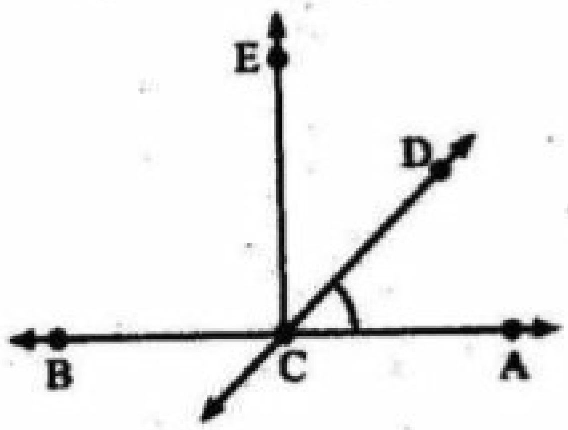
UNIT 10

DEMONSTRATIVE GEOMETRY

THEOREM 10.2.i

Statement

If a straight line stands on another straight line, the sum of measures of two angles so formed is equal to two right angles (180^o).



Given

AB is a straight line. CD is another straight line stands on AB at C.

To Prove

$m\angle ACD + m\angle DCB$ = two right angles (180^o).

Construction

Draw CE perpendicular to AB.

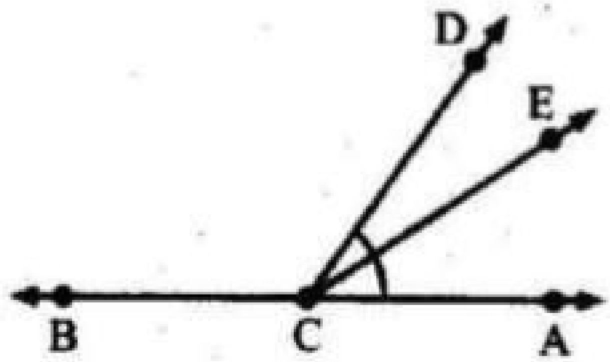
Proof

$m\angle ACD = m\angle ACE - m\angle DCE$	(i)
$m\angle DCB = m\angle DCE + m\angle ECB$	(ii)
$m\angle ACD + m\angle DCB = m\angle ACE -$	
$- m\angle DCE + m\angle DCE +$	by adding (i) and (ii)
$m\angle ECB = m\angle ACE + m\angle ECB$	
$= 90^o + 90^o$	Construction
$= 180^o$	
= two right angles	

THEOREM 10.2.ii

Statement

If sum of measures of two adjacent angles is equal to two right angles, the external arms of the angles are in a straight line.



Given

m $\angle$ACD + m $\angle$DCB = two right angles (180°).

To Prove

CA and CB are in a straight line.

Construction

Suppose CA and CB are not in a straight line, then produce $\overline{BC}$ to E.

Proof

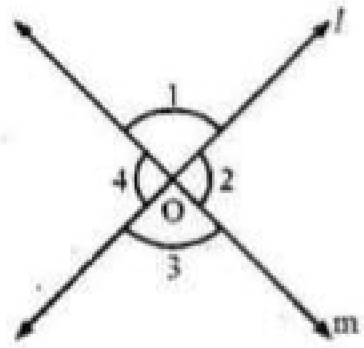
BCE is a straight line on CD stands On line BCE	Construction
$\therefore$ m $\angle$ECD + m $\angle$DCB = two right again (i)	
But m $\angle$ACD + m $\angle$DCB = two right angles	Given
So m $\angle$ECD + m $\angle$DCB = m $\angle$ACD + (ii) m $\angle$DCB	From (i) and (ii)
m $\angle$ECD = m $\angle$ACD It is only possible if CA and CE coincide.	cancellation property of equality.

$\therefore$ CA and CB are in a straight line.

THEOREM 10.2.iii

Statement

If two lines intersects each other, the opposite vertical angles are congruent.



Given

Two lines l and m intersect each other at O. ($\angle 1$, $\angle 3$) and ($\angle 2$, $\angle 4$) are two pairs of opposite vertical angles.

To Prove

$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

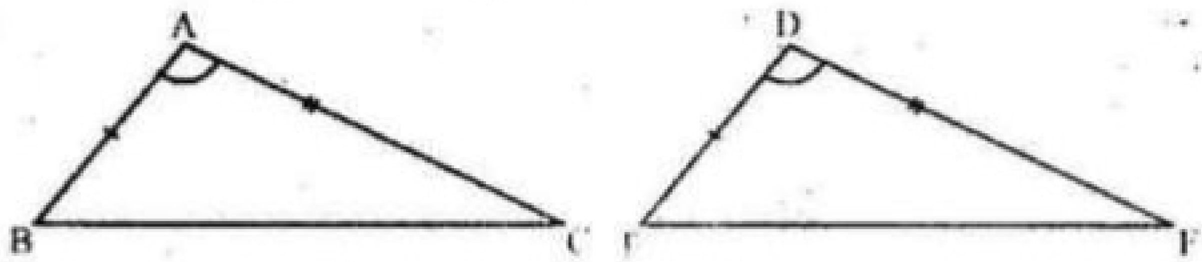
Proof

$m\angle 1 + m\angle 2 = 180^o$	(i)	supplementary angles
$m\angle 2 + m\angle 3 = 180^o$	(ii)	supplementary angles
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$		from (i) and (ii)
$m\angle 1 = m\angle 3$		But m (2 in b/s)
i.e. $\angle 1 \cong \angle 3$		
Similarly $\angle 2 \cong \angle 4$		

THEOREM 10.2. iv

Statement

If any correspondence of two triangles, if two sides and included angle of one triangle are congruent to the corresponding sides and included angle of other triangle, the two triangles are congruent,



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$AB = DE$

$m\angle A = m\angle D$

and $AC = DF$

Required to Prove

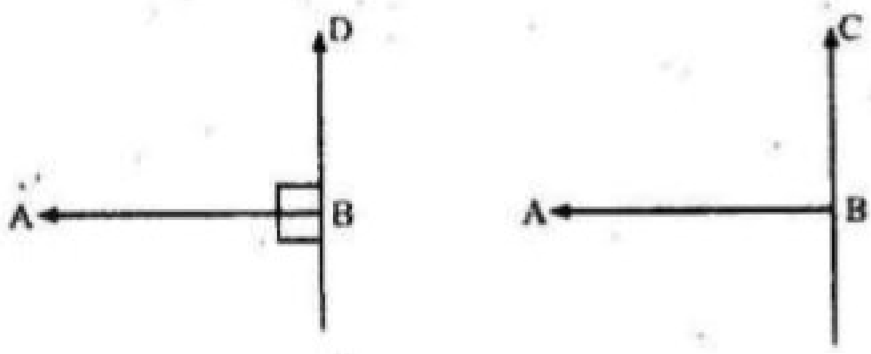
$\Delta ABC \cong \Delta DEF$

Proof

Place ΔABC on ΔDEF in such a way that	
(i) AB coincides with DE	
(ii) Points A and D coincide	
(iii) Points B and F coincide	
$\because$ $\angle A = \angle D$	given
$\therefore$ $\overline{AC}$ will coincide with $\overline{DF}$.	
$\because$ $AC = DF$.	given
$\therefore$ point C will coincides with point F	
You will see that $\overline{BC}$ coincide with $\overline{EF}$.	
$\therefore$ $\Delta ABC \cong \Delta DEF$	

EXERCISE 10.1

1. ***$\overline{AB}$ is a line segment, $\overline{BC}$ and $\overline{BD}$ are two perpendiculars to $\overline{AB}$ on its opposite sides. Show that $\overline{CD}$ is a straight line.***



Given

$\overline{AB}$, $\overline{BC} \perp \overline{AB}$, $\overline{BD}$ $\angle B = 90^\circ$

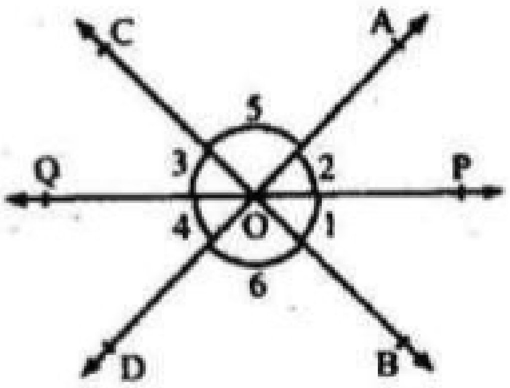
To Prove

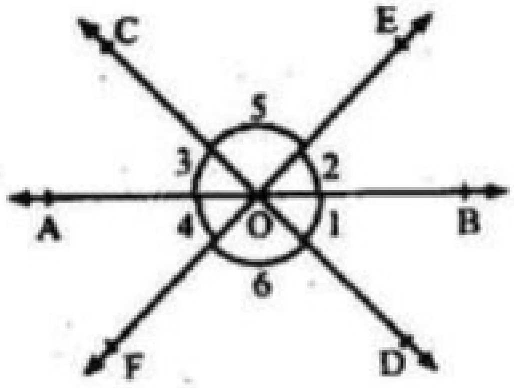
$\overline{CD}$ is a straight line.

Proof

Statement	Reason
Since,	
$\angle ABC + \angle ABD = 180^\circ$	supplementary angle
$\therefore$ C, B and D are collinear.	

2. ***If two lines intersect each other, then show that bisectors of opposite vertical angles are collinear.***





Given

$\angle EOD \cong \angle EOF$ have a common vertex O.

$\overline{OB}$ is bisector $\angle EOD$

$\overline{OA}$ is bisector $\angle COF$

Construction

Name the angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$.

To Prove

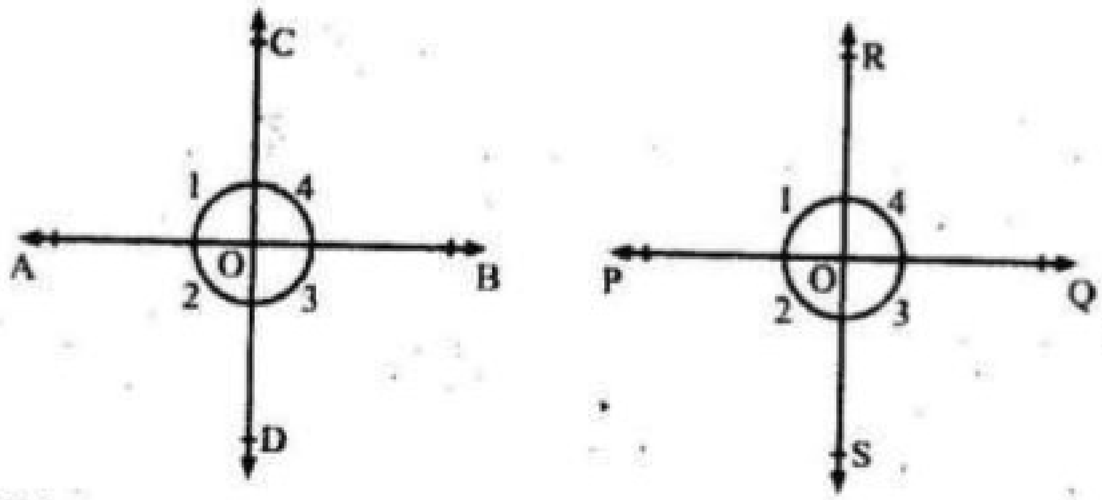
$\overline{OA}$ and $\overline{OB}$ are collinear.

Proof

Statement	Reason
$m\angle 1 = m\angle 2$	Given

$m\angle 2 = m\angle 4$	(i)	Given
$m\angle EOD = \angle EOF$	(ii)	Given
$m\angle EOD = m\angle EOF$	(iii)	Given
$\frac{1}{2} m\angle EOB = \frac{1}{2} m\angle EOF$		Dividing by 2
$m\angle 1 = m\angle 3$	(iv)	$\angle AOB$
Now $m\angle EOD + m\angle EOC = 180^\circ$		$\angle COD$
$m\angle 1 + m\angle 2 + m\angle EOC = 180^\circ$		Supplementary angle
$m\angle 3 + m\angle 2 + m\angle ACO = 180^\circ$		as $m\angle 1 = m\angle 3$ from(iv)
$\Rightarrow$ $m\angle 2 + m\angle AOQ = 180^\circ$		
$\therefore$ AOB is as from		
OB and OA are collinear.		

3. ***If two lines intersect each other such that one of the four angles is right angle, then show that the remaining three angles are also right angles.***



Given

$\overline{AQ}$ and $\overline{RS}$ intersect at O.

Forming angle is 90°.

$\angle 1 = 90^\circ$

To Prove

$m\angle 2 = 90^\circ$

$m\angle 3 = 90^\circ$

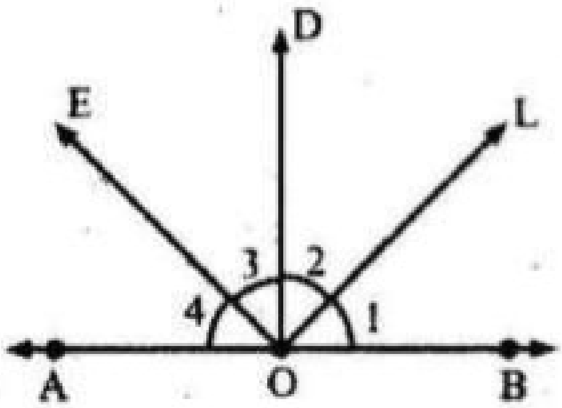
$m\angle 4 = 90^\circ$

Proof

Statement

	$m\angle 1 = 90^\circ$	(i)	Given
	$m\angle 3 = m\angle 1$		Vertical angles
So,	$m\angle 3 = 90^\circ$		
	$m\angle 1 + m\angle 4 = 180^\circ$		Supplementary angle
	$m\angle 4 = 180^\circ - 90^\circ$	(ii)	
$\therefore$	$m\angle 4 = 90^\circ$		
	$m\angle 4 = m\angle 2$		Vertical angles
	$90^\circ = m\angle 2$		
$\therefore$	$m\angle 2 = 90^\circ$	(iii)	
	Hence $m\angle 2 = m\angle 3 = m\angle 4 = 90^\circ$		

4. ***Prove that bisectors of adjacent supplementary angles are perpendicular to each other.***



Given

$\angle BOD$, AOD are two adjacent supplementary angles.

$\overline{OC}$ is bisector of $\angle BOD$ i.e. $m\angle 1 = m\angle 2$

$\overline{OC}$ is bisector of $\angle AOD$ i.e. $m\angle 3 = m\angle 4$

To Prove

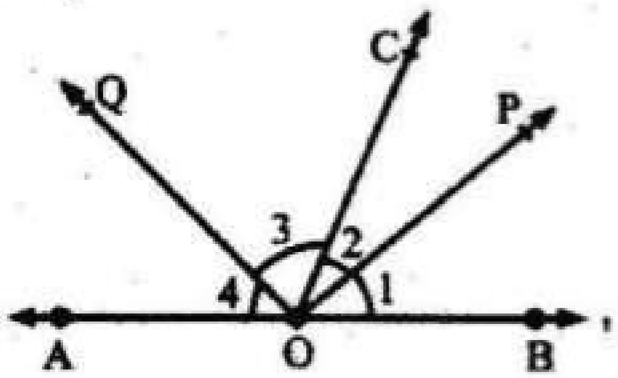
OE, OD = 0 $\perp$ AB = $\angle EOC = 90^\circ$

Proof

$m\angle AOD + m\angle BOD = 180^\circ$	Adjacent angle Given
$\frac{1}{2} m\angle AOD + \frac{1}{2} m\angle BOD = \frac{1}{2}(180^\circ)$	

$m\angle 3 + m\angle 2 = 90^o$	
$\therefore$ $m\angle COL = 90^o$	
Hence $\overline{CQ} \perp \overline{OP}$	

5. ***If bisectors of two adjacent angles are perpendicular to each other, then show that the angles are supplementary.***



Given

$\angle AOC$, $\angle COB$ are adjacent angles.

$\overline{OQ}$ is bisector of $\angle AOC$

$\overline{OP}$ is bisector of $\angle BOC$ that $\overline{OQ} \perp \overline{OP}$.

i.e. $m\angle QOP = 90^o$.

To Prove

$m\angle AOC + m\angle COB = 90^o$

or $\overline{OA}$, $\overline{OB}$ are opposite rays.

i.e. $\overline{AOB}$ is straight line.

Proof

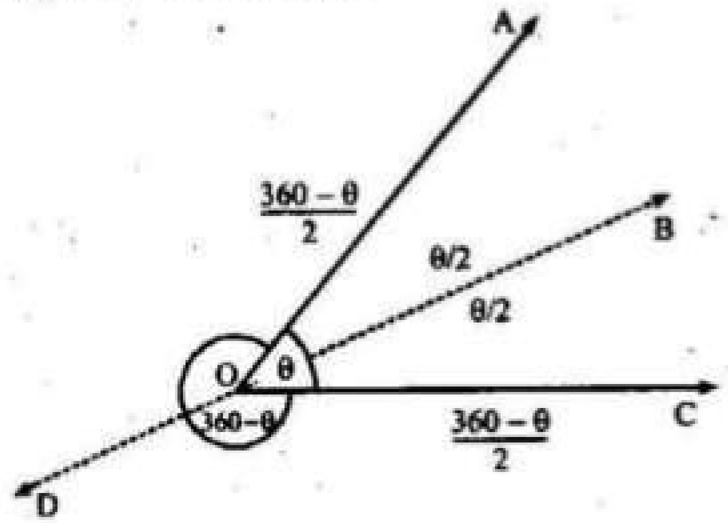
Statement	Reason
$m\angle QOP = 90^o$	Given
$m\angle QOC + m\angle COP = 2 \times 90^o$	Division postulate
$m\angle AOC + m\angle COP = 180^o$	of angle.
$2m\angle QOC = 2m\angle 3$	$= m\angle 3 + m\angle 4$
	$=$ as $\overline{OQ}$ is bisector of

$\angle AOC$, $\angle COP$ are adjacent angles

Hence $\overline{OA}$, $\overline{OB}$ are opposite rays or

$A\overline{O}B$ is straight line.

6. ***Prove that bisectors of an acute angle and its respective reflex angle are collinear.***



To Prove

Bisector of acute angle and bisector of its reflex angle and collinear.

Construction

Draw bisectors of angle θ and angle $360-\theta$ as in figure.

Proof

$m\angle AOB = \frac{\theta}{2}$ (i)	
$m\angle AOD = \frac{360-\theta}{2}$ (ii)	
$m\angle AOB + \angle AOD = \frac{\theta}{2} + \frac{360-\theta}{2}$	Adding (i) and (ii)
$= \frac{\theta + 360 - \theta}{2}$	
$= \frac{360}{2} = 180^{\circ}$	
$m\angle AOB + m\angle AOD = 180^{\circ}$	Bisectors are collinear.

7. *In the adjoining figure the sides of triangle ABC are produced. Write six pairs of congruent angles.*

$\angle IAH \cong \angle BAC$

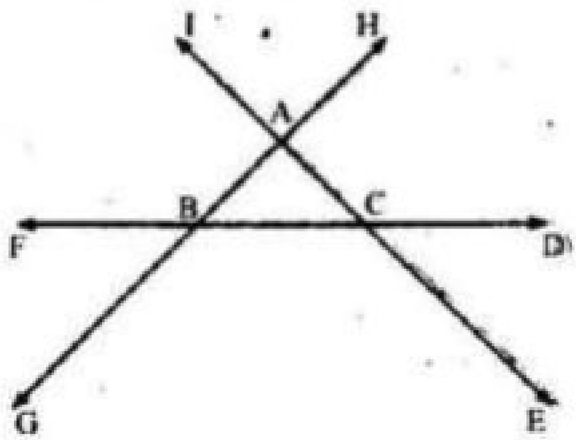
$\angle GBF \cong \angle ABC$

$\angle ECD \cong \angle ACB$

$\angle IAB \cong \angle GAC$

$\angle ABC \cong \angle CBG$

$\angle BCE \cong \angle ACB$

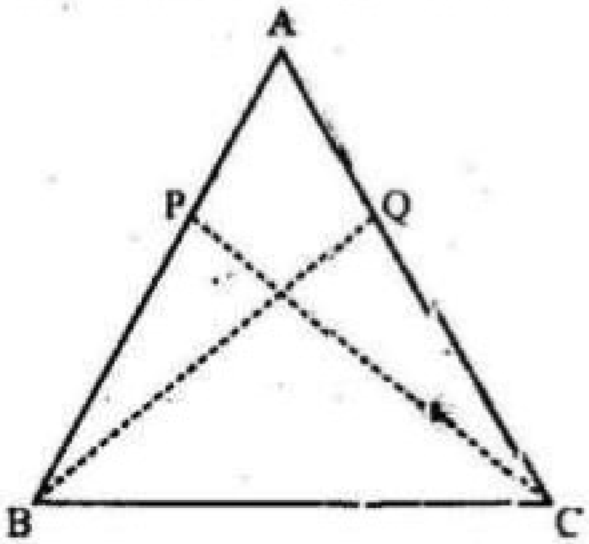


8. *In an equilateral triangle ABC, P and Q are mid points of $\overline{AB}$ and $\overline{AC}$ respectively. Show that $\Delta ABQ \cong \Delta ACP$.*

Given

In ΔABC m AC = m $\overline{AB}$ = m $\overline{BC}$

Also m $\angle A$ = m $\angle B$ = m $\angle C$ = 60° and P, Q are mid points A of $\overline{AB}$ and $\overline{AC}$ respectively.



To Prove

$\Delta ABQ \cong \Delta ACP$

Construction

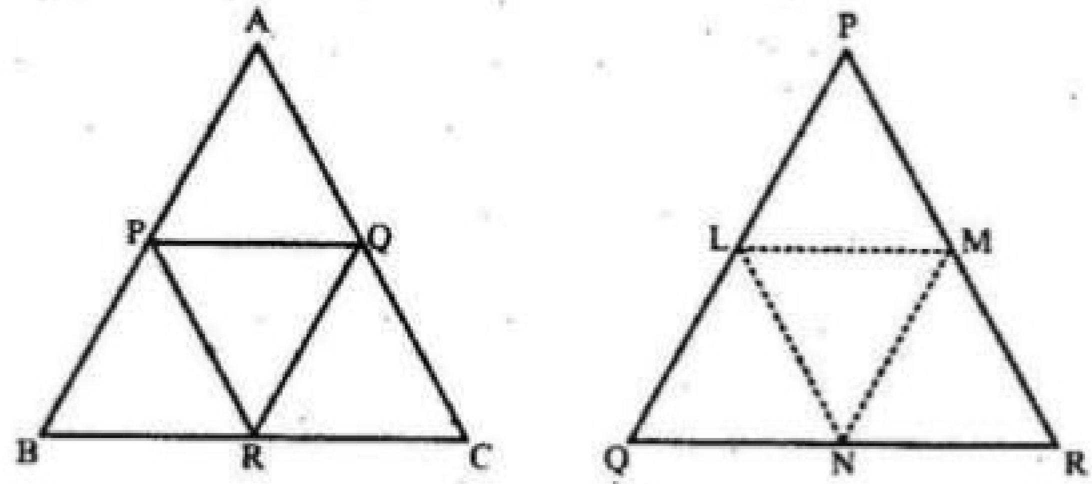
Join P to C and Q to B to get two triangles.

Proof

$\Delta ABQ \leftrightarrow \Delta ACP$

$\overline{AB} \cong \overline{AC}$	Given
$\overline{AQ} \cong \overline{AP}$	Common
$\angle A \cong \angle A$	
$\Delta ABQ \cong \Delta ACP$	S.A.S. postulates

9. ***Prove that the triangle formed by joining mid points of sides of an equilateral triangle is also an equilateral triangle.***



Given

ΔPQR, $m\,\overline{PQ} \cong m\,\overline{PR} \cong m\,\overline{QR}$

L, M and N are mid point of $\overline{PQ}$, $\overline{PR}$ and $\overline{QR}$.

To Prove

$\Delta\,LMN$ is equilateral.

Construction

Join L, M, N to obtain ΔLMN.

Proof

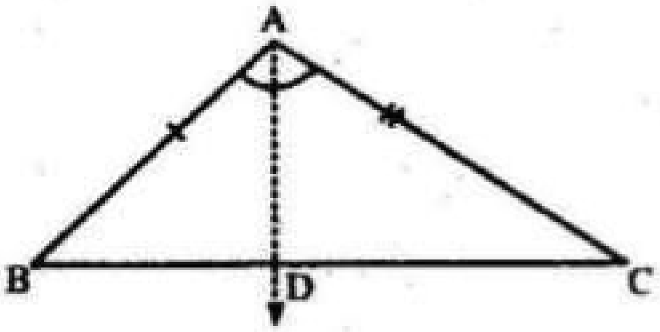
In $\Delta PQR \leftrightarrow LMN$	Both are half part of
$m\,\overline{PL} \cong m\,\overline{PM}$	same line segment.
$m\,\overline{PM} \cong m\,\overline{MN}$	Given
$m\,\overline{PM} \cong m\,\angle Q$	S.A.S.
$\Delta APQ \cong \Delta BPR$	corresponding sides
So, $m\,\overline{PL} = \overline{PM}$ (i)	of congruent triangles.

Similarly, m $\overline{EN}$ = $\overline{MN}$ (ii)	
Thus m $\overline{PQ}$ ≅ m $\overline{PR}$ ≅ $\overline{QR}$	from (i) and (ii)
So, ∠ APQ is an equilateral triangle.	proved.

THEOREM 10.2. v

Statement

If two sides of a triangle are congruent, the angles opposite to these sides are also congruent.



Given

In ΔABC, AB = AC.

Required to Prove

m ∠B = m ∠C

Construction

Draw bisector of ∠A to meet BC at D.

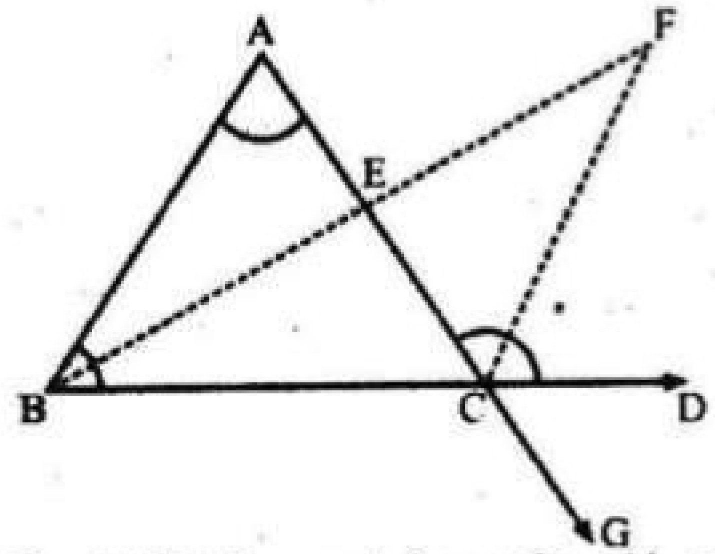
Proof

In ΔABD ΔACD	
AD = AD	Common
m ∠BAD = m ∠CAD	Construction
AB = AC	given
ΔABD ≅ ΔACD	S.A.S. ≅ S.A.S.
m ∠B ≅ m ∠C	corresponding angles of congruent triangles

THEOREM 10.2. vi

Statement

An exterior angle of a triangle is greater in measure then either of its opposite interior angles.



Given

In ΔABC, $\angle ACD$ is an exterior angle. $\angle A$ and $\angle CBA$ are its opposite interior angles.

To Prove

$m\angle ACD > m\angle A$ and $m\angle ACD > m\angle CBA$.

Construction

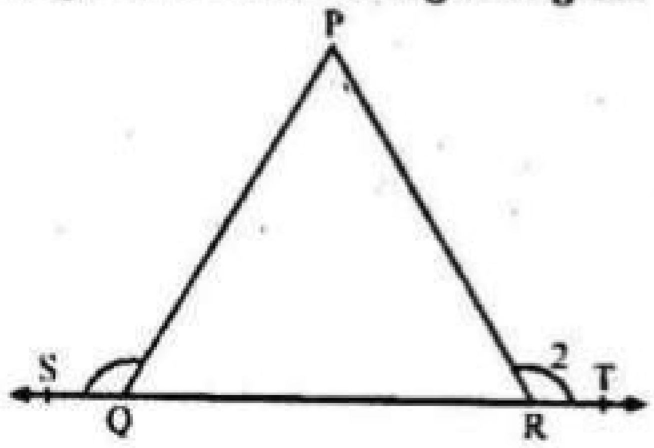
Bisect $\overline{AC}$ at E. Join B to E and produce it to F such that $BE = EF$. Join F with C. $\angle BCG$ is also an exterior angle.

Proof

In $\Delta ABE \leftrightarrow \Delta CFE$		
$AB = EC$		Construction
$m\angle AEB = m\angle CEF$		Opposite vertical angles
$BE = FE$		Construction
$\Delta ABE \cong \Delta CFE$		S.A.S. $\cong$ S.A.S.
$m\angle A = m\angle ECF$	(i)	corresponding angles
$m\angle ACD = m\angle ECD = m\angle ECE + m\angle FCD$		of congruent triangles addition of angles
$m\angle ACD > m\angle ECF$	(ii)	postulate
$m\angle ACD > m\angle A$	(iii)	from (i) and (ii)
Similarly $m\angle BCG > m\angle CBA$	(iv)	
But $m\angle ACD = m\angle BCG$	(v)	from (iv) and (v)
$m\angle ACD > m\angle CBA$	(vi)	

EXERCISE 10.2

1. ***If a side of a triangle is produced on both sides, then show that the sum of two exterior angles at different verticals is greater than two right angles.***



Given

In ΔPQR $\angle Q$ $\angle S$ are form.

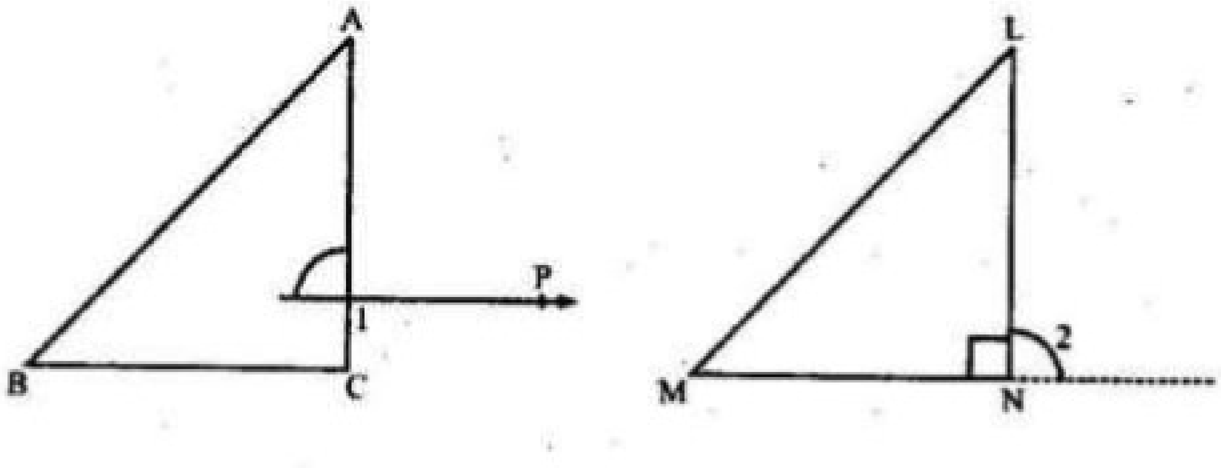
To Prove

$\angle Q + \angle R$ is greater than right angle $\angle Q + \angle R > 180^o$.

Proof

$\angle PRT + 2 = 180^o$	(i)	$\angle C$ of $\angle ACE$ are supplementary angles
$m\angle ABD = 180 - m\angle B$	(ii)	$\angle B$ of $\angle ABD$ are supplementary angles
$m\angle ACE + m\angle ABD$ $= 360 - (m\angle B + m\angle C)$		Adding (i) and (ii)
$m\angle ACE + m\angle ABD > 180$		$\therefore$ $180 - (m\angle B + m\angle C) > 180$

2. ***Prove that in a right angled triangle the other two angles are acute.***



Given

In ΔLMN.

$m\angle N = 90^o$

To Prove

$m\angle M = 90^o$

$m\angle L = 90^o$

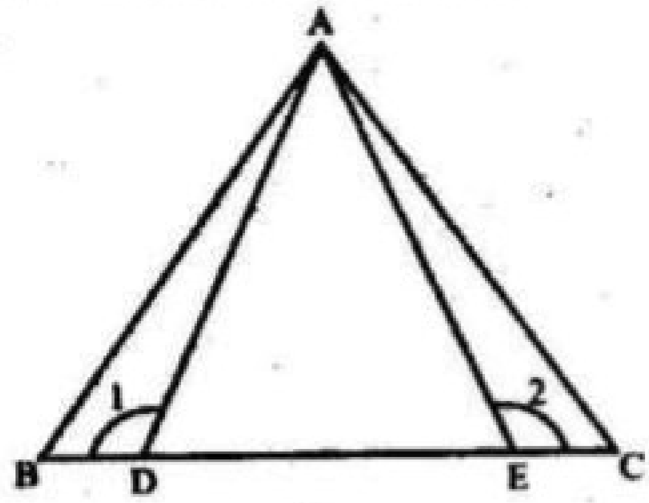
Construction

Extends $\overline{MN} < 1$ is exterior angle.

Proof

In ΔLMN	Given
$m\angle N = 90^o$	Adjacent supp. Angle
$m\angle 1 = 90^o\ \angle L,\ m\angle N = 1$	of $\angle 2$ $\angle 1$ is exterior
Now $m\angle M < m\angle 1$	as $m\angle = 90^o$
and $m\angle M < m\angle 1$	
$m\angle L < 90^o$	
$m\angle M < 90^o$	
$m\angle L,\ m\angle N$	
are active angle.	

3. ***In the following figure $AB = AC$ and $BD = CE$ Prove that $AD = AE$ and $m\angle 1 = m\angle 2$.***



Given

In ΔABC

$AB \cong AC$

$BD \cong EC$

To Prove

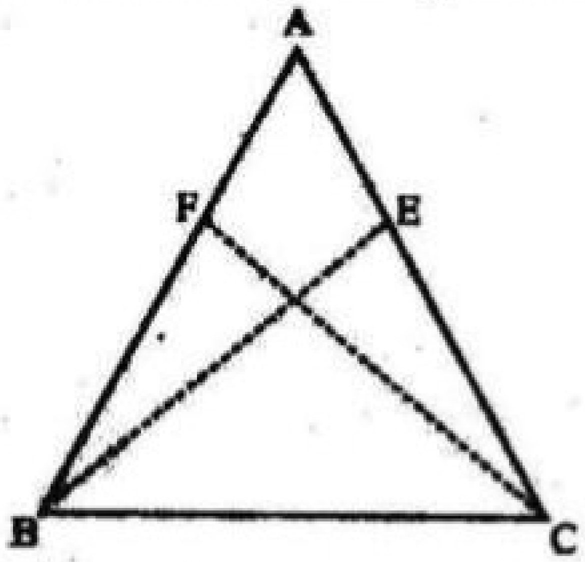
$AD \cong AE$

$m\angle 1 \cong m\angle 2$

Proof

In ΔABC, $\overline{AB} \cong \overline{AC}$		given
$\therefore$ $\angle B \cong \angle C$	(i)	
$\Delta ABD \leftrightarrow \Delta ACE$		
$AB \cong AC$		opp. Angles of long
$BD \cong EC$		sides
$\angle B \cong \angle C$	(ii)	
Hence $\overline{AD} \cong \overline{AE}$		Proved in (i)
$m\angle AEC \cong ADB$		corresponding sides of
$m\angle AEC + m\angle 1 = 180$	(iii)	congruent triangles.
$m\angle ADB + m\angle 2 = 180$	(iv)	supplementary angles
Now $m\angle AEC + \angle 1 = m\angle ADB + m\angle 2$		Each sum = 180°
$m\angle AEC \cong m\angle ADB$		From (i) and (iv)
Hence $m\angle 1 \cong m\angle 2$		as $m\angle AEC \cong m\angle ADB$

4. ***In the following figure $AB = AC$, E and F are mid points of $\overline{AC}$ and $\overline{AB}$ respectively. Show that $BE = CF$.***



Given

$AB \cong AC$

To Prove

$BE \cong CF$

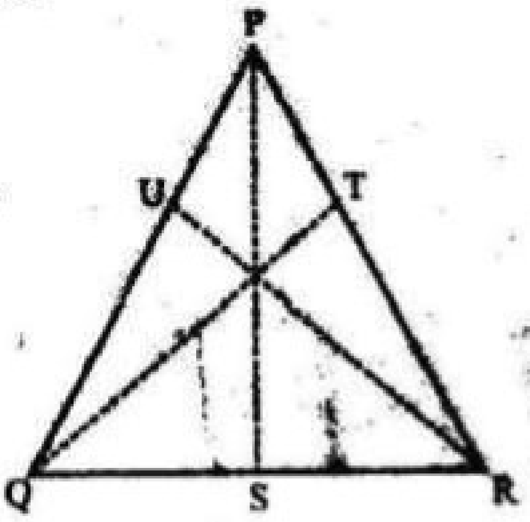
Construction

Join B to E and C to F.

Proof

In $BEC \leftrightarrow \Delta CFB$	Both are half parts of two equal lines segments.
$m\overline{EC} \cong m\overline{FB}$	common
$m\angle C \cong m\angle B$	opposite angles to equal sides are also equal.
$\Delta BEF \cong \Delta CBF$	S.A.S. postulate
$\therefore$ $m\overline{BE} \cong m\overline{CF}$	corresponding sides of congruent triangles.
Proved.	

5. ***Prove that in an equilateral triangle the three medians are congruent.***



Given

ΔPQR an equilateral triangle.

$\overline{PQ} \cong \overline{PR}$

To Prove

$\overline{RU} \cong \overline{PS} \cong \overline{QT}$

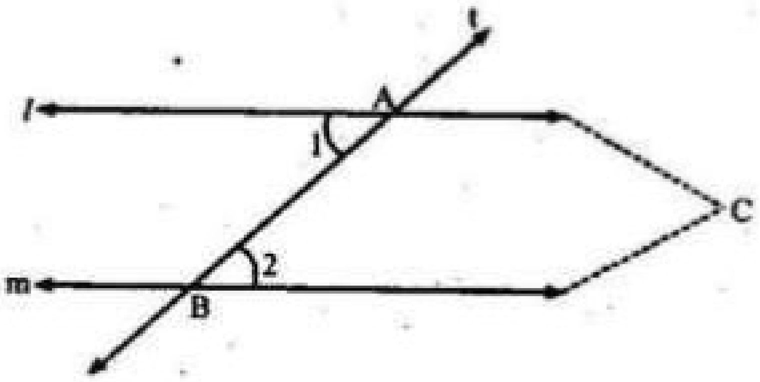
Proof

$\Delta QRU \leftrightarrow \Delta RQT$	
$PS \cong PS$	
$\angle B \cong \angle R$	Common
$\overline{QU} \cong \overline{RT}$	S.A.S. $\cong$ S.A.S.
$\Delta QRU \cong \Delta RQT$	
So, $\Delta PQS \cong \Delta PQT$	
$\therefore$ $\overline{RU} \cong \overline{QU}$	
and so, $PS \cong QT$	

THEOREM 10.2.vii

Statement

If a transversal intersects two lines such that the pair of alternate angles are congruent, then the lines are parallel.



Given

A transversal t intersects two coplanar lines l and m at A and B respectively such that the alternate angles $\angle 1$ and $\angle 2$ are congruent i.e. $m\angle 1 = m\angle 2$.

Required to Prove

$l \parallel$

Construction

Suppose $l \nparallel m$, then l and m intersect each other at a point C and they from a triangle ABC. $\angle 1$ is an exterior angle and $\angle 2$ is one of its opposite interior angle.

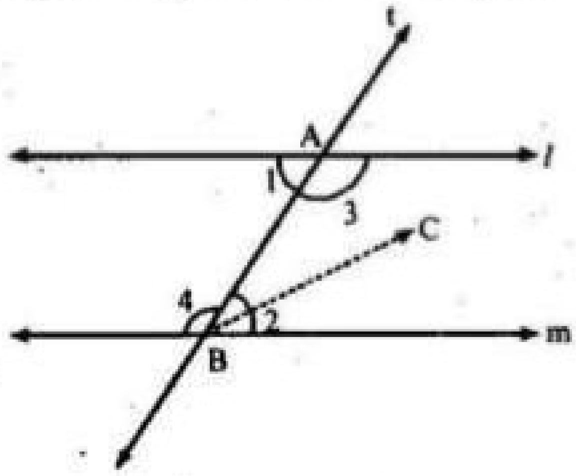
Proof

$m\angle 1 > m\angle 2$	(i)	an exterior angle of a triangle is greater than opposite interior angles.
$m\angle 1 = m\angle 2$		Given
$m\angle 1 > m\angle 2$		not possible
		tracheotomy proper
$l \parallel m$		

THEOREM 10.2. viii

Statement

If a transversal intersects two parallel lines, the alternate angles so formed are congruent.



Given

A transversal t intersects two parallel lines l and m at points A and B respectively. $(\angle 1, \angle 2)$ and $(\angle 3, \angle 4)$ are two pairs of alternate angle.

Required to Prove

$m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$

Construction

Suppose $m\angle 1 \neq m\angle 2$, then a line BC intersects the line m such that $m\angle 1 = m\angle ABC$.

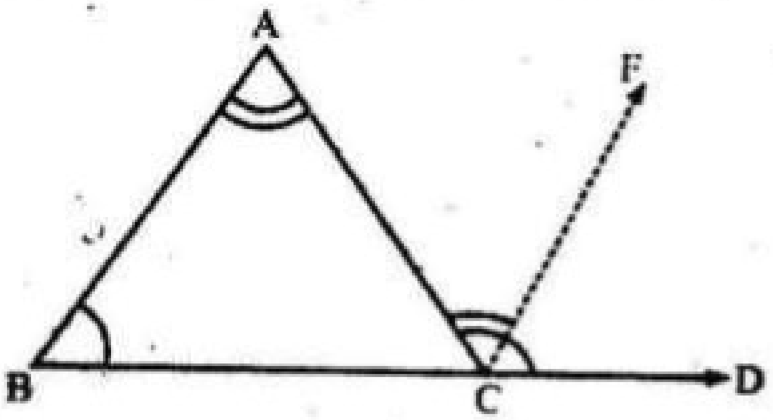
Proof

Statements		Reasons
So, $l \parallel BC$	(i)	alternate angles $m\angle 1 = m\angle ABC$
But $l \parallel m$	(ii)	given
(i) and (ii) can not be true at same time.		
Hence our supposition $m\angle 1 \neq m\angle 2$ is wrong		
So, $m\angle 1 = m\angle 2$	(iii)	
$m\angle 1 + m\angle 3 = 180^{o}$	(iv)	
$m\angle 2 + m\angle 4 = 180^{o}$	(v)	
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$		from (iv) and (v)
$m\angle 1 + m\angle 3 = m\angle 1 + m\angle 4$		$(\because m\angle 1 = m\angle 2)$
$m\angle 3 = m\angle 4$	(vi)	Subtracting $\angle 1$ from sides.
Statements (iii) and (vi) are Required results.		

THEOREM 10.2.ix

Statement

The sum of measures of three angles of a triangle is 180^{o}.



Given

ΔABC.

To Prove

$m\angle A + m\angle B + m\angle C = 180^\circ$

Construction

Produce $\overline{BC}$ to D. $\angle ACD$ is an exterior angle.

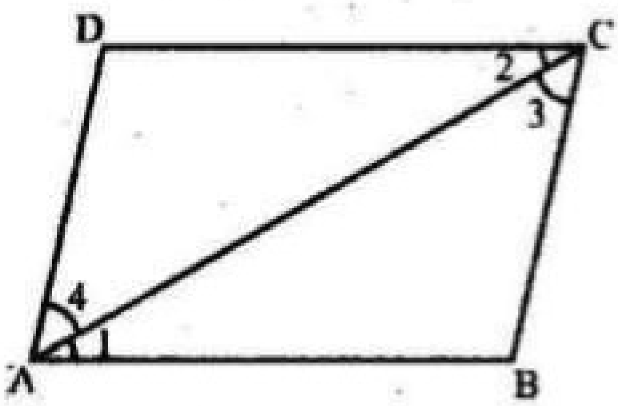
Draw $\overline{CF}$ parallel to $\overline{BA}$.

Proof

$m\angle A = m\angle FCD$	(i)	corresponding angles of parallel lines.
$m\angle A = m\angle ACF$	(ii)	Alternate angles of parallel lines.
$m\angle A + m\angle B = m\angle ACF + m\angle FCD$		adding (i) and (ii)
$m\angle A + m\angle B = m\angle ACD$	(iii)	$m\angle ACF + m\angle FCD = m\angle ACD$
$m\angle A + m\angle B + m\angle BCA = m\angle ACD + m\angle BCA$		adding $m\angle BCA$ on both sides
$m\angle A + m\angle B + m\angle BCA = 180^\circ$		
$\therefore$ $m\angle A + m\angle B + m\angle C = 180^\circ$		$= m\angle ACD + m\angle BCA = 180^\circ$

EXERCISE 10.3

1. In the following figure $AB = CD$, $AD = BC$ and $m\angle B = m\angle D$. Show that $\overline{AB} \parallel \overline{BC}$.



Given

$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$ and $m\angle D \cong m\angle D$.

To Prove

$\overline{AB} \parallel \overline{BC}$

and $\overline{AD} \parallel \overline{BC}$

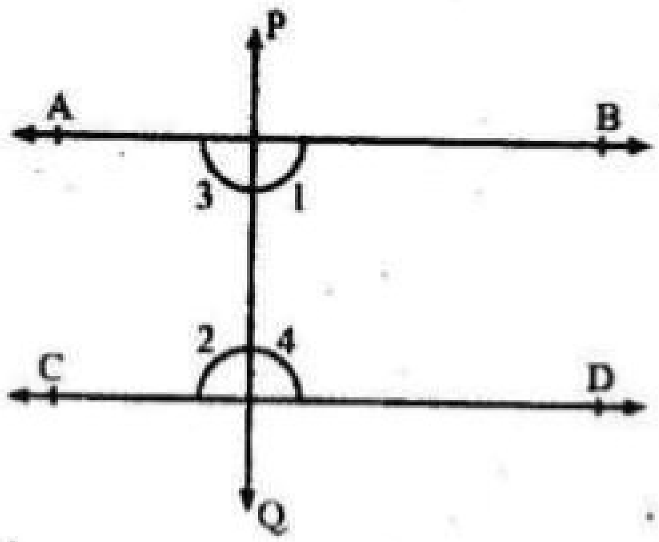
Proof

	In $\Delta ABC \leftrightarrow \Delta CDA$	
	$m\angle\overline{AB} \cong m\angle\overline{CD}$	given
	$m\angle\overline{AD} \cong m\angle\overline{BC}$	given
	$m\angle B \cong m\angle D$	given
$\therefore$	$\Delta ABC \cong \Delta CDA$	S.A.S. $\cong$ S.A.S.
	$m\angle 1 \cong m\angle 2$ (i)	corresponding angles
	$m\angle 3 \cong m\angle 4$ (ii)	of congruent triangles
$\therefore$	$\overline{AD} \parallel \overline{BC}$	
	$m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$	
	$m\angle A = m\angle C$	Adding equal quantity
	Now, similarly	$m\angle 1 + m\angle 4 = m\angle A$
		$m\angle 2 + m\angle 3 = m\angle C$
	Prove that $\overline{AB} \parallel \overline{DC}$	

2. ***Prove that two lines perpendicular to a same line are parallel to each other.***

Given

Two lines $\overline{AB}$ and $\overline{CD}$ are $\perp$ to the same line $\overline{PQ}$.



To Prove

$AB \parallel CD$

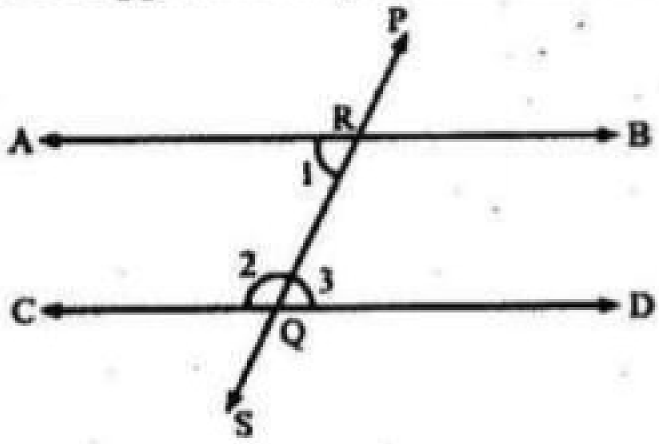
Construction

Name some angles in figure as $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Proof

	$m\angle 1 = m\angle 3 = 90^o$	(i)	Given
	$m\angle 2 = m\angle 4 = 90^o$	(ii)	Given
	$m\angle 1 = m\angle 2$		From (i) and (ii)
$\therefore$	$\overline{AB} \parallel \overline{CD}$		alternate angles are equal, then lines are parallel.

3. ***In the following figure transversal PQ intersects two lines AB and CD at L and M such that $\angle PLA$ and $\angle CMQ$ are supplementary. Show that $\overline{AB} \parallel \overline{CD}$.***



Given

$\overline{AB}$ and $\overline{CD}$ are two st. lines on the same plane $\overline{PS}$ cuts then at points R, Q that $m\angle 1 + \angle 2 = 180^o$.

To Prove

$\overline{AB} \parallel \overline{CD}$

Proof

	$m\angle 1 + m\angle 2 = 180^o$	(i)	Given
	$m\angle 3 + m\angle 2 = 180^o$	(ii)	Adj. supplementary angles.
$\therefore$	$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$		From (i), (ii)
	$m\angle 1 = m\angle 3$		Subtracting $m\angle 2$ from both sides.
$\Rightarrow$	$m\angle 1 \cong m\angle 3$		
$\therefore$	$\overline{AB} \parallel \overline{CD}$		$\angle 1$, $\angle 3$ are alternate angles.

4. *If a transversal intersects two coplanar lines such that the alternate angles are not congruent, then show that the lines are not parallel.*

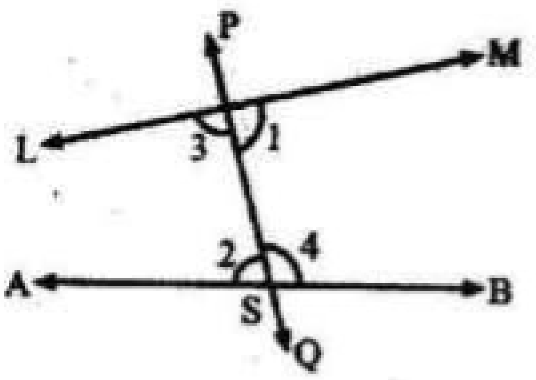
Given

A line $\overline{PQ}$ intersects two coplanar lines $\overline{AB}$ and $\overline{CD}$ at points S and T respectively.

Also $m\angle 1 \neq m\angle 2$ and $m\angle 3 \neq m\angle 4$

To Prove

$\overline{AB} \parallel \overline{CD}$

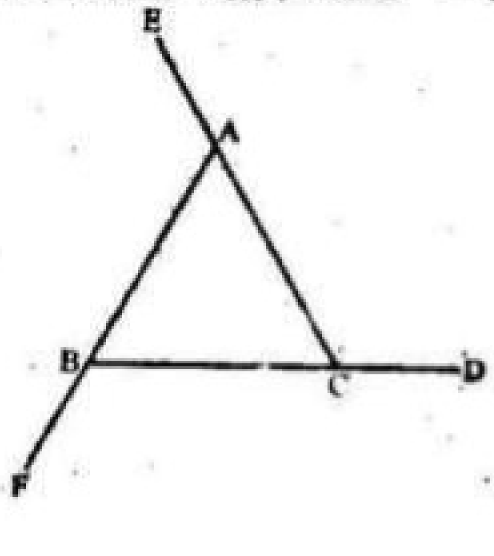


Proof

Suppose, $\overline{AB} \parallel \overline{CD}$	
$m\angle 1 = m\angle 2$	alternate angles
$m\angle 3 = m\angle 4$	alternate angles
But this is given.	at angle
So, our supposition is wrong	
$\overline{AB} \parallel \overline{CD}$	
At times are not parallel.	

5. *In the following figure show that*

$$m\angle ACD + m\angle BAE + m\angle CBF = 360^\circ$$



Given

The figure as shown.

To Prove

$m\angle ACD + m\angle BAE + m\angle CBF = 360^\circ$

Proof

$m\angle ACD = 180 - m\angle C$	(i)	$m\angle ACD$ and $\angle C$
$m\angle BAE = 180 - m\angle A$	(ii)	are supplementary
$m\angle CBF = 180 - m\angle B$	(iii)	angles.
$m\angle ACD + m\angle BAE +$		Adding (i),(ii) and (iii)
$m\angle CBF = 540 -$		
$(m\angle A + m\angle B + m\angle C)$		Sum of three interior
$m\angle ACD + m\angle BAE +$		angles of a
$m\angle CBF = 540 - 180$		triangle 180°
$m\angle ACD + m\angle BAE +$		
$m\angle CBF = 360^\circ$		Proved.

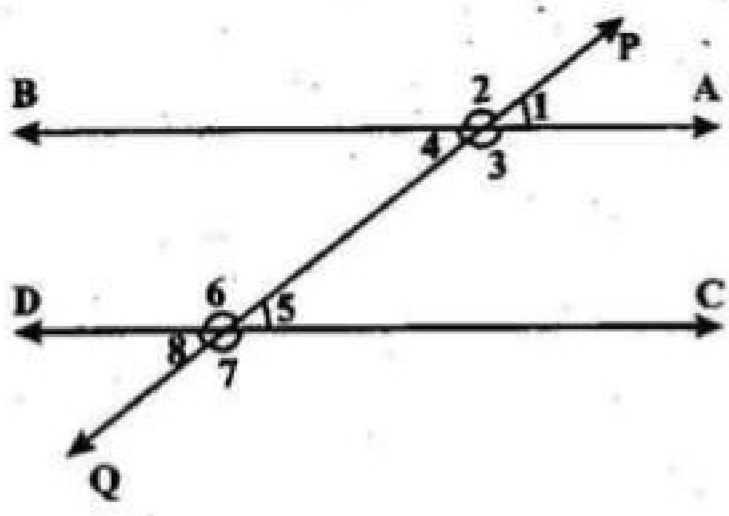
6. ***According to the fig in which of the following cases $\overline{AB}$ is not parallel to $\overline{CD}$.***

(i) $m\angle 4 = 50^\circ$, $m\angle 5 = 50^\circ$

(ii) $m\angle 3 = 130^\circ$, $m\angle 7 = 130^\circ$

(iii) $m\angle 6 = 130^\circ$, $m\angle 4 = 50^\circ$

(iv) $m\angle 6 = 120^\circ$, $m\angle 4 = 50^\circ$



(i) Since $m\angle 4 = m\angle 5 = 50^\circ$

$AB \parallel CD$

(alternate angles are equal)

(ii) $m\angle 3 = m\angle 7 = 130^\circ$

$AB \parallel CD$

($\therefore$ corresponding angles are equal)

(iii) $m\angle 6 = 130^\circ, m\angle 4 = 50^\circ$

Since $m\angle 6 + m\angle 4 = 130^\circ + 50^\circ$

$= 180^\circ$

$AB \parallel CD$ ($\therefore$ sum of interior angles $= 180^\circ$)

(iv) $m\angle 6 = 120^\circ, m\angle 4 = 50^\circ$

As $m\angle 6 + m\angle 4 = 120^\circ + 50^\circ = 170^\circ \neq 180^\circ$

therefore $AB \nparallel CD$