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ASAN Math For Class 8th 120   Polynomials

UNIT 5

POLYNOMIALS

EXERCISE 5.1

1. *Which of the following algebraic expressions are polynomials?*

(i) $6y + 9$ = Polynomial

(ii) $x^2 + 2x + \dfrac{7}{x}$ = non polynomial

(iii) $6y + \dfrac{1}{y}$ = non polynomial

(iv) $x^{3/2} + 3x + 2$ = non polynomial

(v) $x^2 + \dfrac{x}{4}$ = Polynomials

(vi) $y^2 - 2 + \dfrac{y^3}{y^2}$ = Polynomials

2. *Write the degree of the following polynomials and also recognize number of variables.*

(i) $9x + 7$

    Degree = 1, Variables = 1

(ii) $6x^2 - 3x + 2$

    Degree = 2, Variables = 1

(iii) 7

    Degree = 0, Variables = 0

(iv) $10x^4 + 6x^2 + 10$

    Degree = 4, Variables = 1

(v) $x^3 - 10xy^2$

    Degree = 3, Variables = 2

(vi) $6x + 7xy + 12$

    Degree = 2, Variables = 2

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(vii) $y^2 - 6x^2z^3 + y$

    Degree = 5,   Variables = 3

(viii) $x^2yz + 6y^4z + x^3z^2$

    Degree = 5,   Variables = 3

3.   Which of the following are linear, quadratic and cubic polynomials.

(i) $9x + 7$

    = Linear

(ii) $6x^2 - 3x + 2$

    = Quadratic

(iii) $2x^3 + 2x^2 - 4$

    = Cubic

(iv) $x + 7x^2 + 1$

    = Quadratic

(v) $x^3 - 6x$

    = Cubic

(vi) x

    = Linear

EXERCISE 5.2

1.   Find the sum of following polynomials.

(i) $6x^4 + 10x^2 + 5, \; 8x^4 - 7x^2 - 3$

    $= [6x^4 + 10x^2 + 5] + [8x^4 - 7x^2 - 3]$

    $= 6x^4 + 10x^2 + 5 + 8x^4 - 7x^2 - 3$

    $= 6x^4 + 8x^4 + 10x^2 - 7x^2 + 5 - 3$

    $= 14x4 + 3x2 + 2$ Ans.

(ii) $5x - 6x^2 + x^3, \; -8x + 6x^2 - 10x^3 + x^4$

    $= [5x - 6x^2 + x^3] + [-8x + 6x^2 - 10x^3 + x^4]$

    $= 5x - 6x^2 + x^3 - 8x + 6x^2 - 10x^3 + x^4$

    $= 5x - 8x - 6x^2 + 6x^2 + x^3 - 10x^3 + x^4$

    $= 3x - 9x^3 + x^4$

    $= x^4 - 9x^3 + 3x$

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(iii) $2x^4 - 2x^3 + 6x - 1, \ 3x^4 - 4x^3 - 5x + 3$

$= [2x^4 - 2x^3 + 6x - 1] + [3x^4 - 4x^3 - 5x + 3]$

$= 2x^4 - 2x^3 + 6x - 1 + 3x^4 - 4x^3 - 5x + 3$

$= 2x^4 + 3x^4 - 2x^3 - 4x^3 + 6x - 5x - 1 + 3$

$= 5x^4 - 6x^3 + x + 2$

(iv) $7x^4 - 6x^3 + 10x - 9, \ 6x^4 + 10x^3 - 7x + 11$

$= [7x^4 - 6x^3 + 10x - 9] + [6x^4 + 10x^3 - 7x + 11]$

$= 7x^4 - 6x^3 + 10x - 9 + 6x^4 + 10x^3 - 7x + 11$

$= 7x^4 + 6x^4 - 6x^3 + 10x^3 + 10x - 7x - 9 + 11$

$= 13x^4 + 4x^3 + 3x + 2$

(v) $10x^4 - 10x^3 + 5x^2 - 6x + 3, \ 5x^4 + 8x^3 - 3x^2 + 7x - 10$

$= [10x^4 - 10x^3 + 5x^2 - 6x + 3] + [5x^4 + 8x^3 - 3x^2 + 7x - 10]$

$= 10x^4 - 10x^3 + 5x^2 - 6x + 3 + 5x^4 + 8x^3 - 3x^2 + 7x - 10$

$= 10x^4 + 5x^4 - 10x^3 + 8x^3 + 5x^2 - 3x^2 - 6x + 7x + 3 - 10$

$= 15x^4 - 2x^3 + 2x^2 + x - 7$

2. *If $A = 4x^3 - 3y^2 + 5z$*

    $B = 6x^3 - 5y^2 + 8z$

    $C = -2x^3 + 6y^2 - 9z$

    then find $A + B + C$.

$= [4x^3 - 3y^2 + 5z] + [6x^3 - 5y^2 + 8z] + [-2x^3 + 6y^2 - 9z]$

$= 4x^3 - 3y^2 + 5z + 6x^3 - 5y^2 + 8z - 2x^3 + 6y^2 - 9z$

$= 4x^3 + 6x^3 - 2x^3 - 3y^2 - 5y^2 + 6y^2 + 5z + 8z - 9z$

$= 8x^3 - 2y^2 + 4z$

3. *If $P = 3x^2 + 4xy - 4y^2$*

    $Q = x^2 - 3xy + 5y^2$

    $R = -4x^2 + xy + y^2$

    then find $P + 2Q - R$.

$= [3x^2 + 4xy - 4y^2] + 2[x^2 - 3xy + 5y^2] - [-4x^2 + xy + y^2]$

$= 3x^2 + 4xy - 4y^2 + 2x^2 - 6xy + 10y^2 + 4x^2 - xy - y^2$

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$$= 3x^2 + 2x^2 + 4x^2 + 4xy - xy - 4y^2 + 10y^2 - y^2 - 6xy$$
$$= 9x^2 - 3xy + 5y^2$$

4. **Subtract**

(i) $6x^4 + 3x^2 - x + 1$ from $8x^4 + 4x^2 + x - 1$

$$= [8x^4 + 4x^2 + x - 1] - [6x^4 + 3x^2 - x + 1]$$
$$= 8x^4 + 4x^2 + x - 1 - 6x^4 - 3x^2 + x - 1$$
$$= 8x^4 - 6x^4 + 4x^2 - 3x^2 + x + x - 1 - 1$$
$$= 2x^4 + x^2 + 2x - 2$$

(ii) $x^4 + 9x^3 + 3x + 2$ from $x^4 + 7x^3 + 2x + 9$

$$= [x^4 + 7x^3 + 2x + 9] - [x^4 + 9x^3 + 3x + 2]$$
$$= x^4 + 7x^3 + 2x + 9 - x^4 - 9x^3 - 3x - 2$$
$$= \cancel{x^4} - \cancel{x^4} + 7x^3 - 9x^3 + 2x - 3x + 9 - 2$$
$$= -2x^3 - x + 7$$

(iii) $2y^3 + 5y^2 - 9y - 10$ from $4y^3 + 9y^2 + 6y + 13$

$$= [4y^3 + 9y^2 + 6y + 13] - [2y^3 + 5y^2 - 9y - 10]$$
$$= 4y^3 + 9y^2 + 6y + 13 - 2y^3 - 5y^2 + 9y + 10$$
$$= 4y^3 - 2y^3 + 9y^2 + 5y^2 + 6y - 9y + 13 + 10$$
$$= 2y^3 + 14y^2 - 3y + 23$$

5.   *The sum of two quantities are* $4x^3 + 6x^2y^2 + 9y^3$. *If one of them is* $2x^3 + 6x^2y^2 + 6y^3$. *Find the other.*

Sum of two quantities $= 4x^3 + 6x^2y^2 + 9y^3$

One of them $= 2x^3 + 6x^2y^2 + 6y^3$

2nd quantity $= ?$

$$= [4x^3 + 6x^2y^2 + 9y^3] - [2x^3 + 6x^2y^2 + 6y^3]$$
$$= 4x^3 + 6x^2y^2 + 9y^3 - 2x^3 - 6x^2y^2 - 6y^3$$
$$= 4x^3 - 2x^3 + 6x^2y^2 - 6x^2y^2 + 9y^3 - 6y^3$$
$$= 2x^3 + 3y^3$$

6.   *If the two sides of a triangle are* $x - y + 4z$ *and* $x + 2y - 5z$ *Find the third side when perimeter is* $3x + 2y + z$

Two sides of a triangle $= x - y + 4z$ and $x + 2y - 5z$

Perimeter $= 3x + 2y + z$

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Third side of a triangle $=$?

$$= [x - y + 4z] + [x + 2y - 5z]$$
$$= x + x - y + 2y + 4z - 5z$$
$$= 2x + y - z$$
$$[3x + 2y + z] - [2x + y - z]$$
$$= 3x + 2y + z - 2x - y + z$$
$$= 3x - 2x + 2y - y + z + z$$
$$= x + y + 2z$$

EXERCISE 5.3

Evaluate

1. $(x - 4)(x^2 - 5x - 6)$
$$= x(x^2 - 5x - 6) - 4(x^2 - 5x - 6)$$
$$= (x^3 - 5x^2 - 6x) - 4x^2 + 20x + 24$$
$$= x^3 - 5x^2 - 4x^2 - 6x + 20x + 24$$
$$= x^3 - 9x^2 + 14x + 24$$

2. $(a + b)(a^2 - ab + b^2)$
$$= a(a^2 - ab + b^2) + b(a^2 - ab + b^2)$$
$$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3$$
$$= a^3 + b^3$$

3. $(2x - 1)(x^2 - 3x - 2)$
$$= 2x(x^2 - 3x - 2) - 1(x^2 - 3x - 2)$$
$$= 2x^3 - 6x^2 - 4x - x^2 + 3x + 2$$
$$= 2x^3 - 6x^2 - x^2 - 4x + 3x + 2$$
$$= 2x^3 - 7x^2 - x + 2$$

4. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
$$= x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$$
$$= x^6 - x^4y^2 + x^2y^2 + x^4y^2 - x^2y^4 + y^6$$
$$= x^6 - x^4y^2 + x^4y^2 + x^2y^4 - x^2y^4 + y^6$$
$$= x^6 + y^6$$

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5. $(x^2 + xy + y^2)(x^2 - xy + y^2)$

$= x^2(x^2 - xy + y^2) + xy(x^2 - xy + y^2) + y^2(x^2 - xy + y^2)$

$= x^4 - x^3y - x^2y^2 + x^3y - x^2y^2 + xy^3 + x^2y^2 - xy^3 + y^4$

$= x^4 - x^3y + x^3y - x^2y^2 - x^2y^3 + x^3y - xy^3 + y^4$

$= x^4 - 2x^2y^2 + y^4$

6. $(x^2 + 5x + 4)(x^2 - 2x - 6)$

$= x^2(x^2 - 2x - 6) + 5x(x^2 - 2x - 6) + 4(x^2 - 2x - 6)$

$= x^4 - 2x^3 - 6x^2 + 5x^3 - 10x^2 - 30x + 4x^2 - 8x - 24$

$= x^4 - 2x^3 + 5x^3 - 6x^2 - 10x^2 + 4x^2 - 30x - 8x - 24$

$= x^4 + 3x^3 - 12x^2 - 38x - 24$

7. $(3y^4 - 2y^2 + 4y - 5)(y^2 + 1)$

$= 3y^4(y^2 + 1) - 2y^2(y^2 + 1) + 4y(y^2 + 1) - 5(y^2 + 1)$

$= 3y^6 + 3y^4 - 2y^4 - 2y^2 + 4y^3 + 4y - 5y^2 - 5$

$= 3y^6 + 3y^4 - 2y^4 + 4y^3 - 2y^2 - 5y^2 + 4y - 5$

$= 3y^6 + y^4 + 4y^3 - 7y^2 + 4y - 5$

8. $(4x^3 + 6x^2 - 4x + 3)(3x^3 - 4x + 3)$

$= 4x^3(3x^2 - 4x + 3) + 6x^2(3x^3 - 4x + 3) - 4x(3x^3 - 4x + 3)$
$\qquad + 3(3x^3 - 4x + 3)$

$= 12x^6 - 16x^4 + 12x^3 + 18x^5 - 24x^3 + 18x^2 - 12x^4 + 16x^2$
$\qquad - 12x + 9x^3 - 12x + 3$

$= 12x^6 + 18x^5 - 16x^4 - 12x^4 + 12x^3 - 24x^3 + 9x^3 + 18x^2$
$\qquad + 16x^2 - 12x - 12x + 3$

$= 12x^6 + 18x^5 - 28x^4 - 3x^3 + 34x^2 - 24x + 3$

Simplify the following.

(9) $(x^2 + 2x - 1)(x - 1) + (x^2 - 2x + 1)(x + 1)$

$= (x^3 - x^2 + 2x^2 - 2x - x + 1) + (x^3 + x^2 - 2x^2 - 2x + x + 1)$

$= (x^3 + x^2 - 3x + 1) + (x^3 - x^2 - x + 1)$

$= x^3 + x^2 - 3x + 1 + x^3 - x^2 - x + 1$

$= x^3 + x^3 + x^2 - x^2 - 3x - x + 1 + 1$

$= 2x^3 - 4x + 2$

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(10) $(y^2 + 5y)(y - 1) - (y^2 + 2y)(y - 1)$

     $= (y^3 - y^2 + 5y^2 - 5y) - (y^3 + y^2 + 2y^2 - 2y)$

     $= y^3 - y^2 + 5y^2 - 5y - y^3 - y^2 - 2y^2 + 2y$

     $= y^3 - y^3 - y^2 + 5y^2 - y^2 - 2y^2 - 5y + 2y$

     $= y^2 - 3y$

(11) $(5x - 1)(6x + 7) + (7x - 2)(2x + 5)$

     $= (30x^2 + 35x - 6x - 7) + (14x^2 + 35x - 4x - 10)$

     $= 30x^2 + 35x - 6x - 7 + 14x^2 + 35x - 4x - 10$

     $= 30x^2 + 14x^2 + 29x + 35x - 7 - 10$

     $= 34x^2 + 60x - 17$

EXERCISE 5.4

Divide :

(1) $x^2 + 7x + 12$ by $x + 3$

$$
\begin{array}{r}
x + 4 \\
x + 3 \overline{\smash{\big)}\, x^2 + 7x + 12} \\
\underline{\pm x^2 \pm 3x} \\
4x + 12 \\
\underline{\pm 4x \pm 12} \\
0
\end{array}
$$

     $= x + 4$

(2) $9x^2 + 6x + 1$ by $3x + 1$

$$
\begin{array}{r}
3x + 1 \\
3x + 1 \overline{\smash{\big)}\, 9x^2 + 6x + 1} \\
\underline{\pm 9x^2 \pm 3x} \\
3x + 1 \\
\underline{\pm 3x \pm 1} \\
0
\end{array}
$$

     $= 3x + 1$

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(3) $3x^2 + 4x + 1$ b $x + 1$

$$\begin{array}{r} 3x + 1 \\ x + 1 \overline{)\,3x^2 + 4x + 1} \\ \underline{\pm\,3x^2 \pm 3x} \\ x + 1 \\ \underline{\pm\, x \pm 1} \\ 0 \end{array}$$

$= 3x + 1$

(4) $a^2 - 8ab + 16b^2$ by $a - 4b$

$$\begin{array}{r} a - 4b \\ a - 4b \overline{)\,a^2 - 8ab + 16b^2} \\ \underline{\pm\,a^2 - 4ab} \\ -\quad+ \\ -4ab + 16b^2 \\ -4ab + 16b^2 \\ +\quad- \\ \hline 0 \end{array}$$

(5) $6x^2 - 5x + 1$ by $2x - 1$

$$\begin{array}{r} 3x - 1 \\ 2x - 1 \overline{)\,6x^2 - 5x + 1} \\ +\,6x^2 - 3x \\ -\quad+ \\ -2x + 1 \\ -2x + 1 \\ +\quad- \\ \hline 0 \end{array}$$

$= 3x - 1$

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ASAN Math For Class 8th **128**   **Polynomials**

(6) $5x^2 + 7x - 12$ by $x - 1$

$$
\require{enclose}
\begin{array}{r}
5x + 12 \\
x - 1 \enclose{longdiv}{5x^2 + 7x - 12} \\
+5x^2 - 5x \\
\underline{\quad - \qquad + \quad} \\
12x - 12 \\
+4x - 12 \\
\underline{\quad - \qquad + \quad} \\
0
\end{array}
$$

$= 5x + 12$

(7) $x^3 + 1$ by $x + 1$

$$
\begin{array}{r}
x^2 - x + 1 \\
x + 1 \enclose{longdiv}{x^3 + 1} \\
\pm x^3 \quad \pm x^2 \\
\underline{\qquad\qquad\qquad} \\
x^2 + 1 \\
-x^2 \quad -x \\
\underline{\qquad\qquad\qquad} \\
-x + 1 \\
\pm x \pm 1 \\
\underline{\qquad\qquad\qquad} \\
0
\end{array}
$$

$= x^2 - x + 1$

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(8) $5x^3 - 7x^2y - xy^2 + 3y^3$ by $5x + 3y$

$$x^2 - 2xy + y^2$$

$$5x + 3y \sqrt{5x^3 - 7x^2y - xy^2 + 3y^3}$$

$$\underline{\pm 5x^3 \pm 3x^2y}$$

$$-10x^2y - xy^2$$
$$\underline{-10x^2y - 6xy^2}$$
$$+ \qquad +$$

$$5xy^2 + 3y^3$$
$$\underline{\pm 5xy^2 \pm 3y^3}$$

$$0$$

(9) $x^3 - 3x^2y + 3xy^2 - y^3$ by $x - y$

$$x^2 - 2xy + y^2$$

$$x - y \sqrt{x^3 - 3x^2y + 3xy^2 - y^3}$$

$$\underline{\pm x^3 - x^2y}$$

$$-2x^2y + 3xy^2$$
$$\underline{-2x^2y + 2xy^2}$$
$$+ \qquad -$$

$$xy^2 - y^3$$
$$\underline{+ xy^2 - y^3}$$
$$- \qquad +$$

$$0$$

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