CSE551: Advanced Computer Security

4. Asymmetric-key Cryptography

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Project

- 1~2 persons for one team
- The topics must be related to the computer security
  - I recommend linking this to your research!

- Submit your proposal by **9/16**

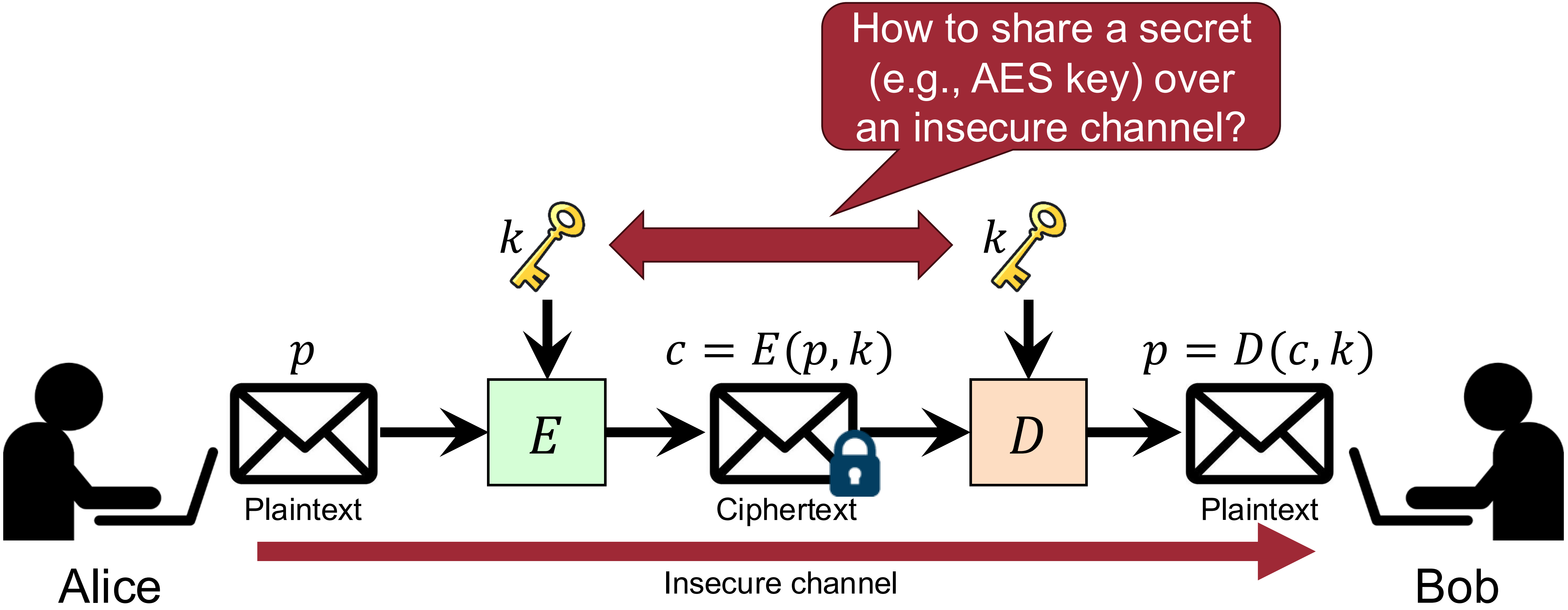
Proposal Submission Guidelines

- You should upload a single PDF file on BlackBored.
- The name of the PDF file should have the following format: `[your ID-last name.pdf]`
  - If your name is Gil-dong Hong, and your ID is 20231234, then you should submit a file named “20231234-Hong.pdf”
  - If your team consists of two people, each member must submit a PDF file

- **Your proposal must follow the following format:**
  - Template: Double-Column ACM format (Sigconf style) – provided on BlackBored
  - 2 pages maximum (reference is excluded)
  - Format: Background, Motivation, Proposed Idea, Expected Results, Research Timeline, (+Role and Responsibility, if the team has two members), Reference

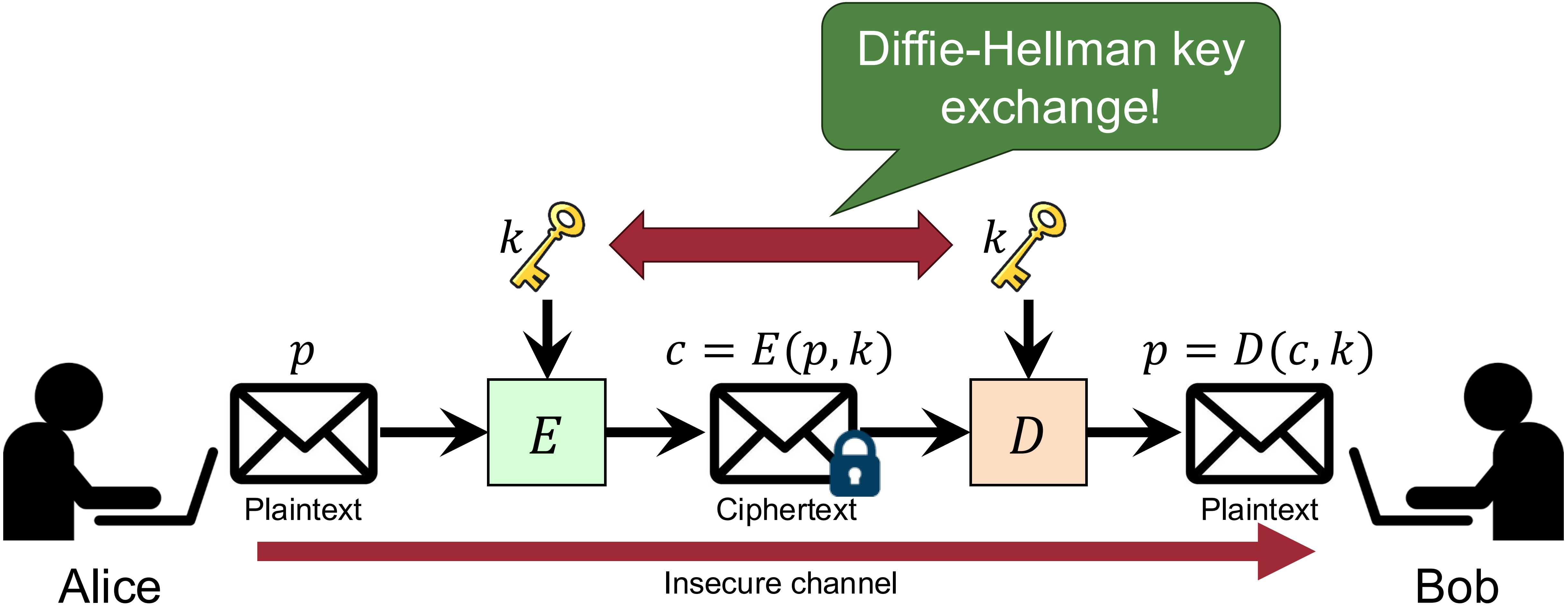
Motivation

- **Symmetric**: the encryption and decryption keys *are the same*



Motivation of the Diffie-Hellman Key Exchange

- **Symmetric**: the encryption and decryption keys *are the same*



Diffie-Hellman key exchange

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

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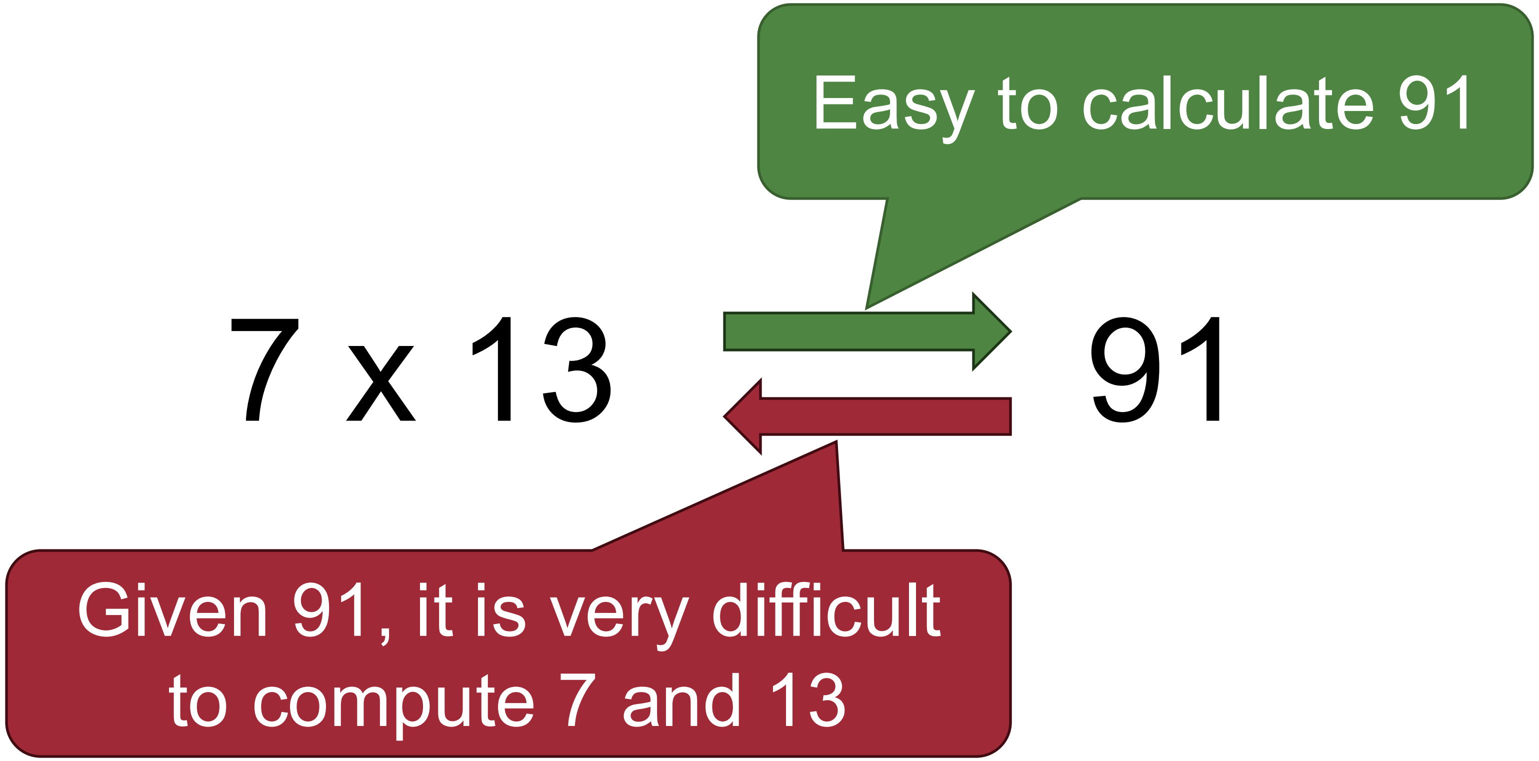


Integer Factorization Problem

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

Given $\underline{g, p, x}$, it is easy to calculate y

$$g^x \bmod p \longrightarrow y$$

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

$$g = 3$$
$$p = 5$$
$$x = 2$$

$$g^x \bmod p \longrightarrow y = ?$$

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

$g = 3$
$p = 5$
$x = 2$

Given $\underline{g, p, x}$, it is easy to calculate y

$$g^x \bmod p \longrightarrow y = 4$$

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

Given $\underline{g, p, x}$, it is easy to calculate y

$$g^x \bmod p \rightleftarrows y$$

Given $\underline{g, p, y}$, it is very difficult to compute x

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

$$g = 3$$
$$p = 5$$
$$x = ?$$

$$g^{x} \, mod \, p \longleftarrow y = 4$$

Given $\underline{g, p, y}$, it is very difficult to compute x

Discrete Logarithm Problem

Core Idea: One-way Function

- Easy in one direction, but hard in the reverse direction
  - f is easy to compute, but f^{-1} is difficult to compute

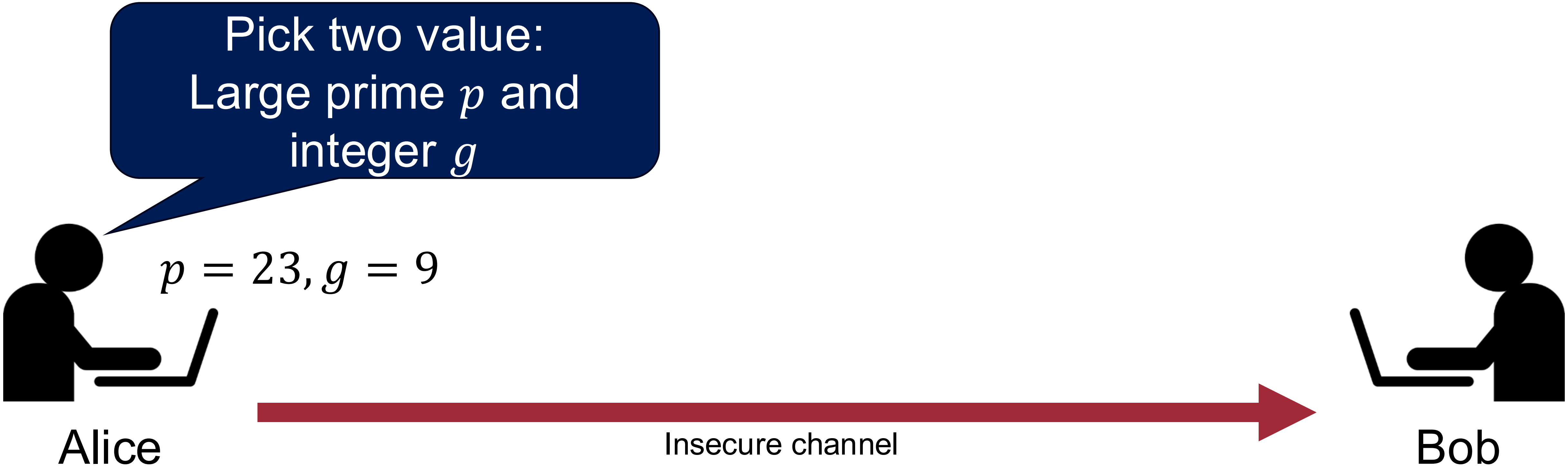
$$g = 3$$
$$p = 5$$
$$x = ?$$

$$g^x \bmod p \longleftarrow y = 4$$

There is no efficient algorithm known for computing discrete logarithms in general

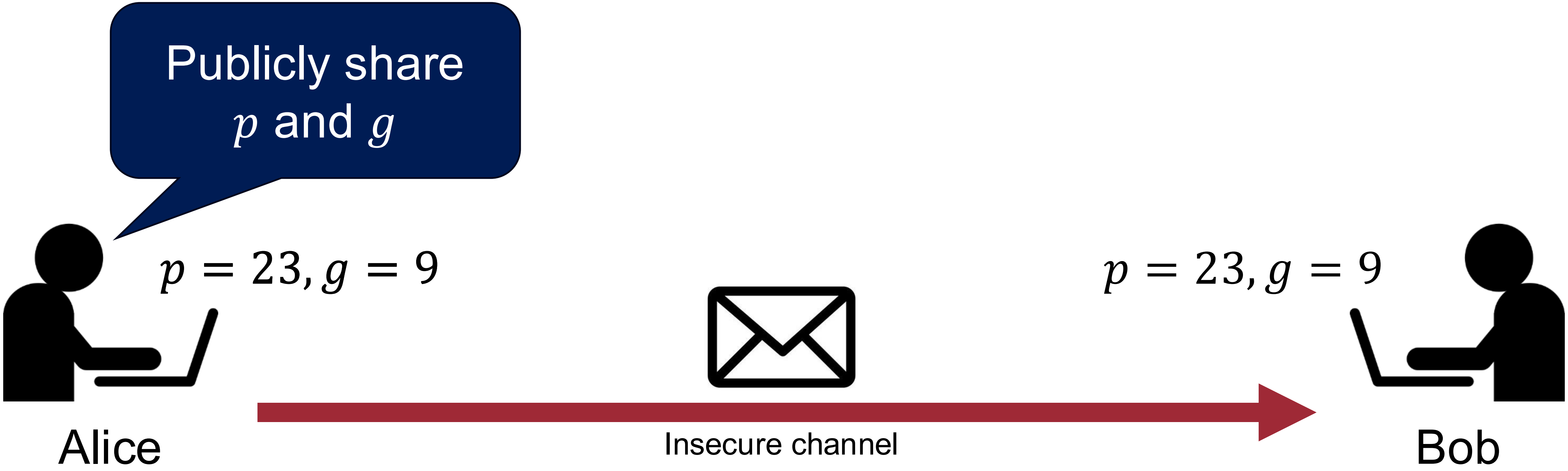
Diffie-Hellman Key Exchange (1)

$$g^x \bmod p \rightleftarrows y$$



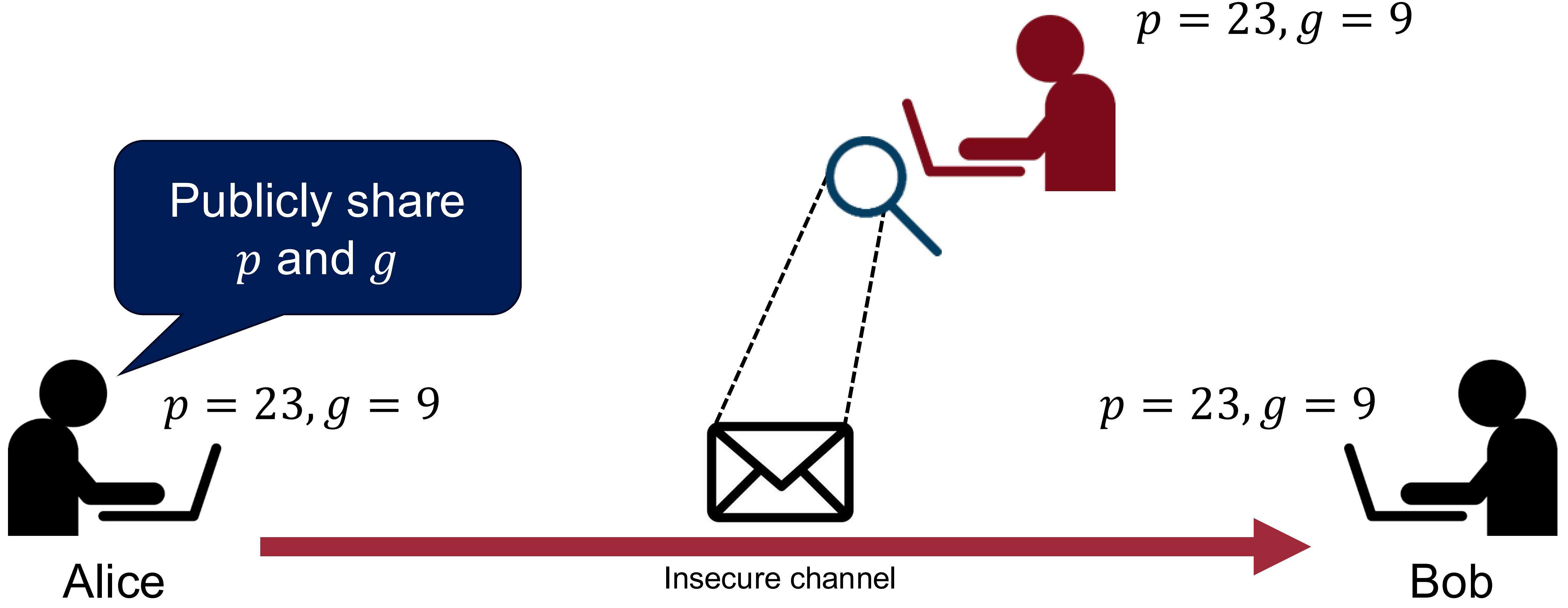
Diffie-Hellman Key Exchange (2)

$$g^x \bmod p \rightleftarrows y$$



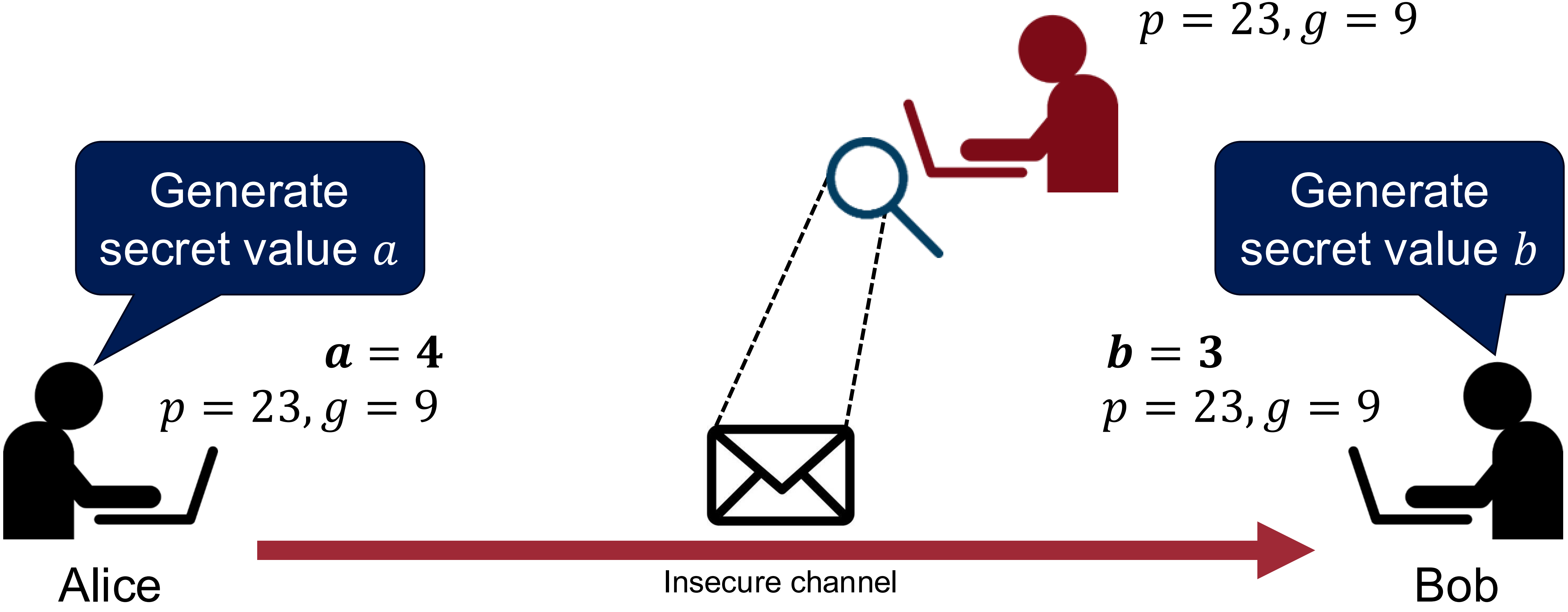
Diffie-Hellman Key Exchange (2)

$$g^x \bmod p \rightleftarrows y$$

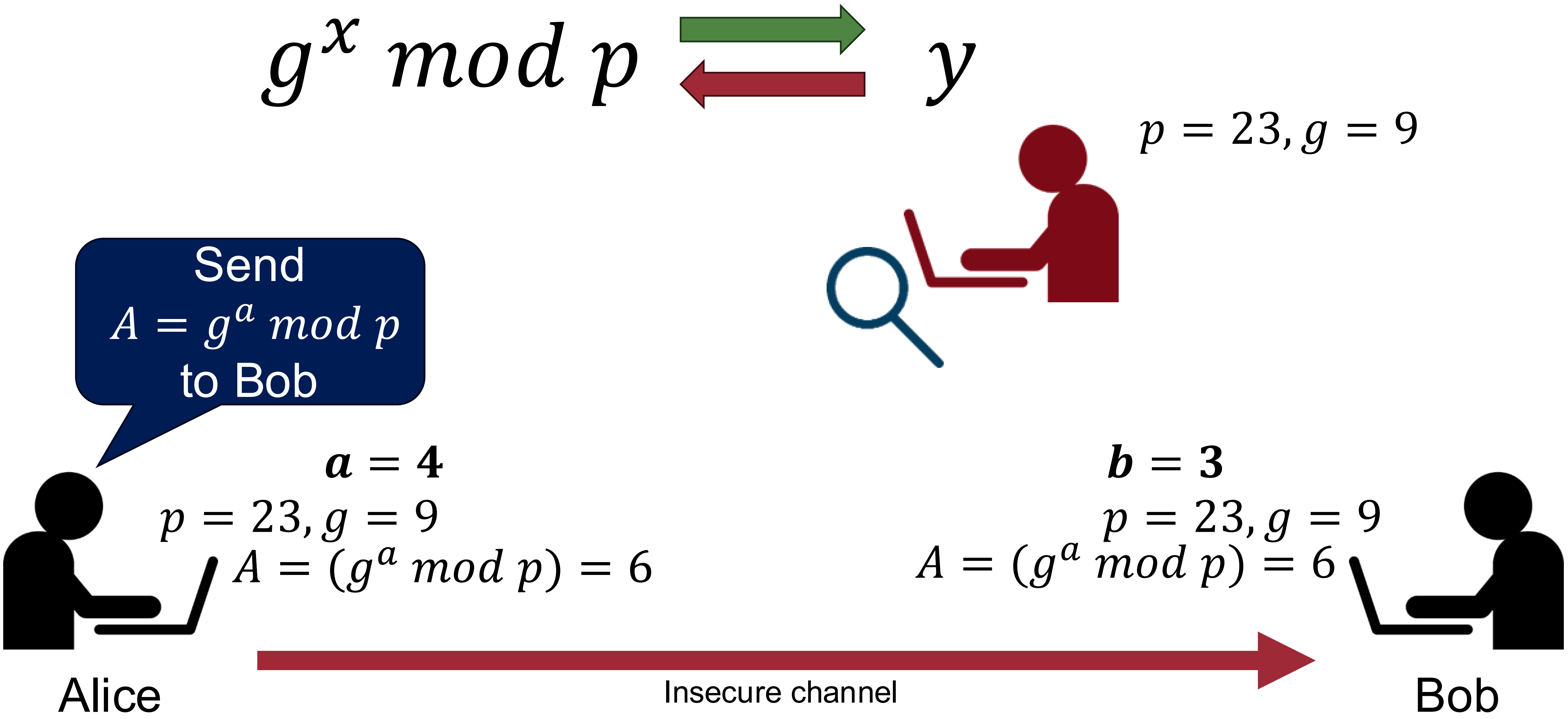


Diffie-Hellman Key Exchange (3)

$$g^x \bmod p \rightleftarrows y$$



Diffie-Hellman Key Exchange (4)



Diffie-Hellman Key Exchange (4)

$$g^x \; mod \; p \rightleftarrows y$$

$p = 23, g = 9$

$\mathrm{A} = (g^a \; mod \; p) = 6$

$\boldsymbol{a = 4}$

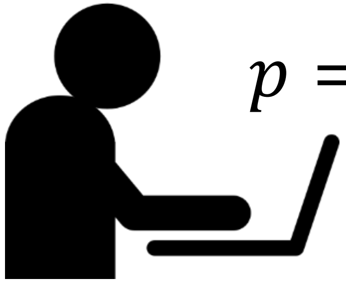
$p = 23, g = 9$

$A = (g^a \; mod \; p) = 6$

$\boldsymbol{b = 3}$

$p = 23, g = 9$

$A = (g^a \; mod \; p) = 6$



Alice

Insecure channel

Bob

Diffie-Hellman Key Exchange (4)

$$g^x \bmod p \;\leftarrow\; y$$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

Given g, p, y, it is very difficult to compute a

$a = 4$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

Alice

Insecure cha

Discrete Logarithm Problem

Diffie-Hellman Key Exchange (4)

$$g^x \bmod p \rightleftarrows y$$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

Send
$B = g^b \bmod p$
to Alice

$\boldsymbol{a = 4}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

$\boldsymbol{b = 3}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

Alice

Insecure channel

Bob

Diffie-Hellman Key Exchange (4)

$$g^x \bmod p \rightleftarrows y$$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

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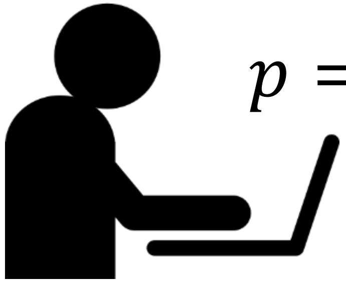

Alice

$\boldsymbol{a = 4}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

Bob

$\boldsymbol{b = 3}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

Insecure channel

Diffie-Hellman Key Exchange (5)

Symmetric key:

$$K = g^{ab} \bmod p$$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

$\boldsymbol{a = 4}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

$\boldsymbol{b = 3}$

$p = 23, g = 9$

$A = (g^a \bmod p) = 6$

$B = (g^b \bmod p) = 16$

Alice

Insecure channel

Bob

Diffie-Hellman Key Exchange (5)

Symmetric key:

$$K = g^{ab} \; mod \; p$$

$p = 23, g = 9$

$A = (g^a \; mod \; p) = 6$

$B = (g^b \; mod \; p) = 16$

$K = (B^a \; mod \; p) = (g^{ab} mod \; p)$

$= (16^4 \; mod \; 23) = 9$

Theorem:

$((X \; mod \; p)^k \; mod \; p) = (X^k \; mod \; p)$

$a = 4$

$p = 23, g = 9$

$A = (g^a \; mod \; p) = 6$

$B = (g^b \; mod \; p) = 16$

$b = 3$

$p = 23, g = 9$

$A = (g^a \; mod \; p) = 6$

$B = (g^b \; mod \; p) = 16$

Alice

Insecure channel

Bob

Diffie-Hellman Key Exchange (5)

Symmetric key:

$$K = g^{ab} \bmod p$$

$$p = 23, g = 9$$
$$A = (g^a \bmod p) = 6$$
$$B = (g^b \bmod p) = 16$$

$$K = (B^a \bmod p) = (g^{ab} \bmod p)$$
$$= (16^4 \bmod 23) = 9$$

$$K = (A^b \bmod p) = (g^{ab} \bmod p)$$
$$= (6^3 \bmod 23) = 9$$

$$\boldsymbol{a = 4}$$
$$p = 23, g = 9$$
$$A = (g^a \bmod p) = 6$$
$$B = (g^b \bmod p) = 16$$

$$\boldsymbol{b = 3}$$
$$p = 23, g = 9$$
$$A = (g^a \bmod p) = 6$$
$$B = (g^b \bmod p) = 16$$

Alice

Insecure channel

Bob

Diffi… ge (5)

The attacker cannot efficiently compute $(g^{ab} \ mod \ p)$
without knowing a and b

Sym…

$$K = g^{ab} \ mod \ p$$

$p = 23, g = 9$
$A = (g^{a} \ mod \ p) = 6$
$B = (g^{b} \ mod \ p) = 16$

$K = (B^{a} \ mod \ p) = (g^{ab} mod \ p)$
$= (16^{4} \ mod \ 23) = 9$

$=$

$K = (A^{b} \ mod \ p) = (g^{ab} mod \ p)$
$= (6^{3} \ mod \ 23) = 9$

$\boldsymbol{a = 4}$
$p = 23, g = 9$
$A = (g^{a} \ mod \ p) = 6$
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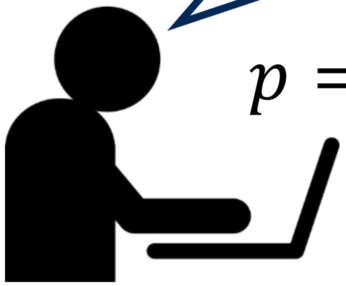
$B = (g^{b} \ mod \ p) = 16$



Alice

Insecure channel

Bob

Why should p be Prime?

Symmetric key:

$K = g^{ab} \; mod \; p$

$g = 2$

Too simple key pattern that can be inferred

$p = 11$

- $2^0 \; mod \; 11 = 1$
- $2^1 \; mod \; 11 = 2$
- $2^2 \; mod \; 11 = 4$
- $2^3 \; mod \; 11 = 8$
- $2^4 \; mod \; 11 = 5$
- $2^5 \; mod \; 11 = 10$
- $2^6 \; mod \; 11 = 9$
- $2^7 \; mod \; 11 = 7$
- $2^8 \; mod \; 11 = 3$
- $2^9 \; mod \; 11 = 6$
- $2^{10} \; mod \; 11 = 1$

$p = 12$

- $2^0 \; mod \; 12 = 1$
- $2^1 \; mod \; 12 = 2$
- $2^2 \; mod \; 12 = 4$
- $2^3 \; mod \; 12 = 8$
- $2^4 \; mod \; 12 = 4$
- $2^5 \; mod \; 12 = 8$
- $2^6 \; mod \; 12 = 4$
- $2^7 \; mod \; 12 = 8$
- $2^8 \; mod \; 12 = 4$
- $2^9 \; mod \; 12 = 8$
- $2^{10} \; mod \; 12 = 4$

Diffie-Hellman Key Exchange

Symmetric key:

$$K = g^{ab} \ mod \ p$$

Problems?

$$K = (B^{a} \ mod \ p) = (g^{ab} mod \ p)$$
$$= (16^4 \ mod \ 23) = 9$$

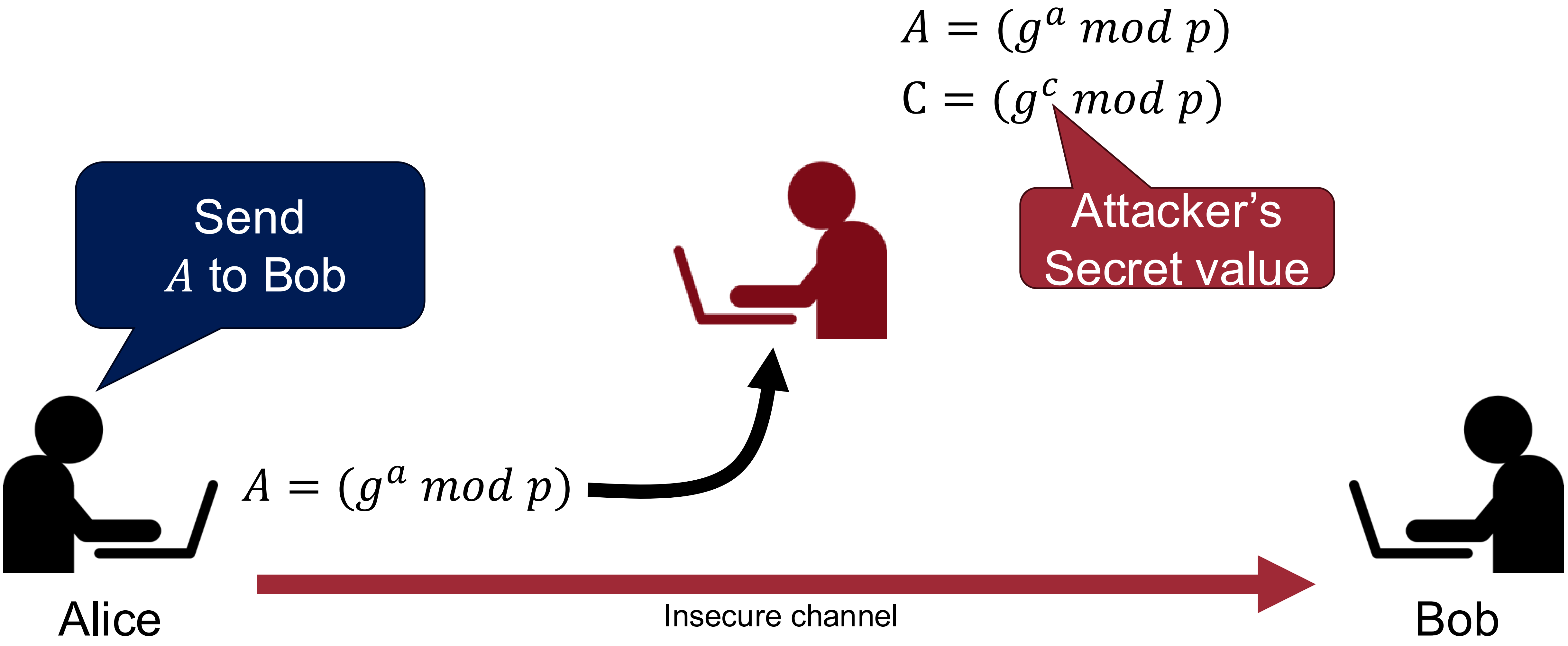
$$K = (A^{b} \ mod \ p) = (g^{ab} mod \ p)$$
$$= (6^3 \ mod \ 23) = 9$$

Alice

Insecure channel

Bob

Problem (1): Man-in-the-Middle Attack



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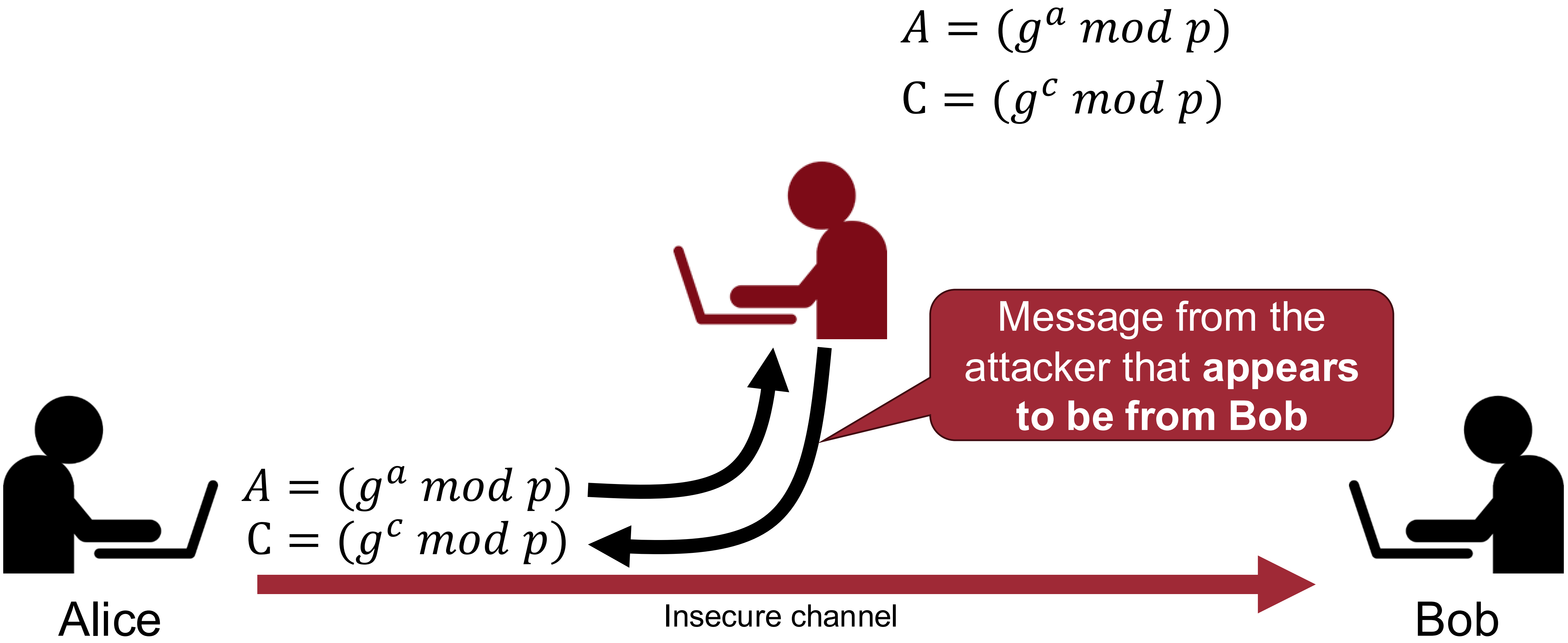

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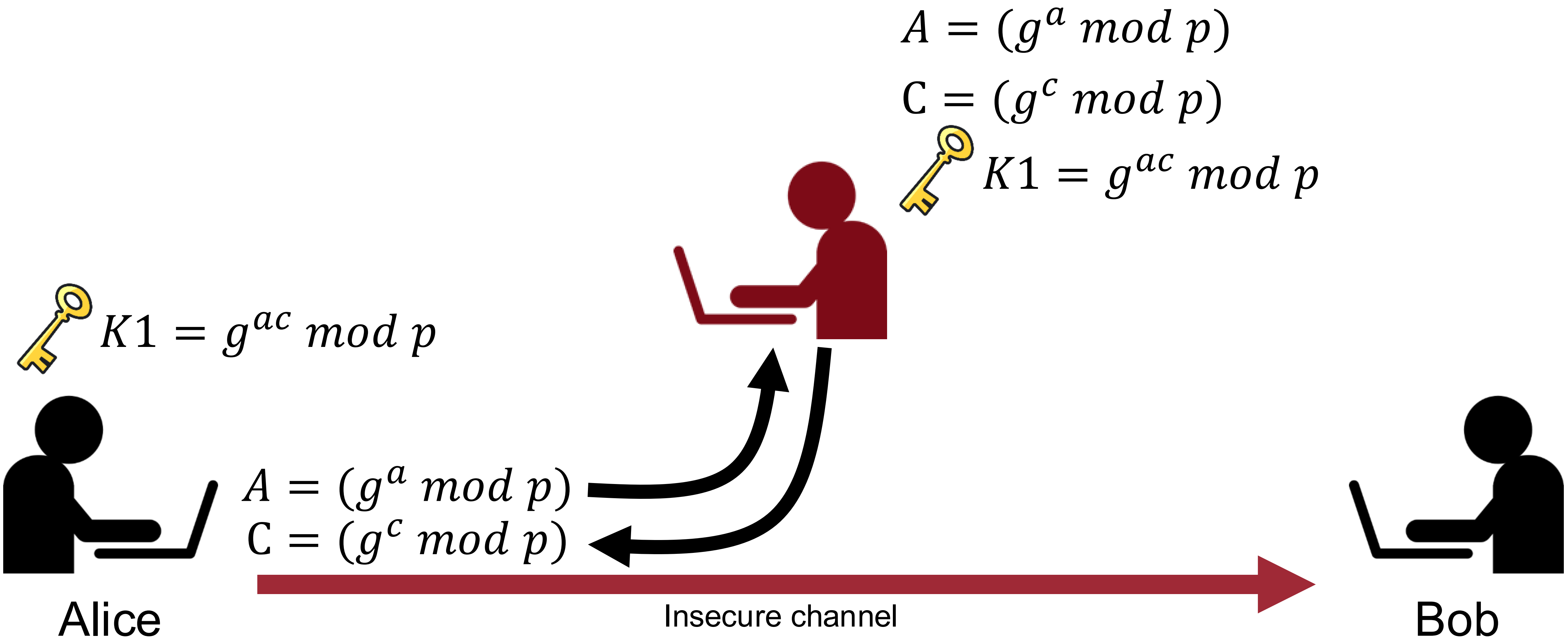

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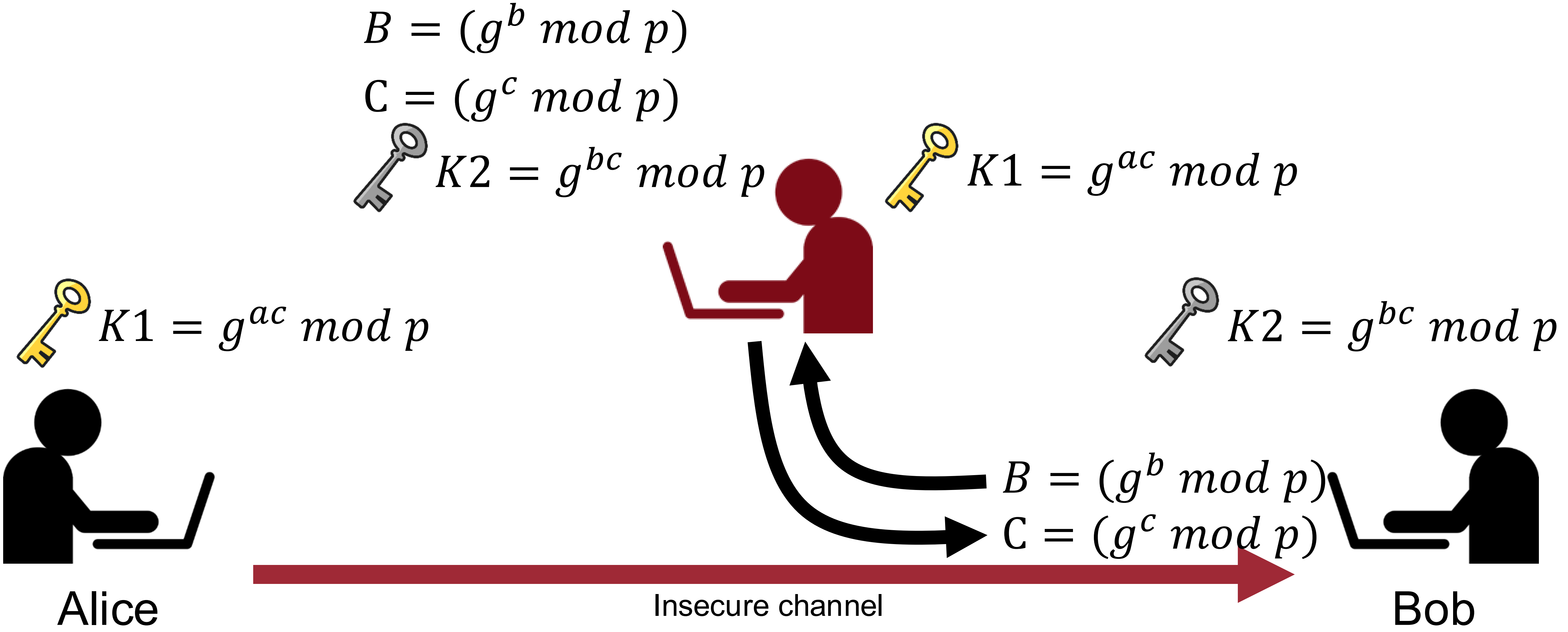
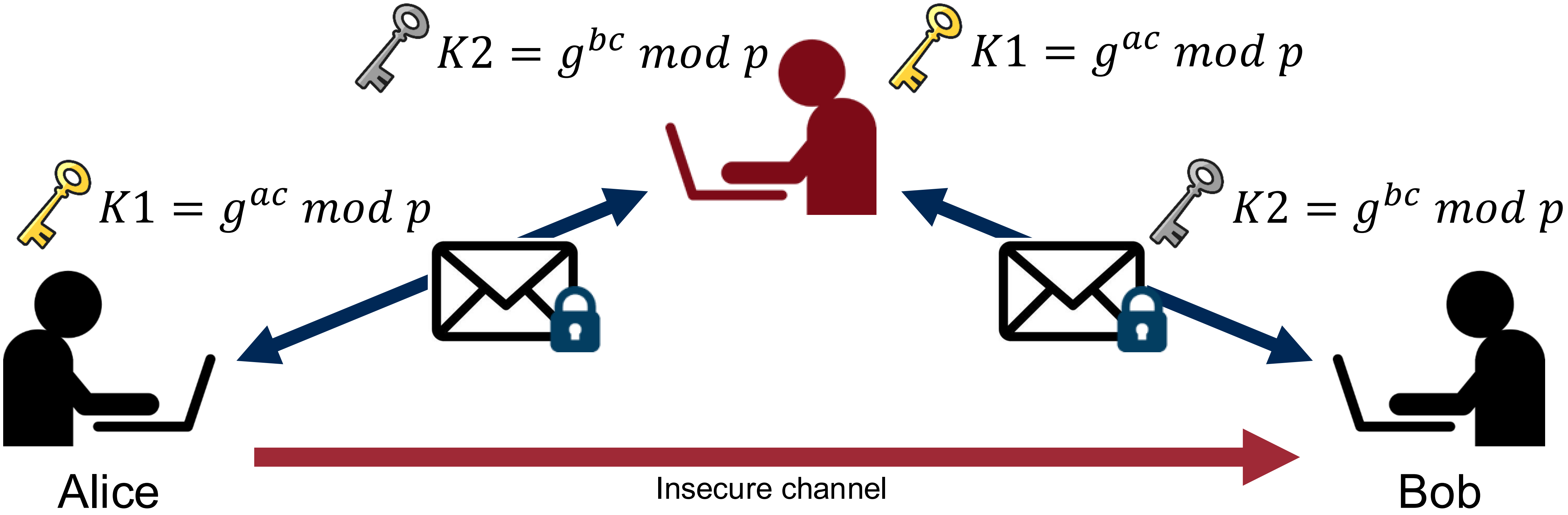


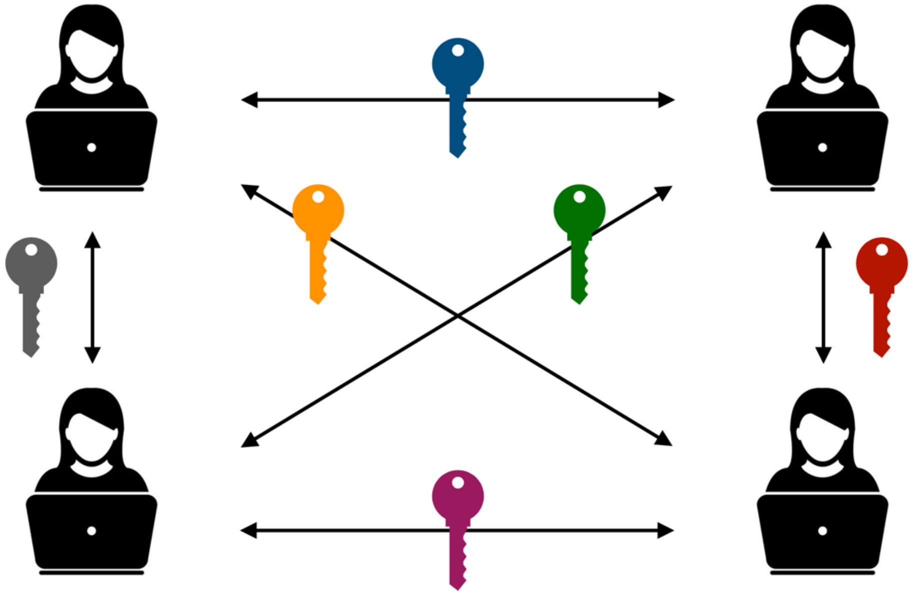
Problem (1): Man-in-the-Middle Attack



Problem (2): Maintenance Problems

- Recap: the same key shared between two parties
- What happens if there are many users?
  - n users: $\binom{n}{2} = n(n-1)/2$
  - Example: 100 users → 4,950 keys
- Key distribution and maintenance problem

How to solve this issue?



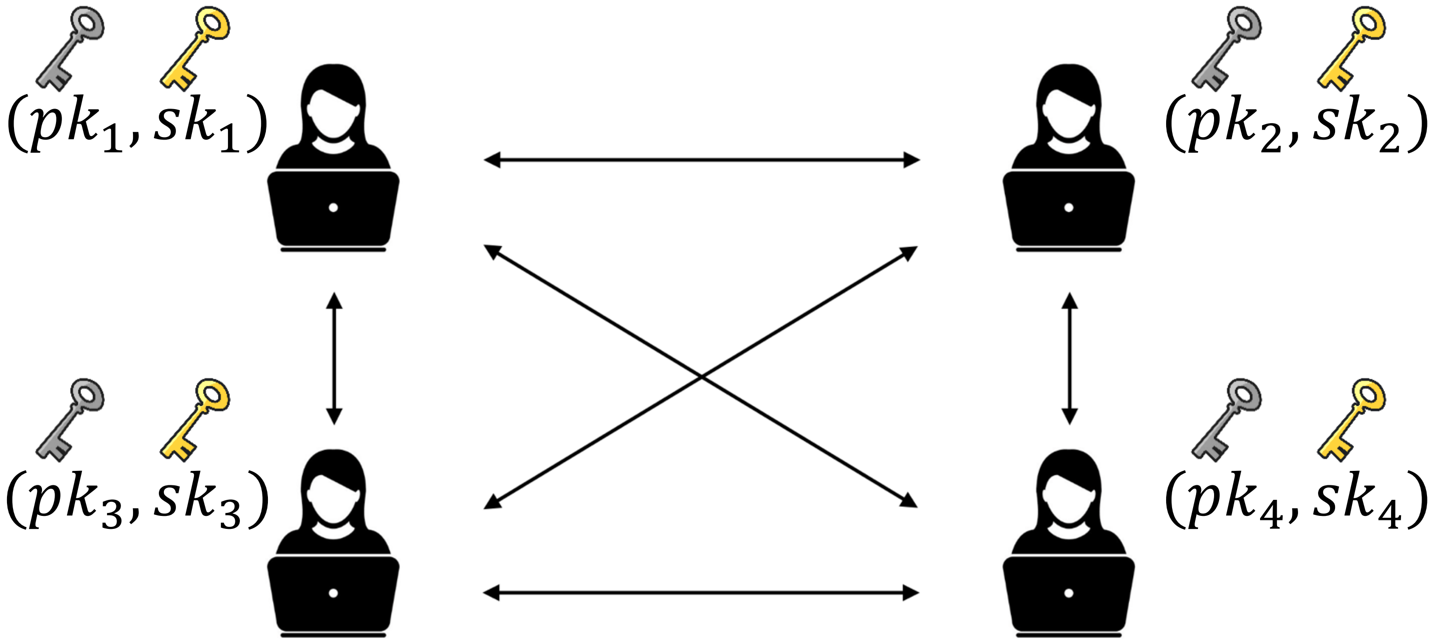
Asymmetric-key Cryptography

Asymmetric-key Cryptography

- Each party has two distinct keys: public key and private key
  - Also known as public-key algorithm
- Invented in 1976 by Diffie and Hellman (ACM Turing Award 2015)

Asymmetric-key Cryptography

- pk: public key, widely disseminated, used for encryption
- sk: private key kept secretly, used for decryption
- **More robust against man-in-the-middle attack**
- **Good maintenance**: n users $\rightarrow$ $2n$ keys



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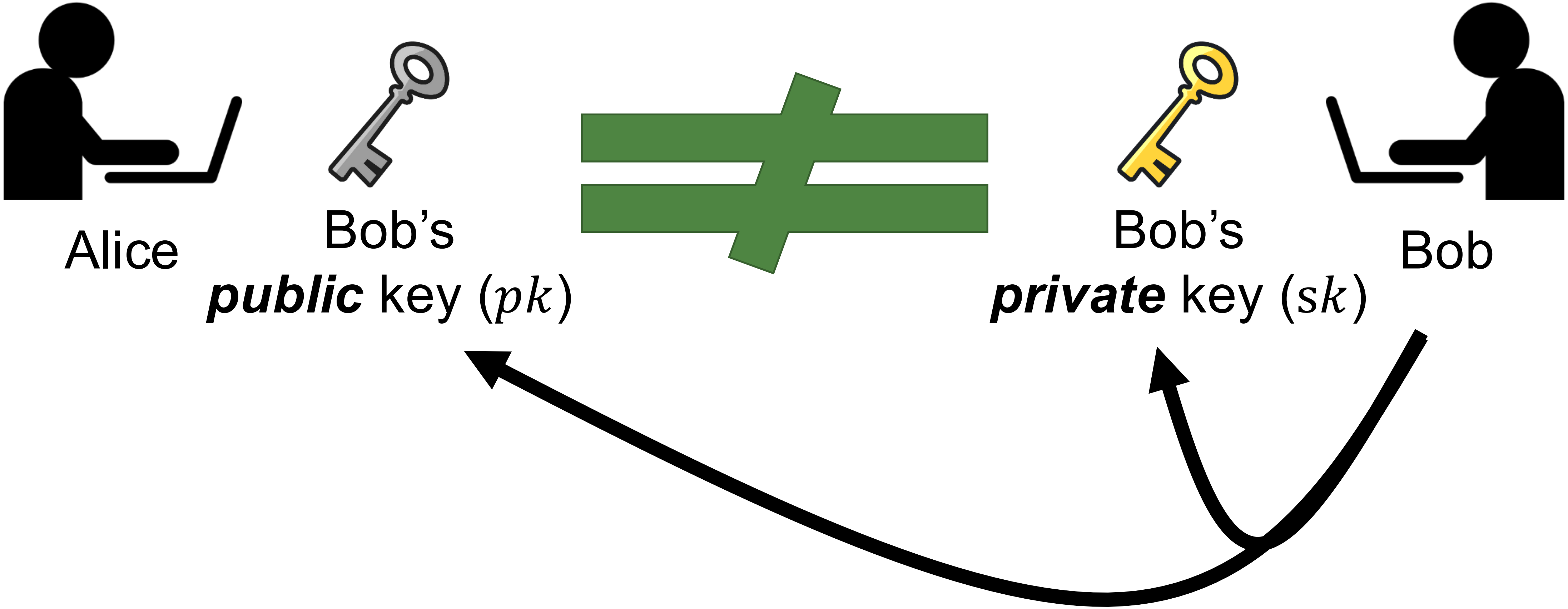

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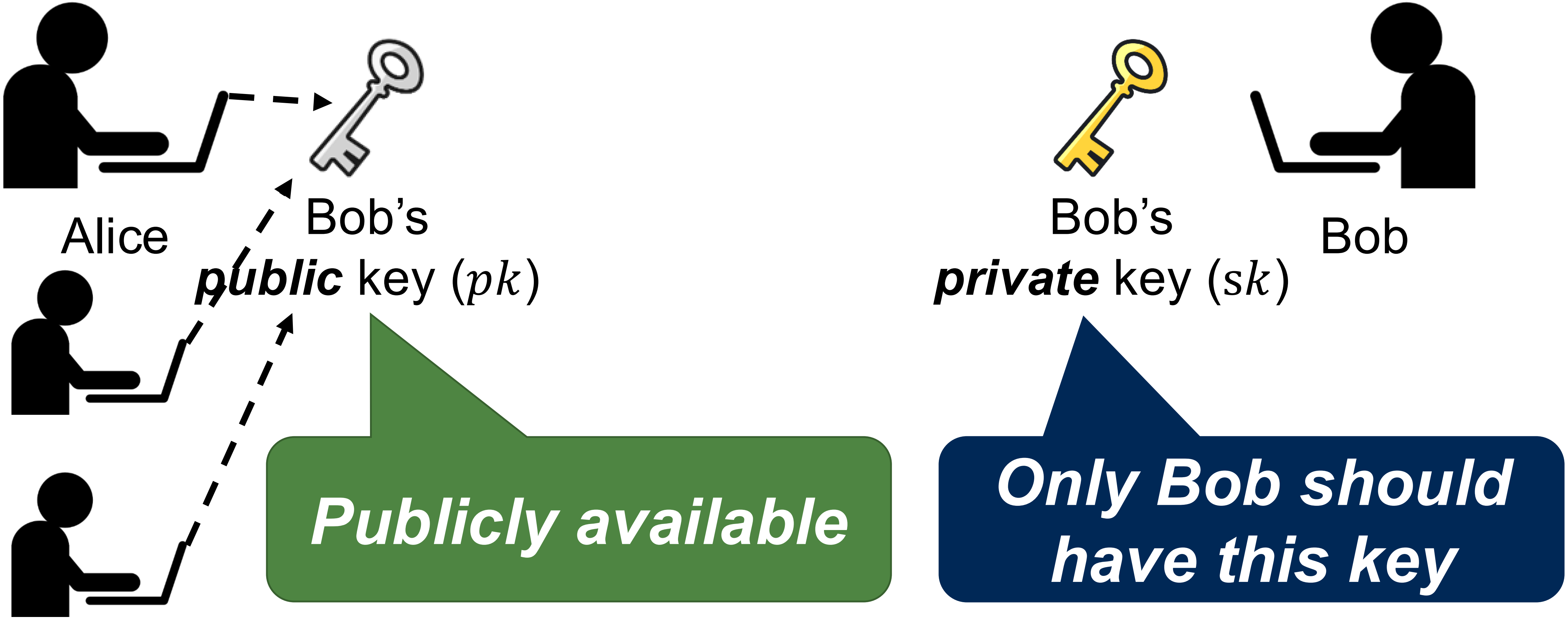

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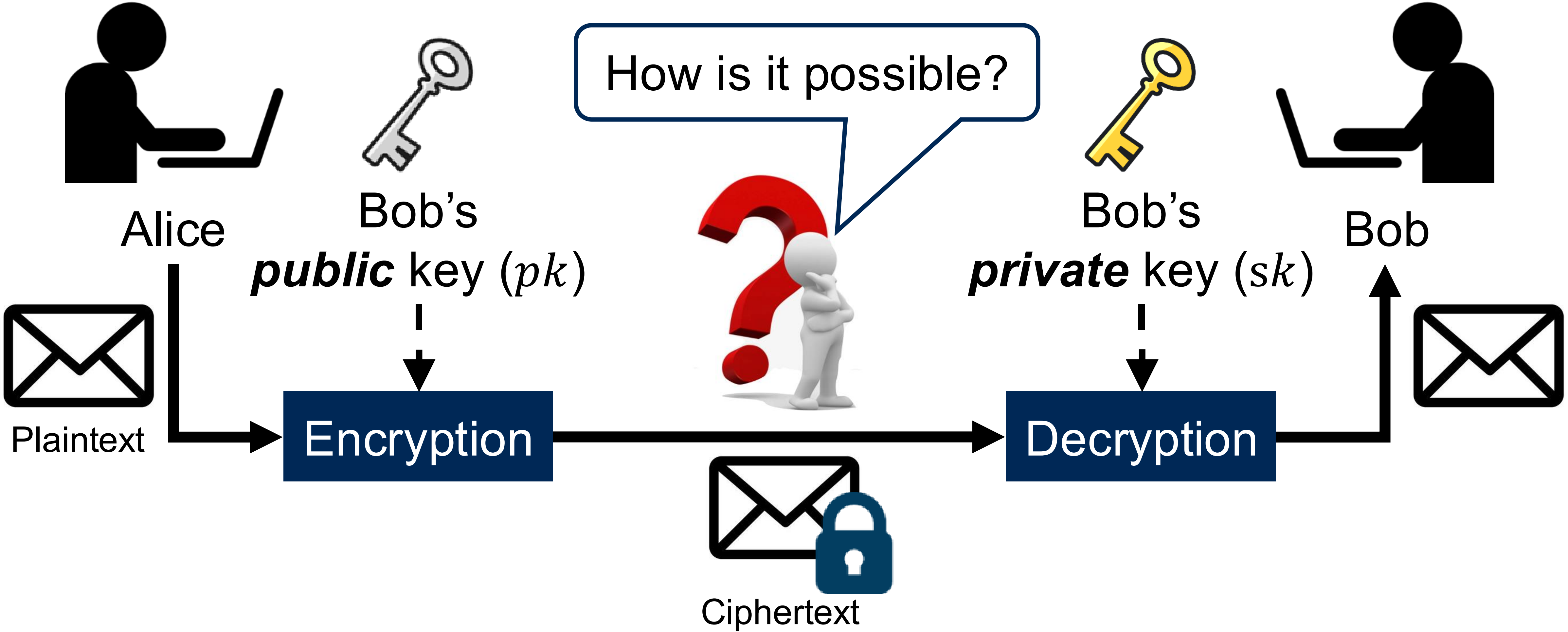


RSA Cryptosystem

RSA Cryptosystem

- Invented by Rivest, Shamir, and Adleman (MIT) in 1977
  - ACM Turing award in 2002

- Rely on the practical ***difficulty of factoring the product of two large prime numbers***
  - Security based on *Prime Factorization Problem*

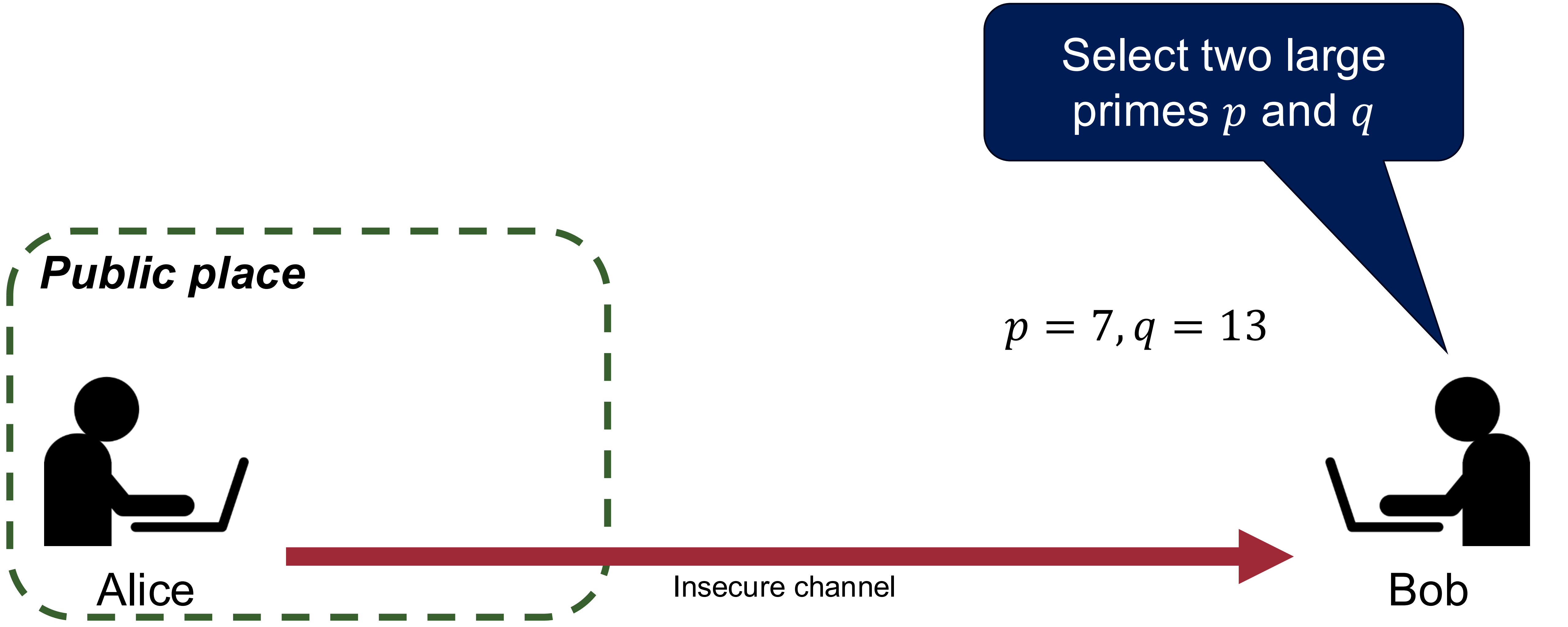
Prime Factorization Problem

Given large prime p and q, it is easy to calculate n

$$p \times q \; \rightleftarrows \; n$$

Given $\underline{n}$, it is very difficult to compute p and q

RSA Algorithm (1): Key Generation



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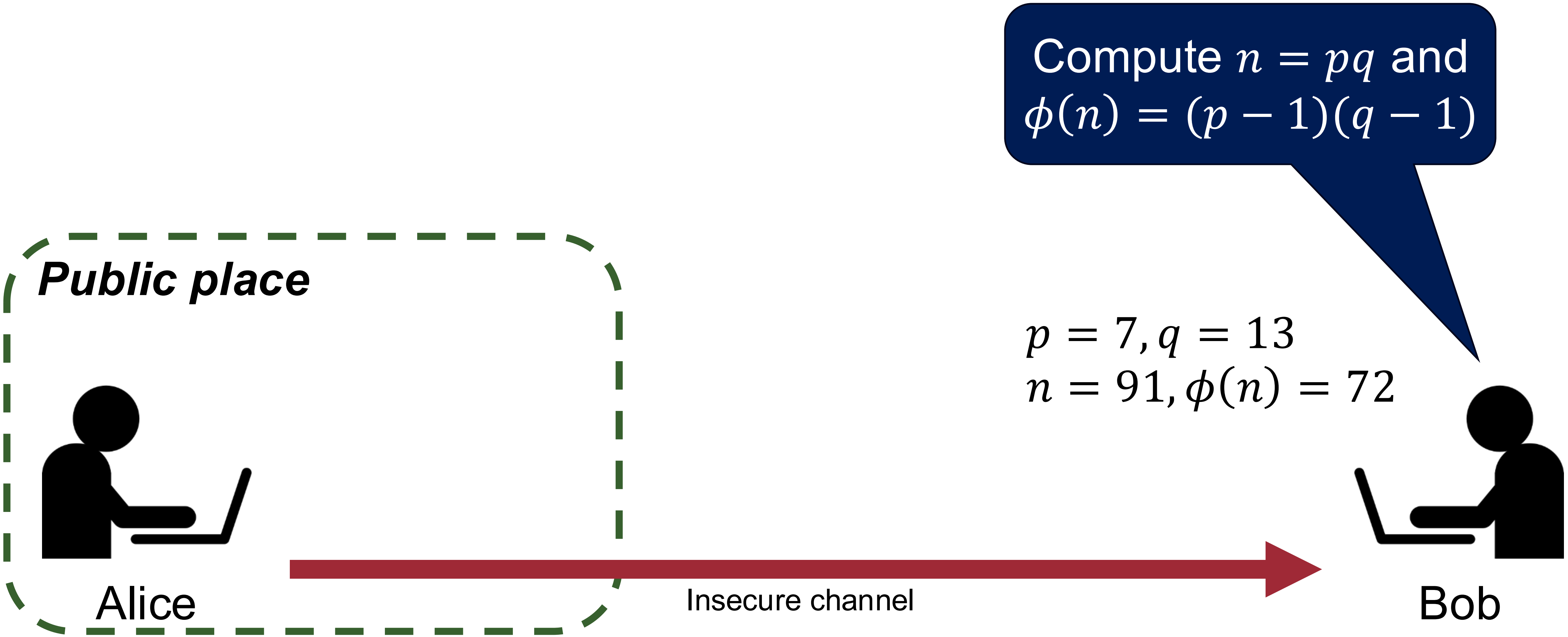
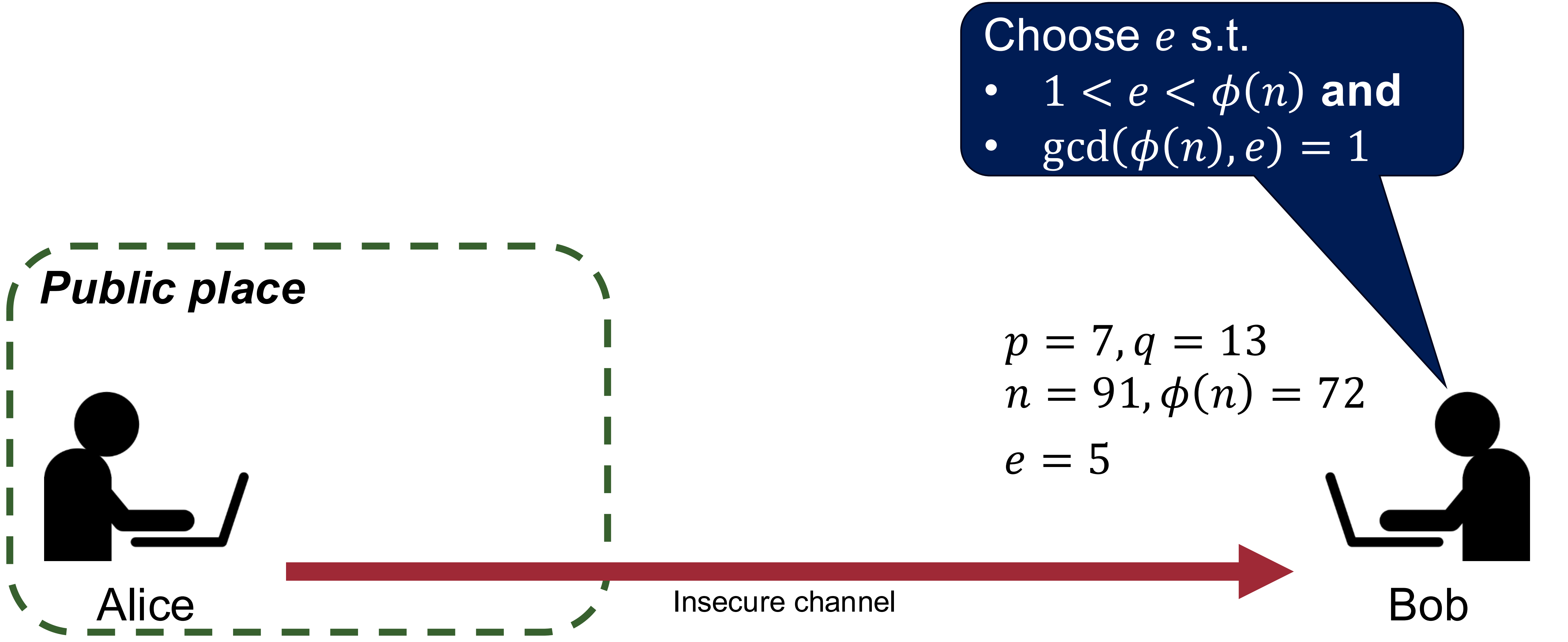


RSA Algorithm (1): Key Generation



RSA Algorithm (1): Key Generation

How to find d?
→ Extended Euclidean Algorithm!

Choose d s.t.
- $1 < d < \phi(n)$ **and**
- $(ed \bmod \phi(n)) = 1$

Public place

Alice

$p = 7, q = 13$
$n = 91, \phi(n) = 72$

$e = 5$
$d = 29$

Insecure channel

Bob

Euclidean Algorithm

Goal: Finding Greatest Common Divisor (GCD)

Fact 1: $\gcd(a, 0) = a$
Fact 2: $\gcd(a, b) = \gcd(b, r)$, where r is the remainder of dividing a by b $(a > b)$

Example

$\gcd(72, 5)$

Euclidean Algorithm

Goal: Finding Greatest Common Divisor (GCD)

Fact 1: $\gcd(a, 0) = a$
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Example

$\gcd(72, 5)$ $\quad$ $72 = (5 * 14) + 2$

Euclidean Algorithm

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Fact 1: $\gcd(a, 0) = a$
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Example

b r

$\gcd(72, 5)$ $72 = (5 * 14) + 2$

$\gcd(5, 2)$ $5 = (2 * 2) + 1$

Euclidean Algorithm

Goal: Finding Greatest Common Divisor (GCD)

Fact 1: $\gcd(a, 0) = a$
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Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

Euclidean Algorithm

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Fact 1: $\gcd(a, 0) = a$
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Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

$\gcd(1, 0)$

Euclidean Algorithm

Goal: Finding Greatest Common Divisor (GCD)

Fact 1: $\gcd(a, 0) = a$
Fact 2: $\gcd(a, b) = \gcd(b, r)$, where r is the remainder of dividing a by b ($a > b$)

Example

$\gcd(72, 5) \qquad 72 = (5 * 14) + 2$

$\gcd(5, 2) \qquad 5 = (2 * 2) + 1$

$\gcd(2, 1) \qquad 2 = (2 * 1) + 0$

$\gcd(1, 0) = 1$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

Choose e s.t.
- $1 < e < \phi(n)$ **and**
- $\gcd(\phi(n), e) = 1$

Choose d s.t.
- $1 < d < \phi(n)$ **and**
- $(ed \ mod \ \phi(n)) = 1$

$p = 7, q = 13$
$n = 91, \phi(n) = 72$

$e = 5$
$d = 29$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

$$ed + \phi(n)(-k) = \gcd(\phi(n), e) = 1$$

Choose e s.t.

- $1 < e < \phi(n)$ **and**
- $\gcd(\phi(n), e) = 1$

Choose d s.t.

- $1 < d < \phi(n)$ **and**
- $(ed \bmod \phi(n)) = 1$

$p = 7, q = 13$
$n = 91, \phi(n) = 72$

$e = 5$
$d = 29$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

We can find the value d!

$$ax + by = \gcd(a, b)$$

$$ed + \phi(n)(-k) = \gcd(\phi(n), e) = 1$$

Choose e s.t.
- $1 < e < \phi(n)$ **and**
- $\gcd(\phi(n), e) = 1$

Choose d s.t.
- $1 < d < \phi(n)$ **and**
- $(ed \bmod \phi(n)) = 1$

$p = 7, q = 13$
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Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

$$ed + \phi(n)(-k) = \gcd(\phi(n), e) = 1$$

$$(e = 5, \phi(n) = 72)$$

Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

$\gcd(1, 0) = 1$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

$$ed + \phi(n)(-k) = \gcd(\phi(n), e) = 1$$

$$(e = 5, \phi(n) = 72)$$

Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1 \;\Rightarrow\; 5 - (2 * 2) = 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

$\gcd(1, 0) = 1$

$x = 1$
$y = -2$

Extended Euclidean Algorithm

- **Goal:** Computing integers x and y s.t.

$$ax + by = \gcd(a, b)$$

$$ed + \phi(n)(-k) = \gcd(\phi(n), e) = 1$$

$$(e = 5, \phi(n) = 72)$$

Example

$$2 = 72 - (5 * 14)$$

$\gcd(72, 5)$ $\quad$ $72 = (5 * 14) + 2$

$\gcd(5, 2)$ $\quad$ $5 = (2 * 2) + 1$ ➡ $5 - (2 * 2) = 1$

$\gcd(2, 1)$ $\quad$ $2 = (2 * 1) + 0$

$\gcd(1, 0) = 1$

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Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2 \Rightarrow 5 - ((72 - 5 * 14) * 2) = 1$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1 \Rightarrow 5 - (2 * 2) = 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

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Example

$\gcd(72, 5)$ $\quad 72 = (5 * 14) + 2 \Rightarrow 5 * 29 + 72(-2) = 1$

$\gcd(5, 2)$ $\quad 5 = (2 * 2) + 1 \Rightarrow 5 - (2 * 2) = 1$

$\gcd(2, 1)$ $\quad 2 = (2 * 1) + 0$

$\gcd(1, 0) = 1$

$x = \mathrm{d} = 29$
$\mathrm{y} = -\mathrm{k} = -2$

Extended Euclidean Algorithm: Logic Flow

$r_1 \leftarrow a; \quad r_2 \leftarrow b;$ (Initialization)

while ($r_2 > 0$)

{

$q \leftarrow r_1 / r_2;$

$r \leftarrow r_1 - q \times r_2;$

$r_1 \leftarrow r_2; \quad r_2 \leftarrow r;$

}

gcd (a, b) $\leftarrow r_1$

Euclidean Algorithm

$r_1 \leftarrow a; \quad r_2 \leftarrow b;$
$s_1 \leftarrow 1; \quad s_2 \leftarrow 0;$ (Initialization)
$t_1 \leftarrow 0; \quad t_2 \leftarrow 1;$

while ($r_2 > 0$)

{

$q \leftarrow r_1 / r_2;$

$r \leftarrow r_1 - q \times r_2;$
$r_1 \leftarrow r_2; \quad r_2 \leftarrow r;$ (Updating r's)

$s \leftarrow s_1 - q \times s_2;$
$s_1 \leftarrow s_2; \quad s_2 \leftarrow s;$ (Updating s's)

$t \leftarrow t_1 - q \times t_2;$
$t_1 \leftarrow t_2; \quad t_2 \leftarrow$ t; (Updating t's)

}

gcd (a , b) $\leftarrow r_1; \quad s \leftarrow s_1; \quad t \leftarrow t_1$

Extended Euclidean Algorithm

RSA Algorithm (1): Key Generation

How to find d?
→ Extended Euclidean Algorithm!

Choose d s.t.

- $1 < d < \phi(n)$ **and**
- $(ed \ mod \ \phi(n)) = 1$

Public place

$p = 7, q = 13$
$n = 91, \phi(n) = 72$

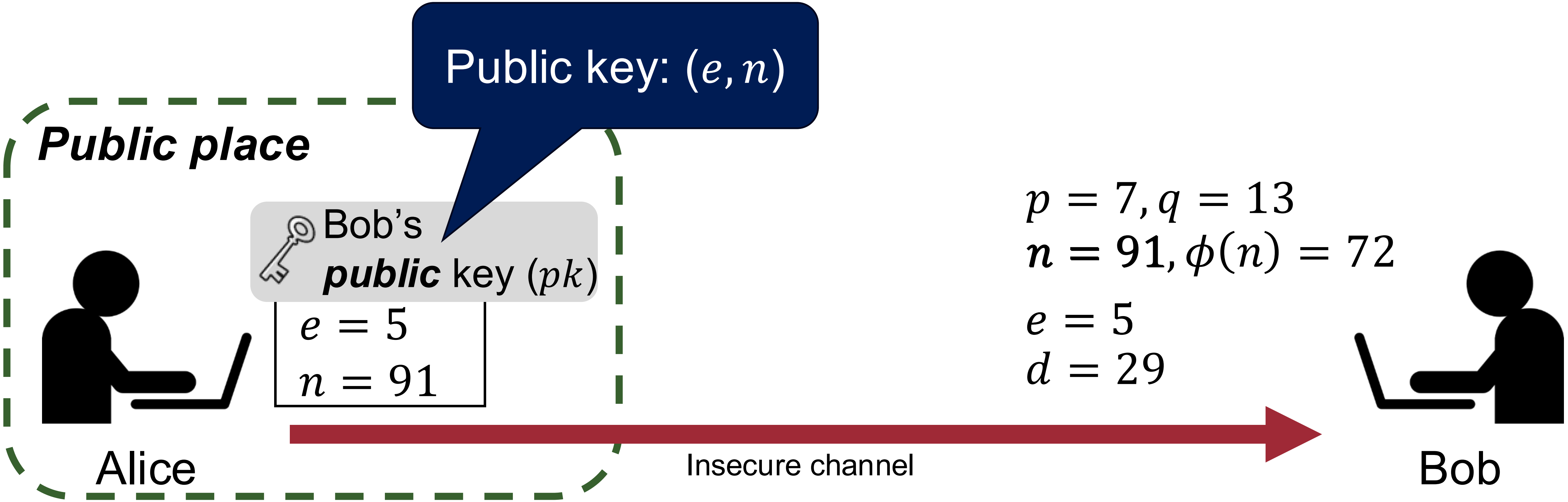
$e = 5$
$d = 29$

Alice

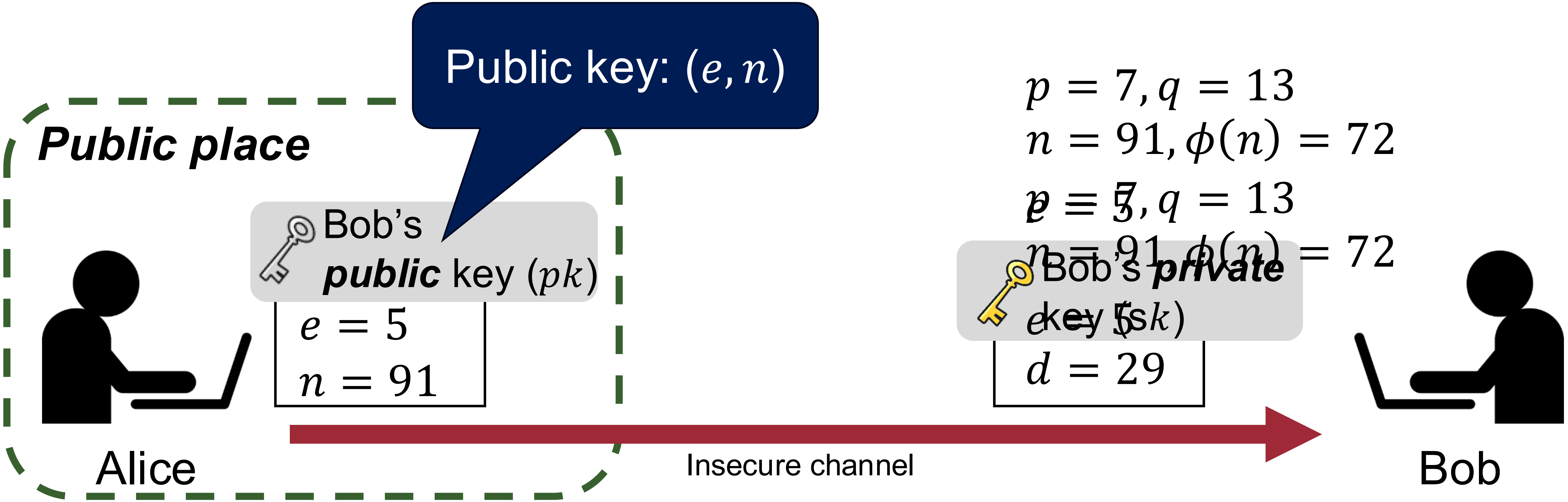
Insecure channel

Bob

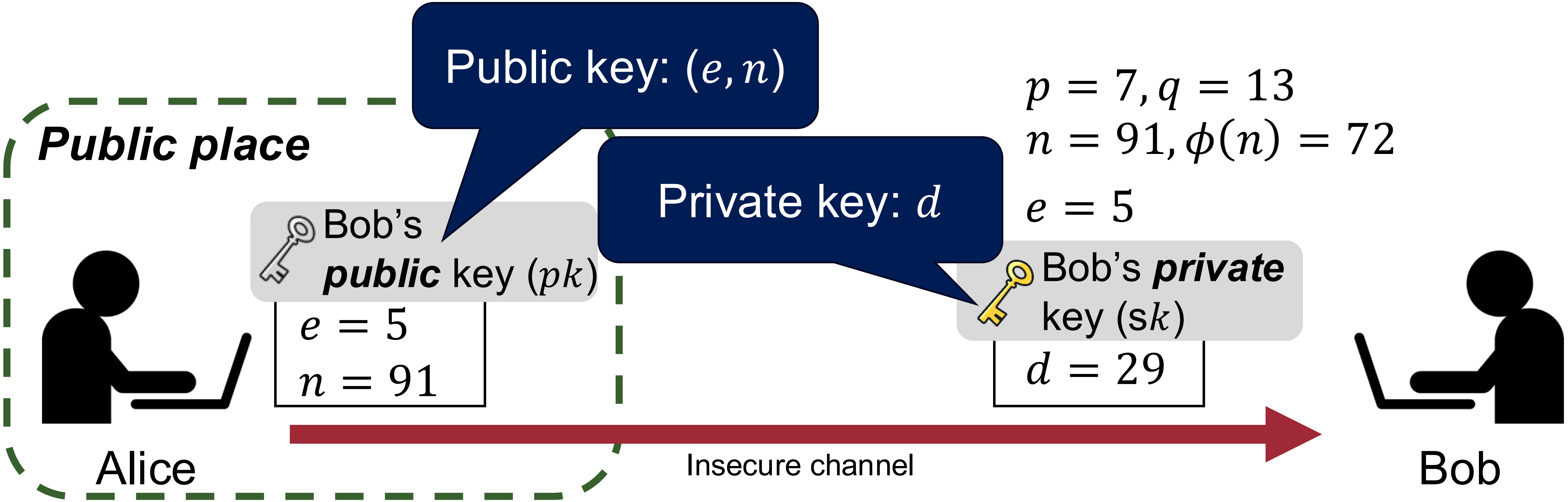
RSA Algorithm (1): Key Generation



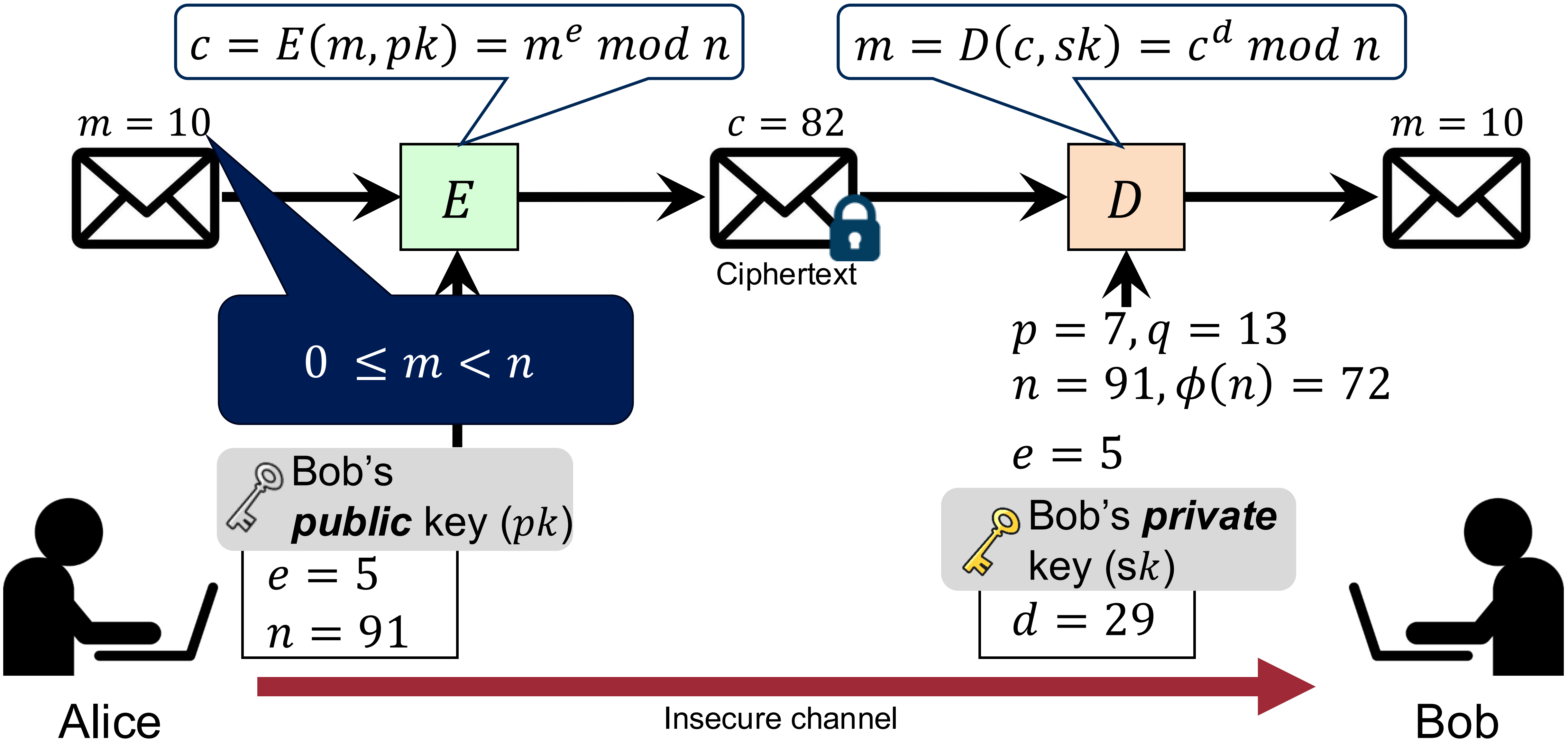
RSA Algorithm (1): Key Generation



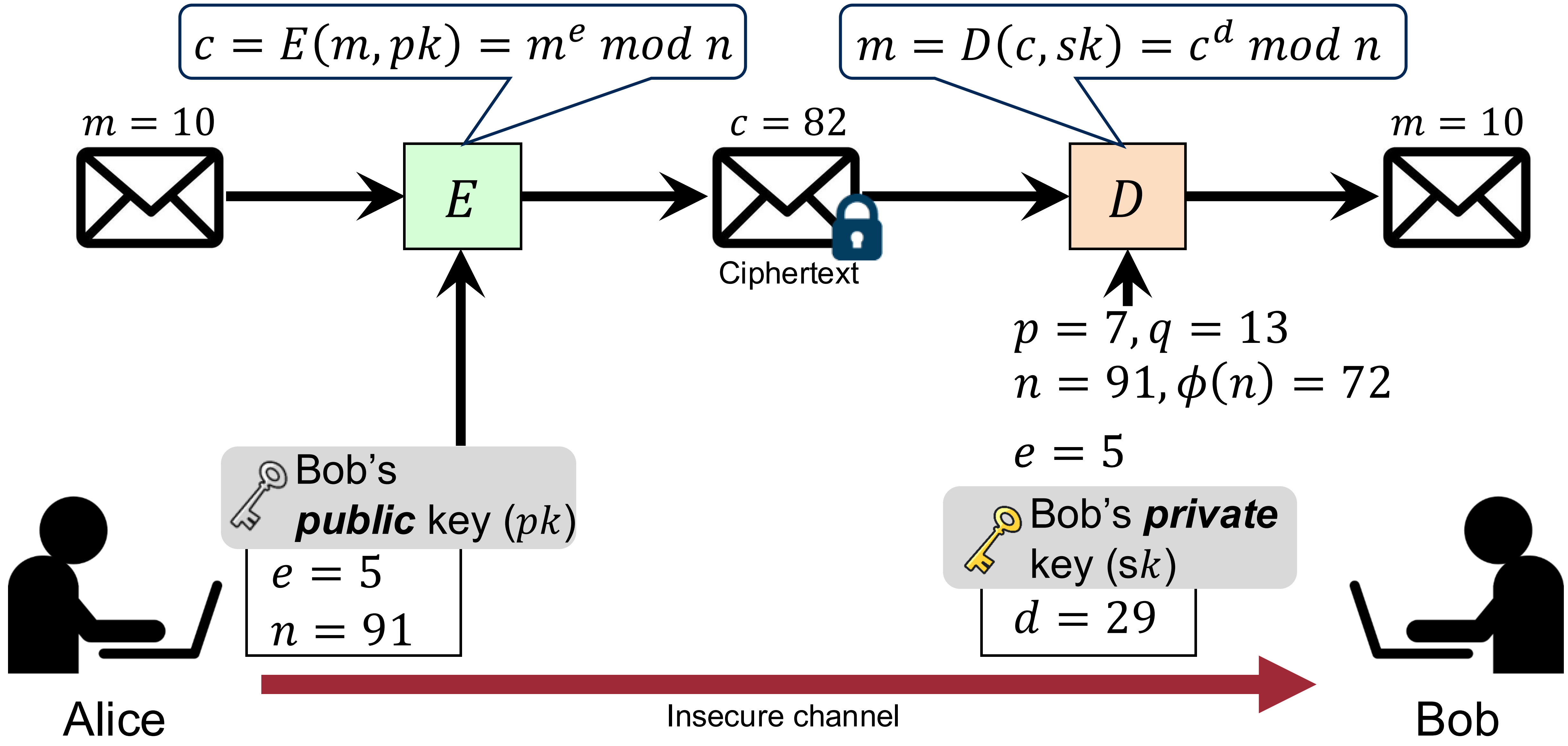
RSA Algorithm (1): Key Generation



RSA Algorithm (2): Encryption and Decryption



RSA Algorithm (2): Encryption and Decryption



Correctness of the RSA Algorithm

$c = E(m, pk) = m^e \bmod n$

$m = D(c, sk) = c^d \bmod n$

Correctness: $m = (m^e \bmod n)^d \bmod n$
$= m^{ed} \bmod n$

Theorem:

$((X \bmod p)^k \bmod p) = (X^k \bmod p)$

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$= m^{1+k\cdot\phi(n)} \bmod n$

We choose d s.t. $(ed \bmod \phi(n)) = 1$

Correctness of the RSA Algorithm

$c = E(m, pk) = m^e \; mod \; n$

$m = D(c, sk) = c^d \; mod \; n$

Correctness: $m = (m^e \; mod \; n)^d \; mod \; n$

$= m^{ed} \; mod \; n$

$= m^{1+k\cdot\phi(n)} \; mod \; n$

$= m \cdot \left(m^{\phi(n)}\right)^k \; mod \; n$

$= m \; mod \; n$

$= m$

Theorem:

$((X \; mod \; p)^k \; mod \; p) = (X^k \; mod \; p)$

We choose d s.t. $(ed \; mod \; \phi(n)) = 1$

Euler's Theorem:

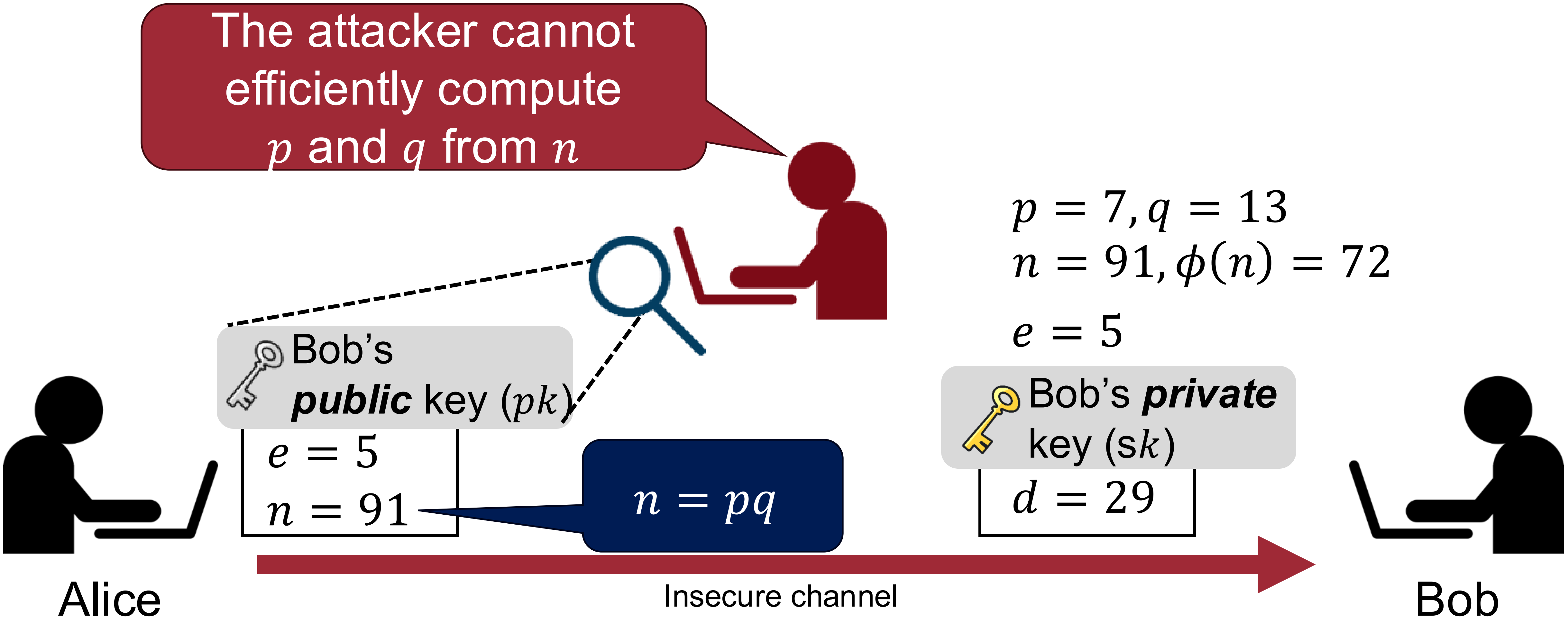
$\left(X^{\phi(n)} \; mod \; n\right) = 1$ where $\gcd(X, n) = 1$

Also, refer to *Fermat's little theorem* ☺

Security of the RSA Algorithm

$$c = E(m, pk) = m^e \bmod n$$

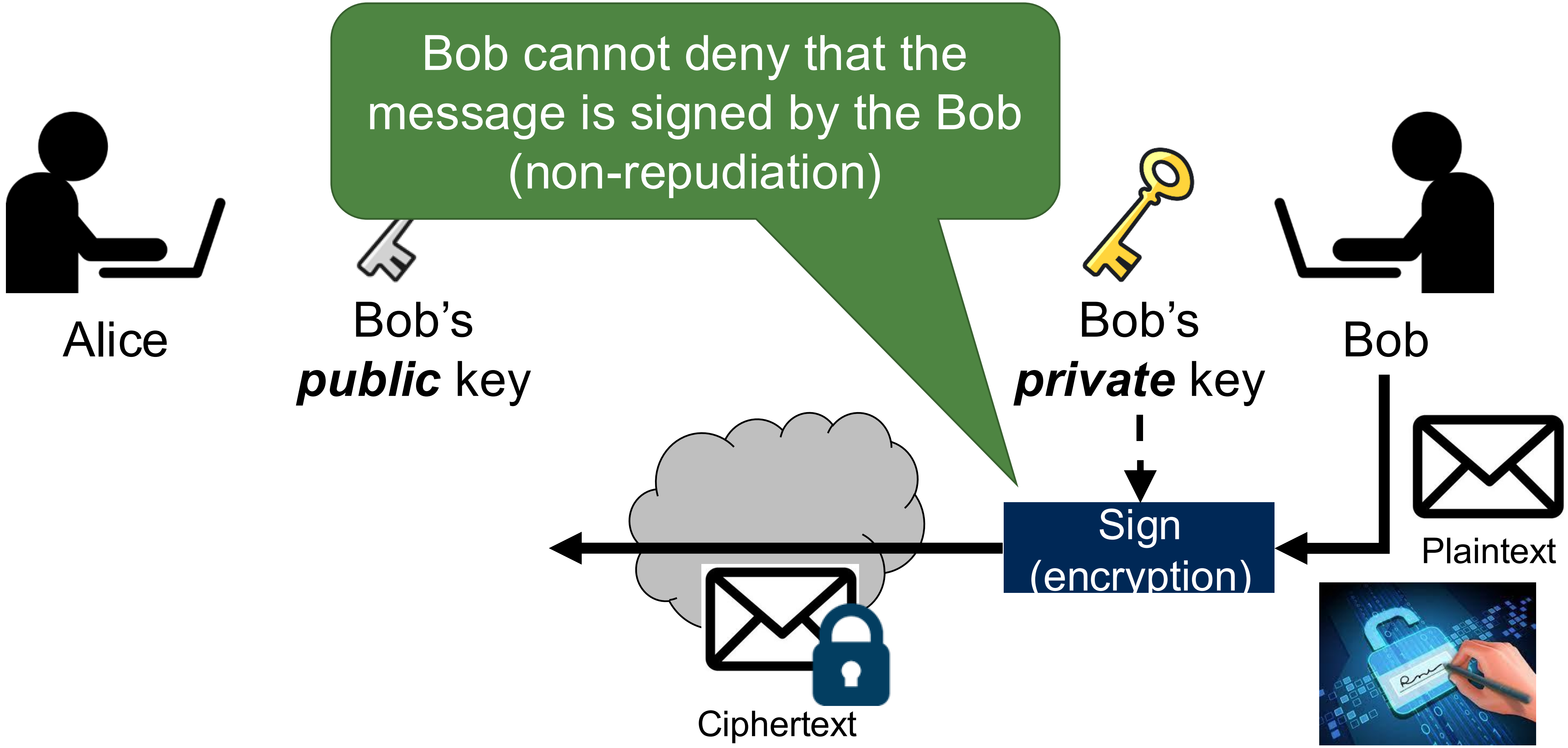
$$m = D(c, sk) = c^d \bmod n$$



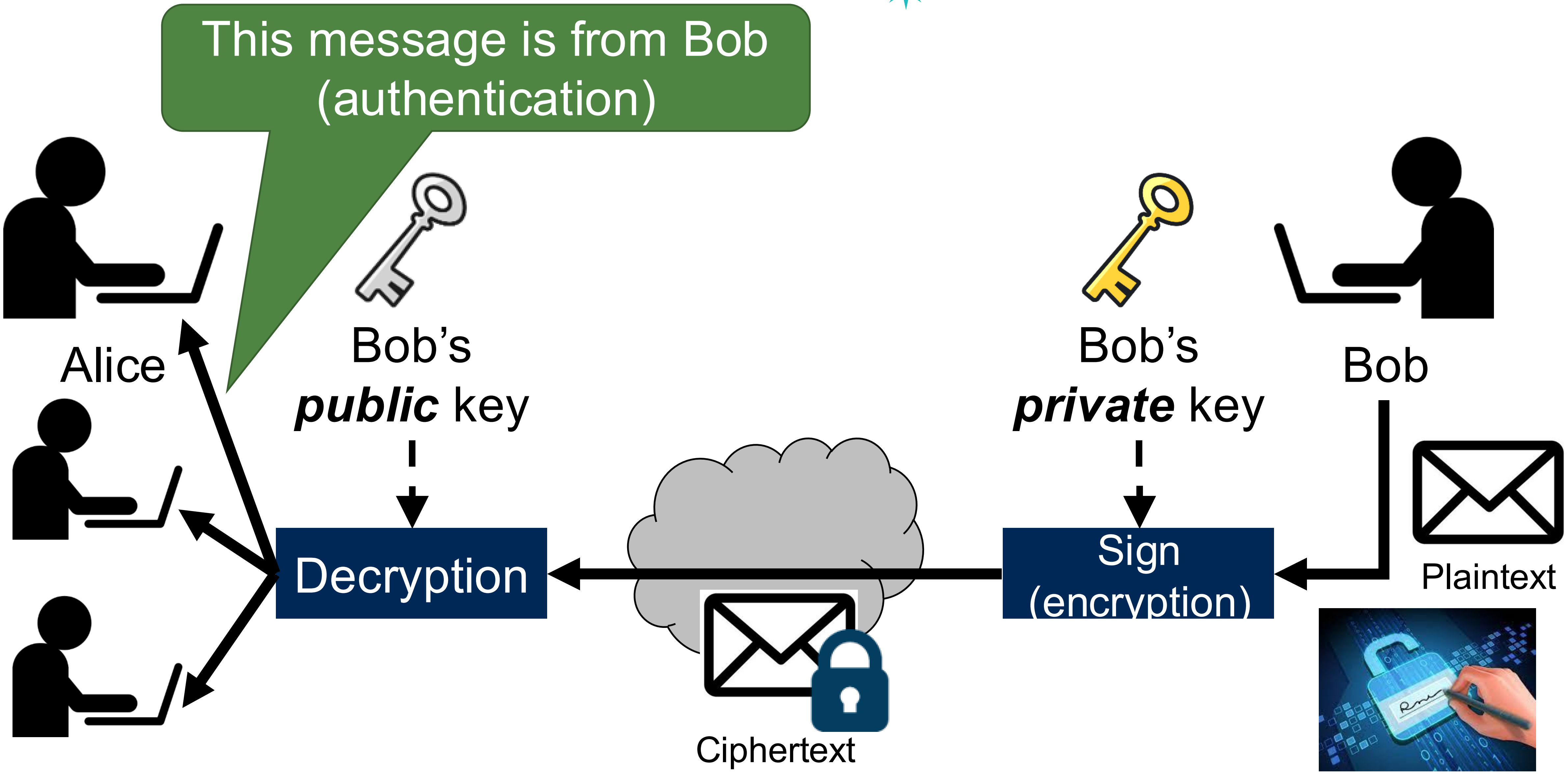
Comparison with Symmetric-Key Cryptography

- Pros
  - No need to share a secret
  - Enable multiple senders to communicate privately with a single receiver
  - More applications: Digital sign

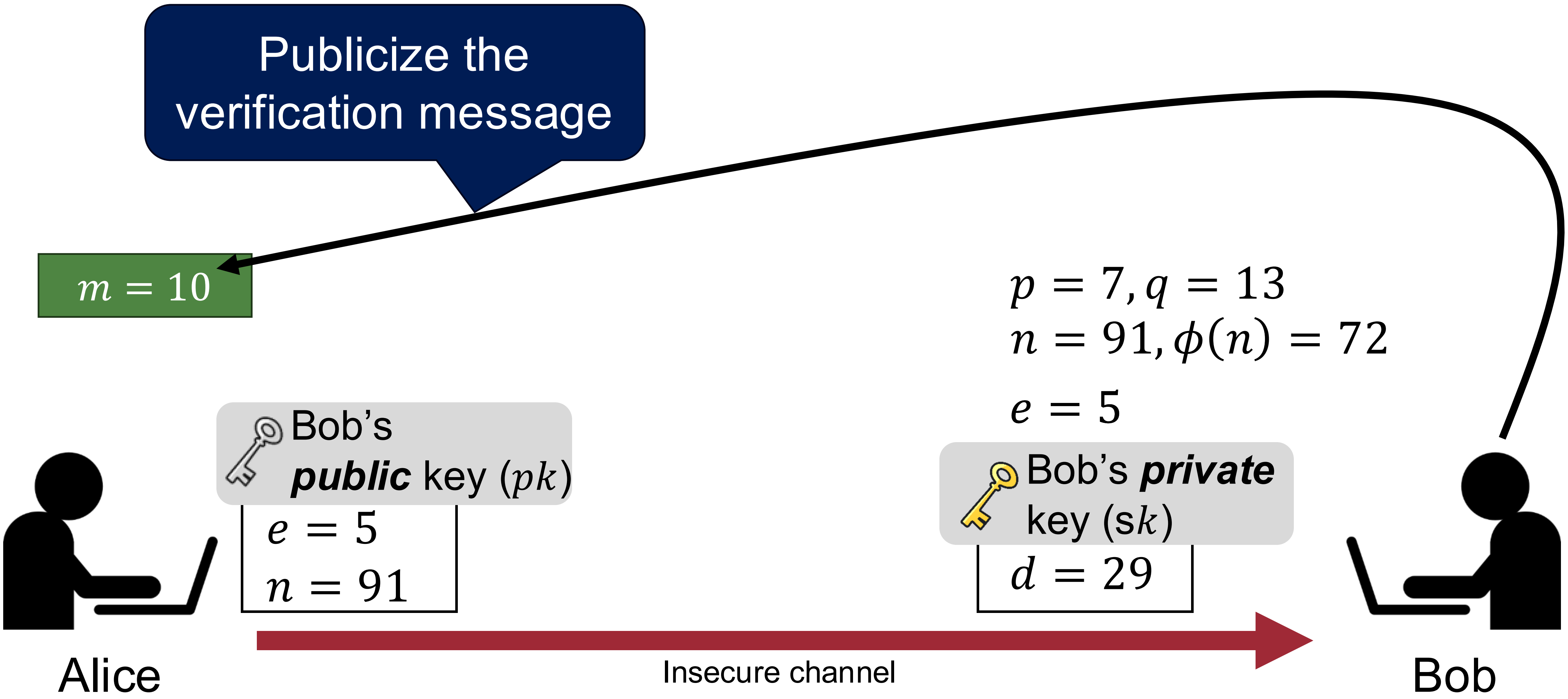
Digital Signature



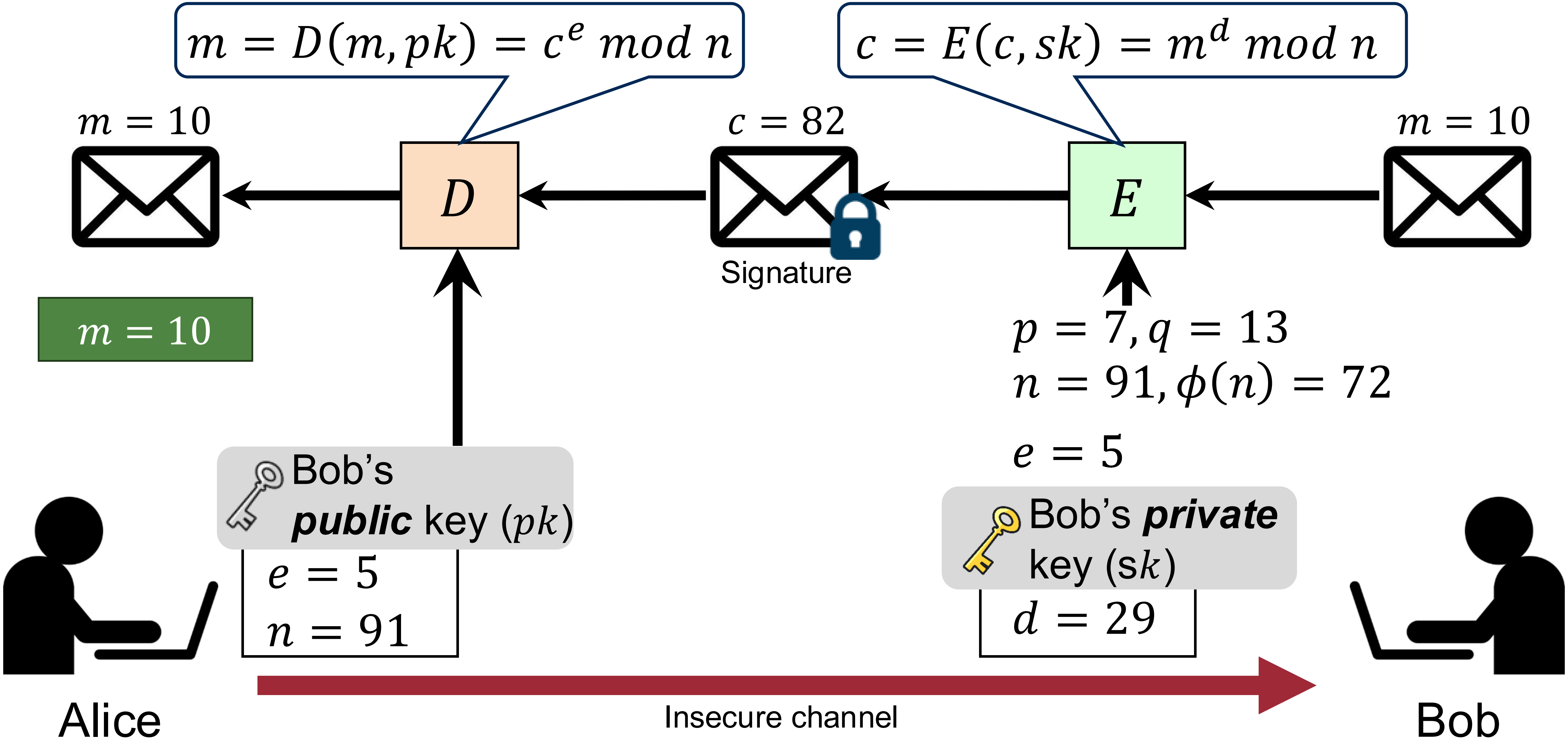
Digital Signature



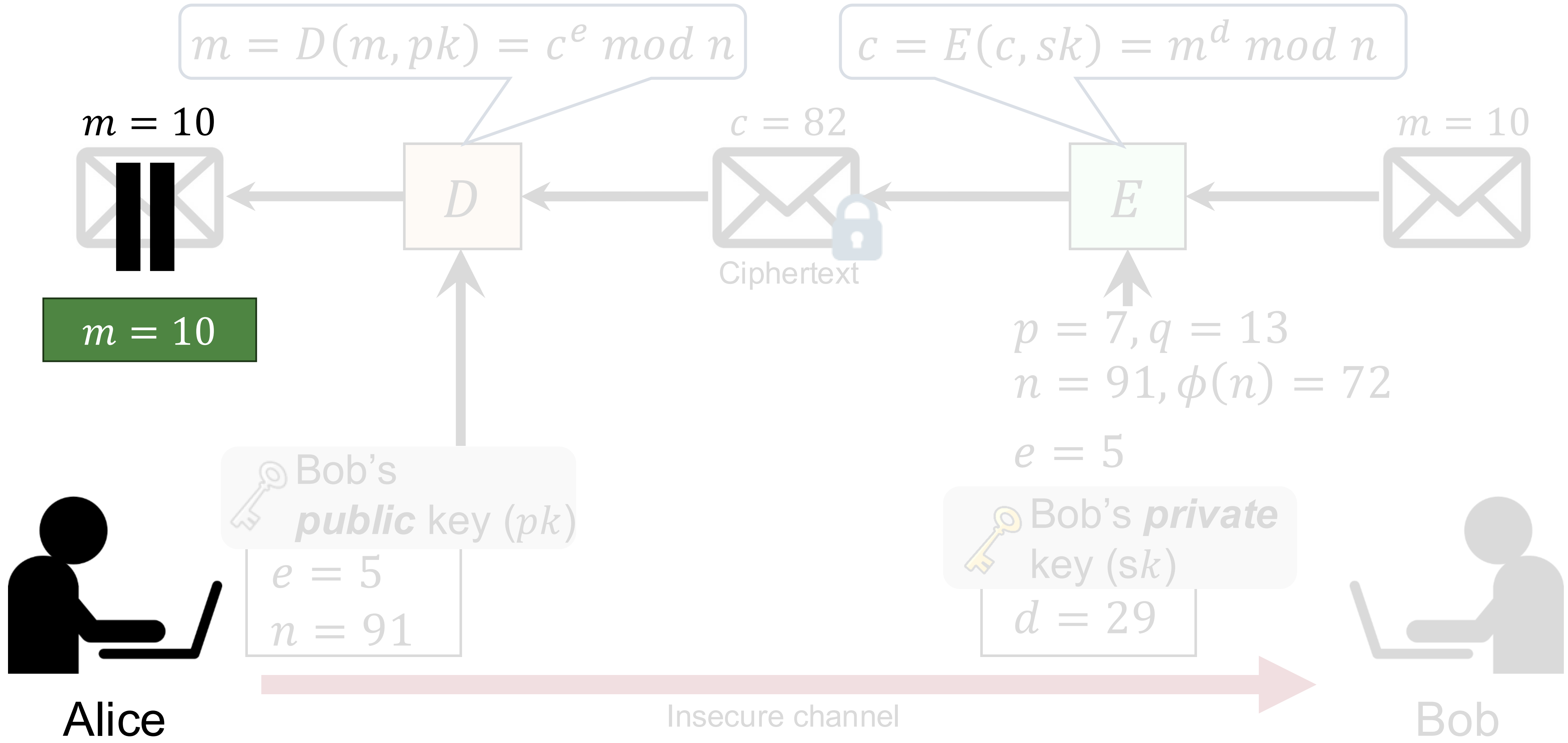
Digital Signature in Detail (1)



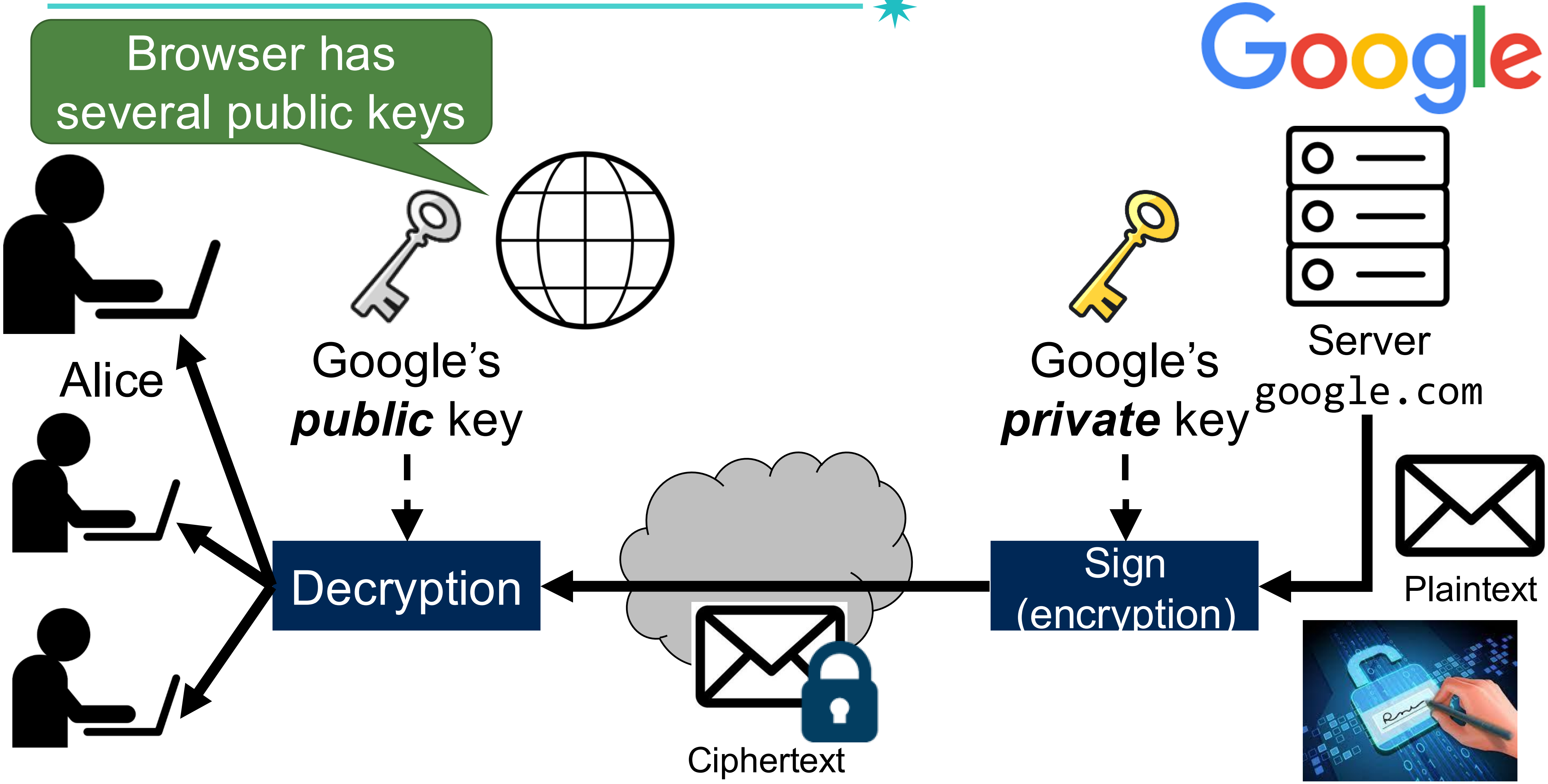
Digital Signature in Detail (2)



Digital Signature in Detail (3)

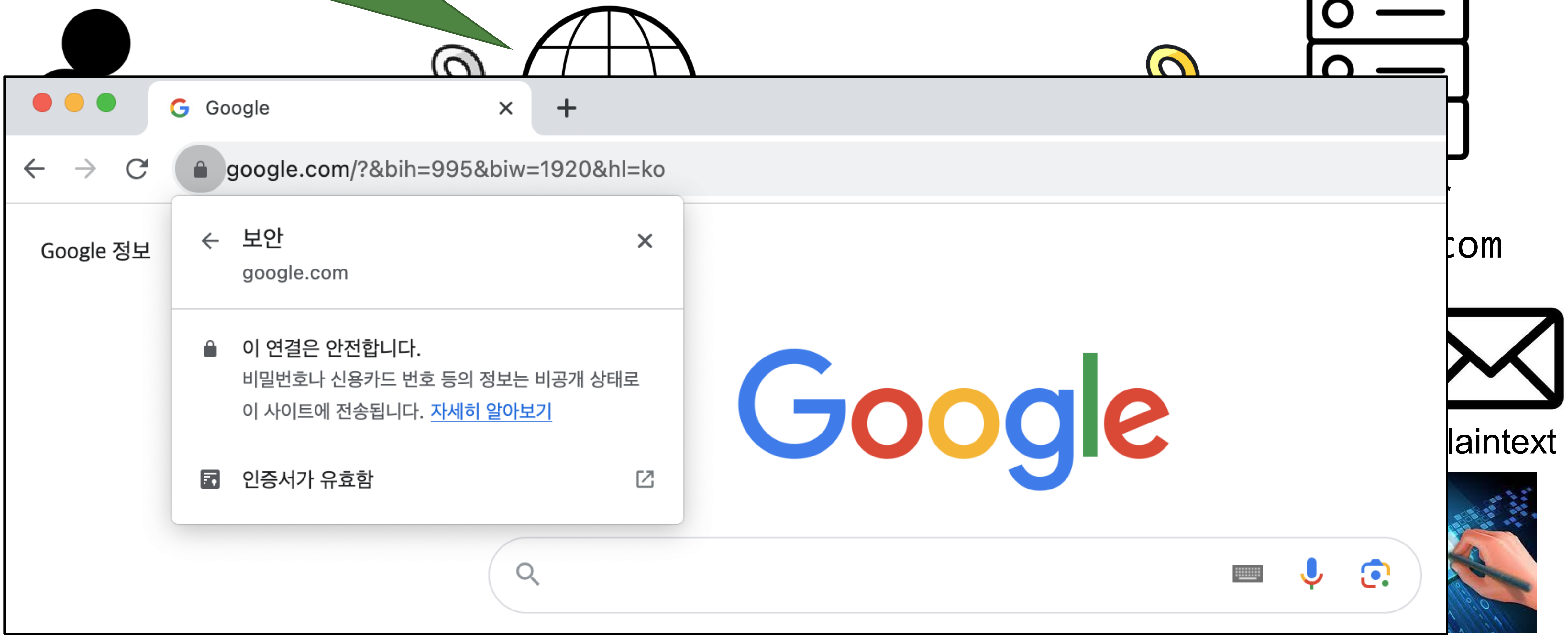


Application of Digital Signature in HTTPs



Application of Digital Signature in HTTPs

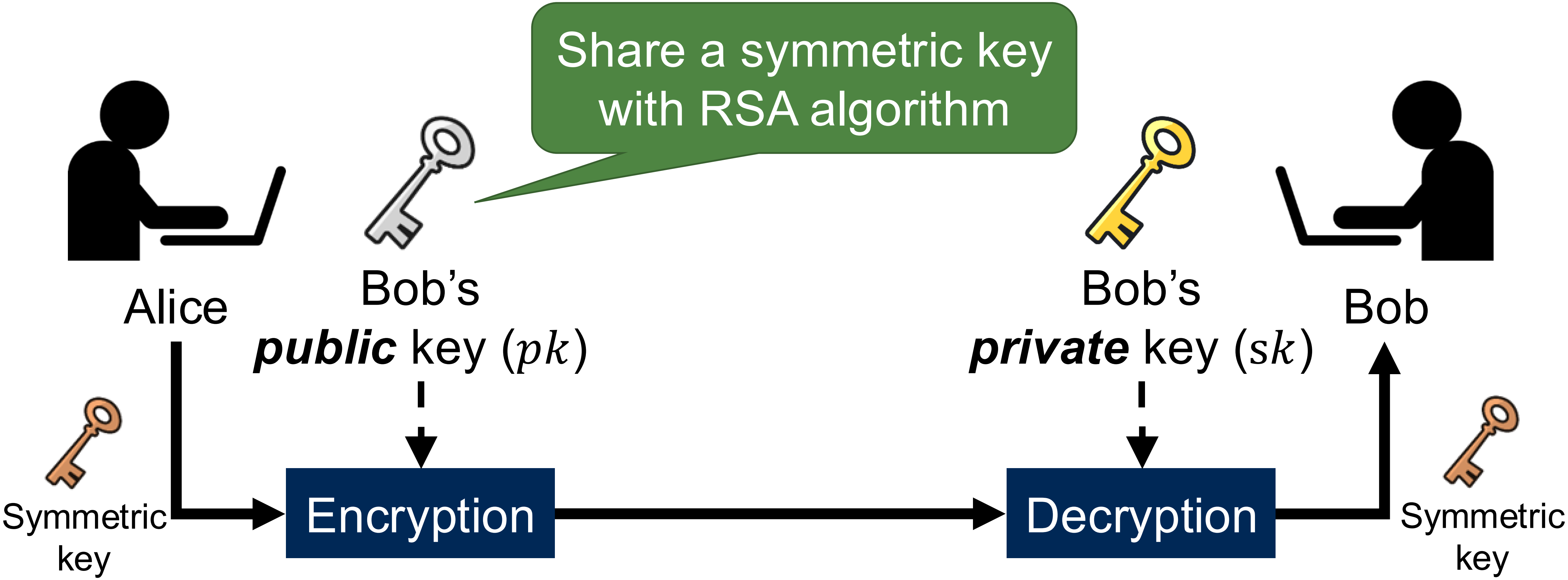
Browser has several public keys



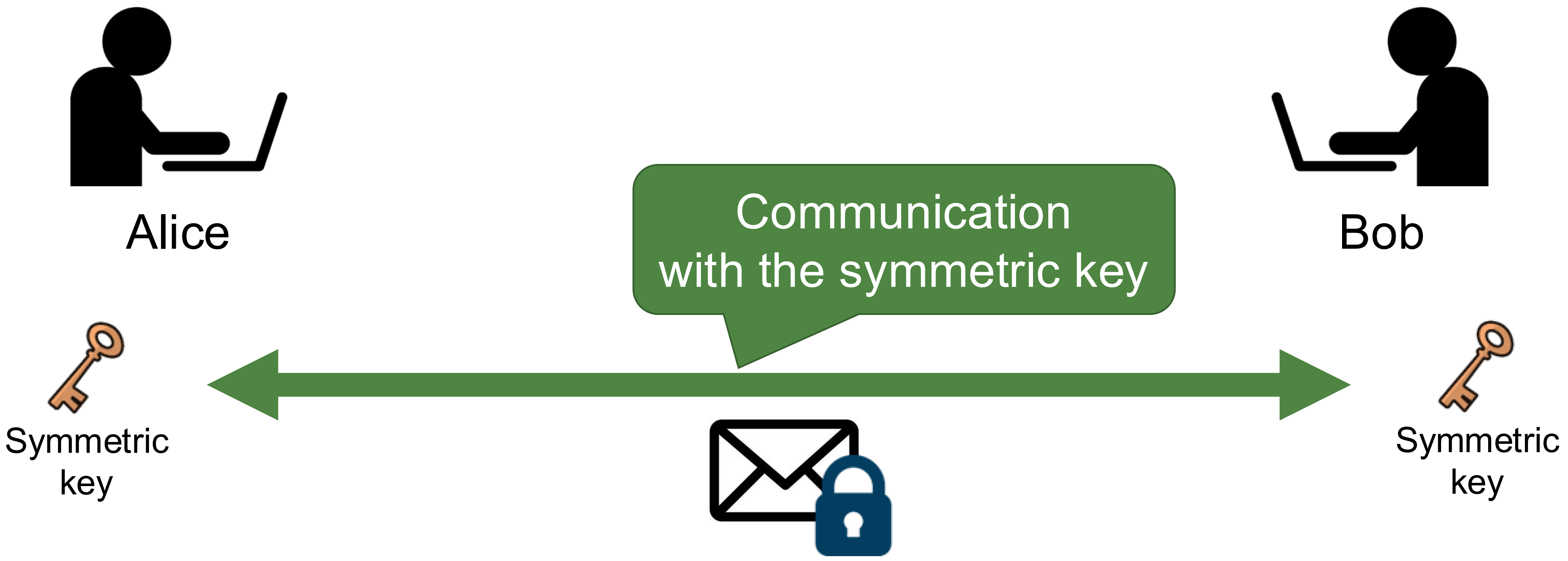
Comparison with Symmetric-Key Cryptography

- Pros
  - No need to share a secret
  - Enable multiple senders to communicate privately with a single receiver
  - More applications: Digital sign
- Cons
  - Slower in general: due to the larger key
    - Roughly 2-3 orders of magnitude slower

In Practice: Combination of Two Schemes



In Practice: Combination of Two Schemes



Summary

- Public-key revolution: solve key distribution and maintenance problem
  - Diffie-Hellman key exchange
  - Public-key encryption
  - Digital signature

Question?