UNIT 6

FACTORIZATION, SIMULTANEOUS EQUATIONS

EXERCISE 6.1

1. Expand the following by using formula.

(i) $(3a+5)^2$ $\quad(\because (a+b)^2 = a^2+2ab+b^2)$

$= (3a)^2 + 2(3a)(5) + (5)^2$

$= 9a^2 + 30a + 25$

(ii) $(6x-8)^2$ $\quad(\because (a-b)^2 = a^2-2ab+b^2)$

$= (6x)^2 - 2(6x)(8) + (8)^2$

$= 36x^2 - 96x + 64$

(iii) $(9c+2d)^2$ $\quad(\because (a+b)^2 = a^2+2ab+b^2)$

$= (9c)^2 + 2(9c)(2d) + (2d)^2$

$= 81c^2 + 26cd + 4d^2$

(iv) $(x^2-y^2)^2$ $\quad(\because (a-b)^2 = a^2-2ab+b^2)$

$= (x^2)^2 - 2(x^2)(y^2) - (y^2)^2$

$= x^4 - 2x^2y^2 + y^4$

(v) $(7x+9y)^2$ $\quad(\because (a+b)^2 = a^2+2ab+b^2)$

$= (7x)^2 + 2(7x)(9y) + (9y)^2$

$= 49x^2 + 126xy + 81y^2$

(vi) $(7x-8y)^2$ $\quad(\because (a-b)^2 = a^2-2ab+b^2)$

$= (7x)^2 - 2(7x)(8y) + (8y)^2$

$= 49x^2 - 112xy + 64y^2$

(vii) $\left(\frac{5}{3}x + \frac{3}{4}y\right)^2$

$= \left(\frac{5}{3}x\right)^2 + 2\left(\frac{5}{3}x\right)\left(\frac{3}{4}y\right) + \left(\frac{3}{4}y\right)^2$

$= \frac{25}{9}x^2 + \frac{5}{2}xy + \frac{9}{16}y^2$

(viii) $\left(x^2-\frac{2}{x^2}\right)^2$

$= (x^2)^2 - 2(x^2)\left(\frac{2}{x^2}\right) + \left(\frac{2}{x^2}\right)^2$

$= x^4 - 4 + \frac{4}{x^4}$

2. *Find the product without actual multiplication.*

(i) $(3x+4y)(3x-4y)$ $\quad (a+b)(a-b) = a^2-b^2$

$= (3x)^2 - (4y)^2$

$= 9x^2 - 16y^2$

(ii) $\left(x^2+\frac{2}{x^2}\right)\left(x^2-\frac{2}{x^2}\right)$ $\quad (a+b)(a-b) = a^2-b^2$

$= (x^2)^2 - \left(\frac{2}{x^2}\right)^2$

$= x^4 - \frac{4}{x^4}$

(iii) $\left(6x+\frac{1}{7}y\right)\left(6x-\frac{1}{7}y\right)$ $\quad (a+b)(a-b) = a^2-b^2$

$= (6x)^2 - \left(\frac{1}{7}y\right)^2$

$= 36x^2 - \frac{1}{49}y^2$

(iv) $(x+5y)(x-5y)(x^2+25y^2)$

$= (x)^2 - (5y)^2 (x^2+25y^2)$

$= (x^2-25y^2)(x^2+25y^2)$

$= (x^2)^2 - (25y^2)^2$

$= x^4 - 625y^4$

(v) $((\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)$

$= (\sqrt{x})^2 - (\sqrt{y^2})(x+y)$

$= (x-y)(x+y)$

$= (x)^2 - (y)^2$

$= x^2 - y^2$

3. ***Find the value of the following by using appropriate formula.***

(i) $(198)^2$

$= (200 - 2)^2$

$= (200)^2 - 2(2)(100) + (2)^2$

$= 40000 - 400 + 4$

$= 39604$

(ii) $(98)^2$

$= (100 - 2)^2$

$= (100)^2 - 2(100)(2) + (2)^2$

$= 10000 - 400 + 4$

$= 9604$

(iii) $(1.02)^2$

$= (1 + 0.02)^2$

$= (1)^2 + 2(1)(0.02) + (0.02)^2$

$= 1 + 0.04 + 0.0004$

$= 1.0404$

(iv) $(195)^2$

$= (200 - 5)^2$

$= (200)^2 - 2(200)(5) + (5)^2$

$= 40000 - 2000 + 25$

$= 38025$

(v) $(999)^2$

$= (1000 - 1)^2$

$= (1000)^2 - 2(1000)(1) + (1)^2$

$= 1000000 - 2000 + 1$

$= 998001$

4. ***Evaluate with the help of formula.***

(i) 101×99

$= (100 + 1) \times (100 - 1)$

$= (100)^2 - (1)^2$

$= 10000 - 1$

$= 9999$

(ii) 303×297

$= (300 + 3)(300 - 3)$

$= (300)^2 - (3)^2$

$= 90000 - 9$

$= 89991$

(iii) $(102) \times (98)$

$= (100 + 2)(100 - 2)$

$= (100)^2 - (2)^2$

$= 1000 - 4$

$= 9996$

(iv) $(502) \times (498)$

$= (500 + 2)(500 - 2)$

$= (500)^2 - (2)^2$

$= 250000 - 4$

$= 249996$

4. ***If $a + b = 4$ and $ab = 3$, then find the value of $a^2 + b^2$.***

$a + b = 4$, $a^2 + b^2 = ?$

Taking square of both side

$(a + b)^2 = (4)^2$

$a^2 + b^2 + 2ab = 16$

$a^2 + b^2 + 2(3) = 16$

$a^2 + b^2 + 6 = 16$

$a^2 + b^2 = 16 - 6$

$a^2 + b^2 = 10$

MATHEMATICS FOR 8TH CLASS (UNIT 6)

6. *If* $x+\frac{1}{x}=4$, *then find the value of* $x^2+\frac{1}{x^2}$

$\left(x+\frac{1}{x}\right)=4$

Taking square of both side

$\left(x+\frac{1}{x}\right)^2=(4)^2$

$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=16$

$x^2+\frac{1}{x^2}=16-2$

$x^2+\frac{1}{x^2}=14$

7. *If* $a-b=2$ *and* $ab=1$, *then find the value of* a^2+b^2.

$ab=1 \quad a-b=2$

As $a-b=2$

Taking square on both sides

$(a-b)^2=(2)^2$

$a^2+b^2-2ab=4$

$a^2+b^2-2(1)=4$

$a^2+b^2-2=4$

$a^2+b^2=4+2$

$a^2+b^2=6$

8. *If* $x-\frac{1}{x}=7$, *then find the value of* $x^2+\frac{1}{x^2}$.

$\left(x-\frac{1}{x}\right)=7 \quad x^2+\frac{1}{x^2}=?$

As $x-\frac{1}{x}=7$

Taking square of both side

$\left(x+\frac{1}{x}\right)^2=(7)^2$

$$x^2+\frac{1}{x^2}+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=49$$

$$x^2+\frac{1}{x^2}-2=49$$

$$x^2+\frac{1}{x^2}=49+2$$

$$x^2+\frac{1}{x^2}=51$$

9. ***If*** $x-\frac{1}{x}=5$, ***then find the value of*** $x^2+\frac{1}{x^2}$, $x^4+\frac{1}{x^4}$.

$\left(x-\frac{1}{x}\right)=5$ $x^2+\frac{1}{x^2}=?$, $x^4+\frac{1}{x^4}=?$

As $x-\frac{1}{x}=5$

Taking square of both side

$$\left(x-\frac{1}{x}\right)^2=(5)^2$$

$$x^2+\frac{1}{x^2}+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=25$$

$$x^2+\frac{1}{x^2}-2=25$$

$$x^2+\frac{1}{x^2}=25+2$$

$$x^2+\frac{1}{x^2}=27$$

Again taking square on both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=(27)^2$$

$$x^4+\frac{1}{x^4}+\frac{1}{x^4}-2(x^2)\left(\frac{1}{x^2}\right)=729$$

$$x^4+\frac{1}{x^4}+2=729$$

$$x^4 + \frac{1}{x^4} = 729 - 2$$

$$x^4 + \frac{1}{x^4} = 727$$

10. If $a + b = 6$, and $ab = 4$, then find the value of $a^2 + b^2$.

$(a + b) = 6$, $ab = 4$, $a^2 + b^2 = ?$

As $a + b = 6$

Taking square on both sides

$(a + b)^2 = (6)^2$

$a^2 + b^2 + 2ab = 36$

$a^2 + b^2 + 2(4) = 36$

$a^2 + b^2 + 8 = 36$

$a^2 + b^2 = 36 - 8$

$a^2 + b^2 = 28$

EXERCISE 6.2

1. Factorize the following polynomials.

(i) $7a - 7a^3 + 14a^4$

$= 7a(1 - a^2 + 2a^3)$

(ii) $x + 6xy - 9xz$

$= x(1 + 6y - 9z)$

(iii) $2x^2y^3 - 6x^2y^2 + 2xy^3$

$= 2xy^2(xy - 3x + y)$

(iv) $x^2y^3 - x^3y^2$

$= x^2y^2(y - x)$

(v) $a^2 + ab + ac + bc$

$= a^2 + ac + ab + bc$

$= a(a + c) + b(a + c)$

$= (a + b)(a + c)$

(vi) $3nx - 3x - 3ny + 3y$

$= 3(nx - x - ny + y)$

$= 3[x(n - 1) - y(n - 1)]$

$= 3(x - y)(n - 1)$

(vii) $ad + dc + df$

$= d(a + c + f)$

(viii) $7ab + 7ac + b + c$

$= 7ab + b + 7ac + b + c$

$= (7a + a) + c(7a + 1)$

$= (7a + 1)(b + c)$

(ix) $t^2 + 4t + st - 4s$

$= t(t + 4) - s(t + 4)$

(x) $8v^2 - 12vy + 14v - 21y$

$= 8v^2 + 14v - 12vy - 21y$

$= (t-s)(t+4)$

$= 2v(4v+7) - 3y(4v+7)$

$= (2v-3y)(4v+7)$

2. *Factorize the following.*

(i) $4x^2 + 4xy + y^2$

$= (2x)^2 + 2(2x)(y) + (y)^2$

(ii) $p^4 - 8p^2q + 16q^2$

$= (p^2)^2 - 2(4q)(p^2) + (4q)^2$

$= (p^2 - 4q)^2$

(iii) $4x^2 - 20x + 25$

$= (2x)^2 - 2(2x)(5) + (5)^2$

$= (2x-5)^2$

(iv) $49x^2 + 42x^2y^2 + 9y^4$

$= (7x^2)^2 + 2(7x^2)(3y^2) + (3y^2)^2$

$= (7x^2 + 3y^2)$

(v) $25x^2 - 10xy^4 + y^8$

$= (5x)^2 - 2(5x)(y^4) + (y^4)^2$

$= (5x - y^4)^2$

(vi) $(a-b)^2 - 4c^2$

$= (a-b)^2 - (2c)^2$

$= (a-b-2c)(a-b+2c)$

(vii) $25x^2 - 1$

$= (5x)^2 - (1)^2$

$= (5x-1)(5x+1)$

(viii) $\frac{1}{9}a^2 - \frac{1}{16}$

$= \left(\frac{1}{3}a\right)^2 - \left(\frac{1}{4}\right)^2$

$= \left(\frac{1}{3}a - \frac{1}{4}\right) - \left(\frac{1}{3}a + \frac{1}{4}\right)$

(ix) $x^4 - 16b^4$

$= (x^2)^2 - (4b^2)^2$

$= (x^2 - 4b^3)(x^2 + 4b^2)$

(x) $p^2q^2 - 64a^4$

$= (pq)^2 - (8a^2)^2$

$= (pq - 8a^2)(pq + 8a^2)$

(xi) $4x^2 - 4xy + y^2$

$= (2x)^2 - 2(2x)(y) + (y)^2$

$= (2x - y)^2$

(xii) $x^8 - y^8$

$= (x^4)^2 - (y^4)^2$

$= (x^4 - y^4)(x^4 + y^4)$

$= (x^2 - y^2)(x^2 + y^2)(x^4 - y^4)$

$= (x + y)(x - y)(x^2 + y^2)(x^4 - y^4)$

(xiii) $25a^2 - 20a + 4$

$= (5a)^2 - 2(5a)(2) + (2)^2$

$= (5a - 2)^2$

(xiv) $4x^2 + 4xy + y^2 - 9z$

$= (2x)^2 - 2(2x)(y) + (y)^2 - 9z^2$

$= (2x + y)^2 - 9z^2$

$= (2x + y)^2 - (3z)^2$

$= (2x + y - 3z)(2x + y + 3z)$

EXERCISE 6.3

1. Expand the following by using formula.

(i) $(a - 5)^3$ $\left(\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right)$

$= (a)^3 - (5)^3 - (a)(5)(a - 5)$

$= a^3 - 125 - 15a(a - 5)$

(ii) $(2x + 1)^3$ $\left(\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right)$

$= (2x)^3 + (1)^3 + 2(2x)(1)(2x + 1)$

2nd method

$(2x + 1)^3$ $\left(\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right)$

$= (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$

$= 8x^3 + 12x^2 + 6x + 1$

(iii) $(4c + 3d)^3$

$= (4c)^3 + 3(4c)^2(3d) + 3(4c)(3d)^3 + (3d)^3$

$= 64c^3 + 3(16c^2)(d) + 3(4c)(9d^2) + 27d^3$

$= 64c^3 + 144c^2d + 108cd^2 + 27d^3$

(iv) $(3x - 4y)^3$ $\left(\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right)$

$= (3x)^3 - 3(3x)^2(4y) - 3(3x)(4y)^2 - (4y)^3$

$= 27x^3 - 3(9x^2)(4y) - 3(3x)(16y^2) - 64y^3$

$= 27x^3 - 108x^2y - 144xy^2 - 64y^3$

(v) $(4x - 5y)^3$

$= (4x)^3 - 3(4x)^2(5y) - 3(4x)(5y)^2 - (5y)^3$

$= 16x^3 - 3(16x^2)(5y) - 3(4x)(25y^2) - 125y^3$

$= 16x^3 - 240x^2y - 300xy^2 - 125y^3$

(vi) $(7x - 8y)^3$

$= (7x)^3 - 3(7x)^2(8y) - 3(7x)(8y)^2 - (8y)^3$

$= 343x^3 - 3(49x^2)(8y) - 3(7x)(64y^2) - 512y^3$

$= 343x^3 - 117x^2y - 1344xy^2 - 512y^3$

(vii) $(3m + 7n)^3$

$= (3m)^3 + 3(3m)^2(7n) + 3(3m)(7n)^2 + (7n)^3$

$= 27m^3 + 3(9m^2)(7n) + 3(3m)(49n^2) + 232n^2$

$= 27m^3 + 189m^2n + 441mn^2 + 343n^2$

(viii) $(p - 6q)^3$

$= (p)^3 - 3(p^2)(6q) - 3(p)(pq) - (6q)^3$

$= p^3 - 18p^2q - 108pq^2 - 216q^3$

2. ***If $a+b=7$ and $ab=5$ find the value of a^3+b^3.***

$ab = 5 \quad a+b = 7 \quad a^3+b^3 = ?$

As $a+b = 7$

Taking cube on both sides

$(a+b)^3 = (7)^3$

$a^3+b^3+3(ab)(a+b) = 343$

$a^3+b^3+3(5)(7) = 343$

$a^3+b^3+105 = 343$

$a^3+b^3 = 343-105$

$a^3+b^3 = 238$

3. ***If $x-\frac{1}{x}=3$, then find the value of $x^3-\frac{1}{x^3}$.***

$\left(x-\frac{1}{x}\right) = 3 \quad x^3-\frac{1}{x^3} = ?$

As $x-\frac{1}{x} = 3$

Taking cube on both side

$\left(x-\frac{1}{x}\right)^3 = (3)^3$

$x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = 27$

$x^3-\frac{1}{x^3}-3(3) = 27$

$x^3-\frac{1}{x^3}-9 = 27+9$

$x^3-\frac{1}{x^3} = 36$

4. ***If $a-b=8$ and $ab=5$ find the value of a^3-b^3.***

$a-b = 8, \quad ab = 5 \quad a^3+b^3 = ?$

As $a-b = 8$

Taking cube on both sides

$(a-b)^3 = (8)^3$

$a^3-b^3+3(ab)(a-b) = 512$

$a^3-b^3-3(5)(8) = 512$

$a^3 - b^3 = 512 + 120$

$a^3 + b^3 = 632$ Ans.

5. *If* $x + \frac{1}{x} = 3$, *then find the value of* $x^3 + \frac{1}{x^3}$.

$x + \frac{1}{x} = 3 \quad x^3 + \frac{1}{x^3} = ?$

As $x + \frac{1}{x} = 3$

Taking cube on both side

$\left(x + \frac{1}{x}\right)^3 = (3)^3$

$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 27$

$x^3 + \frac{1}{x^3} + 3(3) = 27$

$x^3 + \frac{1}{x^3} + 9 = 27$

$x^3 + \frac{1}{x^3} = 27 - 9$

$x^3 + \frac{1}{x^3} = 18$

EXERCISE 6.4

1. ***Solve the following system of equations by elimination method.***

(i) $2x + y = 3$ (i)

$3x - 2y = -6$ (ii)

Multiply equation (i) by 2

$4x + 2y = 6$ (iii)

By adding equation (ii) and (iii)

$4x + 2y = 6$

$3x - 2y = -6$

$$x = \frac{0}{7}$$

$$x = 0$$

Now put the value of x in equation (i)

$$2x + y = 3$$

$$2(0) + y = 3$$

$$0 + y = 3$$

$$y = 3 - 0$$

$$y = 3$$

(ii) $7x - 2y = 1$ (i)

$3x + 4y = 15$ (ii)

Multiply equation (i) by 2

$$2(7x - 2y = 1)$$

$14x - 4y = 2$ (iii)

By adding equation (ii) and (iii)

$$14x - 4y = 2$$

$$3x + 4y = 15$$

$$17x = 17$$

$$x = \frac{17}{17}$$

$$x = 1$$

Now put the value of x in equation (i)

$$7(1) - 2y = 1$$

$$7 - 2y = 1$$

$$-2y = 1 - 7$$

$$2y = -6$$

$$y = \frac{-6}{-2}$$

$$y = \frac{6}{2}$$

$$y = 3$$

(iii) $9x + 4y = 5$ (i)

$3x - 5y = 8$ (ii)

Multiply equation (ii) by 3

$3(3x - 5y = 8)$

$9x - 15y = 24$ (iii)

Subtracting equation (i) from (iii)

$$\begin{array}{r} 9x - 15y = 24 \\ +9x + 4y = +15 \\ - \quad - \quad - \\ \hline -19y = 19 \end{array}$$

Dividing by '19' on both sides

$$x = \frac{19}{-19}$$

$x = -1$

By putting value of x in equation (ii)

$3x - 5(-1) = 8$

$3x + 5 = 8$

$3x = 8 - 5 = 3 \Rightarrow 3x = 3$

$$x = \frac{3}{3} = 1$$

(iv) $2x + 3y = 5$ (i)

$3x + 2y = 10$ (ii)

Multiply equation (i) by 3 and equation (ii) by 2

$3(2x + 3y = 5)$

$6x + 9y = 5$ (iii)

$2(3x + 2y = 10)$

$6x + 4y = 20$ (iv)

Subtracting equation (iv) from (iii)

$$\begin{array}{r} 6x + 9y = 5 \\ +6x + 4y = +20 \\ - \quad - \quad - \\ \hline 5y = -15 \end{array}$$

$$y = \frac{-15}{5}$$

$$y = -3$$

By putting the value of y in equation (i)

$$2x + 3y = 5$$
$$2x + 3(-3) = 5$$
$$2x - 9 = 5$$
$$2x = 5 + 9$$
$$2x = 14$$
$$x = \frac{14}{2}$$
$$x = 7$$

2. ***Solve the following system of equation by substitution method.***

(i) $x + 4y = 2$ (i)

$5x - 4y = 10$ (ii)

By solving equation (i)

$x + 4y = 2$

$x = 2 - 4y$ (iii)

By putting value of 'x' in equation (i)

$5x - 4y = 10$
$5(2 - 4y) - 4y = 10$
$10 - 20y - 4y = 10$
$10 - 24y = 10$
$-24y = 10 - 10$
$-24y = 0$
$y = \frac{0}{24} = 0$

Putting the value of 'y' in equation (i)

$x + 4y = 2$
$x + 4(0) = 2$
$x + 0 = 2$
$x = 2 - 0 = 2$

(ii) $x - y = 5$ (i)

$x + y = 19$ (ii)

$x - y = 5$

$x = 5 + y$ (iii)

Putting equation (iii) in equation (i)

$x + y = 19$

$(5 + y) + y + 19$

$5 + 2y = 19$

$2y = 19 - 5$

$2y = 14$

$y = \frac{14}{2}$

$y = 7$

Putting value of y in equation (i)

$x - 7 = 5$

$x = 5 + 7$

$x = 12$

(iii) $3x - 5y = -1$ (i)

$x - y = -1$ (ii)

By sub (i)

$x - y = -1$

$x = -1 + y$

$x = y - 1$

$x = y - 1$ (iii)

Putting equation (iii) in equation (i)

$3x - 5y = -1$

$3(y - 1) - 5y = -1$

$3y - 3 - 5y = -1$

$-2y = -1 + 3$

$-2y = 2$

$y = \frac{2}{-2}$

$y = -1$

Putting the value of 'y' in equation (ii)

$x = y - 1$

$x = 1 - 1$

$x = -2$

(iv) $3x - 7y = -10$ (i)

$y - 2x = 3$ (ii)

By solving equation (ii)

$y - 2x = 3$

$-2x = 3 - y$

$x = \frac{3-y}{-2}$ (iii)

Putting value of equation (iii) in equation (i)

$3x - 7y = -10$

$3\left(\frac{3-y}{-2}\right) - 7y = -10$

$\frac{9-3y}{-2} - 7y = -10$

$\frac{9-3y+14y}{-2} = -10$

$\frac{9+11y}{-2} = -10$

$9 + 11y = -10x - 2$

$9 + 11y = 20$

$11y = 20 - 9$

$11y = 11$

$y = \frac{11}{11} = 1$

putting the value of 'y' in equation (iii)

$x = \frac{3-y}{-2} \Rightarrow \frac{3-1}{-2} = \frac{2}{-2} = -1$

$x = -1$

3. Solve the following simultaneous equations by the method of cross multiplication.

(i) $2x + y = 1$

$x + y = 3$

$2x + y - 1 = 0$ (i)

$x + y - 3 = 0$ (ii)

General simultaneous linear equations are

$a_1x + b_1y + c_1 = 0$ (iii)

$a_2x + b_2y + c_2 = 0$ (iv)

By comparing eq (i), (ii), (iii) and (iv)

$a_1 = 2,\ b_1 = 1,\ c_1 = -1$

$a_2 = 1,\ b_2 = 1,\ c_2 = -3$

By applying formula

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \Rightarrow \frac{(1)(-3)-(1)(-1)}{(2)(1)-(1)(1)} = \frac{-3+1}{2-1} = \frac{-2}{1} = -2$$

$x = -2$

Now $$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \Rightarrow \frac{(1)(-1)-(2)(-3)}{(1)(1)-(1)(1)} = \frac{-1+6}{1-1} = 5$$

$y = 5$

(ii) $2x + y = 3$

$3x - 2y = 1$

$2x + y - 3 = 0$ (i)

$3x - 2y - 1 = 0$ (ii)

General simultaneous linear equations are

$a_1x + b_1y + c_1 = 0$ (iii)

$a_2x + b_2y + c_2 = 0$ (iv)

By comparing eq (i), (ii), (iii) and (iv)

$a_1 = 2,\ b_1 = 1,\ c_1 = -3$

$a_2 = 3,\ b_2 = -2,\ c_2 = -1$

By applying formula

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \Rightarrow \frac{(1)(-1)-(-2)(-3)}{(2)(-2)-(3)(1)} = \frac{-1-6}{-4-3} = \frac{-7}{7} = 1$$

$x = 1$

Now $$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \Rightarrow \frac{(3)(-3)-(2)(-1)}{(2)(-2)-(3)(1)} = \frac{9+2}{-4-3} = \frac{7}{-7} = -1$$

$y = -1$

(iii) $2x + y = 1$

$3x - y = 4$

$2x + y - 1 = 0$. (i)

$3x - y - 4 = 0$ (ii)

General simultaneous linear equations are

$a_1x + b_1y + c_1 = 0$ (iii)

$a_2x + b_2y + c_2 = 0$ (iv)

By comparing eq (i), (ii), (iii) and (iv)

$a_1 = 2,\ b_1 = 1,\ c_1 = -1$

$a_2 = 3,\ b_2 = -1,\ c_2 = -4$

By applying formula

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \Rightarrow \frac{(1)(-4) - (-1)(-1)}{(2)(-1) - (3)(1)} = \frac{-4-1}{-2-3} = \frac{-5}{-5} = 1$$

$x = 1$

Now $$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \Rightarrow \frac{(3)(-1) - (2)(-4)}{(2)(-1) - (3)(1)} = \frac{-3+8}{-2-3} = \frac{5}{-5} = -1$$

$y = -1$

(iv) $3x - 2y - 9 = 0$

$2x - 2y - 7 = 0$

$3x - 2y - 9 = 0$ (i)

$2x - 2y - 7 = 0$ (ii)

General simultaneous linear equations are

$a_1x + b_1y + c_1 = 0$ (iii)

$a_2x + b_2y + c_2 = 0$ (iv)

By comparing eq (i), (ii), (iii) and (iv)

$a_1 = 3,\ b_1 = -2,\ c_1 = -9$

$a_2 = 2,\ b_2 = -2,\ c_2 = -7$

By applying formula

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \Rightarrow \frac{(-2)(-7) - (-2)(-9)}{(3)(-2) - (2)(-2)} = \frac{14-18}{-6+4} = \frac{-4}{-2} = 2$$

$x = 1$

Now $y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \Rightarrow \frac{(2)(-9)-(3)(-7)}{(3)(-2)-(2)(-2)}$

$= \frac{-18+21}{-6+4} = \frac{3}{-2} = -\frac{3}{2}$

$y = -\frac{3}{2}$

EXERCISE 6.5

1. 5 pens and 6 note books together cost Rs. 9 and 3 pens and 2 note books cost Rs. 5. Find the cost of 1 pen and 1 note book.

Let the pen = x

Note book = y

$5x + 6y = 9$ (i)

$3x + 2y = 5$ (ii)

Multiply equation (i) by 3 and equation (ii) by 5 to equate the coefficient of variable x

$3(5x + 6y = 9)$

$5(3x + 2y = 5)$

$15x + 18y = 27$ (iii)

$15x + 10y = 25$ (iv)

Subtracting equation (iii) from (iv)

$15x + 18y = 27$

$+15x + 10y = 25$

$- \quad - \quad -$

$8y = 2$

$y = \frac{2}{8}$

$y = 0.25$ put in eq (i)

$5x + 6(0.25) = 9$

$5x + 1.50 = 9$

$5x = 9 - 1.50$

$x = \frac{7.50}{5}$

$x = 1.50$

Hence Pen = Rs. 1.50

Note book = Rs. 0.25

2. ***The sum of two numbers is 30 and their difference is 4. Find the numbers.***

Let the first number = x

Let the 2nd number = y

$x + y = 30$ (iii)

$x - y = 4$ (iv)

Adding equation (i) and (ii)

$x + y = 30$

$x - y = 4$

$2x = 34$

$x = \frac{34}{2}$

$x = 17$

Put the value of x in equation (ii)

$x - y = 4$

$17 - y = 4$

$-y = 4 - 17$

$-y = -13$

$y = 13$

First number = x = 17

2nd number = y = 13

3. ***A fraction becomes $\frac{4}{5}$, if 1 is added to both the numerator and denominator, if however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction.***

Let the numerator = x

Denominator = y

According to the given condition

$$\frac{x+1}{y+1} = \frac{4}{5} \quad \text{(i)}$$

According to 2nd condition

$$\frac{x-5}{y-5} = \frac{1}{2} \quad \text{(ii)}$$

By solving equation (i)

$$\frac{x+1}{y+1} = \frac{4}{5}$$

$$5(x+1) = 4(y+1)$$

$$5x+5 = 4y+4$$

$$5x-4y+5 = +4$$

$$5x-4y = 4-5$$

$$5x-4y = -1 \quad \text{(iii)}$$

By solving equation (ii)

$$\frac{x-5}{y-5} = \frac{1}{2}$$

$$2(x-5) = 1(y-5)$$

$$2x-10 = y-5$$

$$2x-y = -5+10$$

$$2x-y = 5 \quad \text{(iv)}$$

Now multiply by 4

$$4(2x-y = 5)$$

$$8x-4y = 20 \quad \text{(v)}$$

By subtracting equation (ii) from (i)

$$\begin{array}{r} 8x-4y = 20 \\ +5x-4y = -1 \\ - \quad + \quad + \\ \hline 3x \quad = 21 \end{array}$$

$$x = \frac{21}{3} = 7$$

Now putting the value of 'x' in equation (iv)

$$2x-y = 5$$

$$2(1)-y = 5$$

$14 - y = 5$

$-y = 5 - 14$

$-y = -9 \Rightarrow y = 9$

So, numerator $= x = 7$

Denominator $= y = 9$

The fraction $= \frac{x}{y} = \frac{7}{9}$

4. ***Think of a pair of number. If added 9 to the first, obtain a number which is twice the second. If 20 is added to the second, obtain a number which is 4 times the first. What are the numbers?***

Let first number $= x$

2nd number $= y$

$x + 9 = 2y$ (i)

$y + 20 = 4x$ (ii)

By solving equation (i)

$x + 9 = 2y$

$x = 2y - 9$ (iii)

Now putting value of equation (iii) in equation (i)

$y + 20 = 4x$

$y + 20 = 4(2y - 9)$

$y + 20 = 8y - 36$

$8y - y = 20 + 36$

$7y = \frac{56}{y} = 8$

Put the value of y in equation (i)

$x + 9 = 2(8)$

$x + 9 = 16$

$x = 16 - 9$

$x = 7$

5. ***I am three times as old as my son. Five years later, I shall be two and half times as old as my son. How old am I and how old my son?***

Let my age $= x$

According to the first condition

$y = 3x$

$3x - y = 0$ (i)

according to 2nd condition

$y + 5 = 2(x + 5)$

$y + 5 = 2x + 10$

$y - 2x = 10 - 5$

By adding equation (i) and (ii)

$3x - y = 0$

$-2x + y = 5$

$x = 5$

$3x + 5 = 2x + 10$

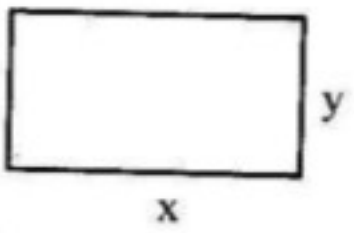
$3x - 2x = 10 - 5$

$x = 5$

Put the value of x in equation (i)

$y = 3 \times 5 = 15$

6. ***The difference between the length and the breath of a rectangle is 12m. The perimeter of a rectangle is 144m. Find its dimensions.***



Let the length = x

Width = y

$x - y = 12m$ (i)

$2(x + y) = 144$

$2x + 2y = 144$ (ii)

multiply equation (i) by 2

$2(x - y = 12m)$

$2x - 2y = 24m$ (iii)

Adding equation (i) and (ii)

$2x + 2y = 144$
$2x - 2y = 24$

$4x = 120$

$x = \frac{120}{4} = 30$

Putting the value of x in equation (i)

$x - y = 12$
$30 - y = 12$
$-y = 12 - 30 \Rightarrow -y = -18$
$y = 18$

EXERCISE 6.6

1. Eliminate x from the following equations by substitution method.

(i) $x + my = 0$ (i)
$3xy - n = 0$ (ii)
solving equation (i)
$x + my = 0$
$x = -my$
substituting equation (i) in equation (ii)
$3(-my)y - n = 0$
$-3my^2 - n = 0$
$-(3my^2 + n) = 0$
$3my^2 + n = 0$

(ii) $nx - 2y = 0$ (i)
$3x + 4y = 2$ (ii)
solving equation (i)
$nx - 2y = 0$
$nx = 2y$
$x = \frac{2y}{n}$
substituting equation (i) in (ii)
$3x + 4y = 2$

$$3\left(\frac{2y}{n}\right)+4y=2.$$

$$\frac{6y}{n}+4y=2$$

$$\frac{6y+4ny}{n}=2$$

$$6y+4ny=2n$$

(iii) $px-s=0$ (i)

$rx-5=0$

solving equation (i)

$rx-5=0$

$rx=5$

$x=\frac{5}{r}$

Substitute equation (i) in equation (ii)

$px-s=0$

$p\left(\frac{5}{r}\right)-s=0$

Multiply by 'r'

$5p-rs=0 \Rightarrow 5p=rs$. Ans.

(iv) $x-pq=0$ (i)

$4x+2y=pq$ (ii)

solving equation (i)

$x-pq=0$

$x=pq$

substituting equation (i) in (ii)

$4x+2y=pq$

$4pq+2y=pq$

$4pq-pq+2y=0$

$3pq+2y=0$

(v) $ax+3ay=0$ (i)

$3x=b-4y$ (ii)

By solving equation (ii)

$3x=b-4y$

$$x = \frac{b-4y}{3}$$

substitute equation (i) in (ii)

$$ax + 3ay = 0$$

$$a\left(\frac{b-4y}{3}\right) + 3ay = 0$$

$$\frac{ab-4ay}{3} + 3ay = 0$$

Multiply by '3'

$$ab - 4ay + 9ay = 0$$

$$ab + 5ay = 0$$

Divided by 'a'

$$by + 5a = 0$$

$$b = -5y$$

(vi) $7x + 8y = 0$ (i)

$4x + 4y = 2$ (ii)

solving equation (i)

$$7x + 8y = 0$$

$$7x = -8y$$

$$x = \frac{-8y}{7}$$

Substitute equation (i) in (ii)

$$4\left(\frac{-8y}{7}\right) + 4y = 2$$

$$\frac{-32}{7}y + 4y = 2$$

Multiply by '7'

$$-32y + 28y = 14$$

$$-4y = 14$$

$$y = \frac{-14}{4} = \frac{-7}{2}$$

2. ***Eliminate x from the following equations by using formula.***

(i) $x+\frac{1}{x}=a$ (i)

$x^2+\frac{1}{x^2}=b$ (ii)

from (i)

$x+\frac{1}{x}=a$

taking square of both sides

$\left(x+\frac{1}{x}\right)^2=(a)^2$

$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=a^2$

$x^2+\frac{1}{x^2}+2=a^2$

$x^2+\frac{1}{x^2}=a^2-2$ (iii)

Comparing equation (ii) and (iii)

$a^2-2=b$

$a^2-b=2$

(ii) $x-\frac{1}{x}=l+2$ (i)

$x^2+\frac{1}{x^2}=m$ (ii)

solving equation (i)

$x-\frac{1}{x}=l+2$

taking square of both sides

$\left(x+\frac{1}{x}\right)^2=(l+2)^2$

$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=l^2+4+2(2)(l)$

$$x^2+\frac{1}{x^2}-2 = l^2+4+4l$$

$$x^2+\frac{1}{x^2} = l^2+4l+4+2$$

$$x^2+\frac{1}{x^2} = l^2+4l+6 \qquad \text{(iii)}$$

Comparing the left hand side of equations (ii) and (iii)

$$l^2+4l+6 = m$$

(iii) $x+\frac{1}{x} = 3$ (i)

$$x^2-\frac{1}{x^2} = m \qquad \text{(ii)}$$

Solving equation (i)

$$x+\frac{1}{x} = 3$$

taking square of both side

$$\left(x+\frac{1}{x}\right)^2 = (3)^2$$

$$x^2+\frac{1}{x^2}+2 = 9$$

$$x^2+\frac{1}{x^2} = 9-2$$

$$x^2+\frac{1}{x^2} = 7$$

Again taking square root

$$\left(x^2+\frac{1}{x^2}\right)^2 = (7)^2$$

$$x^4+\frac{1}{x^4}+2 = 49$$

$$x^4+\frac{1}{x^4} = 49-2$$

$$x^4+\frac{1}{x^4} = 47 \qquad \text{(iii)}$$

By solving (ii)

$$x^2 - \frac{1}{x^2} = m$$

Taking square root of both side

$$\left(x^2 - \frac{1}{x^2}\right)^2 = (m)^2$$

$$x^4 + \frac{1}{x^4} - 2 = 49$$

$$x^4 + \frac{1}{x^4} = m^2 + 2 \qquad \text{(iv)}$$

Comparing equation (iii) and (iv)

$$47 = m^2 + 2$$

$$m^2 + 2 = 47$$

$$m^2 + 2 - 47 = 0$$

$$m^2 - 45 = 0$$

(iv) $$x - \frac{1}{x} - 4 = 0 \qquad \text{(i)}$$

$$x^2 - \frac{1}{x^2} = y \qquad \text{(ii)}$$

Solving equation (i)

$$x - \frac{1}{x} - 4 = 0$$

$$x - \frac{1}{x} = 4$$

Taking square of both side

$$\left(x - \frac{1}{x}\right)^2 = (4)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2$$