Investigating Power Amplification for Quadruped Robotic Jumping

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Abstract - **Jumping is a common mode of locomotion for various animals. Different animals have various physiologic features that affect their ability for locomotion, all of which can be modeled with a simple lever. Most jumping animals have a segmented leg and primarily use muscles in the thigh and calf to jump. These muscle systems rely on the muscle being attached to a biomechanical lever system. The goal of this investigation is exploring the connection between the inlever and outlever ratio on a leg to better understand the dynamics behind power amplification in legged animals. To test this, we constructed an electromechanical quadruped robot with two-segment legs, and applied in-lever force using springs representing each of the thigh muscle, while measuring jump height. Our simulation model derives the dynamical behavior of the system from classical mechanics, and we use our model to explore the robust nature of our system. Validation of the system performance by simulating lever ratio and jump height in our model improved the design of our physical model. Our simulation results and literature review helped us understand better the optimal design of mechanical advantages in some of nature's best jumpers.**

Key words: Quadruped, Mechanical advantage, SLIP, Vertical jumping

I. Introduction

Each species' unique physiological design follows divergent evolutionary paths allowing for extreme adaptation to an environment. Jumping is a common mode of locomotion through habitats, and is especially beneficial when traversing complex environments in ways that conventional wheeled based systems are unable to accomplish. Biomechanically inspired robot design allows for the borrowing of design principles from some of nature's excellent jumpers, resulting in unique and often simple solutions by employing systems that exploit fundamental properties. Sato et al. [7] found that adding a passive elastic muscle to the a bi-articular muscle system increases the height of the jump by 3%. Bi-articular muscles are muscular systems that cross two joints. More importantly however, is the finding that there was also a decrease in the peak Ground Reaction Force (GRF) during both landing and extension phases over a rigid ankle. The instantaneous peaks seen represent points at which biological joints would be more likely to fail and excess stress could be put on a motor in a robotic system.

Dynamic locomotion has captured the interest of a myriad of scientific minds. For how fundamental it feels to move, to try to artificially recreate that same dynamic movement has yet to be achieved. The muscles involved in steady repetitive motion such as walking and running allows for near optimal energy conservation stored as elastic potential energy [1]. This state is the result of finely tuned hip, knee, and ankle systems. In general, when walking up slopes, the ankle contributes 42% of the leg power with the knee contributing 16%. In jumping, the contributions of both are much more similar, with significant decrease in the contribution from the ankle [6]. Jumping however, is much less energetically efficient. The significant energy requirement can be seen in the proportional rise in knee contribution in the movement. Stefanyshyn et al. [8] found that there was a much more similar energy contribution between the knee and ankle when jumping. The larger torque on the knee joint is much more volatile, and can result in remarkable feats of locomotion at the cost of injury. This experiment was designed to investigate the individual torque states of lever ratios on a knee joint using a constant spring applied force, allowing us to increase jump height as well as preserve joint stability.

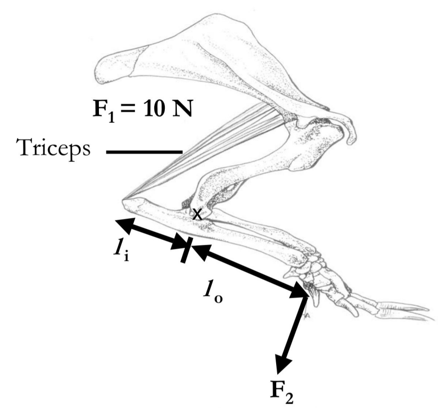

Figure 1. *Armadillo forelimb skeletal structure. The leg shape acts as a lever system, as the humerus-ulna joint pivots the ulna bone. The tricep muscles produce the in-lever force and result in a torque that amplifies the output force which acts on the ground.*

Many animals use catapult mechanisms to produce extremely rapid movements for escape or prey capture,resulting in power outputs far beyond outputs of muscles. These jumping movements can produce joint torques and forces that are very powerful and potentially destructive. Caulfield & Garrett (2002) [4] examined the functional instability of the joint: specifically differences in patterns of ankle and knee movement during a single leg jump. They found that the leg was able to share joint torques upon landing most readily when there was plantar flexion in the ankle (pointing the foot along the direction of the leg), with nearly no elastic potential energy. What is intriguing about this finding is the suggestion of the codependence of the system, where inadequate contributions from one joint has deleterious effects upon all joints.

These breaking points are impossible to be found on a live biologic system, but through biomechanical modeling and known density and strengths, we are able to gain a better understanding. By creating systems that can defy evolutionary time, we can better identify the most important biological systems. Identifying systems that reduce instability and peak force within leg systems during jumping and landing will aid design of future jumping robots to minimize wear.

Jumping is a highly dynamic movement that requires careful execution and coordination of multiple joints in a system. A simpler system of jumping for biomechanical robots follows the same concept as catapults. It is possible to achieve significant energy in a system by building elastic potential energy from torque over time. Animals utilize this through multiple bi-articular muscle-tendon complex consisting of the calf and achilles tendon as well as the hamstring. Jumping animals slowly add elastic potential to their calf and hamstring by stretching the elastic muscle. It is possible to attempt to recreate the elastic potential elements of these muscles through a combination of linear actuators and isoelastic elements.

In studying the lift off dynamics, we look into the specific joint system in the leg that allows tendon muscles to act as lever systems on the bone structure directly affecting the ground reaction force. In particular, the net ground reaction force affects the jumping height. Through simplifying the motion of the system, we aim to investigate how the mechanical advantage produced by in lever out lever ratio on a leg affects the net jump height on a robotic system. This study thus required careful study of leg systems in order to replicate this dynamic in practice.

The animal leg is a complex system of levers and springs with multiple degrees of freedom. Muscles and tendons comprise the locomotive part of the leg with bones creating the structure on which torques can be affected. Due to the mechanical complexity and versatility of muscles, biomechanical models can generally only mimic specific features of these multifaceted tools. One of the most dynamic motions that legs can produce is jumping, the storing of elastic potential energy in a retracted spring with a sudden release.

In designing a jumping biomechanical model, the biomechanical muscles and tendons are replaced by a combination of linear actuators and viscoelastic material, to generate the torques and potential energy that catapult the system for jumping. For some actuators, a common design problem is the need for large energy sources to drive them. Electric rotary motors serve as excellent sources due to their ease of use for controllability, compact size and their light weight. The MIT cheetah, through the use of high density torque motors for hip and knee joints, can run at high speed and jump powerfully like a real cheetah [9].

Through designing an electro-mechanical system that will allow us to control the muscle use ratios, our systematic study for the lift-off dynamics factors seeks to find the optimal combination of muscle use and mechanical advantage to find the maximum possible computed jumping heights. For our design, various viscoelastic materials will represent the muscle power of tendon muscles though the elastic constant (k) and motors apply joint torques to load gears and stretch spring material. The physical structure, mechanical design of our quadruped robot, and the control algorithm code for system control allow us to create a system that emulates the lift off dynamics we aim to investigate. By varying the materials used to represent different tendon muscle strengths, our data will give us insight onto some of the best design choices for leg lift off dynamics.

Different animal muscles have their own maximal muscle output power and joints also have maximal torques they can handle. Through designing a systematic study experiment, our goal is to figure out the factors influencing the peak stresses during jumping motion. Literature review at the end of our experiment seeks to find comparison with some of nature's best jumping terrestrial species.

II. System Design

The model exists as four independently controlled lever based legs. Figure 2 shows the simple nacet design of one of the lever systems. The knee joint had a range of motion of 135º, ranging from 0º to the normal to -135º. Each knee joint was spring loaded and driven in a gear train with a ratio of 4 by an S1213 servo motor with a torque of 25.3 oz*in. This provided enough torque to load the spring tendon with a spring constant, k, of 171 N/cm.

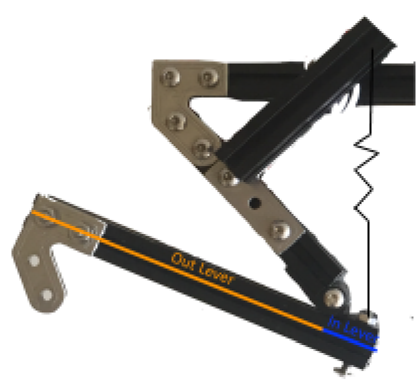

Figure 2. *Robot leg and the joint where the in lever and out lever segments of the system. The leg was loaded by a bi-articulated spring loaded from the body down to to the femur.*

Each leg was in parallel along the sagittal plane with a mirrored leg along the coronal plane. This allowed for the legs to be loaded with movement of only the body as the relative position of each foot stayed constant.

We were able to change the in-lever out lever-ratio by sliding the leg along the hinge that functioned as the joint. The spring anchor position on the body was also moved the equivalent amount to allow for constant spring displacement and applied force to the in-lever.

III. Simulation Modelling

The dynamics of the system were simplified for analysis using classical mechanics concepts. The study of mechanical advantages using in-lever out-lever ratios makes it similar to catapulting problems.

After simplifying the system from multiple trials, this free body diagram was used to derive the governing equations of the biomechanics simulation. Each leg contributes a net force on the hip of the model that accelerates the center of mass upward for a jump.

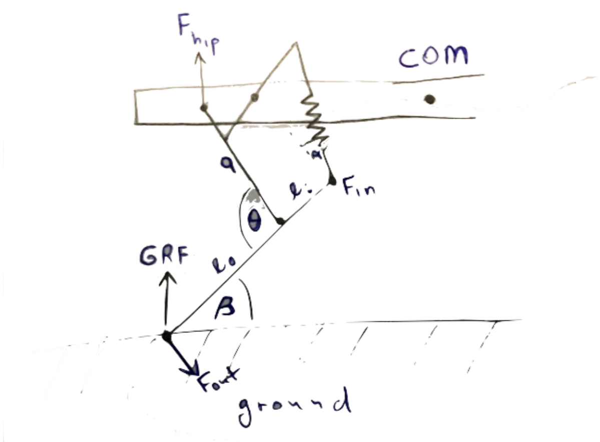

Figure 3: *Free body diagram for one of the quadruped legs including the parameters of the system. The foot angle with ground is important as it affects the ground reaction force*

In deriving the system dynamics, the viscoelastic tendon is vital to the system design, as it provides the in-lever force F_i. For the leg length, an ulna equivalent to the armadillo arm, it acts as the lever arm acted on by the spring and the ground. Mechanical advantage for a lever system is defined as the ratios given below:

$$Mechanical\ Advantage\ =\ \frac{F_{out}}{F_{in}} = \frac{l_{in}}{l_{out}} \quad (1)$$

The force due to the spring is $k\Delta x$, where x is the spring displacement from equilibrium position. The angle α is the angle between the lever and the spring, as it is not always 90^o. The resulting joint torques produce equation (4) below.

$$in - lever\ torque\ =\ out - lever\ torque \quad (2)$$

$$F_{in} l_{in} = F_{out} l_{out} \quad (3)$$

$$k\Delta x\ l_{in} cos\alpha = F_{out} l_{out} \quad (4)$$

The ground reaction force is related to the outlever force by the trigonometric relation

$$GRF = F_{out} cos\beta \quad (5)$$

Combining equations 4 and 5, the relation between the ground reaction force and mechanical advantage is thus:

$$GRF = k\Delta x\ cos\alpha\ cos\beta\ (\frac{l_{in}}{l_{out}}) \quad (6)$$

To compute the acceleration of the system during ground contact, the next step then involves finding the net velocity with which the system leaves the ground in order to find the theoretical jump height.

Both lever arms move at the same angular velocity and the velocity-lever arm length are related by the radial velocity equation for simple harmonic motion $v = \omega r$. The angular velocity is :

$$\omega = \frac{v}{r} = \frac{v_{in}}{l_{in}} = \frac{v_{out}}{l_{out}} \quad (7)$$

The outward velocity is thus v_{out}, also related to initial vertical velocity v_{out-y} by the same trig relation in equation 5, as the ground reaction force equals the force on the hip joint accelerating the center of mass, by Newton's third law of motion. Therefore

$$v_{out} = v_{in} \frac{l_{out}}{l_{int}} cos\alpha \quad (8)$$

$$v_{out-y} = v_{in} \frac{l_{out}}{l_{int}}\ cos\alpha\ cos\beta \quad (9)$$

To calculate the in-lever velocity, we resort back to simple harmonic motion equations for a spring mass system. The initial velocity for a displaced system is given by equation 9, and since 4 legs act on the same mass, this leads to a mass fact of $\frac{m}{4}$ for each equation. This gives us v_{in}

$$v_{in} = \sqrt{\frac{k}{(m/4)}} x\ cos\alpha \quad (10)$$

Substituting into equation 8;

$$v_{out-y} = \sqrt{\frac{k}{(m/4)}} x\ \ (\frac{l_{out}}{l_{int}})\ cos\alpha\ cos\beta \quad (11)$$

At the jump height, the final velocity $v_{jump\ height} = 0$. Simplifying the final systematic study equation to a center of mass dynamic problem, the theoretical jump height is derived from our equations using kinematics. From kinematics

$$(v_{jump\ height})^2 = (v_{out-y})^2 - 2gH \quad (12)$$

Simplifying this equation, the final jump height is thus given by:

$$H = 2\frac{kx^2}{mg}\ (\frac{l_{out}}{l_{int}})^2\ cos^2\ \alpha\ cos^2\beta \quad (13)$$

IV. Results

This experiment sought to better understand the functional relationship between the role that the effective joint torque, as a product of a variable in-lever out-lever ratio affected system performance while jumping. The mechanical advantage was controlled by changing the ratio of the in-lever to out-lever, which in a lever, produces an output force equal to the mechanical advantage and the applied force. The applied force, from a loaded spring, was held constant through a constant spring displacement when loading. The mechanical advantage of the limb was changed with an adjusted lever ratio. In this way, the input energy of the system remained constant by standardizing the elastic potential energy at the start of the jump in the crouched state.

In testing, the beta angle, which is inward foot angle with the ground, was assumed to be a constant for all trials, at $\beta = 30^o$. The spring angle with the end of the in lever was assumed to act at a perpendicular angle, $\alpha = 90^o$. Our system had a total mass of $m = 0.7\ kg$. Our measured Hooke's law spring constant for our viscoelastic material was $k = 171\ kg/s^2$. Plugging in these parameters onto our simulation, we got the fig 4 result shown below for the mechanical advantage against the jump height.

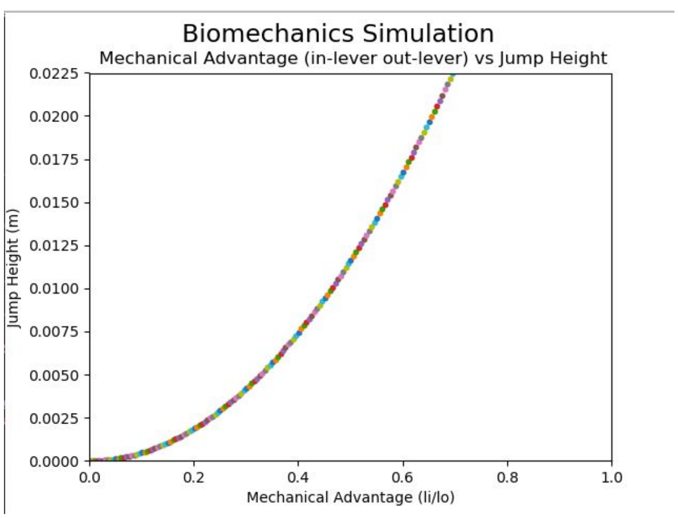

Figure 4. *The mechanical advantage in-lever to out-lever ratio plotted against the simulation computed theoretical jumping height. These results help justify early behaviour of our system design, as the system failed to jump significantly.*

The same behavior as above is plotted again for lever ratio plotted against the the maximum joint torque as well in Figure 5 below. The torque was applied in the anti-clockwise direction, and thus is negative, reaching a maximum torque at a lever ratio of 0.5.

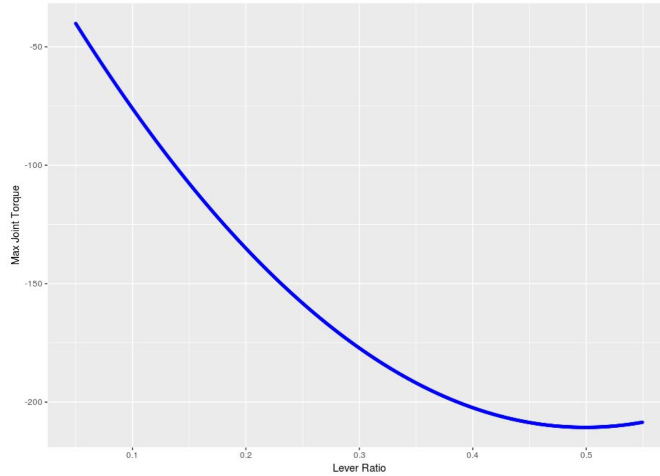

Figure 5. Simulated *maximum joint torque during jump with a total mass of 0.7 and a spring constant, k, of 171. The maximum joint torque always occured at the first time step.*

While the instantaneous forces and velocities are all different, the total work from the loaded spring in the lever system remained constant. It was our goal to better understand the dynamics that occur in a imperfect world where there is friction and lost of energy, and thus jump heights would not be the same. Figure 6. displays the results of estimating the jump height in simulation. The lost of energy through the riemann sums with a timestep of 0.0001 seconds were attributed to the similar imperfect conveying of energy especially complex dynamical movement. This simulation used the same equations except applied them over time.

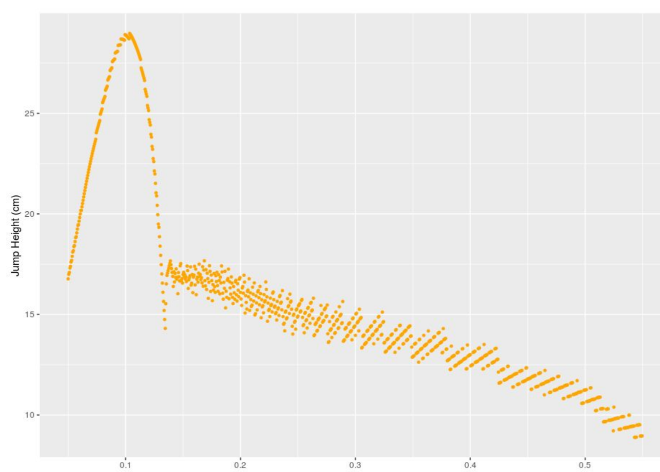

Figure 6. Simulated *jump height of a spring loaded lever leg with a total mass of .7 kg and a spring constant, k, of 171 estimated using a simulated lever model. The x-axis represents the mechanical advantage of the limb. All of the jump heights are estimated from the model.*

VI. Discussion

Modeling a dynamic and rapid motion like jumping is a complex process. Unlike more stable forms of locomotion, it is hard to uncover the loss of energy. When our simulation was run with lossless energy standards, as the mechanical advantage of the lever increased , the effective jump height also increased (Figure 4). This is in line with the noticeably larger maximum joint torque as the mechanical advantage increased (Figure 5).

At high torques however, there is more energy loss than at lower and more stable torques due to increased friction and entropy effects. In order to attempt to create some variability within the context of high torque motion, we simulated the jump over time, using the energy loss with each time step as the stand in for actual energy loss. The results of the simulation are shown in Figure 6. As can be seen, nearly the opposite is true as in the lossless energy system. There is a stable parabola of jmp height between a mechanical advantage of 0.05 and 0.13, with a maximum jump height at 0.12. As the mechanical advantage of the system increased from there, the jump height followed a downward trend. This falls more in line with what we would expect from the real world, where the velocity of the end effector is slower and more stable, resulting in less random energy loss and a higher effective jump height.

In practice, this state is also confirmed as the optimal for a myriad of reasons. Having a smaller mechanical advantage also results in a lower joint torque, which in turn, is less susceptible to injury. Additionally, it would be wildly ineffective to have limbs where half of the limb is only used to generate torque.

We were unable to confirm the results of the simulations with physical measurements due to the variability of the motor control as well as the difficulty loading the spring with enough potential energy to produce a jump. What was interesting about this however was the the most complex part of the movement was attempting to recreate systems as small as muscles that were able to produce as much power

The physical prototype did give us some interesting fundamental properties required for jumping:

(1) Friction of joints and end-effectors plays a significant role in the ability of a terrestrial jump.
(2) Jumping is most readily achieved through the slow loading and releasing of a spring.
(3) Actuation synchrony is important for achieving highly dynamic motion.

Initially we had designed the legs to be mirrored across the coronal plane, but for the squat and elastic load to occur, the relative location of the feed then had to change. In order for this to happen, the feet would have to have little enough friction that they could easily slide in. This however, also meant that they would not have enough friction to create a ground reaction force large enough to jump.

The actuation of the jump in the physical prototype was decided to be achieved using springs. That is because they do not need to be actively driven and can instead be loaded and released. We were unable to find a motor that was able to produce enough torque at a fast enough speed to achieve jumping motion. This changed our initial design but resulted in a system that is more accurate to the biological design, with preloaded viscoelastics used to generate instantaneous application of significant force. In this same vein, for the jump to be effective, all of the legs had to act in synchrony. We had also initially planned on actuating both the knee and ankle joints simultaneously, but were unable to fully synchronize the actuation of just the knee joints. Future work on this topic could include expanding the scope of the study by incorporating multiple muscles, but would require very high torque motors or complex gear systems to drive high strength spring forces.

VII. Conclusion

In nature, the ability to perform highly specialized locomotion must be a weighted function. While there can be systems that are optimized for a specific action, to have a system that can only succeed performing one aspect of locomotion is evolutionarily inefficient. This experiment sought to better understand the specific requirements of a limb to be optimized for jumping.

Our simulation results did agree with relevant literature findings by other studies, such Henry Astley's investigation of elastic loading and recoil impacts on jumping in anural animals. This study found that increasing proximal joint moments early in the jump allowed for high ankle muscle forces and elastic preloading, which in turn resulted in higher jumps. Our simulation in Fig 4 confirmed this as with an increased mechanical advantage, the jump height correlated with the high joint torques as a result.

While a significant amount of the ability to draw conclusions relies on measuring how energy is lost in a physical experiment, we were unable to confirm the expected results from our simulations.

VIII. References

1. Alexander, R. M. (1991). Energy-saving mechanisms in walking and running. *The Journal of Experimental Biology, 160*, 55-69

2. Babic, J., Lim Bokman, Omrcen D., Lenarcic J., and Park F. C. (2009). A Biarticulated Robotic Leg for Jumping Movements: Theory and Experiments. *Journal of Mechanisms and Robotics* pp. 1-9

3. Babic, J., Omrcen, D., and Lenarcic, J., (2006). "Balance and Control of Human Inspired Jumping Robot," *Advances in Robot Kinematics: Mechanisms and Motion*, J. Lenarcic and B. Roth, eds., Springer, Dordrecht, pp. 147–156

4. Caulfield, B & Garrett, M. (2002). Functional Instability of the Ankle: Differences in Patterns of Ankle and Knee Movement Prior To and Post Landing in a Single Leg Jump. International journal of sports medicine. 23. 64-8.

5. Deshmukh, A., Amarnath, C., (2006). Robot Leg Mechanisms. Department of Mechanical Engineering Indian Institute of Technology

6. Montgomery, J. & Grabowski, A., (2018) The contributions of ankle, knee and hip joint work to individual leg work change during uphill and downhill walking over a range of speeds, Royal Society Open Science, 5, 8.

7. Sato, R., Miyamoto, I., Sato, K. Ming, A., and Shimojo, M. (2015) "Development of robot legs inspired by bi-articular muscle-tendon complex of cats," *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Hamburg, pp. 1552-1557.

8. Stefanyshyn, D & Nigg, B. (1998). Contribution of the lower extremity joints to mechanical energy in running vertical jumps and running long jumps. Journal of sports sciences. 16. 177-86

9. Patrick M Wesing (2015) . Proprioceptive Actuator Design in the MIT Cheetah: Impact Mitigation and High-Bandwidth Physical Interaction for Dynamic Legged Robots

10. X. Wan, T. Urakubo and Y. Tada, "Optimization of jumping motion of a legged robot for different take-off postures," *Journal of Mechanical Science and Technology,* vol. 29, *(4),* pp. 1391-1397, 2015.
11. Henry Astley, (2014), The mechanics of elastic loading and recoil in anural jumping, The journal of experimental biology, 4372-4378
12. Ioannis. Poulakakis, and J. W. Grizzle, "The Spring Loaded Inverted Pendulum as the Hybrid Zero Dynamics of an Asymmetric Hopper," in the IEEE Transactions on Automatic Control, Vol. 54, No. 8, pp. 1779-1793, Aug. 2009
13. Jeffrey J Aguillar, (2010), Exploring the lift off dynamics in a jumping robot, Georgia Institute of Technology, Masters Thesis