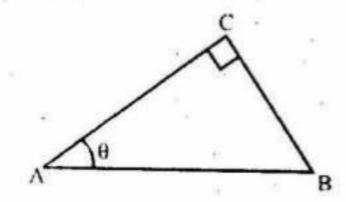
UNIT 11

INTRODUCTION TO TRIGONOMETRY

EXERCISE 11.1

 In the following figures identify hypotenuse, base and perpendicular w.r.t. θ.

(a)

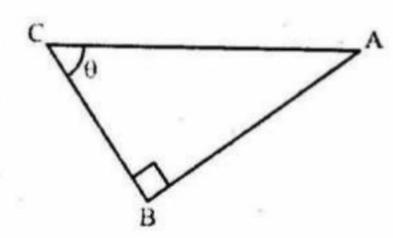


Hypotenuse = AB

Base = AC

Perpendicular = BC

(b)



Hypotenuse = AC

Base = BC

Perpendicular = AB

 Find unknown sides and angles of the following triangles.

In the figure.

$$AC = a$$
, $AB = b$. $BC = c$



$$c^2 = 1 +$$

Taking square root on b/s

$$\sqrt{c^2} = \sqrt{2}$$

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{c}{a} = \frac{\sqrt{2}}{1}$$

$$\tan \theta = 1$$

$$\theta = 45^{\circ}$$

$$m \angle A = 90^{\circ}$$
, $m \angle C = 45^{\circ}$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$90^{\circ} + \text{m} \angle \text{B} + 45^{\circ} = 180^{\circ}$$

$$m \angle B = 180^{\circ} - 90^{\circ} - 45^{\circ}$$

$$m \angle B = 45^{\circ}$$

b)
$$AB = a$$
, $BC = b$, $AC = c$

$$c^2 = a^2 + b^2$$

$$(2)^2 = (1)^2 + a^2$$

$$4 = 1 + a'$$

$$a^2 = 4 - 1$$

$$a^2 = 3$$

$$\sqrt{a^2} = \sqrt{3}$$

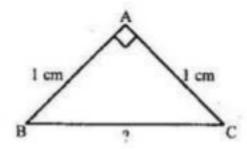
$$a = \sqrt{3}$$

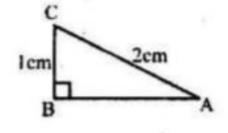
$$m \angle A = \frac{Hypotenuse}{Perpendicular} = \frac{2}{\sqrt{3}}$$

$$\csc \theta = \frac{2}{\sqrt{3}}$$

$$\frac{1}{\cos \theta} = \frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$





$$\theta = 30^{\circ}$$

$$m \angle A = 30^{\circ}$$
, $m \angle B = 90^{\circ}$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$30^{\circ} + 90^{\circ} + m \angle C = 180^{\circ}$$

$$m \angle C = 180^{\circ} - 90^{\circ} - 30^{\circ}$$

$$m \angle C = 60^{\circ}$$

(c)
$$AB = c$$
, $BC = a$, $AC = b$

$$c^2 = a^2 + b^2$$

$$(6)^2 = (3\sqrt{3})^2 + b^2$$

$$36 = 9(3) + b^2$$

$$b^2 = 36 - 27$$

$$b^2 = 9$$

$$\sqrt{(b)^2} = \sqrt{9}$$

$$m \angle B = \frac{Hypotenuse}{Perpendicular}$$

3√3cm

$$\csc \theta = \frac{6^2}{3\sqrt{3}} = \frac{6}{3\sqrt{3}}$$

$$\csc \theta = \frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^{\circ}$$

$$m \angle B = 30^{\circ}$$
, $m \angle C = 90^{\circ}$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$m \angle A + 30^{\circ} + 90^{\circ} = 180^{\circ}$$

$$m \angle A = 180^{\circ} - 90^{\circ} - 30^{\circ}$$

$$m \angle A = 60^{\circ}$$

(d)
$$m \angle B = 90^{\circ}$$
, $m \angle A = 45^{\circ}$, $m \angle C = ?$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$



$$45^{\circ} + 90^{\circ} + m \angle C = 180^{\circ}$$

$$m \angle C = 180^{\circ} - 90^{\circ} - 45^{\circ}$$

$$m \angle C = 45^{\circ}$$

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{b}$$

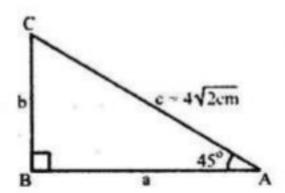
$$\frac{1}{\sqrt{2}} = \frac{b}{4\sqrt{2}}$$

$$b = \frac{4\sqrt{2}}{\sqrt{2}} = 4cm$$

$$\cos \theta = \text{base} = \frac{1}{b}$$

$$\frac{1}{\sqrt{2}} = \frac{d}{4\sqrt{2}}$$

$$a = \frac{4\sqrt{2}}{\sqrt{2}} = 4cm$$



3. If θ and ϕ are acute angles of a right angled triangle then complete the following statements.

(i)
$$\sin \theta = \cos \phi$$

(ii) $Cosec \varphi = sec \theta$

(iii)
$$Tan \theta = cot(\phi)$$

(iv) Sec
$$\phi = \csc(\theta)$$

(v)
$$\cos \theta = \sin \phi$$

(vi) Cot
$$\phi = \tan \theta$$

4. In $\triangle ABC$, $\angle A = 90^{\circ}$, $\angle B = \theta$, AB = c, BC = a and AC = b, then prove the following relations.

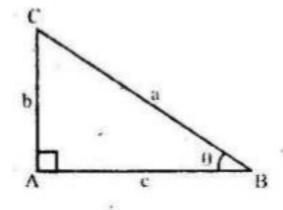
(i)
$$Tan \theta = \frac{Sin \theta}{Cos \theta}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{b}{a} : \frac{c}{a}$$

$$= \frac{b}{a} : \frac{c}{a}$$

$$= \frac{b}{c} = \frac{\text{Perpendiclar}}{\text{base}}$$

$$= \tan \theta = \text{L.H.S.}$$



(ii) $Cot \theta = \frac{Cos \theta}{Sin \theta}$

R.H.S.

$$= \frac{c}{a} \times \frac{a}{b}$$

$$= \frac{c}{b} = \frac{Base}{Perpendicular} = \cot \theta = L.H.S.$$

(iii) $Sin \theta \times Cosec \theta = 1$

L.H.S. =
$$\sin \theta \times \frac{1}{\sin \theta}$$

= $\frac{a}{b} \times \frac{1}{\frac{a}{b}} = \frac{b}{a} \times \frac{a}{b} = 1 = \text{R.H.S.}$

(iv) $\cos \theta \times \sec \theta = 1$

L.H.S. =
$$\cos \theta \times \frac{1}{\cos \theta}$$

= $\frac{c}{a} \times \frac{1}{c} = \frac{c}{a} \times \frac{a}{c} = 1 = \text{R.H.S.}$

(v) $Tan \theta x Cot \theta = 1$

L.H.S. =
$$\tan \theta \times \cot \theta$$

= $\frac{b}{c} \times \frac{c}{b} = 1 = R.H.S.$

Complete the following table.

| θ | sin θ | cos θ | tan θ | cosec θ | sec θ | cot θ |
|-----|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ | 2 | $\frac{2}{\sqrt{3}}$ | $\sqrt{3}$ |
| 45° | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 | √2 | $\sqrt{2}$ | 1 |
| 60° | $\frac{\sqrt{3}}{2}$ | 1 2 | $\sqrt{3}$ | $\frac{2}{\sqrt{3}}$ | 2 | $\frac{1}{\sqrt{3}}$ |

EXERCISE 11.2



(a)
$$AB = b$$
, $BC = c$, $AC = a$

According to Pythagoras theorem

$$a^2 = b^2 + c^2$$

$$c^2 = (1)^2 + (1)^2$$

$$c^2 = 1 +$$

Taking square root on b/s

$$\sqrt{c^2} = \sqrt{2}$$

$$c = \sqrt{2}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{base}}$$

$$\tan \theta = \frac{1}{1}$$

$$\tan \theta = 1$$

$$\theta = 45^{\circ}$$

$$\theta = 1 \tan^{-1}$$

As
$$m \angle B = 45^{\circ}$$
, $m \angle A = 90^{\circ}$

So,
$$m \angle C = ?$$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$m \angle A = 180^{\circ} - 90^{\circ} - 45^{\circ}$$

$$m \angle C = 45^{\circ}$$

(b)
$$AC = b$$
, $AB = c$, $BC = a$

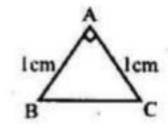
According to Pythagoras theorem

AB =
$$a = 2\sqrt{3}$$
 cm, AC = $c = 4\sqrt{3}$ cm, $b = ?$
 $c^2 = a^2 + b^2$

$$(4\sqrt{3})^2 = (2\sqrt{3})^2 + c^2$$

$$16(3) = 4(3) + c^2$$

$$c^2 = 48 - 12$$



$$c^2 = 36$$

Taking square root on b/s

$$\sqrt{c^2} = \sqrt{36}$$

$$c = 6$$

$$\sin m \angle A = \frac{Perpendicular}{base}$$

$$\sin\theta = \frac{2\sqrt{3}}{4\sqrt{3}}$$

$$\sin \theta = \frac{1}{2}$$

So
$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$(c) \quad AB = c, \quad BC = a, \quad AC = b$$

$$(8)^2 = (4\sqrt{3})^2 + b^2$$

$$64 = 16(3) + b^2$$

$$b^2 = 16$$

Taking square root on b/s

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4cm$$

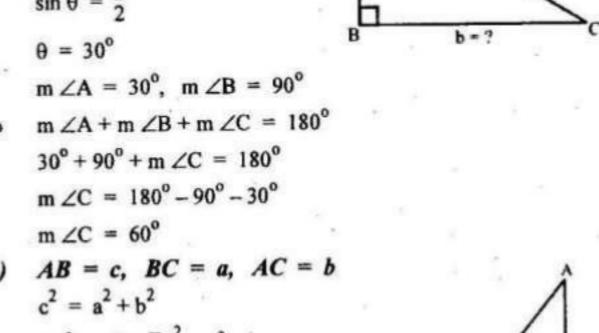
$$\cos m \angle A = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos = \frac{1}{2}$$

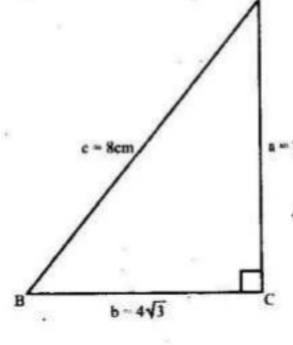
$$\cos \theta = 60^{\circ}$$

$$m \angle A = 60^{\circ}$$
, $m \angle C = 90^{\circ}$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$



a = 2√3cm



$$60^{\circ} + m \angle B + 90^{\circ} = 180^{\circ}$$

 $m \angle B = 180^{\circ} - 90^{\circ} - 60^{\circ}$
 $m \angle B = 30^{\circ}$

(d)
$$m \angle B = 90^{\circ}$$
, $m \angle A = 45^{\circ}$

So,
$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

 $45^{\circ} + 90^{\circ} + m \angle C = 180^{\circ}$
 $m \angle C = 18^{\circ} - 90^{\circ} - 45^{\circ}$
 $m \angle C = 45^{\circ}$
 $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\sin 45^{\circ} = \frac{a}{c}$$

$$\frac{1}{\sqrt{2}} = \frac{a}{4}$$

$$a = \frac{4}{\sqrt{2}}$$

$$\sqrt{2} a = 4$$

$$a = 2\sqrt{2}$$
 cm

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\cos 45^{\circ} = \frac{c}{b}$$

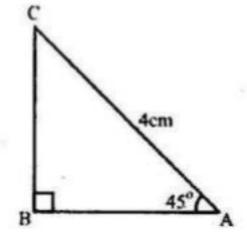
$$\frac{1}{\sqrt{2}} = \frac{c}{4}$$

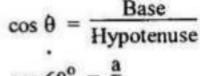
$$c = \frac{4}{\sqrt{2}}$$

$$c = 2\sqrt{2} \text{ cm}$$

Solve ΔABC for the following measurements.

(i)
$$\angle A = 90^{\circ}$$
, $\angle B = 60^{\circ}$, $AB = 4cm$
 $m \angle A + m \angle B + m \angle C = 160^{\circ}$
 $m \angle C = 180^{\circ} - 90^{\circ} - 60^{\circ}$
 $m \angle C = 30^{\circ}$ and $AB = a$, $AC = b$, $BC = c$





$$\cos 60^{\circ} = \frac{1}{3}$$

$$\frac{1}{2} = \frac{4}{6}$$

$$c = 4 \times 2$$

$$c = 8cm$$

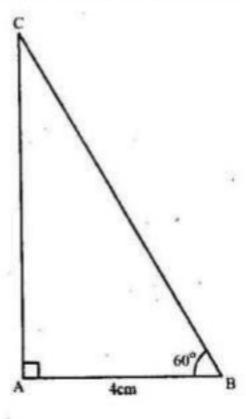
$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin 30^{\circ} = \frac{b}{c}$$

$$\frac{1}{2} = \frac{b}{8}$$

$$b = \frac{8}{2}$$

$$b = 4cm$$



(ii)
$$\angle B = 90^{\circ}$$
, $\angle C = 45^{\circ}$, $BC = 3\sqrt{2}$ cm

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$m \angle A + 90^{\circ} + 45^{\circ} = 180^{\circ}$$

$$m \angle A = 180^{\circ} - 90^{\circ} - 45^{\circ}$$

$$m \angle A = 45^{\circ}$$

$$\cos \theta = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$BC = a$$
, $AB = a$, $AC = 6$

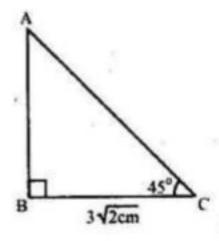
$$\cos 45^{\circ} = \frac{c}{a}$$

$$\frac{1}{\sqrt{2}} = \frac{c}{3\sqrt{2}}$$

$$\sqrt{2} \times c = 3\sqrt{2}$$

$$c = \frac{3\sqrt{2}}{\sqrt{2}}$$

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$



$$\sin 45^{\circ} = \frac{c}{b}$$

$$\frac{1}{\sqrt{2}} = \frac{c}{b}$$

$$c = \frac{6}{\sqrt{2}}$$

$$c = 3\sqrt{2}$$
(iii) $\angle C = 90^{\circ}$, $\angle A = 30^{\circ}$, $AB = 5cm$

$$m \angle A + m \angle B + m \angle C = 180^{\circ}$$

$$30^{\circ} + m \angle B + 90^{\circ} = 180^{\circ}$$

$$m \angle B = 180^{\circ} - 90^{\circ} - 30^{\circ}$$

$$m \angle B = 60^{\circ}$$

$$\cos c \theta = \frac{Base}{Hypotenuse}$$

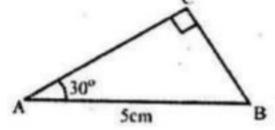
$$BC = a, AC = b, AB = c = 5cm$$

$$\cos 30^{\circ} = \frac{a}{c}$$

$$\frac{\sqrt{3}}{2} = \frac{b}{5}$$

$$b = \frac{5\sqrt{3}}{2}$$

$$AC = \frac{5\sqrt{3}}{2}$$



- 3. Solve AXYZ for the following measurements.
- (i) $\angle X 90^\circ$, XY = 2cm, YZ = 4cm

$$XY = x$$
, $XZ = y$, $ZY = z$

$$z^2 = x^2 + y^2$$

$$(4)^2 = y^2 + (2)^2$$

$$16 = y^2 + 4$$

$$y^2 = 16 - 4$$

$$y = \sqrt{12}$$

$$y = 2\sqrt{3}$$
 cm

$$\tan\theta = \frac{x}{y}$$

$$\tan\theta = \frac{2\sqrt{3}}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60^{\circ}$$

$$m \angle Y = 60^{\circ}$$

$$m \angle X = 90^{\circ}$$
, $m \angle Y = 60^{\circ}$

$$m \angle X + m \angle Y + m \angle Z = 180^{\circ}$$

$$90^{\circ} + 60^{\circ} + m \angle Z = 180^{\circ}$$

$$m \angle Z = 180^{\circ} - 90^{\circ} - 60^{\circ}$$

$$m \angle Z = 30^{\circ}$$

(ii)
$$\angle Y = 90^{\circ}$$
, $XY = 5cm$, $YZ = 5cm$

$$z^2 = x^2 + y^2.$$

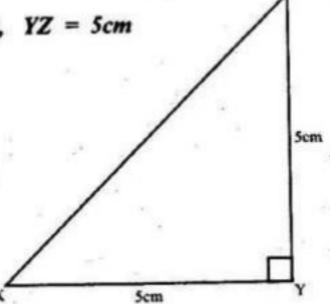
$$z^2 = (5)^2 + (5)^2$$

$$z^2 = 25 + 25$$

$$z^2 = \sqrt{50}$$

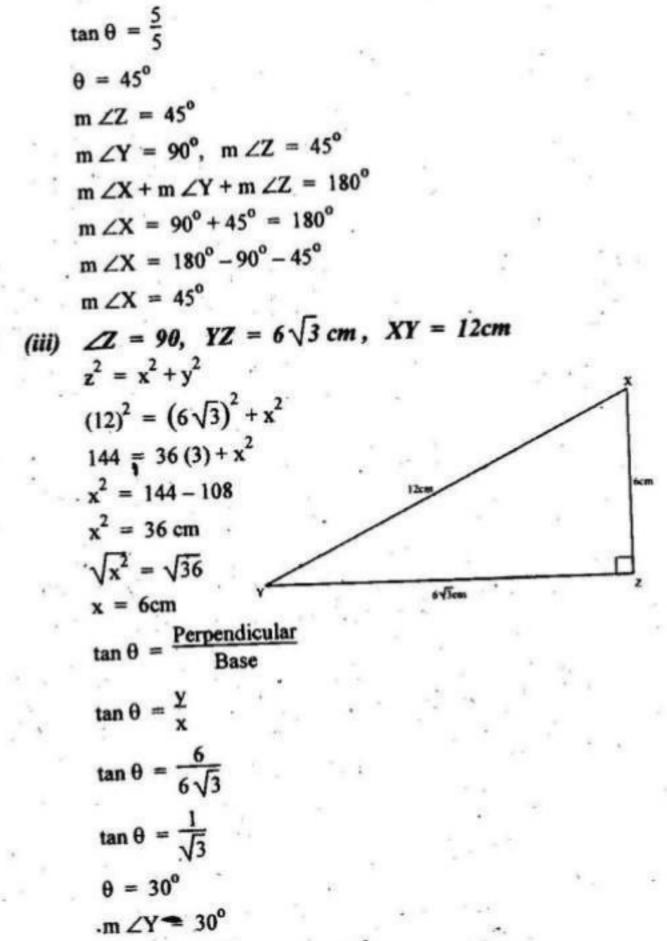
$$z = 5\sqrt{2}$$
 cm

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$



2cm





 $m \angle Z = 90^{\circ}$, $m \angle Y = 30^{\circ}$

 $m \angle X + m \angle Y + m \angle Z = 180^{\circ}$

 $m \angle X = 180^{\circ} - 90^{\circ} - 30^{\circ}$ $m \angle X = 60^{\circ}$

EXERCISE 11.3

1. A man is walking along a straight road. He observes that top of a tower subtends an angles of 30° with ground at the point where he is standing. If distance of tower from the man is loom find height of the tower.

Let the height = x

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\overline{AB}}{\overline{BC}}$$

$$\tan 30^{\circ} = \frac{x}{100}$$

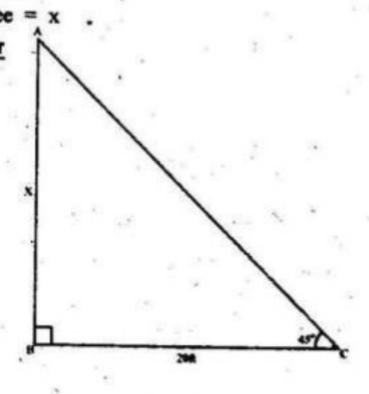
$$\tan \theta = \frac{x}{100}$$

$$\frac{1}{\sqrt{3}} = \frac{X}{100}$$

$$x = \frac{100}{\sqrt{3}} \text{ m}$$

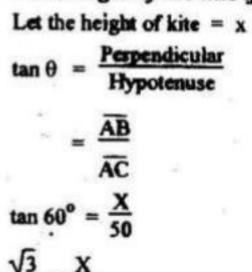
2. A man observes that the top of a tree subtends an angle of 45° at a point on ground 20ft away from foot of tree. Find height of tree.

Let the height of the tree = x $\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$ $= \frac{\overline{AB}}{\overline{BC}}$ $\tan 45^\circ = \frac{x}{20}$ $1 = \frac{x}{20}$ x = 20ft



APPART PLANT STORY STORY INTRODUCTION to Triconometry

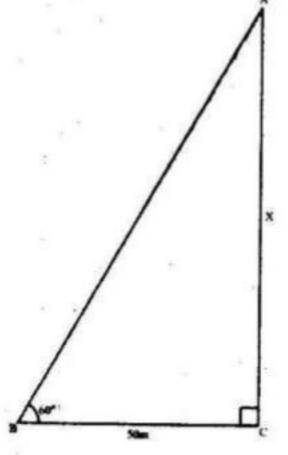
The string of a kite makes an angle of 60° with ground.
 Find height of the kite if length of the string is 50m.



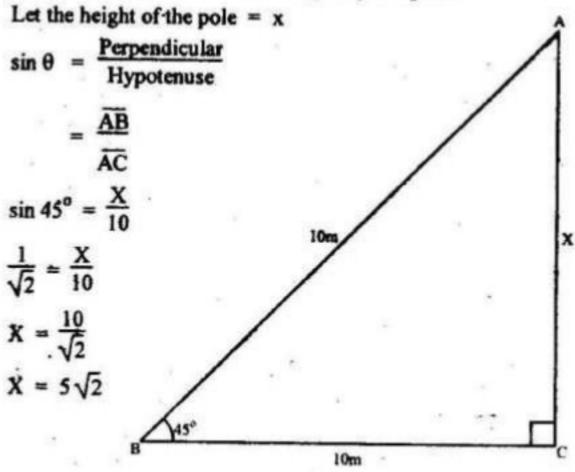
$$\frac{\sqrt{3}}{2} = \frac{X}{20}$$

$$X = 50 \times \frac{\sqrt{3}}{2}$$

$$X = 25\sqrt{3} \text{ m}$$



 A 10m long supporting rope of a pole makes an angle of 45° with ground. Find height of the pole.

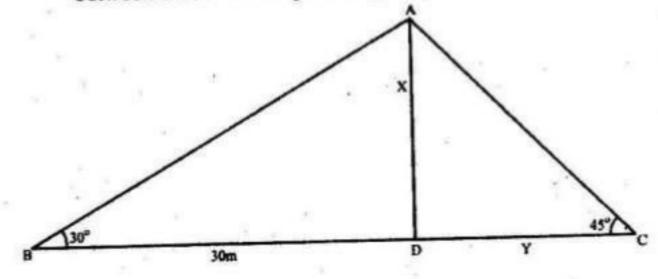


ASAN Math For Class 8th

310

Introduction to Trigonometry

5. Two men on opposite sides of tree observe that the three subtends angle of 45° and 30° on ground at their positions. If men are in same line with tree and distance between them is 30m find height of the tree.



$$\tan \theta = \frac{\text{Perpendicular}}{\text{Perpendicular}} = \frac{\text{AI}}{\text{VI}}$$

$$1 = \frac{x}{y}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{AD}}{\text{BD}}$$

$$\tan 30^{\circ} = \frac{x}{30 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{30 - x}$$

$$x = 0.577(30-x)$$

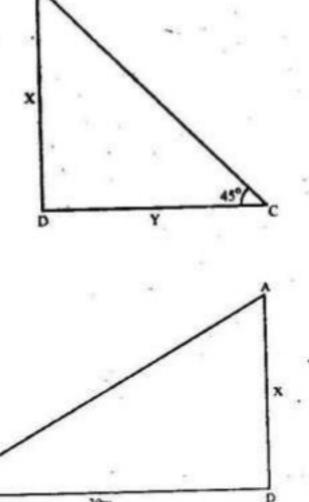
$$x = 17.31 - 0.577x$$

$$x + 0.577x = 17.31$$

$$1.577x = 17.31$$

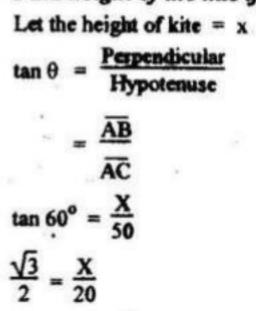
$$x = \frac{17.31}{1.577}$$

$$x = 10.98n$$



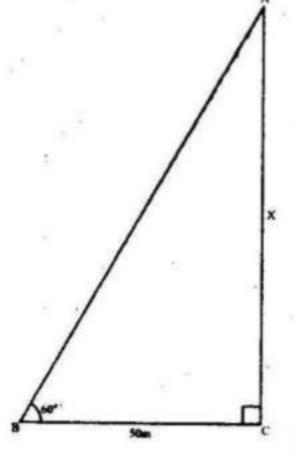
AGON TIME FOR CIEM 5 MIN INTRODUCTION IN Trigonometry

The string of a kite makes an angle of 60° with ground.
 Find height of the kite if length of the string is 50m.

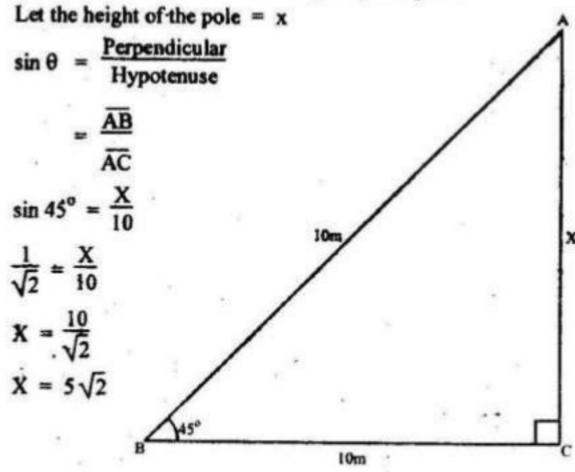


$$X = 50 \times \frac{\sqrt{3}}{2}$$

$$X = 25\sqrt{3} \text{ m}$$



 A 10m long supporting rope of a pole makes an angle of 45° with ground. Find height of the pole.

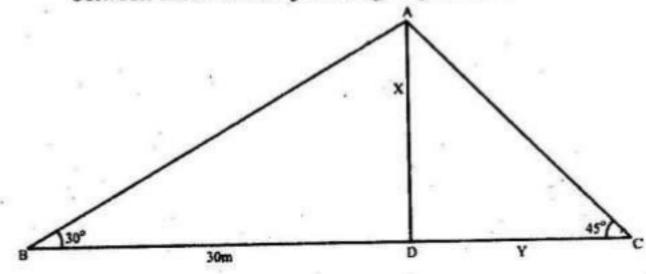


ASAN Math For Class 8th

310

Introduction to Trigonometry

5. Two men on opposite sides of tree observe that the three subtends angle of 45° and 30° on ground at their positions. If men are in same line with tree and distance between them is 30m find height of the tree.



$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{AD}}{\text{KD}}$$

$$1 = \frac{x}{y}$$

$$x = y$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{AD}}{\text{BD}}$$

$$\tan 30^{\circ} = \frac{x}{30 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{30-x}$$

$$x = 0.577(30-x)$$

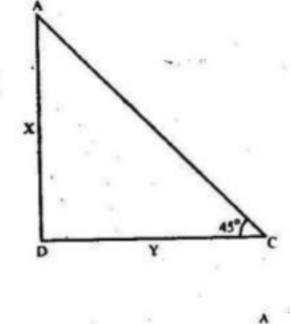
$$x = 17.31 - 0.577x$$

$$x + 0.577x = 17.31$$

$$1.577x = 17.31$$

$$x = \frac{17.31}{1.577}$$

$$x = 10.98m$$





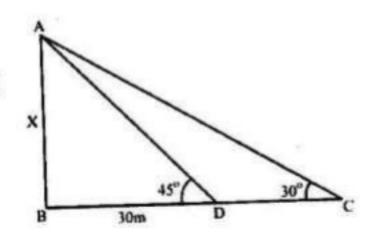
30m

- - 6. Two men on same side of a tree observe that the tree subtends angles of 45° and 30° on ground at their positions. If men are in the same line with tree and distance between them is 30m find height of the tree.

Let the height = x

$$\overline{BC} = y$$
 $\tan \theta = \frac{Perpendicular}{Base}$

$$= \frac{\overline{AB}}{\overline{BD}}$$



$$\tan 45^\circ = \frac{\Delta}{y}$$

$$1 = \frac{x}{y}$$

$$x = y$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$$

$$= \frac{\overline{AB}}{\overline{BC}}$$

$$\tan 30^{\circ} = \frac{x}{x + 30}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{x+30}$$

$$x = \frac{1}{1.732}(X + 30)$$

$$x = 0.577 (X + 30)$$

$$x + 0.577X + 17.31$$

$$x - 0.577X = 17.31$$

$$x = \frac{17.31}{1.423}$$

$$x = 40.9 \text{ m}$$

7. The length of shadow of a tower is equal to its height what is measure of angle subtended by tower along the ground at end point of shadow at that moment?

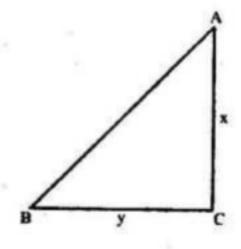
and
$$x = y$$

$$\tan \theta = \frac{1}{2}$$

time =
$$\frac{x}{x}$$

$$\tan \theta = 1$$

$$\theta = 45^{\circ}$$



8. A 4m long ladder makes on angle of 30° with the wall. Find height of top of the ladder along the wall.

Let the height
$$= x$$

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$
$$= \frac{AB}{AC}$$

$$\sin 30^{\circ} = \frac{x}{y}$$

$$\frac{1}{2} = \frac{x}{2}$$

$$4 = 2x$$

$$x = \frac{4}{2}$$

$$x = 2m$$

