UNIT 2

REAL NUMBERS

EXERCISE 2.1

1. Which of the following are rational and which are irrational numbers?

(i) $\sqrt{9}$ = rational number

(ii) 12 = rational number

(iii) $\frac{5}{9}$ = rational number

(iv) $\sqrt{8}$ = irrational number

(v) $\sqrt{100}$ = rational number

(vi) $\frac{13}{2}$ = rational number

(vii) $\sqrt{126}$ = irrational number

(viii) $\frac{25}{9}$ = rational number

(ix) $\sqrt{169}$ = rational number

(x) $\sqrt{26}$ = irrational number

2. Write the following in decimal representation and state which of them are terminating and non-terminating decimals.

(i) $\frac{4}{9} = 0.444\ldots$ = non–terminating decimals

(ii) $\frac{13}{20} = 0.65$ = terminating decimals

(iii) $\frac{1}{6} = 0.1666\ldots$ = non terminating decimals

(iv) $\frac{7}{3} = 2.333\ldots$ = non–terminating decimals

(v) $\frac{9}{8} = 1.125$ = terminating decimals

(vi) $\frac{13}{8}$ = 1.625 = terminating decimals

(vi) $\frac{11}{15}$ = 0.7333..... = non–terminating decimals

(vii) $\frac{7}{11}$ = 0.6363.... = non–terminating decimals

3. Name the type of following decimals. Also state whether they are rational or irrational numbers?

(i) 1.2578 = Terminating, rational

(ii) 0.33333 = Non–terminating, rational

(iii) 1.4142135662 = Non–terminating repeating, rational

(iv) 5.14285714285 7=Non–terminating non–repeating, irrational

(v) 2.236067977 = non–terminating non–repeating, irrational

(vi) 4.36363636.... = Non–terminating repeating, rational

(vii) 4.123105626... = Non–terminating non repeating, irrational

EXERCISE 2.2

1. Find the squares of the following numbers.

(i) 8
$= (8)^2 = 8 \times 8 = 64$

(ii) 12
$= (12)^2$ $12 \times 12 = 144$

(iii) 17
$= (17)^2 = 17 \times 17 = 289$

(iv) 25
$= (25)^2 = 25 \times 25 = 625$

(v) 39
$= (39)^2 = 39 \times 39 = 1521$

(vi) 100
$= (100)^2 = 100 \times 100$
$= 10000$

(vii) 125
$= (125)^2 = 125 \times 125$
$= 15625$

(viii) 200
$= (200)^2 = 200 \times 200$
$= 40000$

(ix) 500
$= (500)^2 = 500 \times 500$
$= 250000$

(x) 900
$= (900)^2 = 900 \times 900$
$= 810000$

2. Tell which of the following are perfect squares.

(i) $64 = (8)^2$
= Perfect square

(ii) $82 = 2 \times$
= Imperfect square

(iii) $99 = 3 \times 3 \times 11$

(iv) $144 = (12)^2$

(v) $900 = (30)^2$
= Perfect square

(vi) $125 = 5 \times 5 \times 5$
= Perfect square

(vi) $169 = (13)^2$
= Perfect square

(viii) $250 = 2 \times 5 \times 5 \times 5$
= Imperfect square

3. Write the patterns of the square of the following numbers.

(i) $3^2 = 9 = 1+2+3+2+1$

(ii) $8^2 = 64$
$= 1+2+3+4+5+6+7+8+7+6+5+4+3+2+1$

(iii) $9^2 = 81$
$= 1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1$

(iv) $10^2 = 100$
$= 1+2+3+4+5+6+7+8+9+10+9+8+7$
$+6+5+4+3+2+1$

(v) $12^2 = 144$
$= 1+2+3+4+5+6+7+8+9+10+11+12+11+10$
$+9+8+7+6+5+4+3+2+1$

(vi) $15^2 = 225$
$= 1+2+3+4+5+6+7+8+9+10+11+12+11+10$
$+11+12+13+14+15+14+13+12+11+10+9+8$
$+7+6+5+4+3+2+1$

(vii) $20^2 = 400$
$= 1+2+3+4+5+6+7+8+9+10+11+12+11+10$
$+11+12+13+14+15+16+17+18+19+20+19$
$+18+17+16+15+14+13+12+11+10+9+8+7$
$+6+5+4+3+2+1$

EXERCISE 2.3

1. Find the square root of the following by factorization.

(i) 256

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$
$= 2^2 \times 2^2 \times 2^2 \times 2^2$

2	256
2	128
2	64

Taking square root of both sides

$\sqrt{256} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{2^2}$

$= 2 \times 2 \times 2 \times 2$

$= 16$

2	32
2	16
2	8
2	4
2	2
	1

(ii) 400

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

$= 2^2 \times 2^2 \times 5^2$

Taking square root of both sides

$\sqrt{400} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{5^2}$

$= 2 \times 2 \times 5$

$= 20$

2	400
2	200
2	100
2	50
5	25
5	5
	1

(iii) 729

$= \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$= 3^2 \times 3^2 \times 3^2$

Taking square root of both sides

$\sqrt{729} = \sqrt{3^2} \times \sqrt{3^2} \times \sqrt{3^2}$

$= 3 \times 3 \times 3$

$= 27$

3	729
3	243
3	81
3	27
3	9
3	3
	1

(iv) 1296

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$= 2^2 \times 2^2 \times 3^2 \times 3^2$

Taking square root of both sides

$\sqrt{1296} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{3^2}$

$= 2 \times 2 \times 3 \times 3$

$= 36$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

(v) 2304

$= \underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}$

$= 2^2\times2^2\times2^2\times2^2\times3^2$

Taking square root of both sides

$\sqrt{256} = \sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{3^2}$

$= 2\times2\times2\times2\times3$

$= 48$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(vi) 20736

$= \underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}$

$\times\underline{3\times3}$

$= 2^2\times2^2\times2^2\times2^2\times3^2\times3^2$

Taking square root of both sides

$\sqrt{256} = \sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}$

$\times\sqrt{3^2}\times\sqrt{3^2}$

$= 2\times2\times2\times2\times3\times3$

$= 144$

2	20736
2	10368
2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

(vii) 38416

$= \underline{2\times2}\times\underline{2\times2}\times\underline{7\times7}\times\underline{7\times7}$

$= 2^2\times2^2\times7^2\times7^2$

Taking square root of both sides

$\sqrt{38416} = \sqrt{2^2}\times\sqrt{2^2}\times\sqrt{7^2}\times\sqrt{7^2}$

2	38416
2	19208
2	9604
2	4802
7	2401

$= 2 \times 2 \times 7 \times 7$

$= 196$

7	343
7	49
7	7
	1

(viii) 50625

$= \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{5 \times 5}$

$= 3^2 \times 3^2 \times 5^2 \times 5^2$

Taking square root of both sides

$\sqrt{50625} = \sqrt{3^2 \times 3^2 \times 5^2 \times 5^2}$

$= \sqrt{3^2} \times \sqrt{3^2} \times \sqrt{5^2} \times \sqrt{5^2}$

$= 3 \times 3 \times 5 \times 5$

$= 225$

3	50625
3	16875
3	5625
3	1875
5	625
5	125
5	25
5	5
	1

2. *Find the square root by division method.*

(i) 324

	18
1	324
	1
28	224
	224
	0

$= 18$

(ii) 4356

	66
6	4356
	36
126	756
	756
	0

$= 66$

(iii) 6561

	81
8	6561
	64
161	161
	161
	0

$= 81$

(iv) 12544

	112
1	12544
	1
21	2544
	21
222	444
	444
	0

$= 112$

(v) 181476

	426
4	181476
	16
82	21476
	164
846	5076
	5076
	0

= 426

(vi) 531441

	729
7	531441
	49
142	41441
	284
1449	13041
	13041
	0

= 729

EXERCISE 2.4

1. Find the square root of the following.

(i) $\frac{25}{49}$

$= \frac{\sqrt{5 \times 5}}{\sqrt{7 \times 7}}$

$= \frac{\sqrt{5^2}}{\sqrt{7^2}}$

$= \frac{\sqrt{5^2}}{\sqrt{7^2}}$

$= \frac{5}{7}$

(ii) $\frac{225}{169}$

$= \frac{\sqrt{225}}{\sqrt{169}}$

$= \frac{\sqrt{3 \times 3 \times 5 \times 5}}{\sqrt{13 \times 13}}$

$= \frac{\sqrt{3^2 \times 5^2}}{\sqrt{13^2}}$

$= \frac{3 \times 5}{13}$

$= \frac{15}{13}$

(iii) $\frac{1681}{841}$

$\frac{\sqrt{1681}}{\sqrt{841}}$

$= \frac{\sqrt{41 \times 41}}{\sqrt{29 \times 29}}$

$= \frac{\sqrt{41^2}}{\sqrt{29^2}}$

(iv) $\frac{361}{625}$

$= \frac{\sqrt{225}}{\sqrt{169}}$

$= \frac{\sqrt{19 \times 19}}{\sqrt{5 \times 5 \times 5 \times 5}}$

$= \frac{\sqrt{19^2}}{\sqrt{5^2 \times 5^2}}$

$= \frac{41}{29}$

(v) $\frac{1296}{1225}$

$= \frac{\sqrt{1296}}{\sqrt{1225}}$

$= \frac{\sqrt{2\times2\times2\times2\times3\times3\times3\times3}}{\sqrt{5\times5\times7\times7}}$

$= \frac{\sqrt{2^2\times2^2\times3^2\times3^2}}{\sqrt{5^2\times7^2}}$

$= \frac{2\times2\times3\times3}{5\times7}$

$= \frac{36}{35}$

(vi) $\frac{3025}{729}$

$= \frac{19}{25}$

$= \frac{\sqrt{3025}}{\sqrt{729}}$

$= \frac{\sqrt{5\times5\times11\times11}}{\sqrt{3\times3\times3\times3\times3\times3}}$

$= \frac{\sqrt{5^2\times11^2}}{\sqrt{3^2\times3^2\times3^2}}$

$= \frac{5\times11}{3\times3\times3}$

$= \frac{55}{27}$

(vii) $\frac{2116}{2601}$

$= \frac{\sqrt{2116}}{\sqrt{2601}}$

$= \frac{\sqrt{2\times2\times23\times23}}{\sqrt{51\times51}}$

$= \frac{\sqrt{2^2\times23^2}}{\sqrt{51^2}}$

$= \frac{2\times23}{51}$

$= \frac{46}{51}$

(viii) $\frac{2025}{1444}$

$= \frac{\sqrt{2025}}{\sqrt{1444}}$

$= \frac{\sqrt{3\times3\times3\times3\times5\times5}}{\sqrt{2\times2\times19\times19}}$

$= \frac{\sqrt{3^2\times3^2\times5^2}}{\sqrt{2^2\times19^2}}$

$= \frac{3\times3\times5}{2\times19}$

$= \frac{45}{38}$

2. *Simplify the following.*

(i) $\sqrt{4\frac{29}{49}} = \sqrt{\frac{225}{49}}$

(ii) $\sqrt{10\frac{6}{25}} = \sqrt{\frac{256}{25}}$

$= \sqrt{\frac{225}{49}}$

$= \frac{\sqrt{3 \times 3 \times 5 \times 5}}{\sqrt{7 \times 7}}$

$= \frac{\sqrt{3^2 \times 5^2}}{\sqrt{7^2}}$

$= \frac{3 \times 5}{7}$

$= \frac{15}{7} = 2\frac{1}{7}$

$= \sqrt{\frac{256}{25}}$

$= \frac{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}}{\sqrt{5 \times 5}}$

$= \frac{\sqrt{2^2 \times 2^2 \times 2^2 \times 2^2}}{\sqrt{5^2}}$

$= \frac{2 \times 2 \times 2 \times 2}{5}$

$= \frac{16}{5} = 3\frac{1}{5}$

(iii) $\sqrt{9\frac{67}{121}} = \sqrt{\frac{1156}{121}}$

$= \frac{\sqrt{2 \times 2 \times 17 \times 17}}{\sqrt{11 \times 11}}$

$= \frac{\sqrt{2^2 \times 17^2}}{\sqrt{11^2}}$

$= \frac{2 \times 17}{11}$

$= \frac{34}{11} = 3\frac{1}{11}$

(iv) $\sqrt{7\frac{21}{25}} = \sqrt{\frac{196}{25}}$

$= \frac{\sqrt{2 \times 2 \times 7 \times 7}}{\sqrt{5 \times 5}}$

$= \frac{\sqrt{2^2 \times 7^2}}{\sqrt{5^2}}$

$= \frac{2 \times 7}{5}$

$= \frac{14}{5} = 2\frac{4}{5}$

EXERCISE 2.5

Find the square root of the following.

(1) 0.16

```
      0.4
   ------------
.4 | 0.16
     0.16
   ------------
        0
```

$= 0.4$

(2) 20.25

```
      4.5
   ------------
 4 | 20.25
     16
   ------------
85 | 4.25
     4.25
   ------------
        0
```

$= 4.5$

(3) 46.24

	0.4
6.	46.24
	36
128	1024
	1024
	0

= 6.8

(4) 0.1296

	0.36
.3	0.1296
	0.09
66	396
	396
	0

= 0.36

(5) 9.8596

	3.14
3	9.8596
	9
61	85
	61
624	2496
	2496
	0

= 3.14

(6) 42.5104

	6.52
6	42.5104
	36
125	651
	625
1302	2604
	2604
	0

= 6.52

(7) 0.000225

	0.015
0	0.000225
	0
1	2
	1
25	125
	125
	0

= 0.015

(8) 727.9204

	26.98
2	727.9204
	4
46	327
	276
529	5192
	4761
5388	43104
	43104
	0

= 26.98

(9) 207.0721

	14.39
1	207.0721
	1
24	107
	96
283	1107
	849
2869	25821
	25821
	0

= 14.39

(10) 460.1025

	21.45
2	460.1025
	1
41	60
	41
424	1910
	1696
4285	21425
	21425
	0

= 21.45

EXERCISE 2.6

Find the square root of the following number up to three places of decimal.

(1) $3 = \sqrt{3}$

	1.732
1	3.000000
	1
27	200
	189
343	1100
	1029
3462	7100
	6924
	176

$\therefore \quad \sqrt{3} = 1.732$

(2) $5 = \sqrt{5}$

	2.236
2	5.000000
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	304

$\therefore \quad \sqrt{5} = 2.236$

(3) $7 = \sqrt{7}$

	2.645
2	7.000000
	4

(4) $2.5 = \sqrt{2.5}$

	1.581
1	2.500000
	1

46	300
	276
524	2400
	2096
5285	30400
	26425
	3965

$\therefore \quad \sqrt{7} = 2.645$

25	150
	125
308	2500
	2464
3161	3600
	3161
	439

$\therefore \quad \sqrt{2.5} = 1.581$

(5) $13 = \sqrt{13}$

	3.605
3	13.000000
	9
66	400
	396
725	40000
	3625
	36375

$\therefore \quad \sqrt{13} = 3.605$

(6) $1.1 = \sqrt{1.1}$

	1.048
1	1.100000
	1
24	100
	96
288	4000
	2304
28214	
	43104
	0

$\therefore \quad \sqrt{1.1} = 1.048$

(7) $20 = \sqrt{20}$

	1.732
4	20.000000
	16
84	400
	336
887	6400
	6209
8942	19100
	17884
	1216

$\therefore \quad \sqrt{20} = 4.472$

(8) $1.7\ \sqrt{1.7}$

	1.303
1	1.700000
	1
23	70
	69
2603	10000
	7809
	2191

$\therefore \quad \sqrt{1.7} = 1.303$

(9) $0.9 = \sqrt{0.9}$

$$\begin{array}{r|l} & 0.948 \\ \hline 9 & 0.900000 \\ & 81 \\ \hline 184 & 900 \\ & 736 \\ \hline 1888 & 16400 \\ & 15104 \\ \hline & 1296 \end{array}$$

$\therefore \quad \sqrt{0.9} = 0.948$

(10) $2\frac{1}{12}$

$= \sqrt{\frac{25}{12}} = \sqrt{2.083}$

EXERCISE 2.7

1. ***The area of a square public park is 19600 square meters. Find the length of the side of the park.***

Solution:

Area of a square public park = 19600 sqr.

Length of the side of a park = ?

$$\begin{array}{r|l} & 140 \\ \hline 1 & 19600 \\ & 1 \\ \hline 24 & 9600 \\ & 9600 \\ \hline & 0 \end{array}$$

= 140 sqr.

2. ***Area of a circular field is $2464m^2$. Find the circumference of the circle.*** $\left(\textit{Take } \pi \approx \frac{22}{7} \right)$

Solution:

Area of circular field = $2464m^2$

Area of a circular field = πr

$\pi r = 2464$

$\frac{22}{7} \times r = 2464$

$r = 2464 \times \frac{22}{7}$

$r = 112 \times 7$

$r = 784$

Taking square root of both sides

$\sqrt{r} = \sqrt{784}$

$C = 2\pi r$

$= 2 \times \frac{22}{7} \times 28$

$= 44 \times 4$

$= 176m$

	28
2	784
	4
48	384
	384
	0

3. ***The students of a school contributed as many rupees as the number of students of a picnic. If the total collection was Rs. 1449616. Then find the number of students and the amount contributed by each.***

Solution:

The students of school contributed = 1449616

The number of students = ?

	1204
1	1449616
	1
22	44
	44
2404	9616
	9616
	0

Number of students = 1204

Each student contributed = 1204

4. ***The area of a square shaped hall is $225m^2$. Find its perimeter.***

Solution:

Area of a square shaped hall = $225m^2$

Perimeter = ?

	15
1	225
	1
25	125
	125
	0

Perimeter = $4 \times L = 4 \times 15 = 60$

5. ***Find the least number which must be subtracted from 3151 to make it a perfect square.***

Solution:

The number is = 3151

The square root = ?

	226
5	3151
	25
106	651
	636
	0

The 15 is least number.

6. ***The area of a rectangle field is 230496 cm^2. The length of field is 6 times its width. Find the length and width of field.***

Solution:

Area of rectangular = $230496m^2$

Width = x

$x \times 6x = 230496$

$x^2 = \frac{230496}{6} = 38416$

Taking square of both sides

	196
1	38416
	1
29	284
	261
386	2316
	2316
	0

$$\sqrt{x^2} = \sqrt{\frac{230496}{6}} = \sqrt{38416}$$

The width = 196

Length = $6 \times 196 = 1176$

7. ***The length of rectangular plot is $2\frac{1}{2}$ times of width. If the area of rectangular plot is $12250m^2$, find its length and width.***

Solution:

Area of a rectangular = 12250

Let the width of rectangular = x

Length of rectangular plot = $2\frac{1}{2}x$

$$x \times \frac{5}{2}x = 12250m^2$$

$$\frac{5}{2}x^2 = 12250$$

$$x^2 = 12250 \times \frac{5}{2}$$

$$x^2 = 4900$$

Taking square root of both sides

$$\sqrt{x^2} = \sqrt{4900}$$

$$\begin{array}{r|l} & 70 \\ \hline 7 & 4900 \\ & 49 \\ \hline & 00 \end{array}$$

Width of rectangular = 70

Length of rectangular = $70 \times \frac{5}{2} = 175m^2$

8. ***The area of a square lawn of a school is $42025m^2$. If you completer 5 rounds of the square lawn. How much distance you traveled?***

Solution:

Ar a of a square lawn = 42025 = $\sqrt{42025}$

$$\begin{array}{r|l} & 205 \\ \hline 2 & 42025 \\ & 4 \\ \hline 405 & 2025 \\ & 2025 \\ \hline & 0 \end{array}$$

Perimeter of lawn = 205 = 205 × 4 = 820

Distance of 1 round = 820

Traveled distance of 5 rounds = 820 × 5 = 4100m

EXERCISE 2.8

1. Find the cube of the following numbers.

(i) $6 = (6)^3 = 6 \times 6 \times 6 = 216$

(ii) $9 = (9)^3 = 9 \times 9 \times 9 = 729$

(iii) $11 = (11)^3 = 11 \times 11 \times 11 = 1331$

(iv) $13 = (13)^3 = 13 \times 13 \times 13 = 2197$

(v) $15 = (15)^3 = 15 \times 15 \times 15 = 3375$

(vi) $16 = (16)^3 = 16 \times 16 \times 16 = 4096$

(vii) $20 = (20)^3 = 20 \times 20 \times 20 = 8000$

(viii) $25 = (25)^3 = 25 \times 25 \times 25 = 15625$

2. Which of the following are perfect cubes?

(i) $21 = 3 \times 7 = 21$ is the not perfect cube.

(ii) $27 = 3 \times 3 \times 3 = 3^3 = 27$ is perfect cube.

(iii) $48 = 2 \times 2 \times 2 \times 2 \times 3 = 4 \times 4 \times 3$ is not perfect cube.

(iv) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4 \times 4 \times 4 = 4^3$

64 is a perfect cube.

(v) $125 = 5 \times 5 \times 5 = 5^3 = 125$ is a perfect cube.

(vi) $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$ is a perfect cube.

(vii) $300 = 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3 \times 5^2$ is not perfect cube.

(viii) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 9 \times 9 \times 9 = 9^3$

729 is a perfect cube.

3. Find the cube root of the following.

(i) $1331 = 11 \times 11 \times 11 = 11^3$

Taking cube root of

$\sqrt[3]{1331} = \sqrt{11^3} = 11$

(ii) $2197 = 13 \times 13 \times 13 = 13^3$

Taking cube root of following

$\sqrt[3]{2197} = \sqrt{13^3} = 13$

(iii) $4096 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 16^3$

Taking cube root of

$\sqrt[3]{4096} = \sqrt{16^3} = 16$

(iv) $5832 = 18 \times 18 \times 18 = 18^3$

$5832 = 18 \times 18 \times 18 = 18^3$

Taking cube root of both sides

$\sqrt[3]{5832} = \sqrt{18^3} = 18$

4. Given that volume of cube is $64m^2$. Find the length of its sides.

Solution:

Volume $= (\text{length})^3$

$64 = (\text{length})^3$

$(\text{length})^3 = (64)^3$

Taking cube root of both sides

$\sqrt{(\text{length})^3} = (\sqrt[3]{64})$

Length = 4m

5. What is the volume of a cube having side 12 cm?

Solution:

Volume of a cube = ?