UNIT 1

OPERATIONS ON SETS

EXERCISE 1.1

1. *Find three subsets of the following sets.*

(i) $\{2, 4\}$
$= \{2\}, \{4\}, \{2, 4\}$

(ii) $\{a, c, e\}$
$= \{a\}, \{c\}, \{a, e\}$

2. *Find all possible subsets of the following sets.*

(i) $\{-1, 0, 1\}$
$= \{\ \}, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}, \{-1, 0, 1\}$

(ii) $\{\ \} = \{\ \}$

(iii) $\{m, n, o, p\}$
$= \{\ \}, \{m\}, \{n\}, \{o\}, \{p\}, \{m, n\}, \{m, o\}, \{m, p\}, \{n, o\}, \{n, p\}, \{o, p\}, \{m, n, o\}, \{m, n, p\}, \{m, o, p\}, \{n, o, p\}, \{m, n, o, p\}$

3. *Write four proper subsets and one improper subsets of the following sets.*

(i) $\{-1, -2, -3\}$
$= \{-1\}, \{-2\}, \{-3\}, \{-1, -2\}, \{-1, -2\}, \{-1, -2, -3\}$

(ii) $\left\{\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$
$= \left\{\frac{1}{2}\right\}, \left\{\frac{1}{2}, \frac{3}{4}\right\}, \left\{\frac{1}{2}, \frac{4}{5}\right\}, \left\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}\right\}, \left\{\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$

(iii) {Pen, Pencil, Copy}
= {Pen}, {Pencil}, {Pen, Copy}, {Pen, Pencil, Copy}

4. *Name the set which has*

(i) Only one subset
= { } empty set is the set which has only one subset.

(ii) Only one proper subset.
= Singleton set

(iii) No proper subset
= { }

5. ***Write the number of elements in the power set of following sets.***

(i) $\{1, 3, 5, 7, 9\}$
$n = 5$
No. of element $= 2^n$
$= 2^5 = 32$

(ii) $\{0, 1, 2, 3, 4, 5\}$
$n = 6$
No. of element $= 2^n$
$2^6 = 64$

(iii) $\phi = n = 0$
No. of element $= 2^n = 2^0 = 1$

6. ***Find the power set of the following sets.***

(i) $\{a, b\}$
$P(A) = \{\{\ \}, \{a\}, \{b\}, \{a, b\}\}$

(ii) $\{0, 2, 4\}$
$P(B) = \{\{\ \}, \{0\}, \{2\}, \{4\}, \{0, 2\}, \{0, 4\}, \{2, 4\}, \{0, 2, 4\}\}$

(iii) $\{1, 2, 3, 4\}$
$P(C) = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$

(iv) $\{101\}$
$P(D) = \{\{\ \}, \{101\}\}$

(v) ϕ
$P(\phi) = \{\phi\}$

EXERCISE 1.2

1. ***Verify commutative property of union for the following pair of sets.***

$A = \{2, 3, 5, 7, 11\}$ $B = \{5, 6, 7, 8, 9, 10\}$

$A \cup B = B \cup A$

L.H.S.

$A \cup B = \{2, 3, 5, 7, 11\} \cup \{5, 6, 7, 8, 9, 10\}$
$= \{2, 3, 5, 6, 7, 8, 9, 10, 11\}$ (i)

R.H.S.

$B \cup A = \{5, 6, 7, 8, 9, 10\} \cup \{2, 3, 5, 7, 11\}$
$= \{2, 3, 5, 6, 7, 8, 9, 10, 11\}$ (ii)

From (i) and (ii)

L.H.S. = R.H.S

2. ***Verify commutative property of intersection for the following pair of sets.***

$X = \{s, c, i, e, n\}$, $Y = \{m, a, t, h, e, i, c, s\}$

$X \cap Y = Y \cap X$

L.H.S.

$X \cap Y = \{s, c, i, e, n\} \cap \{m, a, t, h, e, i, c, s\}$

$= \{i, e\}$ (i)

R.H.S.

$Y \cap X = \{m, a, t, h, e, i, c, s\} \cap \{s, c, i, e, n\}$

$= \{i, e\}$ (ii)

From (i) and (ii)

L.H.S. = R.H.S.

3. ***If M = Set of vowels in English alphabets***

N = Set of consonants in English alphabets.

(i) $M = \{a, e, i, o, u\}$

$N = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

$M \cup N = \{a, e, i, o, u\} \cup \{b, c, d, f, g, \ldots\ldots. y, z\}$

$M \cup N = \{a, b, c, d, e, f, g, h, \ldots\ldots. z\}$ (i)

$N \cup M = \{b, c, d, f, g, h, j, k, \ldots\ldots. y, z\} \cup \{a, e, i, o, u\}$

$N \cup M = \{a, b, c, d, e, f, \ldots\ldots. z\}$ (ii)

From (i) and (ii)

L.H.S. = R.H.S.

(ii) $M \cap N = N \cap M$

$M \cap N = \{a, e, i, o, u\} \cap \{b, c, d, f, g, h, j, \ldots\ldots. x, y, z\}$

$M \cap N = \{\ \}$ (i)

$N \cap M = \{b, c, d, f, g, h, \ldots\ldots. y, z\} \cap \{a, e, i, o, u\}$

$N \cap M = \{\ \}$ (ii)

From (i) and (ii)

Its proved $M \cap N = N \cap M$

4. ***Verify associative property of union for the following sets.***

• (i) $A = \{0, 1, 2, 3, \ldots, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and

$C = \{0, 2, 4, 6, 8\}$

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S.

$$(A \cup B) \cup C = [\{0, 1, 2, 3, ..., 10\} \cup \{1, 2, 3, 4, 5\}] \cup \{0, 2, 4, 6, 8\}$$
$$= \{0, 1, 2, 3, ..., 10\} \cup \{0, 2, 4, 6, 8\}$$
$$= \{0, 1, 2, 3, ..., 10\} \quad \text{(i)}$$

R.H.S.

$$A \cup (B \cup C) = \{0, 1, 2, 3, ..., 10\} \cup \{\{1, 2, 3, 4, 5\} \cup \{0, 2, 4, 6, 8\}\}$$
$$= \{0, 1, 2, 3, ..., 10\} \cup \{0,1,2,3,4, 5, 6, 8\}$$
$$= \{0, 1, 2, 3, ..., 10\} \quad \text{(ii)}$$

From (i) and (ii) its proved L.H.S. = R.H.S.

(ii) $P = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\}$, $Q = \left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}\right\}$

and $R = \left\{\frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}\right\}$

$(P \cup Q) \cup R = P \cup (Q \cup R)$

L.H.S.

$$(P \cup Q) \cup R = \left(\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\} \cup \left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}\right\}\right) \cup \left\{\frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}\right\}$$
$$= \left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\} \cup \left\{\frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}\right\}$$
$$= \left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}, \frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\} \quad \text{(i)}$$

R.H.S.

$$P \cup (Q \cup R) = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\} \cup \left(\left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}\right\} \cup \left\{\frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}\right\}\right)$$
$$= \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right\} \cup \left(\left\{-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4}, -\frac{4}{5}, \frac{2}{3}, \frac{3}{4}, -\frac{2}{3}, -\frac{3}{4}\right\}\right)$$

$$=\left\{-\frac{1}{2},-\frac{2}{3},-\frac{3}{4},-\frac{4}{5},\frac{2}{3},\frac{3}{4},-\frac{2}{3},-\frac{3}{4},\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5}\right\} \quad \text{(ii)}$$

From (i) and (ii). It is proved L.H.S. = R.H.S.

5. ***Verify associative property of intersection for the following sects.***

(i) $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{-5, -4, -3, -2, -1\}$ and $C = \{-2, -1, 0, 1, 2, 3\}$

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S.

$$\begin{aligned}(A \cap B) \cap C &= (\{0, 1, 2, 3, 4, 5\} \cap \{-5, -4, -3, -2, -1\}) \\ &\quad \cap \{-2, -1, 0, 1, 2, 3\} \\ &= \{\ \} \cap \{-2, -1, 0, 1, 2, 3\} \\ &= \{\ \} \qquad \text{(i)}\end{aligned}$$

R.H.S.

$$\begin{aligned}A \cap (B \cap C) &= \{0, 1, 2, 3, 4, 5\} \cap (\{-5, -4, -3, -2, -1\}) \\ &\quad \cap \{-2, -1, 0, 1, 2, 3\}) \\ &= \{0, 1, 2, 3, 4, 5\} \cap \{-2, -1\} \\ &= \{\ \} \qquad \text{(ii)}\end{aligned}$$

From (i) and (ii) L.H.S. = R.H.S.

(ii) $X = \{2, 3, 5, 7, 11, 13, 17, 19\}$, $Y = \{1, 3, 5, \ldots, 19\}$ and $Z = \{2, 4, 6, \ldots, 20\}$

$(X \cap Y) \cap Z = X \cap (Y \cap Z)$

L.H.S.

$$\begin{aligned}(X \cap Y) \cap Z &= (\{2, 3, 5, 7, 11, 13, 17, 19\} \cap \\ &\quad \{(1, 3, 5, \ldots, 19\}) \cap \{2, 4, 6, \ldots, 20\} \\ &= \{3, 5, 7, 11, 13, 17, 19\} \cap \{2, 4, 6, \ldots, 20\} \\ &= \{\ \} \qquad \text{(i)}\end{aligned}$$

R.H.S.

$$\begin{aligned}X \cap (Y \cap Z) &= \{2, 3, 5, 7, 11, 13, 17, 19\} \cap (\{1, 3, 5, \ldots, 19) \\ &\quad \cap (2, 4, 6, \ldots, 20\}) \\ &= \{3, 5, 7, 11, 13, 17, 19\} \cap \{\ \} \\ &= \{\ \} \qquad \text{(ii)}\end{aligned}$$

Hence, from (i) and (ii) L.H.S. = R.H.S.

6. *If X = {1, 2, 3, ..., 10}, Y = {0, 2, 4, 6, 8, 10} and Z = {0, 1, 2, 3, ..., 10} Then prove that:*

(i) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$

L.H.S.

$(X \cup Y) \cup Z = (\{1, 2, 3, ..., 10\} \cup \{0, 2, 4, 6, 8, 10\}) \cup \{0, 1, 2, 3, ..., 10\}$

$= \{0, 1, 2, 3, ..., 10\} \cup \{0, 1, 2, 3, ..., 10\}$

$= \{0, 1, 2, 3, ..., 10\}$ (i)

R.H.S.

$X \cup (Y \cup Z) = \{1, 2, 3, ..., 10\} \cup (\{0, 2, 4, 6, 8, 10\} \cup \{0, 1, 2, 3, ..., 10\})$

$= \{1, 2, 3, ..., 10\} \cup \{0, 1, 2, 3, ..., 10\}$

$= \{0, 1, 2, 3, ..., 10\}$ (ii)

(ii) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

L.H.S.

$(X \cap Y) \cap Z = (\{1, 2, 3, ..., 10\} \cap \{0, 2, 4, 6, 8, 10\}) \cap \{0, 1, 2, 3, ..., 10\}$

$= \{2, 4, 6, 8, 10\} \cap \{0, 1, 2, 3, ..., 10\}$

$= \{2, 4, 6, 8, ..., 10\}$ (i)

R.H.S.

$X \cap (Y \cap Z) = \{1, 2, 3, ..., 10\} \cap (\{0, 2, 4, 6, 8, 10\} \cap \{0, 1, 2, 3, ..., 10\})$

$= \{2, 4, 6, 8, ..., 10\} \cap \{0, 2, 4, 6, 8, ..., 10\}$

$= \{2, 4, 6, 8, ..., 10\}$ (ii)

It is proved from (i) and (ii) L.H.S. = R.H.S.

EXERCISE 1.3

1. *Prove that*

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) $A = \{0, 1, 2, 3\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{5, 6, 7, 8, 9, 10\}$

L.H.S.

$A \cup (B \cap C) = \{0, 1, 2, 3\} \cup (\{2, 3, 4,5,6\} \cap \{5,6,7,8,9,10\})$

$= \{0, 1, 2, 3\} \cup \{5, 6\}$

$= \{0, 1, 2, 3, 5, 6\}$ (I)

R.H.S.

$(A \cup B) \cap (A \cup C) = (\{0, 1, 2, 3\} \cup \{2, 3, 4,5,6\}) \cap$
$((0, 1, 2, 3\} \cup \{5, 6, 7, 8, 9, 10\})$
$= \{0,1,2,3,4,5,6\} \cup \{0,1,2,3,5,6,7,8,9,10\}$
$= \{0, 1, 2, 3, 5, 6\}$ (II)

From (I) and (II). It is proved L.H.S. = R.H.S.

(ii) $A = \{l, m, n, o, p, q\}$, $B = \{r, s, t, u\}$, $C = \{t, u, v, w\}$

L.H.S.

$A \cup (B \cap C) = \{l, m, n, o, p, q\} \cup (\{r, s, t, u\} \cap \{t, u, v, w\}$
$= \{l, m, n, o, p, q\} \cup \{t, u)$
$= \{l, m, n, o, p, q, t, u\}$ (I)

R.H.S.

$(A \cup B) \cap (A \cup C) = (\{l, m, n, o, p, q\} \cup \{r, s, t, u\}) \cap$
$(\{l, m, n, o, p, q\} \cup \{t, u, v, w\})$
$= \{l, m, n, o, p, q, r, s, t, u\} \cap$
$\{l, m, n, o, p, q, t, u, v, w\}$
$= \{l, m, n, o, p, q, t, u\}$ (II)

It is proves from (I) and (II) L.H.S. = R.H.S.

(iii) $A = \{+, -, \times\}$, $B = \{-, \times, \div\}$, $C = \{-, \div, \sqrt{\ }\}$

L.H.S.

$A \cup (B \cap C) = \{+, -, \times\} \cup \left(\{-, \times, \div\} \cap \{-, \div, \sqrt{\ }\}\right)$
$= \{+, -, \times\} \cup \{-, \div\}$
$= \{+, -, \times, \div\}$ (i)

R.H.S.

$(A \cup B) \cap (A \cup C) = \left(\{+, -, \times\} \cup \{-, \times, \div\}\right) \cap \{+, -, \times\}$
$\cup \{-, \div, \sqrt{\ }\}$
$= \{+, -, \times, \div\} \cap \{+, -, \times, \div, \sqrt{\ }\}$
$= \{+, -, \times, \div\}$ (ii)

It is proved from (i) and (ii).

L.H.S. = R.H.S.

(b) $\mathbf{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

(i) $A = \{0, 1, 2, 3\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{5, 6, 7, 8, 9, 10\}$

L.H.S.

$$A \cap (B \cup C) = \{0, 1, 2, 3\} \cap (\{2,3,4,5,6\} \cup \{5,6,7,8,9,10\})$$
$$= \{0, 1, 2, 3\} \cap \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$$
$$= \{2, 3\} \quad \text{(i)}$$

R.H.S.

$$(A \cap B) \cup (A \cup C) = (\{0, 1, 2, 3\} \cap \{2, 3, 4, 5, 6\}) \cup (\{0, 1, 2, 3\} \cap \{5, 6, 7, 8, 9, 10\})$$
$$= \{2, 3\} \cup \{\ \}$$
$$= \{2, 3\} \quad \text{(ii)}$$

From (i) and (ii).

It is proved L.H.S. = R.H.S.

(ii) $A = \{l, m, n, o, p, q\}$, $B = \{r, s, t, u\}$, $C = \{t, u, v, w\}$

L.H.S.

$$A \cap (B \cup C) = \{l, m, n, o, p, q\} \cap (\{r, s, t, u\} \cup \{t, u, v, w\})$$
$$= \{l, m, n, o, p, q\} \cap \{r, s, t, u, v, w\}$$
$$= \{\ \} \quad \text{(i)}$$

R.H.S.

$$(A \cap B) \cup (A \cap C) = (\{l, m, n, o, p, q\} \cap \{r, s, t, u\}) \cup (\{l, m, n, o, p, q\} \cap \{t, u, v, w\})$$
$$= \{\ \} \cup \{\ \}$$
$$= \{\ \} \quad \text{(ii)}$$

It is proved from (i) and (ii).

L.H.S. = R.H.S.

(iii) $A = \{+, -, \times\}$, $B = \{-, \times, \div\}$, $C = \{-, \div, \sqrt{\ }\}$

L.H.S.

$$A \cap (B \cup C) = \{+,-,\times\} \cap (\{-,\times,\div\}) \cup \{+,-,\times\} \cap \{-,\div,\sqrt{\ }\})$$
$$= \{+, -, \times\} \cap \{-, \times, \div, \sqrt{\ }\}$$
$$= \{-, \times\} \quad \text{(i)}$$

R.H.S.

$$(A \cap B) \cup (A \cap C) = (\{+, -, \times\} \cap \{-, \times, \div\}) \cup \{+, -, \times\} \cap \{-, \div, \sqrt{\ }\})$$
$$= \{-, \times\} \cup \{-\}$$

$= \{-, \times\}$ (ii)

From (i) and (ii).

It proved L.H.S. = R.H.S.

2. ***Verify distributive law of union over intersection for the following sets.***

$P = \{1, 2, 3, \ldots\}$, $Q = \{0, 1, 2, 3, \ldots\}$, $R = \{0, \pm, 1, \pm 2, \ldots\}$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S.

$$P \cup (Q \cap R) = \{1,2,3,\ldots\} \cup (\{0,1,2,3,\ldots\} \cap \{0, \pm 1, \pm 2, \pm 3\})$$
$$= \{1, 2, 3, \ldots\} \cup \{0, 1, 2, 3, \ldots\}$$
$$= \{0, 1, 2, 3, \ldots\} \quad \text{(i)}$$

R.H.S.

$$(P \cup Q) \cap (P \cup R) = (\{1, 2, 3, \ldots\} \cup \{0, 1, 2, 3, \ldots\}) \cap \{1, 2, 3, \ldots\} \cup \{0, \pm 1, \pm 2, \ldots\})$$
$$= \{0, 1, 2, 3, \ldots\} \cap \{0, \pm 1, \pm 2, \ldots\}$$
$$= \{0, 1, 2, 3, \ldots\} \quad \text{(ii)}$$

From (i) and (ii).

It proved L.H.S. = R.H.S.

3. ***Verify distributive law of intersection over union for the following sets.***

$X = \{\}$, $Y = \{0\}$,

Z = Set of natural numbers = $\{1, 2, 3, 4, \ldots\}$

$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

L.H.S.

$$X \cap (Y \cup Z) = \{\} \cap \{0\} \cup \{1, 2, 3, 4, 5, \ldots\}$$
$$= \{\} \cap \{1, 2, 3, 4, 5, \ldots\}$$
$$= \{\} \quad \text{(i)}$$

R.H.S.

$$(X \cap Y) \cup (X \cap Z) = (\{\} \cap \{0\}) \cup (\{\} \cap \{1,2,3,4,5,6, \ldots\}$$
$$= \{\} \cup \{\}$$
$$= \{\} \quad \text{(ii)}$$

It is proved from (i) and (ii).

L.H.S. = R.H.S.

4. ***If $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 3, 5, 7, 9\}$, $C = \{2, 4, 6, 8, 10\}$. Then prove that***

(i) $(A \cup B)' = A' \cap B'$

L.H.S. $= (A \cup B)'$

$(A \cup B) = \{2, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$(A \cup B)' = U - A \cup B$

$= \{1,2,3,4,5,6,7, 8, 9, 10\} - \{2,3,4,5,6,7,8,9,10\}$

$= \{1\}$ (i)

R.H.S.

$A' \cap B'$

$A' = U - A$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 3, 5, 7, 9\}$

$A' = \{1, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$A' \cap B' = \{1, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \{1\}$ (ii)

From (i) and (ii).

It is proved.

L.H.S. = R.H.S.

(ii) $(A \cap B)' = A' \cup B'$

L.H.S.

$(A \cap B)' = ?$

$A \cap B = \{2, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\}$

$= \{2\}$

$(A \cap B)' = U - A \cap B$

$= \{1, 2, 3, 4, \ldots, 10\} - \{2\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$ (i)

R.H.S.

$A' \cup B'$

$A' = U - A$

$= \{1, 2, 3, 4, 5, 6, \ldots, 10\} - \{2, 3, 5, 7, 9\}$

$= \{1, 4, 6, 8, 10\}$

$B' = U - B$

$= \{1, 2, 3, 4, 5, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$A' \cup B' = \{1, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$ (ii)

From (i) and (ii).

It is proved L.H.S. = R.H.S.

5. *Verify De Morgan's laws for the following sets*

$U = \{x : x \in W \cap 0 \leq x \leq 20\} = \{0, 1, 2, 3, 4, 5, \ldots, 20\}$

$C = \{x : x \in E \wedge 0 \leq x \leq 20\} = \{1, 3, 5, 7, 9, \ldots, 19\}$

$D = \{x : x \in O \wedge 1 \leq x \leq 19\}$

De Morgan's Law

(i) $(C \cup D)' = C' \cap D'$

(ii) $(C \cap D)' = C' \cup D'$

(i) $(C \cup D)' = C' \cap D'$

L.H.S.

$C \cup D = \{0, 2, 4, 6, 8, 20\} \cup \{1, 3, 5, 7, \ldots, 19\}$

$C \cup D = \{0, 1, 2, 3, 4, \ldots, 20\}$

$(C \cup D)' = U - C \cup D$

$= \{0, 1, 2, 3, 4, 5, \ldots, 20\} - \{0, 1, 2, 3, 4, \ldots, 20\}$

$= \{\ \}$ (i)

R.H.S. $= C' \cap D'$

$C' = U - C$

$= \{0, 1, 2, 3, 4, \ldots, 20\} - \{0, 2, 4, 6, 8, 20\}$

$= \{1, 3, 5, 7, \ldots, 19\}$ (ii)

$D' = U - D$
$= \{0, 1, 2, 3, 4, \ldots, 20\} - \{1, 3, 5, 7, \ldots, 19\}$
$= \{0, 2, 4, 6, 8, \ldots, 20\}$.

$C' \cap D' = \{1, 3, 5, 7, 9, \ldots, 19\} \cap \{0, 2, 4, 6, \ldots, 20\}$
$= \{\ \}$ (iii)

It is proved from (i) and (ii).

L.H.S. = R.H.S.

(ii) $(C \cap D)' = C' \cup D'$

L.H.S.

$(C \cap D)'$

$C \cap D = \{0, 2, 4, 6, 8, \ldots, 20\} \cap \{1, 2, 3, 5, \ldots, 19\}$
$= \{\ \}$

$(C \cap D)' = U - C \cap D$
$= \{1, 2, 3, 4, 5, \ldots, 20\} - \{\ \}$
$= \{0, 1, 2, 3, 4, 5, \ldots, 20\}$ (i)

R.H.S.

$C' \cup D'$

$C' = \{1, 3, 5, 7, 9, \ldots, 19\}$

$D' = \{0, 2, 4, 0, 8, \ldots, 20\}$

$C' \cup D' = \{1, 3, 5, 7, \ldots, 19\} \cup \{0, 2, 4, 6, 8, \ldots, 20\}$

$C' \cup D' = \{0, 1, 2, 3, \ldots, 20\}$ (ii)

It is proved from (i) and (ii).

L.H.S. = R.H.S.

Distributive Law of Intersection Over Union

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S. $= A \cap (B \cup C)$

$A \cap (B \cup C) = \{1, 3, 5, 7, 9\} \cap (\{2, 4, 6, 8, 10\} \cup \{0, 1, 2, 3, \ldots, 10\})$
$= \{1, 3, 5, 7, 9\}$ (i)

R.H.S. $= (A \cap B) \cup (A \cap C)$

$(A \cap B) \cup (A \cap C) = (\{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\} \cup$

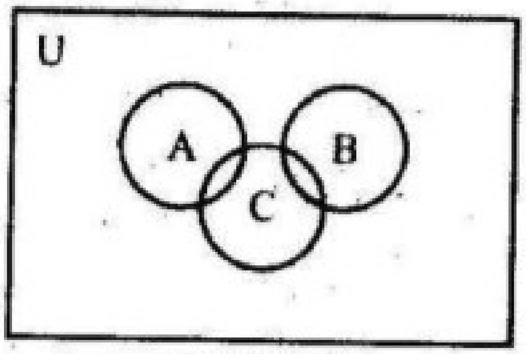
$$(\{1, 3, 5, 7, 9\} \cap \{0, 1, 2, 3, \ldots, 10\})$$
$$= \{1, 4, 8, 10\} \cap \{1, 3, 5, 7, 9, 10\}$$
$$= \{1, 3, 5, 7, 9, 10\} \qquad \text{(ii)}$$

Hence proved from L.H.S. = R.H.S.

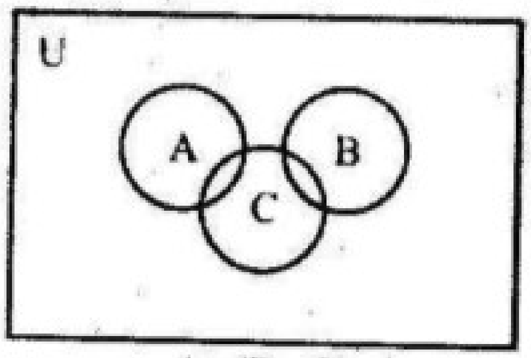
EXERCISE 1.4

1. ***Shade $A \cup (B \cap C)$, $A \cap (B \cup C)$, $(A \cup B) \cup C$ and $A \cap (B \cap C)$ using following Venn diagram.***

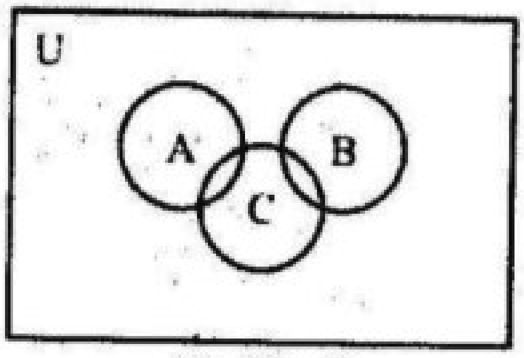
(i)



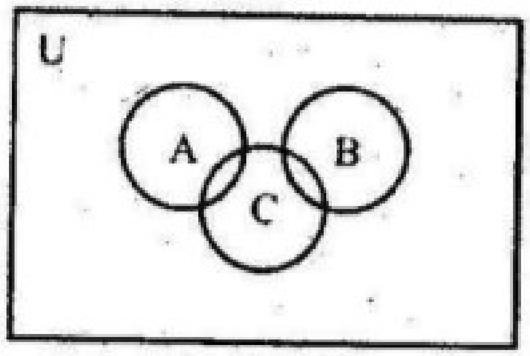
A∪(B∪C)



A∩(B∪C)

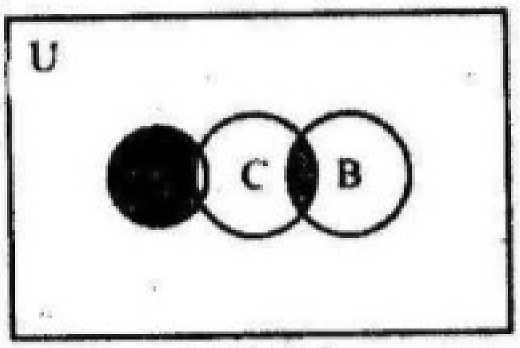


(A∪B)∪C

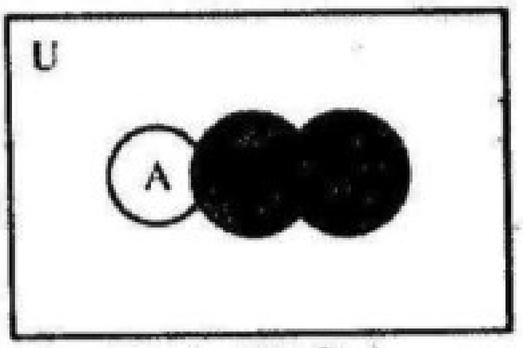


A∩(B∩C)

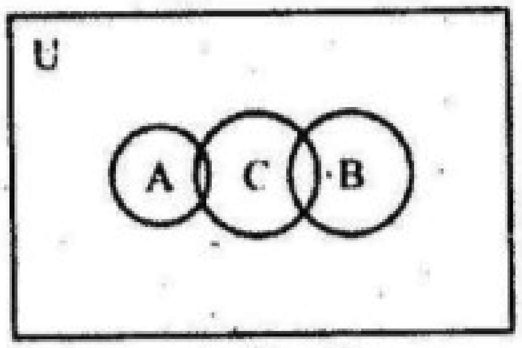
(ii)



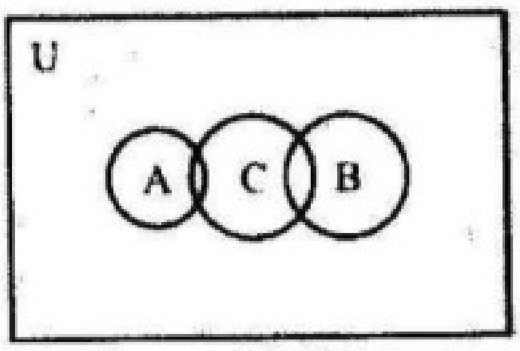
A∪(B∩C)



A∩(B∪C)

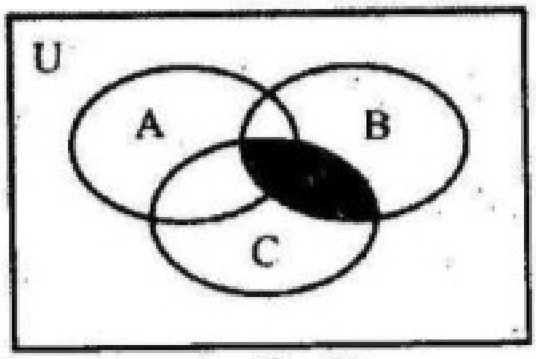


(A∪B)∪C

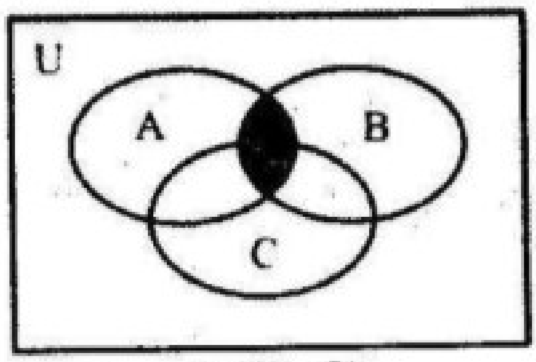


A∩(B∩C)

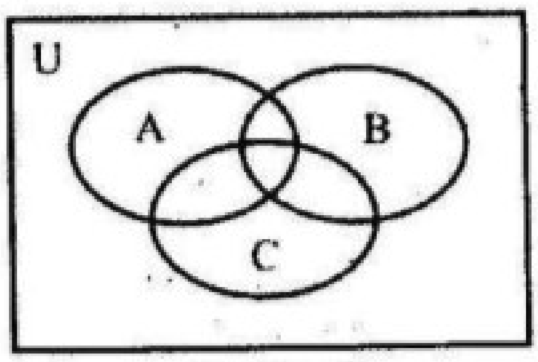
(iii)



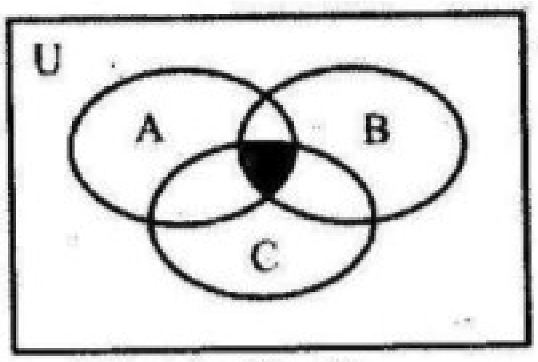
A∪(B∩C)



A∩(B∪C)

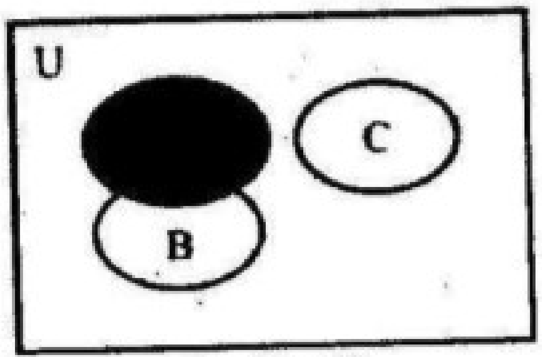


(A∪B)∪C

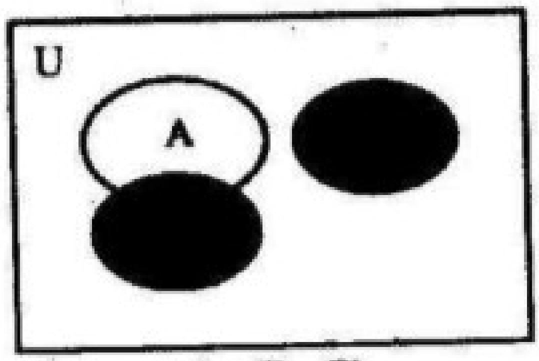


A∩(B∩C)

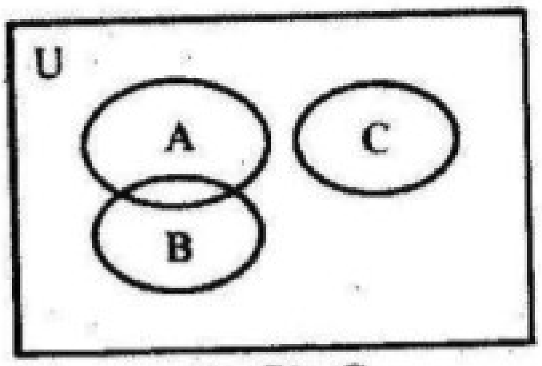
(iv)



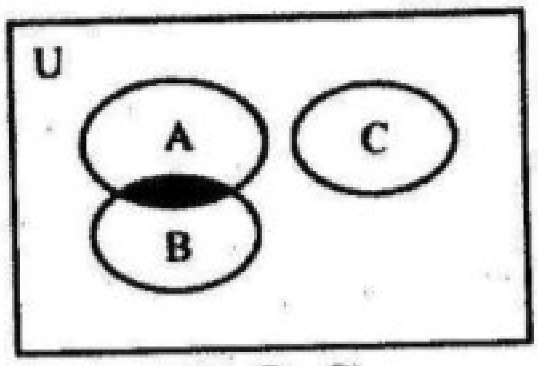
$A \cup (B \cap C)$



$A \cap (B \cup C)$



$(A \cup B) \cup C$

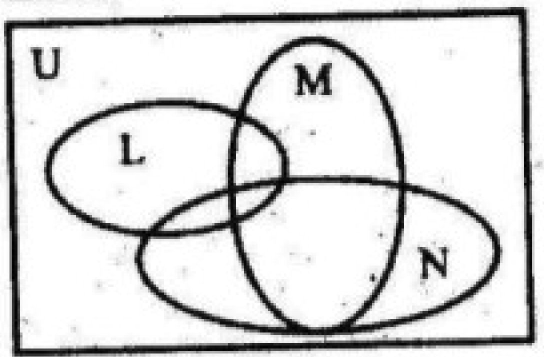


$A \cap (B \cap C)$

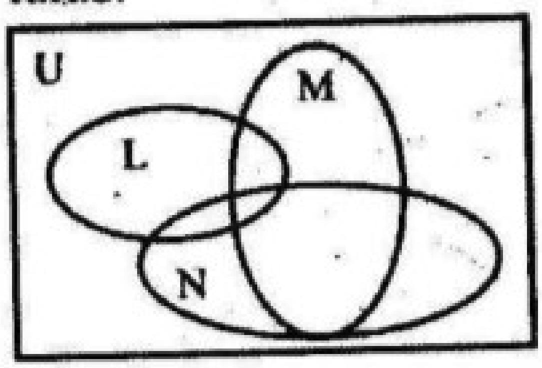
2. ***Verify associative law of union and intersection with the help of adjoining diagram.***

(i) $(L \cup M) \cup N = L \cup (M \cup N)$

L.H.S.

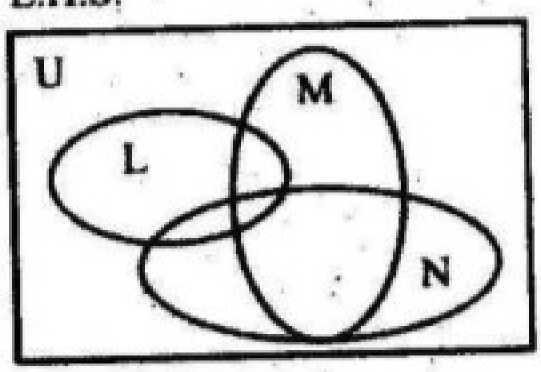


R.H.S.

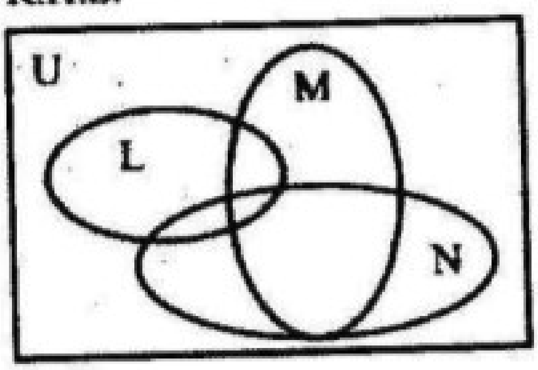


(ii) $(L \cap M) \cap N = L \cap (M \cap N)$

L.H.S.

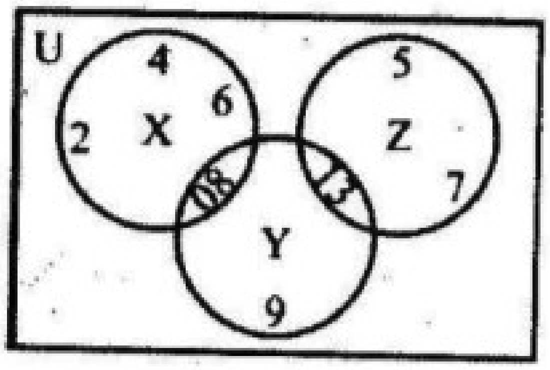


R.H.S.



3. ***Verify the following properties with the help of adjoining figure.***

(i) Distributive property of union over intersection.



$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

$Y \cap Z = \{1, 3\}$

L.H.S.

$X \cup (Y \cap Z) = \{0, 2, 4, 6, 8\} \cup \{1, 3\}$

$= \{0, 1, 2, 3, 4, 6, 8\}$

$\Rightarrow$ $X \cup Y = \{0, 2, 4, 6, 8, 9, 1, 3\}$

$X \cup Z = \{0, 2, 4, 6, 8, 1, 3, 5, 7\}$

R.H.S.

$(X \cup Y) \cap (X \cap Z) = \{0, 2, 4, 6, 89, 1, 3\} \cap \{0, 2, 4, 6, 8, 1, 3,5,7\}$

$= \{0, 2, 4, 6, 8, 1, 3\}$

(ii) **Distributive Property of Intersection Over Union**

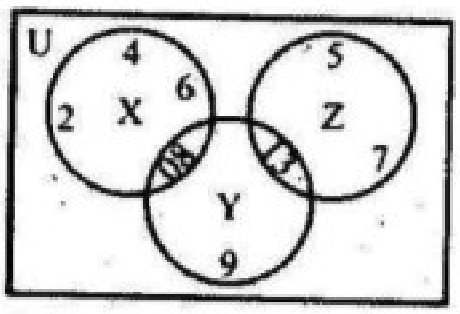
$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

$Y \cup Z = \{0, 1, 3, 8, 5, 7, 9\}$

L.H.S.

$X \cap (Y \cup Z) = \{0, 2, 4, 6, 8\} \cap \{0, 1, 3, 8, 5, 7, 9\}$

$= \{0, 8\}$



R.H.S.

$X \cap Y = \{0, 8\}$

$X \cap Z = \{\ \}$

$(X \cap Y) \cup (X \cap Z) = \{0, 8\} \cup \{\ \}$

$= \{0, 8\}$

4. *Prove by using Venn diagram:*

(a) $(P \cup Q) \cup R = P \cup (Q \cup R)$

(i) $P = \{0, 1, 2, 3\}$, $Q = \{2, 3, 4, 5, 6\}$, $R = \{5, 6, 7, 8, 9\}$

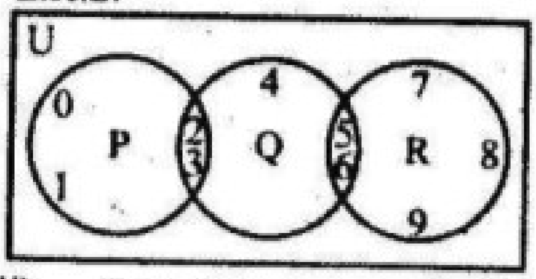
L.H.S.

$$
\begin{aligned}
(P \cup Q) \cup R &= (\{0, 1, 2, 3\} \cup \{2, 3, 4, 5, 6\}) \cup \{5, 6, 7, 8,9\} \\
&= \{0, 1, 2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8, 9\} \\
&= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}
\end{aligned}
$$

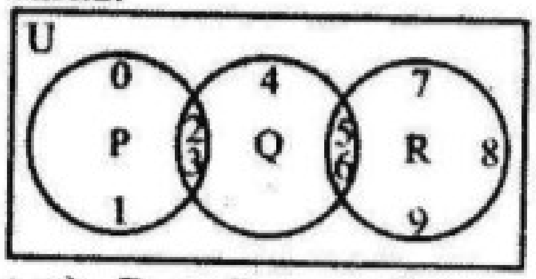
R.H.S.

$$
\begin{aligned}
P \cup (Q \cup R) &= \{0, 1, 2, 3\} \cup (\{2, 3, 4, 5, 6\} \cup \{5, 6, 7, 8 9\}) \\
&= \{0, 1, 2, 3\} \cup (\{2, 3, 4, 5, 6, 7, 8, 9\} \\
&= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}
\end{aligned}
$$

L.H.S.



R.H.S.



(ii) $P = \{m, n, o, p, q\}$, $Q = \{r, s, t, u\}$, $R = \{t, u, v, w\}$

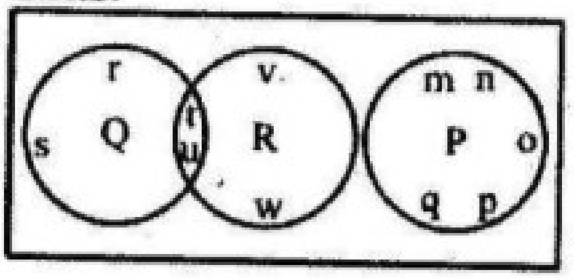
L.H.S.

$$
\begin{aligned}
(P \cup Q) \cup R &= (m, n, o, p, q\} \cup \{r, s, t, u\}) \cup \{t, u, v, w\} \\
&= (\{m, n, o, p, q, r, s, t, u\}) \cup \{t, u, v, w\} \\
&= \{m, n, o, p, q, r, s, t, u, v, w\}
\end{aligned}
$$

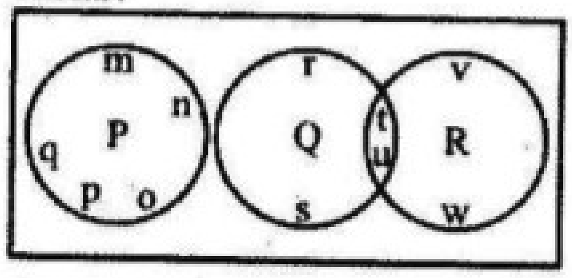
L.H.S.

$$
\begin{aligned}
P \cup (Q \cup R) &= (m, n, o, p, q\} \cup \{r, s, t, u\} \cup \{t, u, v, w\} \\
&= (\{m, n, o, p, q\}) \cup \{r, s, t, u, v, w\} \\
&= (m, n, o, p, q, r, s, t, u, v, w\}
\end{aligned}
$$

L.H.S.



R.H.S.



(b) $(P \cap Q) \cap R = P \cap (Q \cap R)$

(i) $P = \{0, 1, 2, 3\}$, $Q = \{2, 3, 4, 5, 6\}$, $R = \{5, 6, 7, 8, 9\}$

L.H.S.

$(P \cap Q) \cap R = (\{0, 1, 2, 3\} \cap \{2, 3, 4, 5, 6\}) \cap \{5, 6, 7, 8, 9\}$

$= \{2, 3 \{\cap (5, 6, 7, 8, 9\}$

$= \{\ \}$

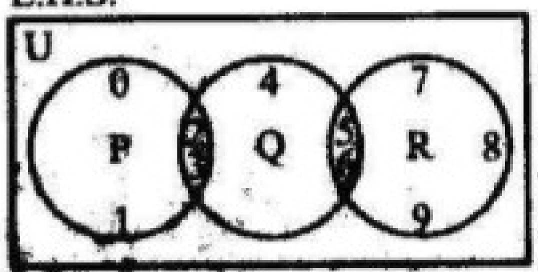
R.H.S.

$P \cap (Q \cap R) = (\{0, 1, 2, 3\} \cap (\{2, 3, 4, 5, 6\}) \cap \{5, 6, 7, 8, 9\})$

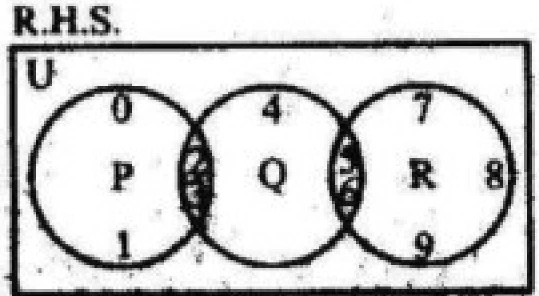
$= \{0, 1, 2, 3\} \cap \{5, 6\}$

$= \{\ \}$

L.H.S.



R.H.S.



(ii) $P = \{m, n, o, p, q\}$, $Q = \{r, s, t, u\}$, $R = \{t, u, v, w\}$

L.H.S.

$(P \cap Q) \cap R = (\{m, n, o, p, q\} \cap \{r, s, t, u\} \cap \{t, u, v, w\}$

$= \{\ \} \cap \{t, u, v, w\}$

$= \{\ \}$

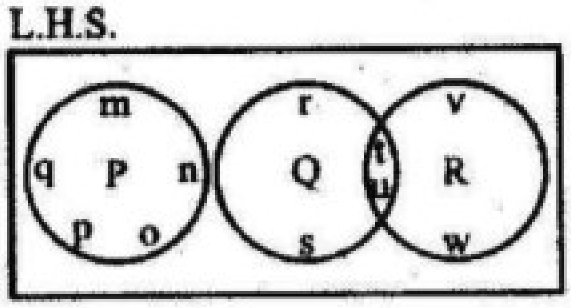
R.H.S.

$P \cap (Q \cap R) = (\{m, n, o, p, q\} \cap \{r, s, t, u\} \cap \{t, u, v, w\}$

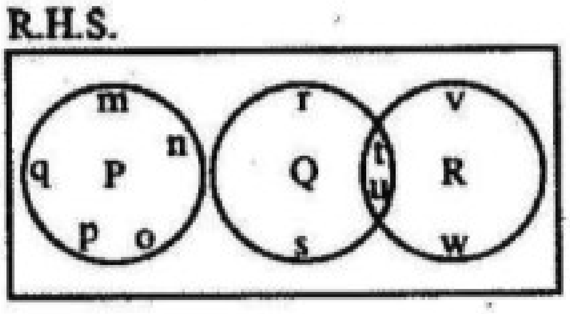
$= (\{m, n, o, p, q\} \cap \{t, u\}$

$= \{\ \}$

L.H.S.



R.H.S.



5. ***Verify*** $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

Using Venn diagram for the following sets.

$X = \{-1, -2, -3\}$, $Y = \{0, 1, 2, 3\}$, $Z = \{0, \pm 1, \pm 2, \pm 3\}$

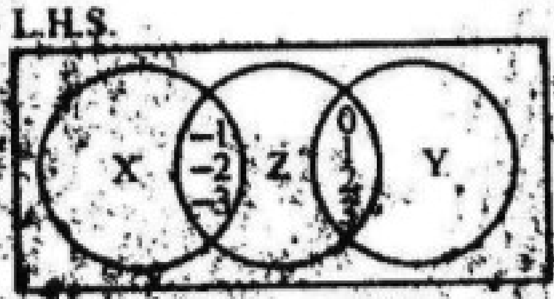
L.H.S.

$$X \cup (Y \cap Z) = \{-1, -2, -3\} \cup \{0,1,2,3\} \cap \{0, \pm 1, \pm 2, \pm 3\}$$
$$= \{-1, -2, -3\} \cup \{0, 1, 2, 3\}$$
$$= \{0, \pm 1, \pm 2, \pm 3\}$$

R.H.S.

$$(X \cup Y) \cap (X \cup Z) = \{-1, -2, -3\} \cup \{0, 1, 2, 3\} \cap \{0, -1, -2, -3\} \cup \{0, \pm 1, \pm 2, \pm 3\}$$
$$= \{0, \pm 1, \pm 2, \pm 3\} \cap \{0, \pm 1, \pm 2, \pm 3\}$$
$$= \{0, \pm 1, \pm 2, \pm 3\}$$

L.H.S.



R.H.S.

6. ***Verify*** $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

$X = \{a, e, i, o, u\}$, $Y = \{e, g, n, r, y\}$, $Z = \{a, b, e, g, l, r\}$

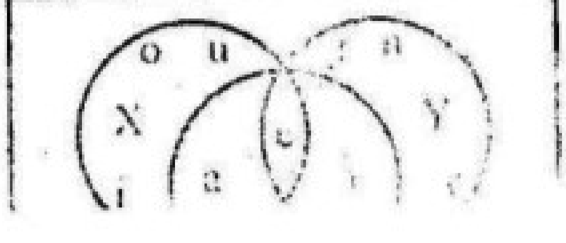
L.H.S.

$$X \cap (Y \cup Z) = \{a, e, i, o, u\} \cap \{e, g, n, r, y\} \cup \{a, b, e, g, l, r\}$$
$$= \{a, e, i, o, u\} \cap \{a, b, e, g, n, l, r, y\}$$
$$= \{a, e\}$$

R.H.S.

$$(X \cap Y) \cup (X \cap Z) = (\{a, e, i, o, u\} \cap \{e, g, n, r, y\}) \cup (\{a, e, i, o, u\} \cap \{a, b, e, g, l, r\})$$
$$= \{e\} \cup \{a, e\}$$
$$= \{a, e\}$$

L.H.S.



R.H.S.