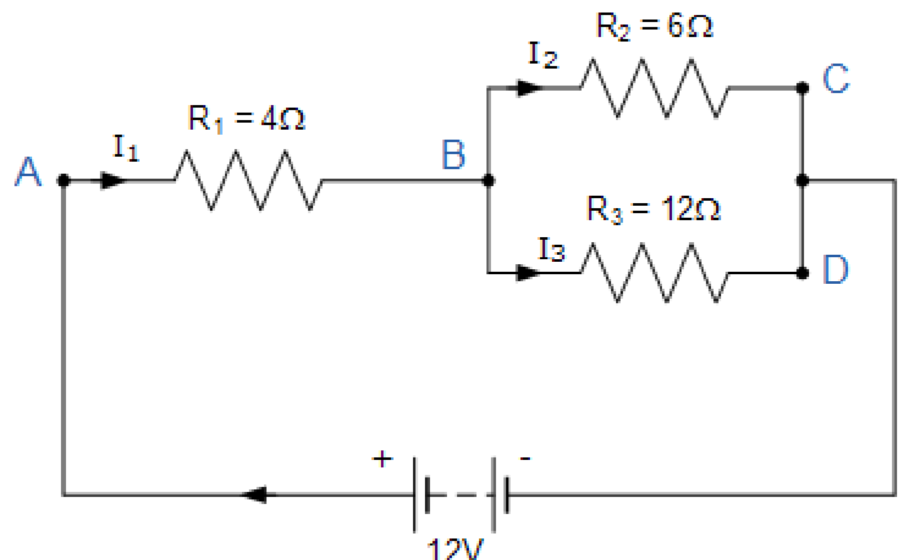
(ii)



(iii)



(b) Determine the current I_1.



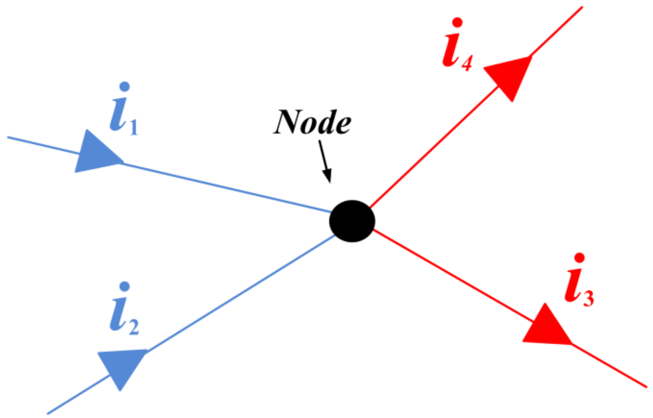
Fundamentals of Electrical Circuits

Kirchhoff's Law

- In solving circuit problems that involve many electromotive forces, resistances, capacitances, inductances, and so on, it is often necessary to use Kirchhoff's laws, of which there are two: the *current law* (node law) and the *voltage law* (loop law).

Kirchhoff's Current Law

- A basic law used to model electrical systems using conservation of charge.
- A *node* in an electrical circuit is a point where three or more wires are joined together.
- The law states that the algebraic sum of all currents entering and leaving a node is zero. (This law can also be stated as follows: The sum of all the currents entering a node is equal to the sum of all the currents leaving the same node.)
- Conversion: Currents going toward a node should be preceded by a plus sign; currents going away from a node should be preceded by a minus sign.



$$i_1 + i_2 - i_3 - i_4 = 0$$

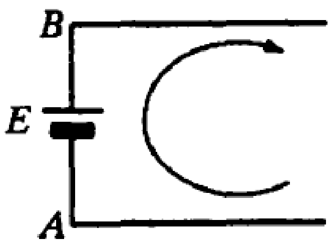
or

$$i_1 + i_2 = i_3 + i_4$$

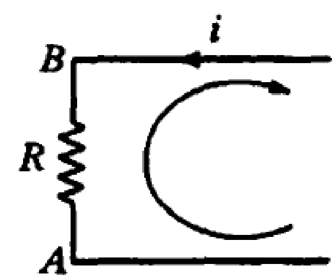
Fundamentals of Electrical Circuits

Kirchhoff's Voltage Law

- This basic law uses conservation of energy.
- Kirchhoff's voltage law states that at any given instant of time the algebraic sum of the voltages around any loop in an electrical circuit is zero: The sum of the voltage drops is equal to the sum of the voltage rises around a loop.
- Conversion: A rise in voltage [which occurs in going through a source of electromotive force from the negative to the positive terminal, or in going through a resistance in opposition to the current flow] should be preceded by a plus sign.

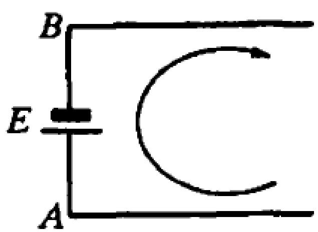


$e_{AB} = +E$

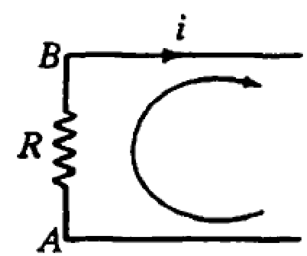


$e_{AB} = +Ri$

- Conversion: A drop in voltage [which occurs in going through a source of electromotive force from the positive to the negative terminal, or in going through a resistance in the direction of the current flow] should be preceded by a minus sign.



$e_{AB} = -E$



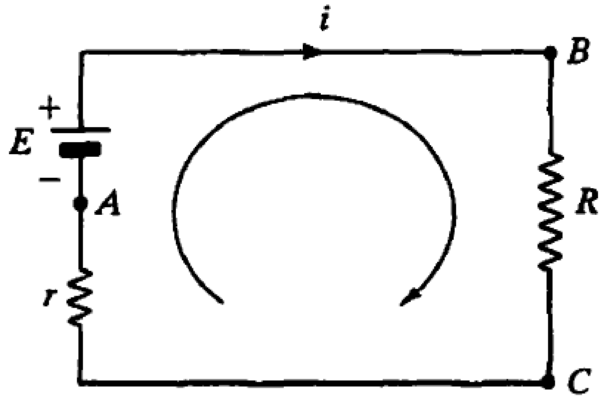
$e_{AB} = -Ri$

Fundamentals of Electrical Circuits

Example 2

Obtain the mathematical model of the following circuits.

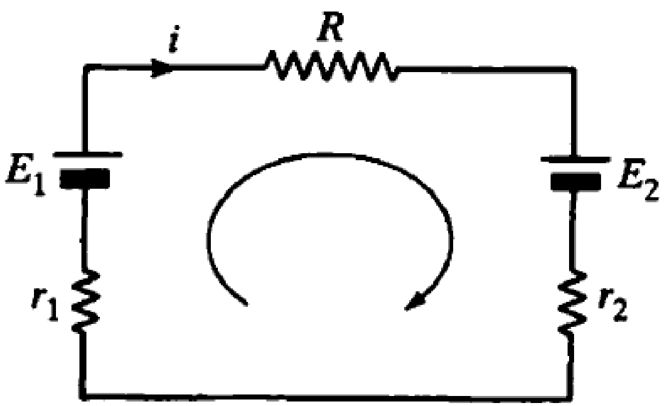
(a)

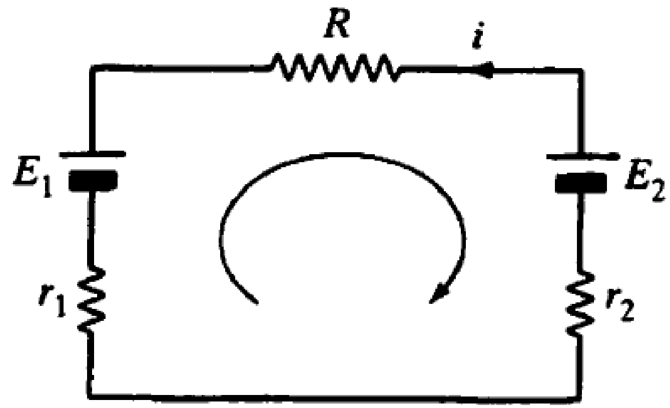


$$E - iR - ir = 0 \qquad Or \qquad E = iR + ir$$

$$i = \frac{E}{R + r}$$

(b)





$$E_1 - iR - E_2 - ir_2 - ir_1 = 0$$

$$i = \frac{E_1 - E_2}{r_1 + r_2 + R}$$

$$E_1 + iR - E_2 + ir_2 + ir_1 = 0$$

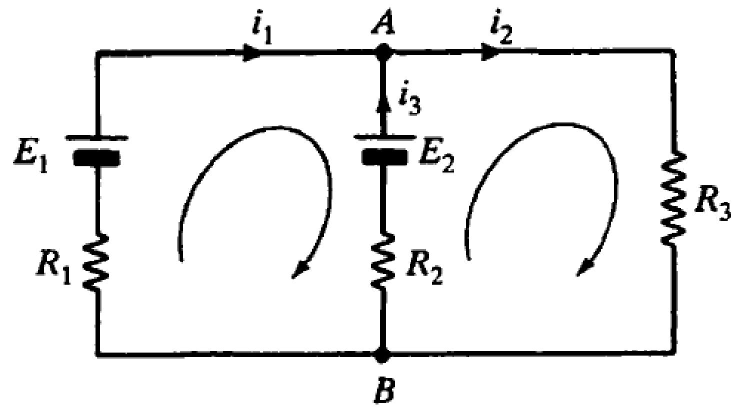
$$i = \frac{E_2 - E_1}{r_1 + r_2 + R}$$

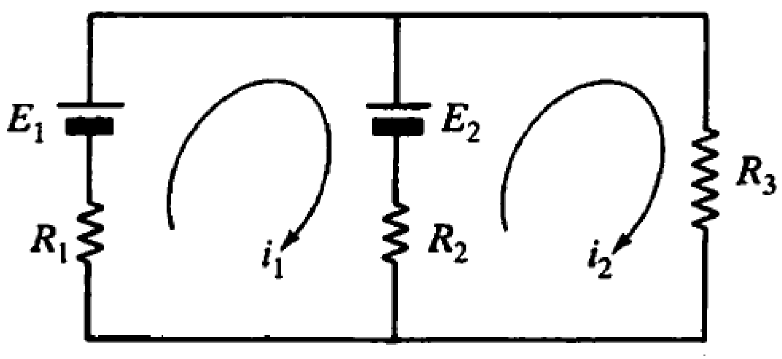
Fundamentals of Electrical Circuits

Example 2

Obtain the mathematical model of the following circuits.

(c)





$\boldsymbol{KCL}$:

$At\ point\ A$: $i_1 + i_3 - i_2 = 0$

$\boldsymbol{KVL}$:

$Left\ loop$: $E_1 - E_2 + i_3R_2 - i_1R_1 = 0$

$Right\ loop$: $E_2 - i_2R_3 - i_3R_2 = 0$

$\boldsymbol{KVL}$:

$Left\ loop$:

$$E_1 - E_2 - R_2(i_1 - i_2) - R_1i_1 = 0$$

$Right\ loop$:

$$E_2 - R_3i_2 - R_2(i_2 - i_1)\ = 0$$

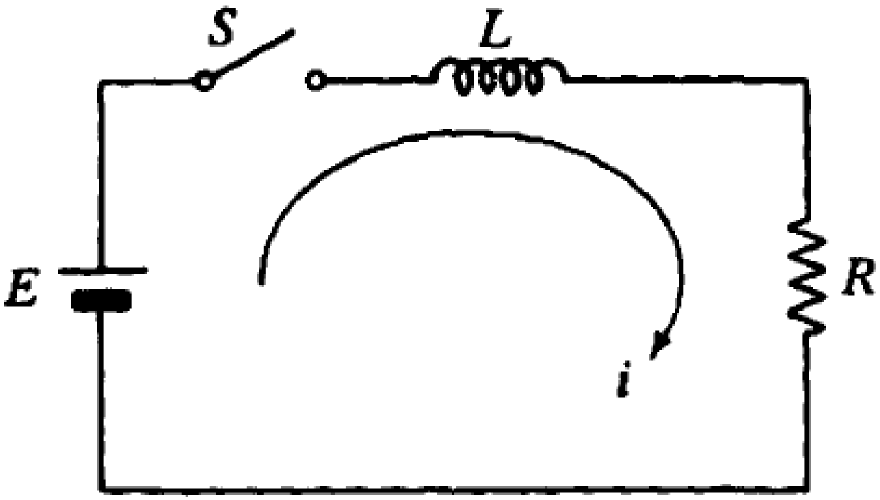
$$i_3 = \frac{E_1(R_1 + R_3) - E_1R_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad i_1 = \frac{E_1(R_2 + R_3) - E_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \qquad i_2 = \frac{E_1R_2 + E_2R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$

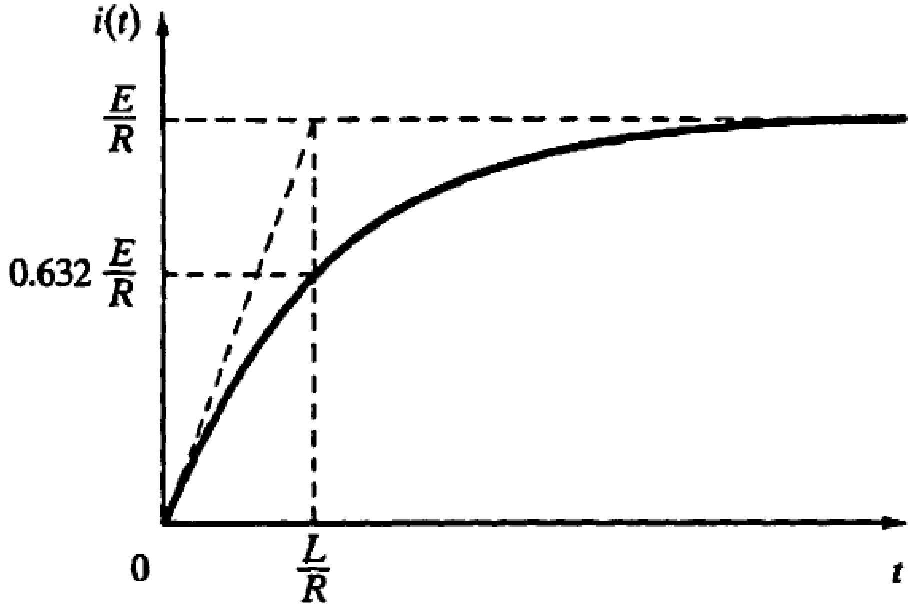
Mathematical Modeling of Electrical Systems

- The first step in analyzing circuit problems is to obtain mathematical models for the circuits. (Although the terms *circuit* and *network* are sometimes used interchangeably, *network* implies a more complicated interconnection than *circuit*.)

First-order Systems: Illustration 1

- Consider the circuit shown in the Figure. Assume that the switch S is open for $t < 0$ and closed at $t = 0$. Obtain a mathematical model for the circuit and obtain an equation for the current $i(t)$.





$$L\frac{di}{dt} + Ri = E \qquad i(0) = 0 \qquad i(t) = \frac{E}{R}[1 - e^{-(R/L)t}]$$

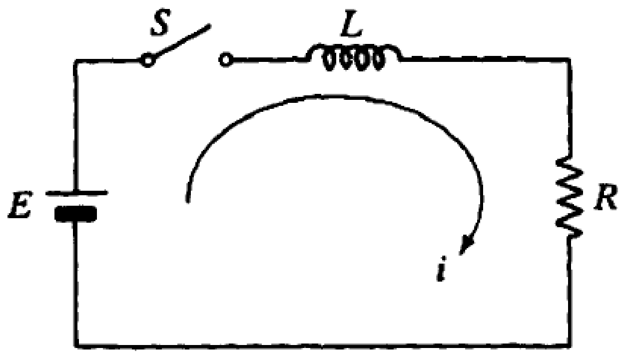
Mathematical Modeling of Electrical Systems

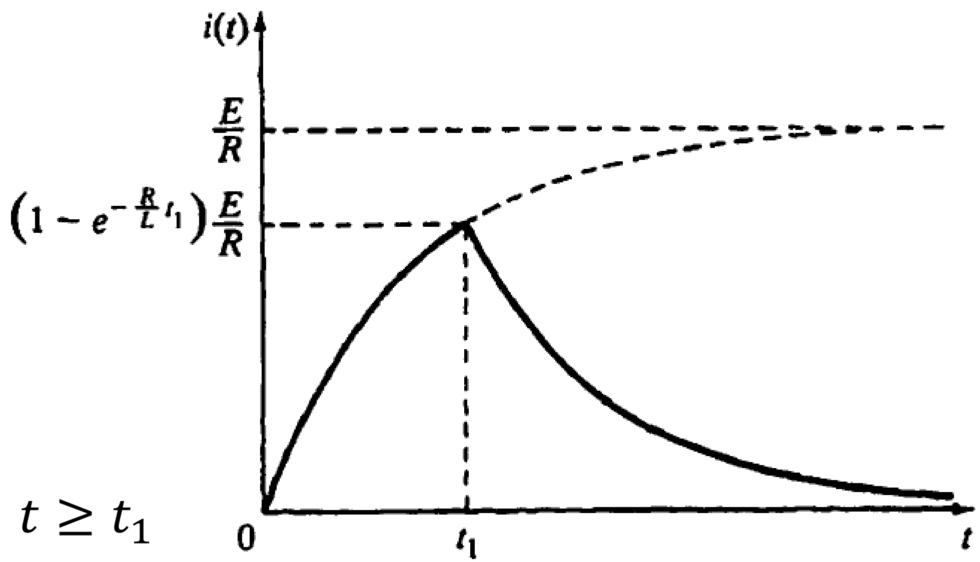
First-order Systems: Illustration 2

- Consider again the circuit shown in the Figure. Assume that switch S is open for $t < 0$, it is closed at $t = 0$, and is open again at $t = t_1 > 0$. Obtain a mathematical model for the system, and find the current $i(t)$ for $t \geq 0$.

$$L\frac{di}{dt} + Ri = E \qquad i(0) = 0 \qquad 0 \leq t < t_1$$

$$L\frac{di}{dt} + Ri = 0 \qquad i(t_1) = \frac{E}{R}\left[1 - e^{-(R/L)t_1}\right] \qquad t \geq t_1$$





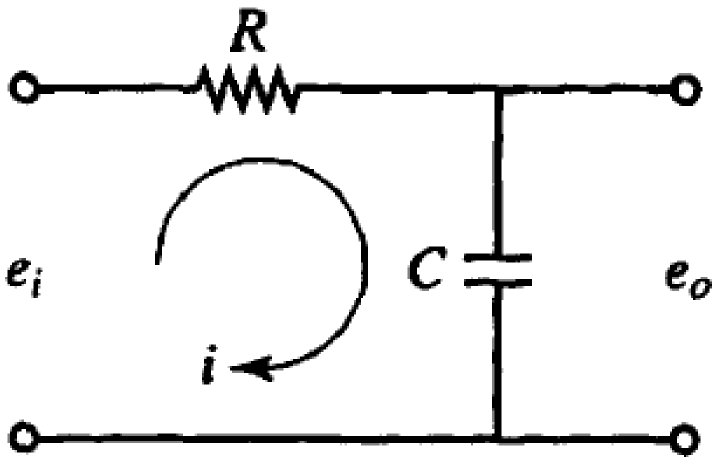
$$i(t) = \frac{E}{R}\left[1 - e^{-(R/L)t}\right], \qquad 0 \leq t < t_1$$

$$= \frac{E}{R}\left[1 - e^{-(R/L)t_1}\right]e^{-(R/L)(t-t_1)}, \qquad t \geq t_1$$

Mathematical Modeling of Electrical Systems

First-order Systems: Illustration 3

- Consider the electrical circuit shown in the Figure. The circuit consists of a resistance R (in ohms) and a capacitance C (in farads). Obtain the transfer function $E_o(s)/E_i(s)$. Also, obtain a state-space representation of the system.



$$Ri + \frac{1}{C}\int i dt = e_i$$

$$\frac{1}{C}\int i dt = e_o \quad \text{or} \quad i = C\frac{de_o}{dt}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{RCs + 1}$$

$$RC\frac{de_o}{dt} + e_o = e_i$$

$$Or \qquad RC\dot{e}_o + e_o = e_i$$

$$\dot{x} = -\frac{1}{RC}x + \frac{1}{RC}u$$

$$y = x$$

$$x = e_o$$

$$u = e_i$$

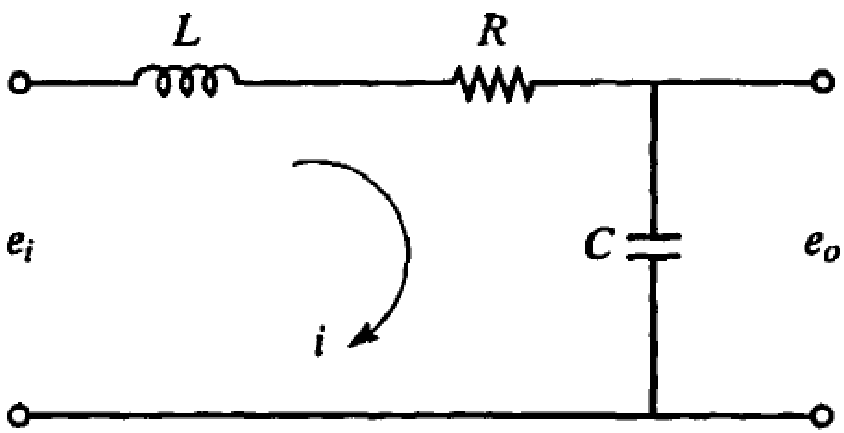
$$A = -\frac{1}{RC} \qquad B = \frac{1}{RC} \qquad C = 1 \qquad D = 0$$

$$y = e_o = x$$

Mathematical Modeling of Electrical Systems

Second-order Systems: Illustration 1

- Consider the electrical circuit shown in the Figure. The circuit consists of an inductance L (in henrys), a resistance R (in ohms), and a capacitance C (in farads). Obtain the transfer function $E_o(s)/E_i(s)$. Also, obtain a state-space representation of the system.



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e_i$$

$$\frac{1}{C}\int idt = e_o \quad \text{or} \quad i = C\frac{de_o}{dt}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1}$$

$$\ddot{e}_o + \frac{R}{L}\dot{e}_o + \frac{1}{LC}e_o = \frac{1}{LC}e_i$$

$$x_1 = e_o$$
$$x_2 = \dot{e}_o$$

$$u = e_i$$

$$y = e_o = x_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{LC} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{LC}\end{bmatrix}u$$

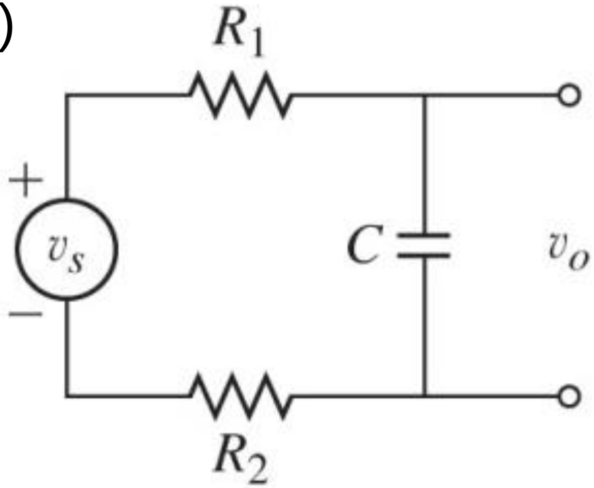
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Mathematical Modeling of Electrical Systems

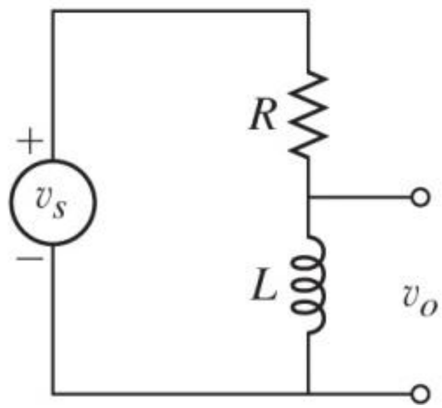
Example 3

Obtain the model of the voltage v_o, given the supply voltage v_s for the circuits shown below.

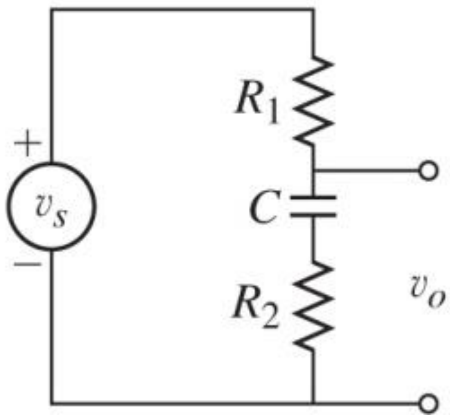
(a)



(b)



(c)



$$(R_1 + R_2)C\frac{dv_o}{dt} + v_o = v_s$$

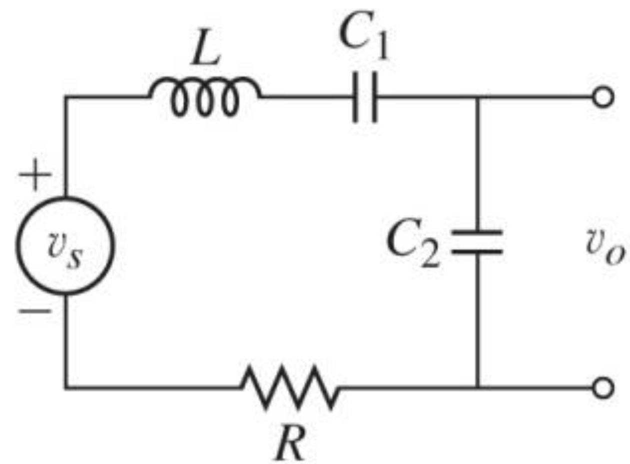
$$\frac{L}{R}\frac{dv_o}{dt} + v_o = \frac{L}{R}\frac{dv_s}{dt}$$

$$(R_1 + R_2)C\frac{dv_o}{dt} + v_o = R_2C\frac{dv_s}{dt} + v_s$$

Mathematical Modeling of Electrical Systems

Example 4

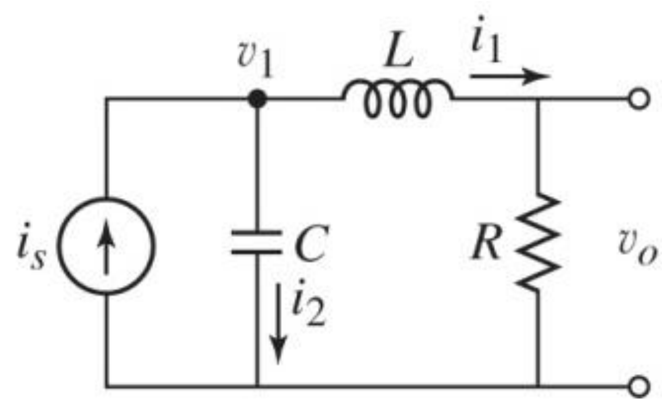
a) Find the transfer function $V_o(s)/V_s(s)$ of the electrical circuit below, given the supply voltage v_s and the output voltage v_o. Obtain the voltage $v_o(t)$ when the input voltage $v_s(t)$ is a impulse signal with strength of $10\ \text{V}$. Assume that initial conditions are zero, and that $C_1 = C_2 = 10\mu F, R = 200\ \Omega,$ and $L = 2\ \text{H}$



$$\frac{V_o(s)}{V_s(s)} = \frac{C_1}{LC_1C_2s^2 + RC_1C_2s + C_1 + C_2}$$

b) Obtain the state-space model of the circuit below, given the supply current i_s and the output voltage v_o.

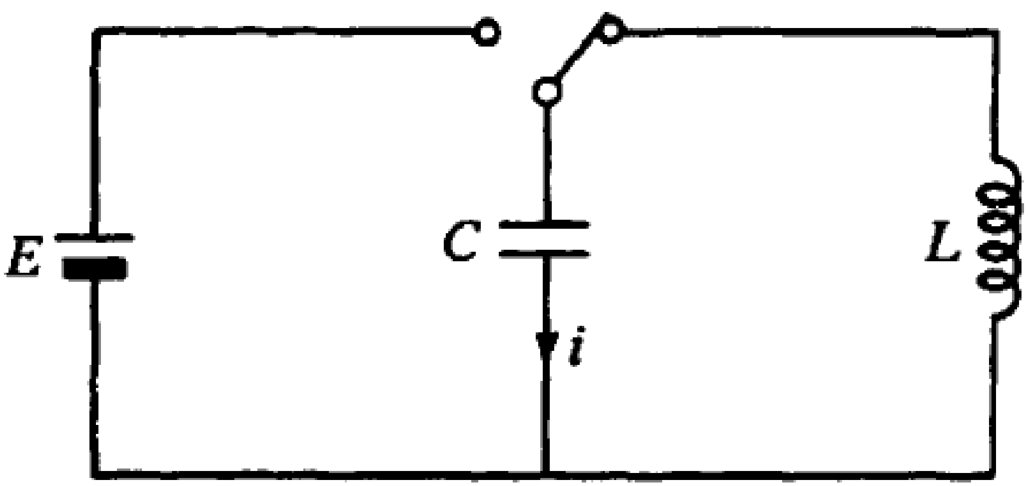
$$LC\frac{d^2v_o}{dt^2} + RC\frac{dv_o}{dt} + v_o = Ri_s$$



Mathematical Modeling of Electrical Systems

Example 5

Consider the circuit shown in the Figure. Assume that capacitor C is initially charged to q_o. At $t = 0$, switch S is disconnected from the battery and simultaneously connected to inductor L. The capacitance has a value of $50\,\mu F$. Calculate the value of the inductance L that will make the oscillation occur at a frequency of 200 Hz.



$$L\frac{di}{dt} + \frac{1}{C}\int i dt = 0$$

$$L\frac{d^2 i}{dt^2} + \frac{1}{C} i = 0$$

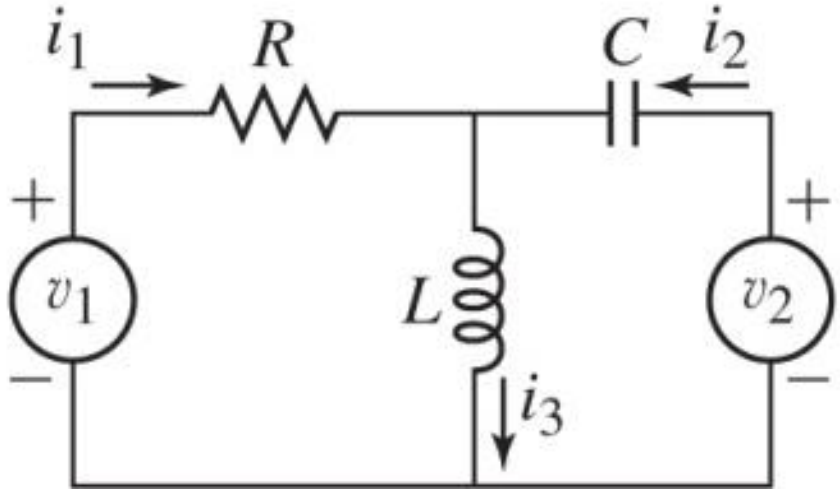
$$L\frac{d^2 q}{dt^2} + \frac{1}{C} q = 0$$

$$L = 0.0127 \text{ H}$$

Mathematical Modeling of Electrical Systems

Example 6

The RLC circuit shown in the Figure has two input voltages. Obtain the differential equation model for the current $i_3(t)$.



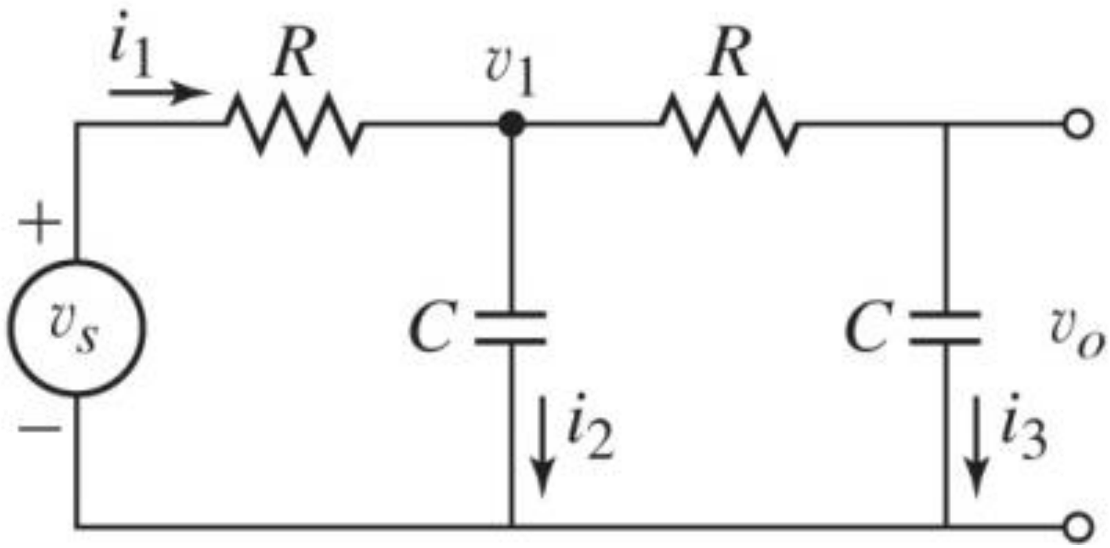
$$LRC\frac{d^2 i_3}{dt^2} + L\frac{di_3}{dt} + Ri_3 = v_1 + RC\frac{dv_2}{dt}$$

Obtain the current response $i_3(t)$ using MATLAB/Simulink. The parameter values are $R = 10^3\ \Omega$, $C = 2 \text{ x } 10^{-6}\ \mathrm{F}$, and $C = 2 \text{ x } 10^{-3}\ \mathrm{H}$. The voltage $v_1(t)$ is a step input of magnitude 5 V, and the voltage $v_2(t)$ is sinusoidal with frequency of 60 Hz and an amplitude of 4 V. The initial conditions are zero.

Mathematical Modeling of Electrical Systems

Example 7

Determine the transfer function $V_o(s)/V_s(s)$ of the circuit below, given the supply voltage v_s and the output voltage v_o. Draw two simulation block diagrams for the system.



$$\frac{V_o(s)}{V_s(s)} = \frac{1}{R^2C^2s^2 + 3RCs + 1}$$

Mathematical Modeling of Electrical Systems

Example 7

Find the transfer function $V_o(s)/V_s(s)$ of the electrical circuit below, given the supply voltage v_s and the output voltage v_o. Draw two simulation block diagrams for the system.