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Brain Dynamics	008
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<b>Abstract</b>	027
An introduction to the idea of brain dynamics in fMRI studies	028
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A fractal structure, either spatial or temporal, is composed of smaller parts that exhibit the same pattern at every scale [1–3]. Classic spatial examples of fractals in nature	043
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047 are coastlines [3], circulatory system [4], and brain anatomy [5], where at each level of  
048 magnification these structures look the same; i.e. *self-similarity*. For a dynamic process,  
049 or in the temporal domain, this is known as *scale invariance*, meaning that both rapid  
050 and slow processes follow the same pattern [6]. Scale-free behavior, characterized by  
051 the absence of any inherent scale or preferred length, is a signature characteristic  
052 of complex systems that can be understood as the collective outcome of numerous  
053 interacting components with weak and random connections [7]. Physiological systems  
054 usually need a mix of randomness and structure to function optimally. When a system  
055 is operating near the critical point, it is expected to exhibit these long range temporal  
056 correlations (LRTCs). Therefore, by analyzing the LRTC of a time-series, valuable  
057 insights can be found into the underlying mechanisms driving system behavior, such  
058 as memory effects, self-organization, and criticality [8].  
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071 “Because pink noise lies between white and Brown(ian) noise, it has been proven  
072 to bring stability and adaptability into dynamic processes, thus, crucial properties of  
073 well-functioning complex systems” [9]  
074  
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076  
077 “as pink noise arises from the interaction of multiple systems and operates over different  
078 scales, it has been shown to contribute to system resiliency and structural integrity if  
079 individual components were lost or interrupted” [10, 11]  
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082 See Herman et al. [12] for a really good review  
083  
084

085 Scale-free dynamics is a hallmark of complexity viewed as an emergent property of biolog-  
086 ical systems composed of numerous elements with a network of stochastic (typically weak)  
087 connections amongst them (Csermely, 2006) from Mukli et al. [13]  
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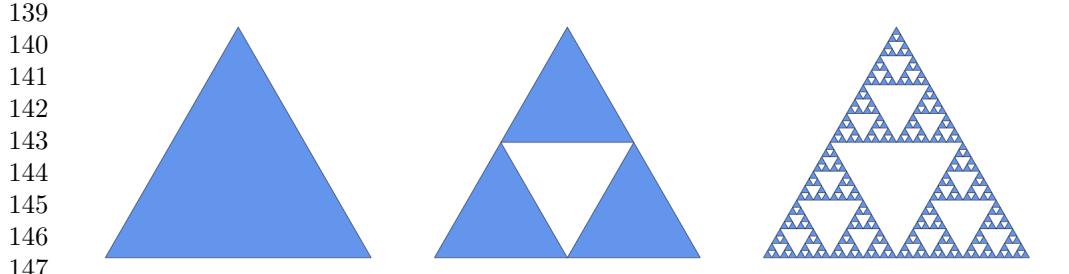
Another good review is by Werner (2010) linking fractals/Hurst and Criticality Werner [14]	093
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## Fractal Dimension and Hurst Exponent

Non-periodic fluctuations are prevalent in physiological systems, and these irregular patterns can be mathematically modeled using stochastic, chaotic, or noisy chaotic methods. Stochastic models assume that the fluctuations result from a large number of weak influences, while chaotic models conceptualize that strong nonlinear interactions between a few factors shape the fluctuations. Among the various stochastic approaches, ‘fractal’ models offer the most accurate representation of reality by considering the self-similar nature of physiological fluctuations over different time scales [15]. Fractal structures were first expressed in the late 19th and early 20th century by mathematicians who generated complex geometrical structures with simple objects (e.g. a triangle) by applying a simple rule of transformation in an infinite number of iterative steps Figure 1. A use-case for these geometric structures was not fully realized until the 1960s, when Benoit Mandelbrot formalized them as a new form of geometry capable of describing the complex shapes and forms of nature [3].

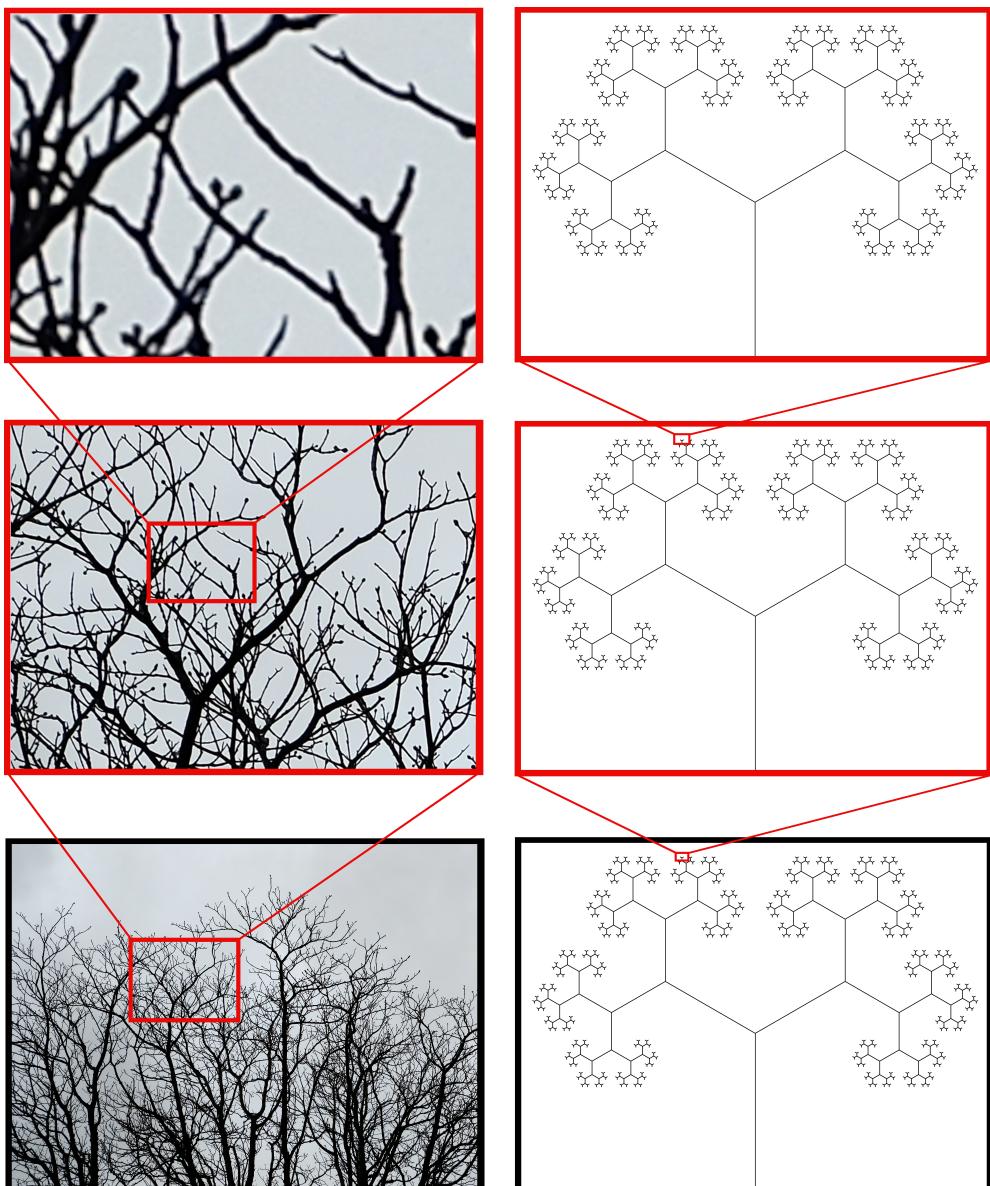
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148 **Figure 1. Ideal mathematical fractal.** The 2D Sierpinski triangle starts with a  
149 simple equilateral triangle (left), and subdivides it recursively into smaller equilateral  
150 triangles. For every iteration, each triangle (in blue) is further subdivided it into four  
151 smaller congruent equilateral triangles with the central triangle removed. The first  
152 such iteration is shown in the centre, with the fifth iteration shown on the right.  
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157 Unlike Euclidean structures that can use axioms and rules to describe an object of  
158 integer dimensions, fractal structures can only be characterized by recursive algo-  
159 rithms that extend the use of dimension to the non-integer range [16]. Recognizing this  
160 difference, Mandelbrot named these complex structures ‘fractals’, and defined their  
161 non-Euclidean dimension as a ‘fractal dimension’ (FD) — that is, a non-integer. For  
162 the one-dimensional time-series, FD will be between 1 and 2. While a structure such  
163 as the Sierpinski triangle is an ‘exact fractal’, meaning it is assembled from pieces  
164 that are an exact replica of the whole, nature is composed of ‘statistical fractals’,  
165 whose self-similarity is found in the power law scaling of the parameters characterizing  
166 their structures at different scales of observation Figure 2. Unlike Euclidean struc-  
167 tures which can be defined by axioms, fractals can only be characterized by a set of  
168 properties that, when present, indicate the structure is indeed fractal (see Section 2).  
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178 The Hurst exponent ( $H$ ) is a statistical measure used to quantify long-range depen-  
179 dence or persistence in time-series data (i.e. LRTCs). First introduced by Harold Hurst  
180 in 1951 for hydrological applications [18], it was Mandelbrot again who helped pop-  
181 ularize its use for studying complex shapes and forms of nature [3, 19]. Since then  
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**Figure 2. A comparison of statistical and exact fractal patterns.** The two basic forms of fractals are demonstrated. Zooming in on tree branches (left), an exact self-similar element cannot be found. Zooming in on an exact fractal (right), exact replica of the whole are found. Photo by author. Branching fractal made in Python. Figure inspired by Taylor (2006) [17]

231 it has been applied across various fields, including finance, geophysics, ecology, and  
232 neuroscience, to characterize complex systems with intricate dynamics [20–23]. H and  
233 FD for time-series can be directly converted, as

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$$H = 2 - D. \quad (1)$$

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241 This means that FD and H are inversely correlated, and that H is a real number  
242 between 0 and 1 (non-inclusive; although it can be extended to  $> 1$ ; see ...). For the  
243 rest of this paper, we will be discussing H.

244

245

246 *Thus, the basic premise behind fractal time-series analysis is that, beneath the seem-*  
247 *ingly chaotic and unpredictable variations in the signal, there lies a stable underlying*  
248 *mechanism which can be effectively described using the fewest possible parameters*  
249 *(i.e. H or FD).*

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## Properties

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255 When  $H = 0.5$  ( $FD = 1.5$ ), the time-series is completely uncorrelated, has no memory,  
256 and is like white noise (pure random). It represents the highest level of unpredictability  
257 and entropy, but not necessarily the most complex state in a physiological sense. When  
258  $H < 0.5$  ( $FD > 1.5$ ), the time-series is said to be anti-persistent, exhibiting negative  
259 correlations: i.e. if the signal increases at one point, it's likely to decrease at the next  
260 point. This is also known as short-term reversal. This tends to make the time-series  
261 more predictable and reduces randomness, often simplifying dynamics. When  $H >$   
262  $0.5$  ( $FD < 1.5$ ), the time-series exhibits positive long-range correlations, and is more  
263 structured. The time-series displays a long-term trend, meaning past states influence  
264 future states.

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There are four main properties that must be present for a time-series to be characterized by H or FD: 1) self-similarity; 2) power law scaling relationship; 3) scale-invariance; and 4) a scaling range (**?@fig-fourprop**). Each of these will be discussed in turn.

## Self-similarity

Self-similarity means that pieces of the structure, when enlarged, resemble larger pieces or the whole (Figure 2 and **?@fig-fourprop A-C**). Technically speaking, physiological time-series are self-affine, meaning their scaling is anisotropic (i.e. the proportions between enlarged pieces are different in one direction from those in the other). This is because in one direction (time) the proportions between the enlarged pieces is different than in the other (amplitude of signal; e.g. fMRI BOLD) [24].

## Power law scaling relationship

Power law scaling means that, for a quantitative property,  $q$ , is measured in quantities of  $s$ , its value depends on  $s$  according to the following scaling relationship:

$$q = f(s). \quad (2)$$

For non-fractal objects, the estimate of  $q$  using progressively smaller units of measure  $s$  will converge to a single value as the size of the measurement units approaches zero. On the other hand, fractals exhibit a power law scaling relationship with  $s$ , whereby the estimated value of  $q$  increases without limit as the size of the  $s$  decreases.

$$q = ps^\epsilon \quad (3)$$

323 where  $p$  is a factor of proportionality (prefactor) and  $\epsilon$  is a negative number, the  
 324 scaling exponent. The value of  $\epsilon$  can be determined as the slope of the linear regression  
 325 fit to the data pairs on the plot of  $\log q$  versus  $\log s$ :  
 326

$$\log q = \log p + \epsilon \log s \quad (4)$$

333 Data points for exact fractals will line up along perfectly with regression slope, while  
334 statistical fractals scatter around it since the two sides of Equation 4 are equal only  
335 in distribution.

## Scale-invariance

The ratio of two estimates of  $q$  measured at two different scales,  $s_1$  and  $s_2$ ,  $q_2/q_1$  depends only on the ratio of scales (relative scale),  $s_2/s_1$ , and not directly on the absolute scale,  $s_1$  or  $s_2$

$$q_2/q_1 = ps_2^\epsilon / ps_1^\epsilon = (s_2/s_1)^\epsilon \quad (5)$$

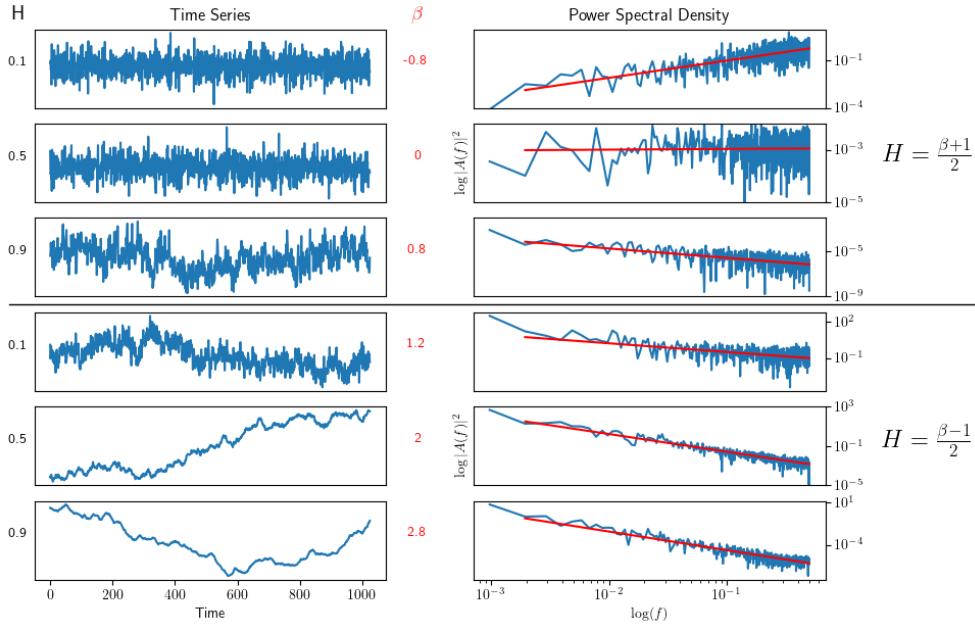
For statistical fractals, like those of nature,  $s_2/s_1$  may change in a continuous fashion still leaving the validity of Equation 5 unaffected. The scale-invariance of fractals arises from the fact that the geometrical structure only depends on the scaling factor (ratio of scales), and not the absolute scale. As a result, quantitative properties of smaller parts are similar to those of larger parts.

## Scaling range

Natural fractals may only display scale-invariance within a restricted range, as they are finite (either by definition or due to the fact that they must be sampled). The upper cut-off point ( $s_{max}$ ) in Equation 6, falls within the size range of the structure itself.

Similarly, the lower cut-off point ( $s_{min}$ ) falls within the dimensions of the smallest structural elements. The scaling range (SR) is defined in decades

$$SR = \log_{10}(s_{max}/s_{min}) \quad (6)$$



**Figure 3. Simulated fractional Gaussian noise and fractional Brownian motion.** Raw simulated time-series with 1,024 time-points and known Hurst values. Signals were created using the Davies-Harte method. a-c) are fractional Gaussian noise while d-f are fractional Brownian motion. a) and d) have H values of 0.1; b) and e) have H values of 0.5; and c) and f) have H values of 0.9. Note how fractional Gaussian noise remain centered around a mean (i.e. stationary), while fractional Brownian motion is free to wander (i.e. non-stationary). Exact fractal time-series were created using the Davies-Harte method.

If the class of signal is not appropriately identified — for example, when  $\beta$  is  $\sim 1$  — it is possible to seriously miscalculate the H value, such that a true  $H \sim 0.9$  will be

415 calculated as 0.1. Therefore, algorithms and research in properly classifying fractal  
416 signals is crucially important.  
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419 **Prerequisites to measuring H**

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421 **Time-points**

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423 Before determining if a time-series exhibits a power-law scaling relationship, one  
424 should first determine if the time-series has enough time-points to do so. The rule  
425 of thumb is that the power law relationship should be present in a range larger  
426 than two decades in the frequency domain of a PSD [24]. Due to the Whittaker–  
427 Nyquist-Shannon sampling theorem [25], one requires two times as many time-points  
428 as frequency samples. Since two decades in the frequency domain is 100 distinct  
429 frequencies, one would required at least 200 time-points to proceed.  
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433 **Power-law scaling relationship**

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435 The next step would be to perform a periodogram or PSD and test for a power-law  
436 scaling relationship. Usually power laws are tested using a probability distribution  
437 [26]. However, as the PSD is not a probability distribution, alternative methods must  
438 be used. A goodness-of-fit test can be devised based on Clauset et al. [26], which can  
439 also help us derive the SR (i.e.  $freq_{min}$  and  $freq_{max}$ ). First, a PSD is produced, and  
440 the variance ( $\sigma^2$ ),  $\beta$ , and H are computed using Equation 8. Next, the Kolmogorov–  
441 Smirnov statistic [27, 28] is used to measure the distance D between the raw PSD data-  
442 points and the best-fit linear-regression line used to calculate H (i.e. the residuals).  
443 The D in this case is the largest residual error. Then, 1,000 time series of either fGn  
444 or fBm (depending on what value of  $\beta$  was found; see Equation 8) with the same  
445 length ( $n > 200$ ),  $\sigma^2$ , and Hurst exponent are generated using one of several methods  
446 for producing exact fractal signals: spectral synthesis method (SSM) [29], Davies–  
447 Harte method (DH) [30], Cholesky method [31], or the Hosking’s method [32]. Each  
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synthetic time-series is converted to a PSD, and the Kolmogorov–Smirnov statistic	461
is used to measure the distance D between the raw PSD data-points and the best-fit	462
linear-regression line. The $p$ -value is defined as the fraction of synthetic time series	463
with Ds that are larger than the original D of the original time-series. The larger the	464
$p$ -value, the more plausible the synthetic model (either fGn or fBm) is for representing	465
the original time-series, and the better the fit of the original data to a scale-free	466
distribution. The null-hypothesis that the time-series is not scale free is rejected if $p$	467
< 0.05 (i.e. if $p > 0.05$ , we say that the time-series is scale-free).	468
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However, this method can be computationally intensive, and prone to false negatives,	475
especially signals with very high or low H values. One solution is, instead of applying	476
this test to every voxel in a 4D fMRI brain scan, to segment the brain into separate	477
ROIs (e.g. anatomically based on an atlas, or functionally by first running ICA to	478
identify RSNs), average the time-series within each ROI, which will improve SNR,	479
and then run the power-law scaling test.	480
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A sample python code is provide in Section 2	486
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<b>Scaling range</b>	489
<b>fGn or fBm</b>	490
<b>Choosing the right method</b>	491
Ideally therefore, a long memory process signal should be categorized as either stationary	492
(fGn) or non-stationary (fBm) and the variance and correlation defined. from [24]	493
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<b>Methods of measuring H</b>	500
Fractal methods are diverse, but their approaches have one thing in common in that	501
they employ equation (2) in fitting their proposed model to data pairs of log feature	502
versus log scale for finding the scaling exponent, , from the regression slope. Each fractal	503
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507 analysis tool has different performance, prerequisite conditions, and limitations, and each  
508 needs thorough evaluation in order to avoid bias or misinterpretation of the derived fractal  
509 parameters [5, 6, 8, 9], especially when applied to physiological signals which may be  
510 contaminated with noise [15, 28]. from [24]  
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## 515 Neuroscience Applications

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517 H has emerged as a valuable tool in neuroscience and clinical research. Typically, H  
518 values reported in adult brains are above 0.5, with higher H values in grey matter  
519 than white matter or cerebrospinal fluid [33, 34]. Some key findings from neuroscience  
520 research include: a decrease in H during task performance [35, 36]; negative corre-  
521 lations with task novelty and difficulty [37]; increases with age in the frontal and  
522 parietal lobes [33], and hippocampus [38]; decreases with age in the insula, and lim-  
523 bic, occipital and temporal lobes [33]; H < 0.5 in preterm infants [39]; and more [40].  
524 In terms of clinical findings, abnormal H values have been identified in Alzheimer's  
525 disease (AD) [41, 42], autism spectrum disorder (ASD) [43–46], mild traumatic brain  
526 injury [47], major depressive disorder [48, 49] and schizophrenia [46, 50].  
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## 530 fMRI preprocessing considerations

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### 533 Nuisance regression

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536 When attempting to regress out non-BOLD signal, it is important to apply the regres-  
537 sion at the same time, and not in succession. Even performing a band-pass filter after  
538 nuisance regression can re-introduce noise components [51].  
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### 543 Detrending

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547 see Tanabe et al. [52]

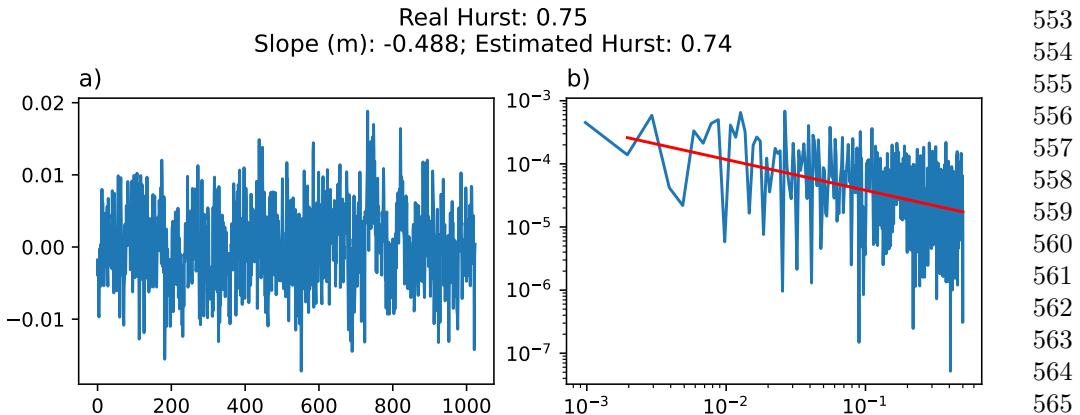
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**Figure 4. Simulated fractional Gaussian noise.** a) Raw simulated time-series with 1,024 time-points and a known Hurst value of 0.75; b) The same signal from a) in a log-log power spectral density plot and a linear-regression used to calculate the estimated Hurst value.

## Hurst Reviews

Nonparametric trend estimation in the presence of fractal noise:

Application to fMRI time-series analysis - Afshinpour et al. (2008)

[53]

In this paper, a method for estimating trend in the presence of fractal noise is proposed and applied to fMRI time-series. To this end, a partly linear model (PLM) is fitted to each time-series. The parametric and nonparametric parts of PLM are considered as contributions of hemodynamic response and trend, respectively. Using the whitening property of wavelet transform, the unknown components of the model are estimated in the wavelet domain. The results of the proposed method are compared to those of other parametric trend-removal approaches such as spline and polynomial models. It is shown that the proposed method improves activation detection and decreases variance of the estimated parameters relative to the other methods.

Notes:

- trend estimation paper
  - 1.5T, 3.9x3.9x6mm, 1.648s TR, 256 time-points
  - Hurst method: Wavelet db4 with 5 scales

## 605 Fractal Analysis of BOLD time-series in a Network Associated With 606

607 Waiting Impulsivity - Akhrif et al. (2018) [54]  
608

examined **103** healthy male students at **rest** and while performing the 5-choice serial reaction time **task**. We addressed fractality in a network associated with waiting impulsivity using the **adaptive fractal analysis (AFA)** approach to determine H. We revealed the fractal nature of the impulsivity network. Furthermore, fractality was influenced by individual impulsivity in terms of decreasing fractality (H) with higher impulsivity in regions of top-down control (left middle frontal gyrus) as well as reward processing (nucleus accumbens and anterior cingulate cortex).

621 Notes:

- fMRI split into low and high frequency components. LFC is the second order polynomial that is a smooth and global fit of the original time course.
  - AFA: variance of fluctuation computed around, in this case, a second order polynomial trend  $v(i)$  fitted to time-series within each segment  $w$ , and its size:

$$F(w) = \sqrt{\frac{1}{N} \sum_{i=1}^N (u(i) - v(i))^2} \sim w^H \quad (7)$$

636 *N*: length of the time-series

$$w = 2n + 1, n = 5, 6, \dots, 13$$

<sup>643</sup> H is determined as the slope of the log-log plot  $\log_2(F(w))$  as a function of  $\log_2(w)$

```

Example code:                                645
                                                646
import numpy as np                         647
                                            648
                                            649
def adaptive_fractal_analysis(signal, n_values=range(5, 14)): 650
    """                                         651
                                                652
                                                653
                                                Perform Adaptive Fractal Analysis (AFA) to compute the Hurst exponent. 654
                                                655
                                                656
Parameters:                                 657
                                                658
                                                659
    signal (array-like): time-series data to analyze.           660
    n_values (iterable): Sequence of `n` values to define window sizes as w = 2n + 1. 661
                                                662
                                                663
Returns:                                    664
                                                665
                                                666
    float: Estimated Hurst exponent (H).                      667
    """
                                                668
# Define window sizes as w = 2n + 1          669
window_sizes = [2 * n + 1 for n in n_values] 670
fluctuations = []                            671
                                                672
                                                673
                                                674
for window_size in window_sizes:            675
    segment_variances = []                  676
    for start in range(0, len(signal) - window_size + 1, window_size): 677
        # Extract the window                678
        window = signal[start:start + window_size] 679
        # Fit a second-order polynomial (quadratic fit) and compute residual 680
        x = np.arange(len(window))           681
        p = np.polyfit(x, window, deg=2)    # Degree 2 polynomial       682
        residual = window - np.polyval(p, x) 683
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691     # Compute variance of the residuals
692     variance = np.var(residual)
693     segment_variances.append(variance)
694
695
696
697     # Compute average variance for this window size
698     fluctuations.append(np.mean(segment_variances))
699
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702     # Fit the scaling law: log(fluctuations) vs. log(window_sizes)
703     log_window_sizes = np.log(window_sizes)
704     log_fluctuations = np.log(fluctuations)
705
706     slope, intercept = np.polyfit(log_window_sizes, log_fluctuations, deg=1)
707
708
709
710     # The slope corresponds to the Hurst exponent
711
712     return slope
713
714
715
716 Endogenous human brain dynamics recover slowly following
717
718 cognitive effort - Barnes et al. (2009) [55]
719
720 1) Does performance of a cognitively effortful task significantly change fractal
721 scaling properties of fMRI time-series compared to their values before task per-
722 formance? 2) If so, can we relate the extent of task-related perturbation to the
723 difficulty of the task? This result supports the model that endogenous low fre-
724 quency oscillatory dynamics are relevant to the brain's response to exogenous
725 stimulation. Moreover, it suggests that large-scale neurocognitive systems mea-
726 sured using fMRI, like the heart and other physiological systems subjected to
727 external demands for enhanced performance, can take a considerable period of
728 time to return to a stable baseline state.
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<b>Notes:</b>	737
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• maximum likelihood in the wavelet domain	740
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<b>Wavelets and functional magnetic resonance imaging of the human brain - Bullmore et al. (2004) [56]</b>	743
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We provide a brief formal introduction to key properties of the DWT and review the growing literature on its application to fMRI. We focus on three applications in particular: (i) wavelet coefficient resampling or “wavestrapping” of 1-D time-series, 2- to 3-D spatial maps and 4-D spatiotemporal processes; (ii) wavelet-based estimators for signal and noise parameters of time-series regression models assuming the errors are fractional Gaussian noise (fGn); and (iii) wavelet shrinkage in frequentist and Bayesian frameworks to support multiresolution hypothesis testing on spatially extended statistic maps.	747
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<b>Notes:</b>	759
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• This paper suggests that motion correction translates many fBm signals to fGn... however, it is not clear where this data comes from.	762
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<b>Fractal-Based Analysis of fMRI BOLD Signal During Naturalistic Viewing Conditions - Campbell et al. (2021) [57]</b>	767
	768
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	770
We performed fractal analysis on Human Connectome Project 7T fMRI data ( $n = 72$ , 41 females, mean age $29.46 \pm 3.76$ years) to compare H across movie-watching and rest. Results: In contrast to previous work using conventional tasks, we found higher H values for movie relative to rest (mean difference = 0.014; $p = 5.279 \times 10^{-7}$ ; 95% CI [0.009, 0.019]). H was significantly higher in movie than rest in the visual, somatomotor and dorsal attention networks, but was significantly lower during movie in the frontoparietal and default networks. We found no cross-condition differences in test-retest reliability of	771
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783 H. Finally, we found that H of movie-derived stimulus properties (e.g., luminance changes)  
784 were fractal whereas H of head motion estimates were non-fractal.  
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787 **Notes:**

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794 **Scale-free brain dynamics under physical and psychological distress:**

795 **Pre-treatment effects in women diagnosed with breast cancer -**

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797 **Churchill et al. (2015) [58]**

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799 In a BOLD functional magnetic resonance imaging study, we scanned three groups during a  
800 working memory task: women scheduled to receive chemotherapy or radiotherapy and aged-  
801 matched controls. Surprisingly, patients' BOLD signal exhibited greater H with increasing  
802 intensity of anticipated treatment. However, an analysis of H and functional connectivity  
803 against self-reported measures of psychological distress (Worry, Anxiety, Depression) and  
804 physical distress (Fatigue, Sleep problems) revealed significant interactions. The modula-  
805 tion of (Worry, Anxiety) versus (Fatigue, Sleep Problems, Depression) showed the strongest  
806 effect, where higher worry and lower fatigue was related to reduced H in regions involved  
807 in visuospatial search, attention, and memory processing. This is also linked to decreased  
808 functional connectivity in these brain regions.

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811 **Notes:**

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819 **The suppression of scale-free fMRI brain dynamics across three**

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821 **different sources of effort: Aging, task novelty and task difficulty -**

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823 **Churchill et al. (2016) [37]**

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Decreases in the Hurst exponent (H), which quantifies scale-free signal, was related to three  
different sources of cognitive effort/task engagement: 1) task difficulty, 2) task novelty,  
and 3) aging effects. These results were consistently observed across multiple datasets

and task paradigms. We also demonstrated that estimates of H are robust across a range of time-window sizes. H was also compared to alternative metrics of BOLD variability (SDBOLD) and global connectivity (Gconn), with effort-related decreases in H producing similar decreases in SDBOLD and Gconn.

**Notes:**

**Interplay between functional connectivity and scale-free dynamics in intrinsic fMRI networks - Ciuciu et al. (2014) [35]**

We applied this framework to fMRI data acquired from healthy young adults at rest and performing a visual detection task. First, we found that scale-invariance existed beyond univariate dynamics, being present also in bivariate cross-temporal dynamics. Second, we observed that frequencies within the scale-free range do not contribute evenly to interregional connectivity, with a systematically stronger contribution of the lowest frequencies, both at rest and during task. Third, in addition to a decrease of the Hurst exponent and inter-regional correlations, task performance modified cross-temporal dynamics, inducing a larger contribution of the highest frequencies within the scale-free range to global correlation. Lastly, we found that across individuals, a weaker task modulation of the frequency contribution to inter-regional connectivity was associated with better task performance manifesting as shorter and less variable reaction times. These findings bring together two related fields that have hitherto been studied separately – resting-state networks and scale-free dynamics, and show that scale-free dynamics of human brain activity manifest in cross-regional interactions as well.

**Notes:**

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875 **Temporal fractal analysis of the rs-BOLD signal identifies brain**  
876 **abnormalities in autism spectrum disorder - Dona et al. (2017) [43]**  
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878     “It is important to mention here that fractal dimension estimation based on a disper-  
879     sional analysis is quite robust with respect to uncorrelated noise and does not require  
880     881     preprocessing”  
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885 **Notes:**

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- 888     • ASD = reduced FD = increased H;  
889     • rare study to properly define fGn vs fBm first?

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892 **Fractal analysis of brain blood oxygenation level dependent (BOLD)**

893 **signals from children with mild traumatic brain injury (mTBI) -**

894     **Dona et al. (2017) [47]**  
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897 **Notes:**

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- 901     • children with mTBI; mTBI = reduced FD = increased H  
902     • rare study to properly define fGn vs fBm first?

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906 **Hurst Exponent Analysis of Resting-State fMRI Signal Complexity**

907 **across the Adult Lifespan - Dong et al. (2018) [33]**

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910     Region-wise and voxel-wise analyses were performed to investigate the effects of age, gen-  
911     der, and their interaction on complexity. In region-wise analysis, we found that the healthy  
912     913     aging is accompanied by a loss of complexity in frontal and parietal lobe and increased  
914     complexity in insula, limbic, and temporal lobe. Meanwhile, differences in HE between gen-  
915     916     ders were found to be significant in parietal lobe ( $p = 0.04$ , corrected). However, there was  
917     918     no interaction between gender and age. In voxel-wise analysis, the significant complexity  
919     decrease with aging was found in frontal and parietal lobe, and complexity increase was  
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found in insula, limbic lobe, occipital lobe, and temporal lobe with aging. Meanwhile, differences in HE between genders were found to be significant in frontal, parietal, and limbic lobe. Furthermore, we found age and sex interaction in right parahippocampal gyrus ( $p = 0.04$ , corrected). Our findings reveal HE variations of the rs-fMRI signal across the human adult lifespan and show that HE may serve as a new parameter to assess healthy aging process.

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- They state that increase in age = decrease in complexity 934  
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**Pitfalls in fractal time-series analysis: fMRI BOLD as an exemplary case - Eke et al. (2012) [59]** 937  
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**Wavelet-Generalized Least Squares: A New BLU Estimator of Linear Regression Models with 1/f Errors - Fadili & Bullmore (2002) [60]** 944  
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**Not in one metric: Neuroticism modulates different resting state metrics within distinctive brain regions - Gentili et al. (2017) [61]** 951  
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Metrics more related to the measurement of regional intrinsic brain activity (fALFF, ALFF and REHO), or that provide a parsimonious index of integrated and segregated brain activity (HE), were more broadly modulated in regions related to emotions and their regulation. Metrics related to connectivity were modulated across a wider network of areas. Overall, these results show that neuroticism affects distinct aspects of brain resting state activity.

967 **Notes:**

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- 970 • “parsimonious index of integrated and segregated brain activity (HE)”

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- 972 • HE was inversely correlated to neuroticism

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975 **Proneness to social anxiety modulates neural complexity in the**

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977 **absence of exposure: A resting state fMRI study using Hurst**

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979 **exponent - Gentili et al. (2015) [62]**

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981 Results from fALFF were highly consistent with those obtained using LSAS and BFNE  
982 to predict HE. Overall our data indicate that spontaneous brain activity is influenced by  
983 the degree of social anxiety, on a continuum and in the absence of social stimuli. These  
984 findings suggest that social anxiety is a trait characteristic that shapes brain activity and  
985 predisposes to different reactions in social contexts.

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988 **Notes:**

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- 993 • “A recent article (Rubin et al., 2013) analyzes the robustness of different algorithms  
994 with respect to possible fMRI artifacts and time-series lengths. In particular, the  
995 relevance of preprocessing steps as motion correction, detrending and filtering were  
996 evaluated both on simulated and real fMRI data, while other preprocessing steps  
997 like segmentation were not evaluated, although they may have an impact on”

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- 999 • “The HE of fMRI time-series is generally higher in gray matter than in white matter  
1000 (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with  
1001 cholinergic transmission enhancement (Wink et al., 2006)”

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- 1003 • “As pointed out by Maxim (Maxim et al., 2005), fMRI noise, after these pre-  
1004 processing steps, can be described as fGn.”

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- “The HE of fMRI time-series is generally higher in gray matter than in white matter (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with cholinergic transmission enhancement (Wink et al., 2006).” 1013  
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- “a reduction of HE has been observed in autistic and schizophrenic patients (Lai et al., 2010; Sokunbi et al., 2014)” 1018  
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- “Since the HE can describe long and short range memory dynamics, it has been proposed as a measure of online information-processing efficiency: higher HEs are related to long memory dynamics and to higher temporal redundancy and less freedom to vary (He, 2011).” 1021  
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### **Real-time fractal signal processing in the time domain. - Hartmann et al. (2013) [63]**

Here we introduce real-time variants of the Detrended Fluctuation Analysis (DFA) and the closely related Signal Summation Conversion (SSC) methods, which are suitable to estimate the fractal exponent in one pass.

### **Altered fractal dynamics of gait: Reduced stride-interval correlations with aging and Huntington’s disease. - Hausdoff et al. (1997) [64]**

#### **Notes:**

- Gait... not fMRI

### **Scale-Free Properties of the Functional Magnetic Resonance Imaging Signal during Rest and Task - He (2011) [36]**

its power-law exponent differentiates between brain networks and correlates with fMRI signal variance and brain glucose metabolism. Importantly, in parallel to brain electrical field potentials, the variance and power-law exponent of the fMRI signal decrease during

task activation, suggesting that the signal contains more long-range memory during rest and conversely is more efficient at online information processing during task. The scale-free properties of the fMRI signal and brain electrical field potentials bespeak their respective stationarity and nonstationarity. This suggests that neurovascular coupling mechanism is likely to contain a transformation from nonstationarity to stationarity.

The fMRI signal time course from each ROI was extracted for each subject and fMRI run. The normalized or non-normalized power spectrum of the fMRI signal was computed using the Bartlett smoothing procedure of deriving the power spectral function from the lagged autocorrelation or auto-covariance function, respectively (Jenkins and Watts, 1998). A Tukey window of 20 fMRI frame width was applied for additional smoothing. The power spectra were then averaged across runs and subjects and across homologous ROIs, resulting in an average power spectrum for each of 21 brain regions (Fig. 2A). Finally, to obtain the power-law exponent  $\beta$ , the  $<0.1$  Hz range of each average power spectrum was fit with a power-law function:  $P(f) \propto f^{-\beta}$  using a least-squares fit. Using the low-frequency range to fit the power-law exponent avoids aliasing artifact in higher-frequency range (we used TR of 2.16 s, hence Nyquist limit is 0.23 Hz) and yields reliable measurement of the scale-free distribution (Eke et al., 2002).

The DFA method has the particular advantage of being applicable to both stationary and nonstationary data. To analyze our fMRI data, window lengths of 5, 10, 19, 38, and 95 fMRI frames were chosen so that the number of frames in each run (190 after discarding the first four frames) is an integer multiple of the window length.

<b>Fractal characterization of complexity in dynamic signals:</b>	1105
<b>Application to cerebral hemodynamics - Herman (2009) [12]</b>	1106
<b>Identification of brain activity from fMRI data: Comparison of three fractal scaling analyses. - Hu (2006) [65]</b>	1107
<b>A shift to randomness of brain oscillations in people with autism.</b>	1108
<b>Lai (2010) [44]</b>	1109
Complex fractal scaling of fMRI time-series was found in both groups but globally there was a significant shift to randomness in the ASC (mean $H = .758$ , $SD = .045$ ) compared with neurotypical volunteers (mean $H = .788$ , $SD = .047$ ).	1110
<b>Extraversion is encoded by scale-free dynamics of default mode network. Lei (2013) [66]</b>	1111
<b>Fractional Gaussian noise, functional MRI and Alzheimer's disease.</b>	1112
<b>Maxim (2005) [41]</b>	1113
we adopted the Davies-Harte algorithm, which is both exact and fast, to generate the fGn simulations used here. For each value of $H = 0.1, \dots 0.9$ , we simulated 1000 realizations of fGn with 512 time-points in each series; we set $\omega^2 = 1$ for all simulations.	1114
<b>NOTES:</b>	1115
• This paper has the figure showing signal goes from fBm to fGn with proper motion regression	1116
<b>Decomposing multifractal crossovers. Nagy (2017) [67]</b>	1117
The NIRS and fMRI-BOLD low-frequency fluctuations were dominated by a multifractal component over an underlying biologically relevant random noise, thus forming a bimodal	1118

1151 signal. The crossover between the EEG signal components was found at the boundary  
1152 between the and bands, suggesting an independent generator for the multifractal  
1153 rhythm. The robust implementation of the SFD method should be regarded as essential  
1154 in the seamless processing of large volumes of bimodal fMRI-BOLD imaging data for the  
1155 topology of multifractal metrics free of the masking effect of the underlying random noise.  
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1160 **Optimizing complexity measures for FMRI data: Algorithm,  
1161 artifact, and sensitivity. Rubin (2013) [68]**  
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1164 Power-spectrum, Higuchi's fractal dimension, and generalized Hurst exponent based esti-  
1165 mates were most successful by all criteria; the poorest-performing measures were wavelet,  
1166 detrended fluctuation analysis, aggregated variance, and rescaled range. Our results clearly  
1167 demonstrate that decisions regarding choice of algorithm, signal processing, time-series  
1168 length, and scanner have a significant impact on the reliability and sensitivity of com-  
1169 plexity estimates. operating on the edge of chaos, complex systems position themselves  
1170 for optimal responsivity to inputs, as well as ability to maintain homeostatic regulation.  
1171 Daubechies wavelet based computations (Hdb) *have long computation times, are not sen-*  
1172 *sitive to spikes, and show poor sensitivity to activation, tissue type, and emotional content;*  
1173 *for these Daubechies wavelet based estimates the overall performance increases with the*  
1174 *wavelet order up to a point (Hdb8), and then deteriorates. HRS and HAV, performed poorly*  
1175 *across the board. In terms of image contrast, overlap with activation, and group differences,*  
1176 *HDFA performed poorly as well, with HDFA-S outperforming HDFA and HDFA-L, sug-*  
1177 *gesting that the bulk of useful information is found at shorter lags. The most consistently*  
1178 *successful measures were the powerspectrum based measures HFFT and HpWelch, with*  
1179 *the latter slightly outperforming the former while taking much longer to compute Second,*  
1180 *it appears that detrending, regressing out the global mean, and excluding low frequencies*  
1181 *improves agreement between complexity and activation. 300-600 time-points Finally, the*  
1182 *best measures to use are either the power-spectrum based ones (HFFT or HpWelch) on a*  
1183 *restricted frequency range (above ,0.01 Hz),*  
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<b>Mutual information identifies spurious Hurst phenomena in resting state EEG and fMRI data. von Wegner (2018) [69]</b>	1197
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1703 **Appendix**

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1706 **Python code**

1707

1708 **Sample python code for testing power-law scaling**

1709

```
1710 print("HELLO")
1711
```

1712

```
1713 , color="r", fontsize=14) ax[1][1].text(0.002, .002, r'$H=0.8
1714
```

1715

1716 **Gaussian noise vs Brownian motion**

1717

1718 Time-series that are scale-invariant come in two categories: fractional Gaussian noise  
1719

1720 (fGn) and fractional Brownian motion (fBm) Figure 7. fGn signals are stationary,  
1721

1722 meaning they tend to center themselves along a mean value over time. In contrast,  
1723

1724 fBm signals are non-stationary, and their values tend to wander away from the mean.  
1725

1726 fGn and fBm can easily be converted from one to the other: fGn to fBm by applying a  
1727

1728 successive summation between elements of the fGn series; and fBm to fGn by applying  
1729 successive differences between elements of a fBm series.  
1730

1731 This introduces the first method of measuring H of a time-series: using a log-log  
1732

1733 plot of its power spectral density (**?@fig-fourprop D and E**), calculating the linear-  
1734

1735 regression slope, and, depending on the slope angle, using one of two equations:  
1736

1737

$$H = \begin{cases} \frac{\beta+1}{2}, & \text{if } -1 < \beta < 1 \\ \frac{\beta-1}{2}, & \text{if } 1 < \beta < 3 \end{cases} \quad (8)$$

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1743

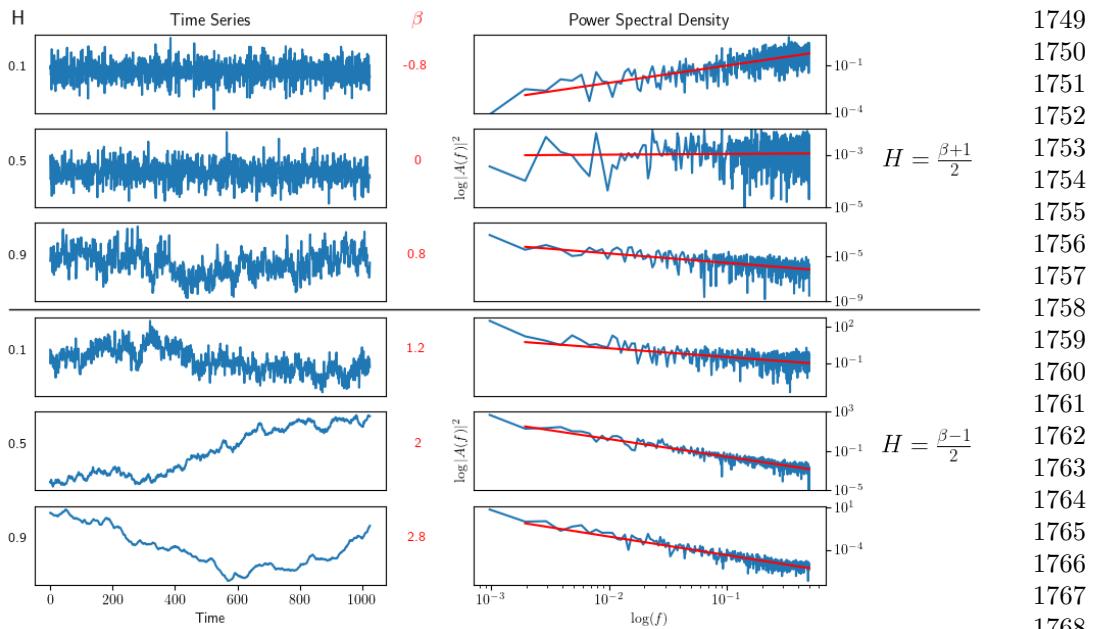
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**Figure 5. Simulated fractional Gaussian noise and fractional Brownian motion.** Raw simulated time-series with 1,024 time-points and known Hurst values. Signals were created using the Davies-Harte method. a-c) are fractional Gaussian noise while d-f are fractional Brownian motion. a) and d) have  $H$  values of 0.1; b) and e) have  $H$  values of 0.5; and c) and f) have  $H$  values of 0.9. Note how fractional Gaussian noise remain centered around a mean (i.e. stationary), while fractional Brownian motion is free to wander (i.e. non-stationary). Exact fractal time-series were created using the Davies-Harte method.

If the class of signal is not appropriately identified — for example, when  $\beta$  is  $\sim 1$  — it is possible to seriously miscalculate the  $H$  value, such that a true  $H \sim 0.9$  will be calculated as 0.1. Therefore, algorithms and research in properly classifying fractal signals is crucially important.

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1795 **Prerequisites to measuring H**

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1797 **Time-points**

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1799 Before determining if a time-series exhibits a power-law scaling relationship, one  
1800 should first determine if the time-series has enough time-points to do so. The rule  
1802  
1803 of thumb is that the power law relationship should be present in a range larger  
1804 than two decades in the frequency domain of a PSD [24]. Due to the Whittaker–  
1805 Nyquist-Shannon sampling theorem [25], one requires two times as many time-points  
1807  
1808 as frequency samples. Since two decades in the frequency domain is 100 distinct  
1809 frequencies, one would required at least 200 time-points to proceed.  
1810

1811

1812 **Power-law scaling relationship**

1813

1814 The next step would be to perform a periodogram or PSD and test for a power-law  
1815  
1816 scaling relationship. Usually power laws are tested using a probability distribution  
1817  
1818 [26]. However, as the PSD is not a probability distribution, alternative methods must  
1819 be used. A goodness-of-fit test can be devised based on Clauset et al. [26], which can  
1820 also help us derive the SR (i.e.  $freq_{min}$  and  $freq_{max}$ ). First, a PSD is produced, and  
1822  
1823 the variance ( $\sigma^2$ ),  $\beta$ , and H are computed using Equation 8. Next, the Kolmogorov–  
1824 Smirnov statistic [27, 28] is used to measure the distance D between the raw PSD data-  
1825  
1826 points and the best-fit linear-regression line used to calculate H (i.e. the residuals).  
1827  
1828 The D in this case is the largest residual error. Then, 1,000 time series of either fGn  
1829  
1830 or fBm (depending on what value of  $\beta$  was found; see Equation 8) with the same  
1831 length ( $n > 200$ ),  $\sigma^2$ , and Hurst exponent are generated using one of several methods  
1832  
1833 for producing exact fractal signals: spectral synthesis method (SSM) [29], Davies–  
1834 Harte method (DH) [30], Cholesky method [31], or the Hosking’s method [32]. Each  
1835  
1836 synthetic time-series is converted to a PSD, and the Kolmogorov–Smirnov statistic  
1837  
1838 is used to measure the distance D between the raw PSD data-points and the best-fit  
1839 linear-regression line. The  $p$ -value is defined as the fraction of synthetic time series  
1840

with Ds that are larger than the original D of the original time-series. The larger the	1841
<i>p</i> -value, the more plausible the synthetic model (either fGn or fBm) is for representing	1842
the original time-series, and the better the fit of the original data to a scale-free	1843
distribution. The null-hypothesis that the time-series is not scale free is rejected if <i>p</i>	1844
< 0.05 (i.e. if <i>p</i> > 0.05, we say that the time-series is scale-free).	1845
	1846
	1847
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	1849
However, this method can be computationally intensive, and prone to false negatives,	1850
especially signals with very high or low H values. One solution is, instead of applying	1851
this test to every voxel in a 4D fMRI brain scan, to segment the brain into separate	1852
ROIs (e.g. anatomically based on an atlas, or functionally by first running ICA to	1853
identify RSNs), average the time-series within each ROI, which will improve SNR,	1854
and then run the power-law scaling test.	1855
	1856
	1857
	1858
	1859
	1860
A sample python code is provide in Section 2	1861
	1862
<b>Scaling range</b>	1863
<b>fGn or fBm</b>	1864
<b>Choosing the right method</b>	1865
Ideally therefore, a long memory process signal should be categorized as either stationary	1866
(fGn) or non-stationary (fBm) and the variance and correlation defined. from [24]	1867
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<b>Methods of measuring H</b>	1875
Fractal methods are diverse, but their approaches have one thing in common in that	1876
they employ equation (2) in fitting their proposed model to data pairs of log feature	1877
versus log scale for finding the scaling exponent, , from the regression slope. Each fractal	1878
analysis tool has different performance, prerequisite conditions, and limitations, and each	1879
needs thorough evaluation in order to avoid bias or misinterpretation of the derived fractal	1880
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1887 parameters [5, 6, 8, 9], especially when applied to physiological signals which may be  
1888 contaminated with noise [15, 28]. from [24]  
1889

1890

1891 **Neuroscience Applications**

1892

1893 H has emerged as a valuable tool in neuroscience and clinical research. Typically, H  
1894 values reported in adult brains are above 0.5, with higher H values in grey matter  
1895 than white matter or cerebrospinal fluid [33, 34]. Some key findings from neuroscience  
1896 research include: a decrease in H during task performance [35, 36]; negative corre-  
1897 lations with task novelty and difficulty [37]; increases with age in the frontal and  
1898 parietal lobes [33], and hippocampus [38]; decreases with age in the insula, and lim-  
1899 bric, occipital and temporal lobes [33]; H < 0.5 in preterm infants [39]; and more [40].  
1900

1901 In terms of clinical findings, abnormal H values have been identified in Alzheimer's  
1902 disease (AD) [41, 42], autism spectrum disorder (ASD) [43–46], mild traumatic brain  
1903 injury [47], major depressive disorder [48, 49] and schizophrenia [46, 50].  
1904

1905

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1907 **fMRI preprocessing considerations**

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1909 **Nuissance regression**

1910

1911 When attempting to regress out non-BOLD signal, it is important to apply the regres-  
1912 sion at the same time, and not in succession. Even performing a band-pass filter after  
1913 nuissance regression can re-introduce noise components [51].  
1914

1915

1916 **Detrending**

1917

1918 see Tanabe et al. [52]

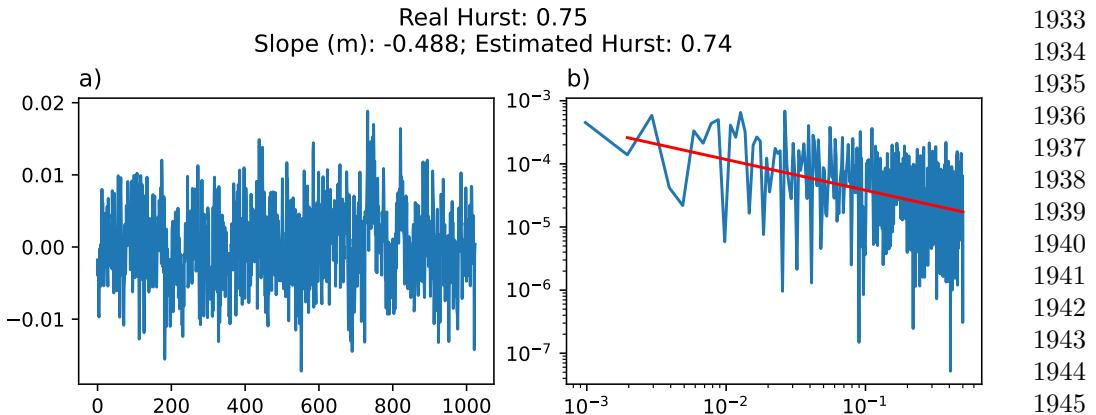
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**Figure 6. Simulated fractional Gaussian noise.** a) Raw simulated time-series with 1,024 time-points and a known Hurst value of 0.75; b) The same signal from a) in a log-log power spectral density plot and a linear-regression used to calculate the estimated Hurst value.

## Hurst Reviews

**Nonparametric trend estimation in the presence of fractal noise:**

**Application to fMRI time-series analysis - Afshinpour et al. (2008)**

[53]

In this paper, a method for estimating trend in the presence of fractal noise is proposed and applied to fMRI time-series. To this end, a partly linear model (PLM) is fitted to each time-series. The parametric and nonparametric parts of PLM are considered as contributions of hemodynamic response and trend, respectively. Using the whitening property of wavelet transform, the unknown components of the model are estimated in the wavelet domain. The results of the proposed method are compared to those of other parametric trend-removal approaches such as spline and polynomial models. It is shown that the proposed method improves activation detection and decreases variance of the estimated parameters relative to the other methods.

**Notes:**

1979 • trend estimation paper  
 1980  
 1981 • 1.5T, 3.9x3.9x6mm, 1.648s TR, 256 time-points  
 1982 • Hurst method: Wavelet db4 with 5 scales  
 1983  
 1984  
 1985 **Fractal Analysis of BOLD time-series in a Network Associated With**  
 1986  
 1987 **Waiting Impulsivity - Akhrif et al. (2018) [54]**  
 1988  
 1989 examined **103** healthy male students at **rest** and while performing the 5-choice serial  
 1990 reaction time **task**. We addressed fractality in a network associated with waiting impulsiv-  
 1991 ity using the **adaptive fractal analysis (AFA)** approach to determine H. We revealed  
 1992 the fractal nature of the impulsivity network. Furthermore, fractality was influenced by  
 1993 individual impulsivity in terms of decreasing fractality (H) with higher impulsivity in  
 1994 regions of top-down control (left middle frontal gyrus) as well as reward processing (nucleus  
 1995 accumbens and anterior cingulate cortex).  
 1996  
 1997  
 1998  
 1999  
 2000  
 2001 **Notes:**  
 2002  
 2003  
 2004 • fMRI split into low and high frequency components. LFC is the second order  
 2005 polynomial that is a smooth and global fit of the original time course.  
 2006  
 2007 • AFA: variance of fluctuation computed around, in this case, a second order  
 2008 polynomial trend  $v(i)$  fitted to time-series within each segment  $w$ , and its size:  
 2009  
 2010  
 2011  
 2012  
 2013 
$$F(w) = \sqrt{\frac{1}{N} \sum_{i=1}^N (u(i) - v(i))^2} \sim w^H \quad (9)$$
  
 2014  
 2015  
 2016  $N$ : length of the time-series  
 2017  
 2018  
 2019  
 2020  $w = 2n + 1, n = 5, 6, \dots, 13$   
 2021  
 2022  
 2023 H is determined as the slope of the log-log plot  $\log_2(F(w))$  as a function of  $\log_2(w)$   
 2024

Example code:

```
import numpy as np  
  
def adaptive_fractal_analysis(signal, n_values=range(5, 14)):  
    """  
        Perform Adaptive Fractal Analysis (AFA) to compute the Hurst exponent.  
  
    Parameters:  
        signal (array-like): time-series data to analyze.  
        n_values (iterable): Sequence of `n` values to define window sizes as  $w = 2n + 1$ .  
  
    Returns:  
        float: Estimated Hurst exponent ( $H$ ).  
    """  
  
    # Define window sizes as  $w = 2n + 1$   
    window_sizes = [2 * n + 1 for n in n_values]  
    fluctuations = []  
  
    for window_size in window_sizes:  
        segment_variances = []  
        for start in range(0, len(signal) - window_size + 1, window_size):  
            # Extract the window  
            window = signal[start:start + window_size]  
            # Fit a second-order polynomial (quadratic fit) and compute residual  
            x = np.arange(len(window))  
            p = np.polyfit(x, window, deg=2)  # Degree 2 polynomial  
            residual = window - np.polyval(p, x)  
            segment_variances.append(np.var(residual))  
        fluctuations.append(np.sqrt(np.var(segment_variances)))  
  
    return fluctuations
```

```

2071     # Compute variance of the residuals
2072     variance = np.var(residual)
2073     segment_variances.append(variance)
2074
2075
2076
2077     # Compute average variance for this window size
2078     fluctuations.append(np.mean(segment_variances))
2079
2080
2081
2082     # Fit the scaling law: log(fluctuations) vs. log(window_sizes)
2083     log_window_sizes = np.log(window_sizes)
2084
2085     log_fluctuations = np.log(fluctuations)
2086
2087     slope, intercept = np.polyfit(log_window_sizes, log_fluctuations, deg=1)
2088
2089
2090
2091     # The slope corresponds to the Hurst exponent
2092
2093     return slope
2094
2095
2096 Endogenous human brain dynamics recover slowly following
2097
2098 cognitive effort - Barnes et al. (2009) [55]
2099
2100 1) Does performance of a cognitively effortful task significantly change fractal
2101 scaling properties of fMRI time-series compared to their values before task per-
2102 formance? 2) If so, can we relate the extent of task-related perturbation to the
2103 difficulty of the task? This result supports the model that endogenous low fre-
2104 quency oscillatory dynamics are relevant to the brain's response to exogenous
2105 stimulation. Moreover, it suggests that large-scale neurocognitive systems mea-
2106 sured using fMRI, like the heart and other physiological systems subjected to
2107 external demands for enhanced performance, can take a considerable period of
2108 time to return to a stable baseline state.
2109
2110
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2116

```

<b>Notes:</b>	2117
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	2119
• maximum likelihood in the wavelet domain	2120
	2121
	2122
<b>Wavelets and functional magnetic resonance imaging of the human brain - Bullmore et al. (2004) [56]</b>	2123
	2124
	2125
	2126
We provide a brief formal introduction to key properties of the DWT and review the growing literature on its application to fMRI. We focus on three applications in particular: (i) wavelet coefficient resampling or “wavestrapping” of 1-D time-series, 2- to 3-D spatial maps and 4-D spatiotemporal processes; (ii) wavelet-based estimators for signal and noise parameters of time-series regression models assuming the errors are fractional Gaussian noise (fGn); and (iii) wavelet shrinkage in frequentist and Bayesian frameworks to support multiresolution hypothesis testing on spatially extended statistic maps.	2127
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<b>Notes:</b>	2139
	2140
	2141
• This paper suggests that motion correction translates many fBm signals to fGn... however, it is not clear where this data comes from.	2142
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	2146
<b>Fractal-Based Analysis of fMRI BOLD Signal During Naturalistic Viewing Conditions - Campbell et al. (2021) [57]</b>	2147
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2163 H. Finally, we found that H of movie-derived stimulus properties (e.g., luminance changes)  
2164 were fractal whereas H of head motion estimates were non-fractal.  
2165

2166

2167 **Notes:**

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2170 •

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2174 **Scale-free brain dynamics under physical and psychological distress:**

2175 **Pre-treatment effects in women diagnosed with breast cancer -**  
2176

2177 **Churchill et al. (2015) [58]**

2178

2179 In a BOLD functional magnetic resonance imaging study, we scanned three groups during a  
2180 working memory task: women scheduled to receive chemotherapy or radiotherapy and aged-  
2181 matched controls. Surprisingly, patients' BOLD signal exhibited greater H with increasing  
2182 intensity of anticipated treatment. However, an analysis of H and functional connectivity  
2183 against self-reported measures of psychological distress (Worry, Anxiety, Depression) and  
2184 physical distress (Fatigue, Sleep problems) revealed significant interactions. The modula-  
2185 tion of (Worry, Anxiety) versus (Fatigue, Sleep Problems, Depression) showed the strongest  
2186 effect, where higher worry and lower fatigue was related to reduced H in regions involved  
2187 in visuospatial search, attention, and memory processing. This is also linked to decreased  
2188 functional connectivity in these brain regions.

2189

2190

2191 **Notes:**

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2198

2199 **The suppression of scale-free fMRI brain dynamics across three**

2200

2201 **different sources of effort: Aging, task novelty and task difficulty -**

2202

2203 **Churchill et al. (2016) [37]**

2204

2205

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2208

Decreases in the Hurst exponent (H), which quantifies scale-free signal, was related to three  
different sources of cognitive effort/task engagement: 1) task difficulty, 2) task novelty,  
and 3) aging effects. These results were consistently observed across multiple datasets

and task paradigms. We also demonstrated that estimates of H are robust across a range  
of time-window sizes. H was also compared to alternative metrics of BOLD variability  
(SDBOLD) and global connectivity (Gconn), with effort-related decreases in H producing  
similar decreases in SDBOLD and Gconn.

**Notes:**

**Interplay between functional connectivity and scale-free dynamics in  
intrinsic fMRI networks - Ciuciu et al. (2014) [35]**

We applied this framework to fMRI data acquired from healthy young adults at rest and  
performing a visual detection task. First, we found that scale-invariance existed beyond  
univariate dynamics, being present also in bivariate cross-temporal dynamics. Second, we  
observed that frequencies within the scale-free range do not contribute evenly to interre-  
gional connectivity, with a systematically stronger contribution of the lowest frequencies,  
both at rest and during task. Third, in addition to a decrease of the Hurst exponent and  
inter-regional correlations, task performance modified cross-temporal dynamics, inducing  
a larger contribution of the highest frequencies within the scale-free range to global corre-  
lation. Lastly, we found that across individuals, a weaker task modulation of the frequency  
contribution to inter-regional connectivity was associated with better task performance  
manifesting as shorter and less variable reaction times. These findings bring together two  
related fields that have hitherto been studied separately – resting-state networks and scale-  
free dynamics, and show that scale-free dynamics of human brain activity manifest in  
cross-regional interactions as well.

**Notes:**

2255 **Temporal fractal analysis of the rs-BOLD signal identifies brain**  
2256  
2257 **abnormalities in autism spectrum disorder - Dona et al. (2017) [43]**  
2258  
2259 "It is important to mention here that fractal dimension estimation based on adisper-  
2260 sional analysis is quite robust with respect to uncorrelated noise and does not require  
2261 preprocessing"  
2263  
2264  
2265 **Notes:**  
2266  
2267  
2268 • ASD = reduced FD = increased H;  
2269 • rare study to properly define fGn vs fBm first?  
2271  
2272  
2273 **Fractal analysis of brain blood oxygenation level dependent (BOLD)**  
2274 **signals from children with mild traumatic brain injury (mTBI) -**  
2275  
2276 **Dona et al. (2017) [47]**  
2277  
2278 **Notes:**  
2279  
2280  
2281 • children with mTBI; mTBI = reduced FD = increased H  
2282  
2283 • rare study to properly define fGn vs fBm first?  
2284  
2285  
2286 **Hurst Exponent Analysis of Resting-State fMRI Signal Complexity**  
2287  
2288 **across the Adult Lifespan - Dong et al. (2018) [33]**  
2289  
2290 Region-wise and voxel-wise analyses were performed to investigate the effects of age, gen-  
2291 der, and their interaction on complexity. In region-wise analysis, we found that the healthy  
2292 aging is accompanied by a loss of complexity in frontal and parietal lobe and increased  
2293 complexity in insula, limbic, and temporal lobe. Meanwhile, differences in HE between gen-  
2294 ders were found to be significant in parietal lobe ( $p = 0.04$ , corrected). However, there was  
2297 no interaction between gender and age. In voxel-wise analysis, the significant complexity  
2298 decrease with aging was found in frontal and parietal lobe, and complexity increase was  
2300

found in insula, limbic lobe, occipital lobe, and temporal lobe with aging. Meanwhile, differences in HE between genders were found to be significant in frontal, parietal, and limbic lobe. Furthermore, we found age and sex interaction in right parahippocampal gyrus ( $p = 0.04$ , corrected). Our findings reveal HE variations of the rs-fMRI signal across the human adult lifespan and show that HE may serve as a new parameter to assess healthy aging process.

**Notes:**

- They state that increase in age = decrease in complexity

**Pitfalls in fractal time-series analysis: fMRI BOLD as an exemplary case - Eke et al. (2012) [59]**

**Wavelet-Generalized Least Squares: A New BLU Estimator of Linear Regression Models with 1/f Errors - Fadili & Bullmore (2002) [60]**

**Not in one metric: Neuroticism modulates different resting state metrics within distinctive brain regions - Gentili et al. (2017) [61]**

Metrics more related to the measurement of regional intrinsic brain activity (fALFF, ALFF and REHO), or that provide a parsimonious index of integrated and segregated brain activity (HE), were more broadly modulated in regions related to emotions and their regulation. Metrics related to connectivity were modulated across a wider network of areas. Overall, these results show that neuroticism affects distinct aspects of brain resting state activity.

2347 **Notes:**

2348

2349

2350 • “parsimonious index of integrated and segregated brain activity (HE)”

2351

2352 • HE was inversely correlated to neuroticism

2353

2354

2355 **Proneness to social anxiety modulates neural complexity in the**

2356

2357 **absence of exposure: A resting state fMRI study using Hurst**

2358

2359 **exponent - Gentili et al. (2015) [62]**

2360

2361 Results from fALFF were highly consistent with those obtained using LSAS and BFNE  
2362 to predict HE. Overall our data indicate that spontaneous brain activity is influenced by  
2363 the degree of social anxiety, on a continuum and in the absence of social stimuli. These  
2364 findings suggest that social anxiety is a trait characteristic that shapes brain activity and  
2365 predisposes to different reactions in social contexts.

2366

2367

2368 **Notes:**

2369

2370

2371

2372

2373 • “A recent article (Rubin et al., 2013) analyzes the robustness of different algorithms  
2374 with respect to possible fMRI artifacts and time-series lengths. In particular, the  
2375 relevance of preprocessing steps as motion correction, detrending and filtering were  
2376 evaluated both on simulated and real fMRI data, while other preprocessing steps  
2377 like segmentation were not evaluated, although they may have an impact on”

2378

2379 • “The HE of fMRI time-series is generally higher in gray matter than in white matter  
2380 (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with  
2381 cholinergic transmission enhancement (Wink et al., 2006)”

2382

2383 • “As pointed out by Maxim (Maxim et al., 2005), fMRI noise, after these pre-  
2384 processing steps, can be described as fGn.”

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• “The HE of fMRI time-series is generally higher in gray matter than in white matter (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with cholinergic transmission enhancement (Wink et al., 2006).”	2393 2394 2395 2396 2397
• “a reduction of HE has been observed in autistic and schizophrenic patients (Lai et al., 2010; Sokunbi et al., 2014)”	2398 2399 2400
• “Since the HE can describe long and short range memory dynamics, it has been proposed as a measure of online information-processing efficiency: higher HEs are related to long memory dynamics and to higher temporal redundancy and less freedom to vary (He, 2011).”	2401 2402 2403 2404 2405 2406 2407 2408
<b>Real-time fractal signal processing in the time domain. - Hartmann et al. (2013) [63]</b>	2409 2410 2411 2412
Here we introduce real-time variants of the Detrended Fluctuation Analysis (DFA) and the closely related Signal Summation Conversion (SSC) methods, which are suitable to estimate the fractal exponent in one pass.	2413 2414 2415 2416 2417 2418 2419 2420
<b>Altered fractal dynamics of gait: Reduced stride-interval correlations with aging and Huntington’s disease. - Hausdoff et al. (1997) [64]</b>	2421 2422
<b>Notes:</b>	2423 2424 2425 2426 2427 2428 2429 2430
• Gait... not fMRI	2431 2432 2433 2434 2435 2436 2437 2438
<b>Scale-Free Properties of the Functional Magnetic Resonance Imaging Signal during Rest and Task - He (2011) [36]</b>	
its power-law exponent differentiates between brain networks and correlates with fMRI signal variance and brain glucose metabolism. Importantly, in parallel to brain electrical field potentials, the variance and power-law exponent of the fMRI signal decrease during	

2439 task activation, suggesting that the signal contains more long-range memory during rest  
2440 and conversely is more efficient at online information processing during task. The scale-free  
2441 properties of the fMRI signal and brain electrical field potentials bespeak their respective  
2442 stationarity and nonstationarity. This suggests that neurovascular coupling mechanism is  
2443 likely to contain a transformation from nonstationarity to stationarity.  
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2448 The fMRI signal time course from each ROI was extracted for each subject and fMRI  
2449 run. The normalized or non-normalized power spectrum of the fMRI signal was computed  
2450 using the Bartlett smoothing procedure of deriving the power spectral function from the  
2451 lagged autocorrelation or auto-covariance function, respectively (Jenkins and Watts, 1998).  
2452 A Tukey window of 20 fMRI frame width was applied for additional smoothing. The power  
2453 spectra were then averaged across runs and subjects and across homologous ROIs, resulting  
2454 in an average power spectrum for each of 21 brain regions (Fig. 2A). Finally, to obtain the  
2455 power-law exponent , the <0.1 Hz range of each average power spectrum was fit with a  
2456 power-law function:  $P(f) \propto 1/f^{\alpha}$  using a least-squares fit. Using the low-frequency range to  
2457 fit the power-law exponent avoids aliasing artifact in higher-frequency range (we used TR  
2458 of 2.16 s, hence Nyquist limit is 0.23 Hz) and yields reliable measurement of the scale-free  
2459 distribution (Eke et al., 2002).  
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2467 The DFA method has the particular advantage of being applicable to both stationary and  
2468 nonstationary data. To analyze our fMRI data, window lengths of 5, 10, 19, 38, and 95  
2469 fMRI frames were chosen so that the number of frames in each run (190 after discarding  
2470 the first four frames) is an integer multiple of the window length.  
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<b>Fractal characterization of complexity in dynamic signals:</b>	2485
<b>Application to cerebral hemodynamics - Herman (2009) [12]</b>	2486
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<b>Identification of brain activity from fMRI data: Comparison of three fractal scaling analyses. - Hu (2006) [65]</b>	2489
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	2492
<b>A shift to randomness of brain oscillations in people with autism.</b>	2493
<b>Lai (2010) [44]</b>	2494
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	2496
Complex fractal scaling of fMRI time-series was found in both groups but globally there was a significant shift to randomness in the ASC (mean $H = .758$ , $SD = .045$ ) compared with neurotypical volunteers (mean $H = .788$ , $SD = .047$ ).	2497
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<b>Extraversion is encoded by scale-free dynamics of default mode network. Lei (2013) [66]</b>	2503
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<b>Fractional Gaussian noise, functional MRI and Alzheimer's disease.</b>	2507
<b>Maxim (2005) [41]</b>	2508
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we adopted the Davies-Harte algorithm, which is both exact and fast, to generate the fGn simulations used here. For each value of $H = 0.1, \dots, 0.9$ , we simulated 1000 realizations of fGn with 512 time-points in each series; we set $\omega^2 = 1$ for all simulations.	2511
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<b>NOTES:</b>	2517
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• This paper has the figure showing signal goes from fBm to fGn with proper motion regression	2520
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<b>Decomposing multifractal crossovers. Nagy (2017) [67]</b>	2525
	2526
The NIRS and fMRI-BOLD low-frequency fluctuations were dominated by a multifractal component over an underlying biologically relevant random noise, thus forming a bimodal	2527
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2531 signal. The crossover between the EEG signal components was found at the boundary  
2532 between the and bands, suggesting an independent generator for the multifractal  
2533 rhythm. The robust implementation of the SFD method should be regarded as essential  
2534 in the seamless processing of large volumes of bimodal fMRI-BOLD imaging data for the  
2535 topology of multifractal metrics free of the masking effect of the underlying random noise.  
2536  
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2540 **Optimizing complexity measures for fMRI data: Algorithm,  
2541 artifact, and sensitivity. Rubin (2013) [68]**  
2543

2544 Power-spectrum, Higuchi's fractal dimension, and generalized Hurst exponent based esti-  
2545 mates were most successful by all criteria; the poorest-performing measures were wavelet,  
2546 detrended fluctuation analysis, aggregated variance, and rescaled range. Our results clearly  
2547 demonstrate that decisions regarding choice of algorithm, signal processing, time-series  
2548 length, and scanner have a significant impact on the reliability and sensitivity of com-  
2549 plexity estimates. operating on the edge of chaos, complex systems position themselves  
2550 for optimal responsivity to inputs, as well as ability to maintain homeostatic regulation.  
2551 Daubechies wavelet based computations (Hdb) *have long computation times, are not sen-*  
2552 *sitive to spikes, and show poor sensitivity to activation, tissue type, and emotional content;*  
2553 *for these Daubechies wavelet based estimates the overall performance increases with the*  
2554 *wavelet order up to a point (Hdb8), and then deteriorates. HRS and HAV, performed poorly*  
2555 *across the board. In terms of image contrast, overlap with activation, and group differences,*  
2556 *HDFA performed poorly as well, with HDFA-S outperforming HDFA and HDFA-L, sug-*  
2557 *gesting that the bulk of useful information is found at shorter lags. The most consistently*  
2558 *successful measures were the powerspectrum based measures HFFT and HpWelch, with*  
2559 *the latter slightly outperforming the former while taking much longer to compute Second,*  
2560 *it appears that detrending, regressing out the global mean, and excluding low frequencies*  
2561 *improves agreement between complexity and activation. 300-600 time-points Finally, the*  
2562 *best measures to use are either the power-spectrum based ones (HFFT or HpWelch) on a*  
2563 *restricted frequency range (above ,0.01 Hz),*  
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<b>Mutual information identifies spurious Hurst phenomena in resting state EEG and fMRI data. von Wegner (2018) [69]</b>	2577
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## 2623 **References**

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<b>Appendix</b>	2669
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<b>Python code</b>	2671
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<b>Sample python code for testing power-law scaling</b>	2675
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<b>print("HELLO")</b>	2677
	2678
,   color="r",   fontsize=14)   ax[1][1].set_title("Log-log plot of D",   loc='left')	2679
ax[1][1].set_xlabel('Frequency (f)') ax[1][1].set_ylabel(r'Amplitude ( $ A(f) ^2$ )')	2680
	2681
newpower=power fp5 = FBM(scale1, hurst=0.5, length=1, method='daviesharte')	2682
fgnp5   =   fp5.fgn()5   p5freq,   p5power   =   signal.periodogram(fgnp5,   1)	2683
p5power=p5power - np.mean(p5power) + predicted_power[-1]/1.4 newpower[:10]=	2684
(np.random.rand(10)+0.5) # 10 was the power value around the low frequency	2685
newpower[312:] = p5power[312:512]    ax[2][1].plot(freq.reshape(-1,1),    newpower)	2686
ax[2][1].plot(freq[1:].reshape(-1,1)[10:312],   predicted_power[10:312],   color='red')	2687
ax[2][1].axvline(x=freq[10],   color='red',   linestyle='--')   ax[2][1].axvline(x=freq[312],	2688
color='red',   linestyle='--')   ax[2][1].set_xscale('log')   ax[2][1].set_yscale('log')	2689
ax[2][1].set_xlabel('Frequency (f)') ax[2][1].set_title("E but with a scaling range",	2690
loc='left')	2691
	2692
text_xy = (freq[10]*1.7,  0.001) ax[2][1].annotate(text, (freq[10],0.001), text_xy,	2693
arrowprops={"arrowstyle": "->", "color": "red"}, color="red") ax[2][1].annotate(text,	2694
(freq[312],0.001),  text_xy,  arrowprops={"arrowstyle": "->",  "color": "red"},	2695
color="red")	2696
	2697
for i   in  range(3):  ax[i,  0].text(-0.1,  1.1,  string.ascii_uppercase[i],  trans-	2698
form=ax[i,  0].transAxes,  size=13,  weight='bold')  ax[i,  1].text(-0.1,  1.1,	2699
string.ascii_uppercase[i+3], transform=ax[i, 1].transAxes, size=13, weight='bold')	2700
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```

2715 plt.savefig('..../image/fourproperties.png', dpi=600, bbox_inches='tight',
2716 format='png') plt.show()
2718
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2720
2721 :::.cell-output .cell-output-display}
2722 ! [I don't get why this isn't working] (index_files/figure-latex/notebooks-Figures-fig-fourp
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2747 ## Gaussian noise vs Brownian motion
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2750
2751 Time-series that are scale-invariant come in two categories: fractional Gaussian noise (fG
2752
2753
2754 <!-- TODO: find where to put this: -->
2755
2756 This introduces the first method of measuring H of a time-series: using a log-log plot of
2757
2758
2759 $$
2760

```

```

H =
2761
\begin{cases}
2762
\frac{\beta + 1}{2}, & \text{if } -1 < \beta < 1 \\
2764
\frac{\beta - 1}{2}, & \text{if } 1 < \beta < 3
2765
\end{cases}
2766
\$ \$ \{\#eq-psdH\}
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:::{.quarto-embed-nb-cell notebook="/home/weberam2/Dropbox/AssistantProf_B2MRI/Projects/Brainl
::: {.cell execution_count=19}
``` {.python .cell-code .hidden}
np.random.seed(1)
plt.rcParams['text.usetex'] = True #latex
def psd(timeseries):
    freq, power = signal.periodogram(timeseries, 1)
    power = power[1:]
    freq = freq[1:]

    log_power = np.log10(power)
    log_freq = np.log10(freq)
    log_power = log_power[1:]

```

```

2807     log_freq = log_freq[1:]
2808
2809
2810     model = LinearRegression()
2811
2812     model.fit(log_freq.reshape(-1,1), log_power)
2813
2814     predicted_power = 10**model.predict(log_freq.reshape(-1,1))
2815
2816
2817     return freq.reshape(-1,1), power, predicted_power
2818
2819
2820 fp5 = FBM(1024, hurst=0.5, length=1, method='daviesharte')
2821
2822 fp1 = FBM(1024, hurst=0.1, length=1, method='daviesharte')
2823
2824 fp9 = FBM(1024, hurst=0.9, length=1, method='daviesharte')
2825
2826 fgnp5 = fp5.fgn()
2827
2828 fgnp1 = fp1.fgn()
2829
2830 fgnp9 = fp9.fgn()
2831
2832 fbmp5 = fp5.fbm()
2833
2834 fbmp1 = fp1.fbm()
2835
2836 fbmp9 = fp9.fbm()
2837
2838
2839 fig, ax = plt.subplots(6,2, figsize=(10,7))
2840
2841 fig.subplots_adjust(wspace=0.25)
2842
2843 ax[0,0].plot(fgnp1)
2844
2845 ax[0,0].set_title("Time Series")
2846
2847 ax[0,0].xaxis.set_visible(False)
2848
2849 ax[0,0].set_ylabel("0.1", rotation=0, labelpad=12)
2850
2851 ax[0,0].text(1.05, 0.55, "-0.8", rotation=0,
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```

```

ax[1,0].plot(fgnp5)                                     2853
ax[1,0].xaxis.set_visible(False)                         2854
ax[1,0].set_yticks([])                                  2855
ax[1,0].set_ylabel("0.5", rotation=0, labelpad=12)       2856
ax[1,0].text(1.08, 0.55, "0", rotation=0,               2857
             transform=ax[1,0].transAxes, color='red')      2858
ax[2,0].plot(fgnp9)                                     2859
ax[2,0].xaxis.set_visible(False)                         2860
ax[2,0].set_yticks([])                                  2861
ax[2,0].set_ylabel("0.9", rotation=0, labelpad=12)       2862
ax[2,0].text(1.06, 0.55, "0.8", rotation=0,              2863
             transform=ax[2,0].transAxes, color='red')      2864
ax[3,0].plot(fbmp1)                                    2865
ax[3,0].xaxis.set_visible(False)                         2866
ax[3,0].set_yticks([])                                  2867
ax[3,0].set_ylabel("0.1", rotation=0, labelpad=12)       2868
ax[3,0].text(1.06, 0.55, "1.2", rotation=0,              2869
             transform=ax[3,0].transAxes, color='red')      2870
ax[4,0].plot(fbmp5)                                    2871
ax[4,0].xaxis.set_visible(False)                         2872
ax[4,0].set_yticks([])                                  2873
ax[4,0].set_ylabel("0.5", rotation=0, labelpad=12)       2874
ax[4,0].text(1.08, 0.55, "2", rotation=0,                2875
             transform=ax[4,0].transAxes, color='red')      2876
ax[5,0].plot(fbmp9)                                    2877
ax[5,0].set_yticks([])                                  2878
ax[5,0].set_ylabel("0.9", rotation=0, labelpad=12)       2879

```

```

2899 ax[5,0].set_xlabel("Time")
2900
2901 ax[5,0].text(1.06, 0.55, "2.8", rotation=0,
2902         transform=ax[5,0].transAxes, color='red')
2903
2904
2905
2906 line = lines.Line2D([0.1, 1.05], [0.495, 0.495], transform=fig.transFigure, color='black',
2907 fig.add_artist(line)
2908
2909 fig.text(1, 0.705, r"$H=\frac{\beta+1}{2}$", ha='center', va='center', fontsize=18, transform=fig.transFigure)
2910 fig.text(1, 0.305, r"$H=\frac{\beta-1}{2}$", ha='center', va='center', fontsize=18, transform=fig.transFigure)
2911
2912
2913
2914 fig.text(0.11, .9, r"H", fontsize=14, ha='center', va='center', transform=fig.transFigure)
2915 fig.text(0.5, .9, r"$\beta$", fontsize=14, ha='center', va='center', transform=fig.transFigure)
2916
2917
2918
2919 freq,power,pred_power = psd(fgnp1)
2920
2921 ax[0,1].plot(freq,power)
2922 ax[0,1].set_title("Power Spectral Density")
2923
2924 ax[0,1].plot(freq[1:], pred_power, color='red')
2925
2926 ax[0,1].set_xscale('log')
2927 ax[0,1].set_yscale('log')
2928
2929 ax[0,1].xaxis.set_visible(False)
2930
2931 ax[0,1].yaxis.tick_right()
2932 ax[0,1].set_ylim(1e-4,10)
2933
2934
2935
2936 freq,power,pred_power = psd(fgnp5)
2937
2938 ax[1,1].plot(freq,power)
2939 ax[1,1].plot(freq[1:], pred_power, color='red')
2940
2941 ax[1,1].set_xscale('log')
2942 ax[1,1].set_yscale('log')
2943
2944

```

```

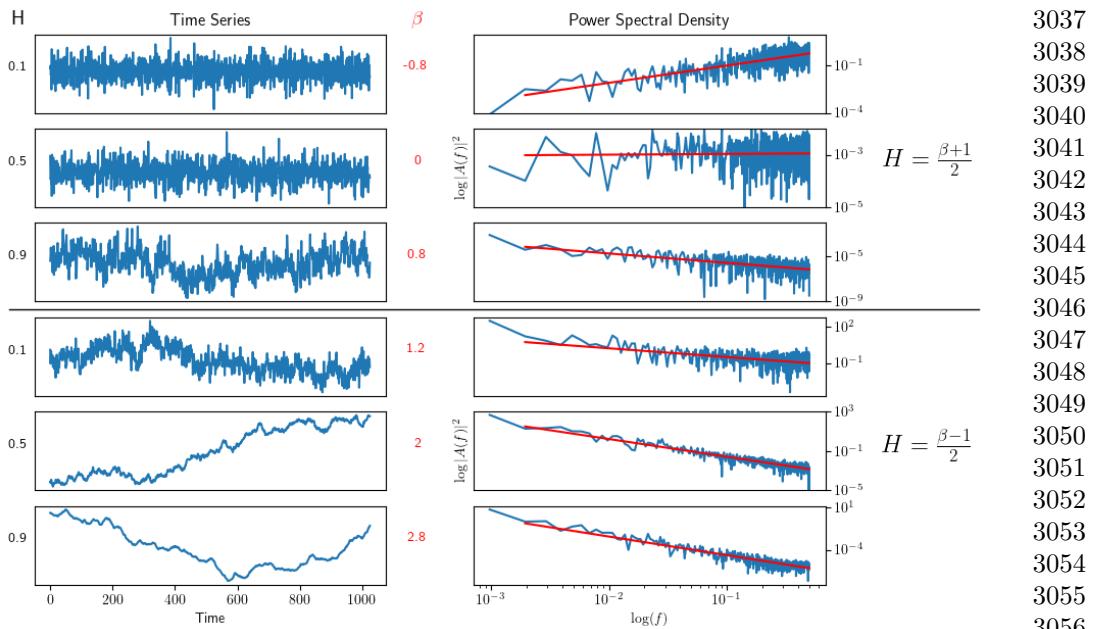
ax[1,1].xaxis.set_visible(False)                                2945
ax[1,1].yaxis.tick_right()                                    2946
ax[1,1].set_ylabel(r"$\log|A(f)|^2$")                         2947
ax[1,1].set_ylim(1e-5,1e-2)                                  2948
   2949
   2950
   2951
   2952
freq,power,pred_power = psd(fgnp9)                           2953
   2954
ax[2,1].plot(freq,power)                                     2955
   2956
ax[2,1].plot(freq[1:], pred_power, color='red')             2957
   2958
ax[2,1].set_xscale('log')                                    2959
   2960
ax[2,1].set_yscale('log')                                    2961
   2962
ax[2,1].xaxis.set_visible(False)                            2963
   2964
ax[2,1].yaxis.tick_right()                                2965
   2966
ax[2,1].set_ylim(1e-9,1e-2)                                2967
   2968
freq,power,pred_power = psd(fbmp1)                          2969
   2970
ax[3,1].plot(freq,power)                                     2971
   2972
ax[3,1].plot(freq[1:], pred_power, color='red')            2973
   2974
ax[3,1].set_xscale('log')                                    2975
   2976
ax[3,1].set_yscale('log')                                    2977
   2978
ax[3,1].xaxis.set_visible(False)                            2979
   2980
ax[3,1].yaxis.tick_right()                                2981
   2982
freq,power,pred_power = psd(fbmp5)                          2983
   2984
ax[4,1].plot(freq,power)                                     2985
   2986
ax[4,1].plot(freq[1:], pred_power, color='red')            2987
   2988
ax[4,1].set_xscale('log')                                    2989
   2990

```

```

2991 ax[4,1].xaxis.set_visible(False)
2992
2993 ax[4,1].set_ylabel(r"$|\log|A(f)|^2$")
2994 ax[4,1].yaxis.tick_right()
2995
2996 ax[4,1].set_ylim(1e-5,1e3)
2997
2998
2999 freq,power,pred_power = psd(fbmp9)
3000
3001 ax[5,1].plot(freq,power)
3002
3003 ax[5,1].plot(freq[1:], pred_power, color='red')
3004 ax[5,1].set_xscale('log')
3005
3006 ax[5,1].set_yscale('log')
3007
3008 #ax[5,1].set_xticklabels([])
3009 ax[5,1].yaxis.tick_right()
3010
3011 ax[5,1].set_xlabel("$\log(f)$")
3012
3013
3014 plt.savefig('../image/typicalsamplepaths.png', dpi=600, bbox_inches='tight', format='png')
3015
3016 plt.show()
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**Figure 7. Simulated fractional Gaussian noise and fractional Brownian motion.** Raw simulated time-series with 1,024 time-points and known Hurst values. Signals were created using the Davies-Harte method. a-c) are fractional Gaussian noise while d-f are fractional Brownian motion. a) and d) have H values of 0.1; b) and e) have H values of 0.5; and c) and f) have H values of 0.9. Note how fractional Gaussian noise remain centered around a mean (i.e. stationary), while fractional Brownian motion is free to wander (i.e. non-stationary). Exact fractal time-series were created using the Davies-Harte method.

⋮

⋮

If the class of signal is not appropriately identified — for example, when  $\beta$  is  $\sim 1$  — it is possible to seriously miscalculate the H value, such that a true  $H \sim 0.9$  will be calculated as 0.1. Therefore, algorithms and research in properly classifying fractal signals is crucially important.

3083 **Prerequisites to measuring H**

3084

3085 **Time-points**

3086

3087 Before determining if a time-series exhibits a power-law scaling relationship, one  
3088 should first determine if the time-series has enough time-points to do so. The rule  
3090 of thumb is that the power law relationship should be present in a range larger  
3092 than two decades in the frequency domain of a PSD [24]. Due to the Whittaker–  
3093 Nyquist-Shannon sampling theorem [25], one requires two times as many time-points  
3095 as frequency samples. Since two decades in the frequency domain is 100 distinct  
3097 frequencies, one would required at least 200 time-points to proceed.  
3098

3099

3100 **Power-law scaling relationship**

3101

3102 The next step would be to perform a periodogram or PSD and test for a power-law  
3103 scaling relationship. Usually power laws are tested using a probability distribution  
3105 [26]. However, as the PSD is not a probability distribution, alternative methods must  
3107 be used. A goodness-of-fit test can be devised based on Clauset et al. [26], which can  
3108 also help us derive the SR (i.e.  $freq_{min}$  and  $freq_{max}$ ). First, a PSD is produced, and  
3110 the variance ( $\sigma^2$ ),  $\beta$ , and H are computed using Equation 8. Next, the Kolmogorov–  
3112 Smirnov statistic [27, 28] is used to measure the distance D between the raw PSD data-  
3113 points and the best-fit linear-regression line used to calculate H (i.e. the residuals).  
3114 The D in this case is the largest residual error. Then, 1,000 time series of either fGn  
3116 or fBm (depending on what value of  $\beta$  was found; see Equation 8) with the same  
3118 length ( $n > 200$ ),  $\sigma^2$ , and Hurst exponent are generated using one of several methods  
3120 for producing exact fractal signals: spectral synthesis method (SSM) [29], Davies–  
3122 Harte method (DH) [30], Cholesky method [31], or the Hosking’s method [32]. Each  
3123 synthetic time-series is converted to a PSD, and the Kolmogorov–Smirnov statistic  
3125 is used to measure the distance D between the raw PSD data-points and the best-fit  
3127 linear-regression line. The  $p$ -value is defined as the fraction of synthetic time series  
3128

with Ds that are larger than the original D of the original time-series. The larger the	3129
<i>p</i> -value, the more plausible the synthetic model (either fGn or fBm) is for representing	3130
the original time-series, and the better the fit of the original data to a scale-free	3131
distribution. The null-hypothesis that the time-series is not scale free is rejected if <i>p</i>	3132
< 0.05 (i.e. if <i>p</i> > 0.05, we say that the time-series is scale-free).	3133
	3134
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	3137
However, this method can be computationally intensive, and prone to false negatives,	3138
especially signals with very high or low H values. One solution is, instead of applying	3139
this test to every voxel in a 4D fMRI brain scan, to segment the brain into separate	3140
ROIs (e.g. anatomically based on an atlas, or functionally by first running ICA to	3141
identify RSNs), average the time-series within each ROI, which will improve SNR,	3142
and then run the power-law scaling test.	3143
	3144
	3145
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	3148
A sample python code is provide in Section 2	3149
	3150
<b>Scaling range</b>	3151
<b>fGn or fBm</b>	3152
<b>Choosing the right method</b>	3153
Ideally therefore, a long memory process signal should be categorized as either stationary	3154
(fGn) or non-stationary (fBm) and the variance and correlation defined. from [24]	3155
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<b>Methods of measuring H</b>	3163
Fractal methods are diverse, but their approaches have one thing in common in that	3164
they employ equation (2) in fitting their proposed model to data pairs of log feature	3165
versus log scale for finding the scaling exponent, , from the regression slope. Each fractal	3166
analysis tool has different performance, prerequisite conditions, and limitations, and each	3167
needs thorough evaluation in order to avoid bias or misinterpretation of the derived fractal	3168
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3175 parameters [5, 6, 8, 9], especially when applied to physiological signals which may be  
3176 contaminated with noise [15, 28]. from [24]  
3177

3178

3179

## 3180 **Neuroscience Applications**

3181

3182 H has emerged as a valuable tool in neuroscience and clinical research. Typically, H  
3183 values reported in adult brains are above 0.5, with higher H values in grey matter  
3184 than white matter or cerebrospinal fluid [33, 34]. Some key findings from neuroscience  
3185 research include: a decrease in H during task performance [35, 36]; negative corre-  
3186 lations with task novelty and difficulty [37]; increases with age in the frontal and  
3187 parietal lobes [33], and hippocampus [38]; decreases with age in the insula, and lim-  
3188 3190 bic, occipital and temporal lobes [33]; H < 0.5 in preterm infants [39]; and more [40].  
3191

3192 In terms of clinical findings, abnormal H values have been identified in Alzheimer's  
3193 disease (AD) [41, 42], autism spectrum disorder (ASD) [43–46], mild traumatic brain  
3194 3196 injury [47], major depressive disorder [48, 49] and schizophrenia [46, 50].  
3197

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3199

## 3200 **fMRI preprocessing considerations**

3201

### 3202 **Nuissance regression**

3203

3204 When attempting to regress out non-BOLD signal, it is important to apply the regres-  
3205 sion at the same time, and not in succession. Even performing a band-pass filter after  
3206 3207 nuissance regression can re-introduce noise components [51].  
3208

3209

### 3210 **Detrending**

3211

3212 see Tanabe et al. [52]

3213

3214

3215

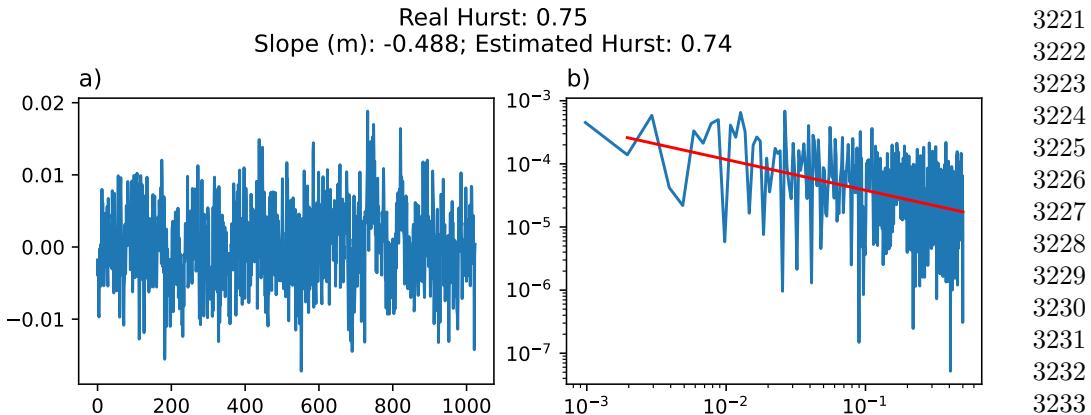
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**Figure 8. Simulated fractional Gaussian noise.** a) Raw simulated time-series with 1,024 time-points and a known Hurst value of 0.75; b) The same signal from a) in a log-log power spectral density plot and a linear-regression used to calculate the estimated Hurst value.

## Hurst Reviews

Nonparametric trend estimation in the presence of fractal noise:

Application to fMRI time-series analysis - Afshinpour et al. (2008)

[53]

In this paper, a method for estimating trend in the presence of fractal noise is proposed and applied to fMRI time-series. To this end, a partly linear model (PLM) is fitted to each time-series. The parametric and nonparametric parts of PLM are considered as contributions of hemodynamic response and trend, respectively. Using the whitening property of wavelet transform, the unknown components of the model are estimated in the wavelet domain. The results of the proposed method are compared to those of other parametric trend-removal approaches such as spline and polynomial models. It is shown that the proposed method improves activation detection and decreases variance of the estimated parameters relative to the other methods.

Notes:

3267 • trend estimation paper

3268 • 1.5T, 3.9x3.9x6mm, 1.648s TR, 256 time-points  
3269

3270 • Hurst method: Wavelet db4 with 5 scales

3271

3272

## 3273 Fractal Analysis of BOLD time-series in a Network Associated With 3274

3275 Waiting Impulsivity - Akhrif et al. (2018) [54]

3276

examined **103** healthy male students at **rest** and while performing the 5-choice serial reaction time **task**. We addressed fractality in a network associated with waiting impulsivity using the **adaptive fractal analysis (AFA)** approach to determine H. We revealed the fractal nature of the impulsivity network. Furthermore, fractality was influenced by individual impulsivity in terms of decreasing fractality (H) with higher impulsivity in regions of top-down control (left middle frontal gyrus) as well as reward processing (nucleus accumbens and anterior cingulate cortex).

3281  
2009

3288

3289 Notes:  
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3290

3291

- fMRI split into low and high frequency components. LFC is the second order polynomial that is a smooth and global fit of the original time course.

• AFA: variance of fluctuation computed around, in this case, a second order polynomial trend  $v(i)$  fitted to time-series within each segment  $w$ , and its size:

3299

$$F(w) = \sqrt{\frac{1}{N} \sum_{i=1}^N (u(i) - v(i))^2} \sim w^H \quad (10)$$

3303

3304 N: length of the time-series

3305

3306

$$w = 2n + 1, n = 5, 6, \dots, 13$$

3309  
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```

Example code:                                3313
  3314
import numpy as np                         3315
  3316
  3317
def adaptive_fractal_analysis(signal, n_values=range(5, 14)): 3318
    """  3319
  3320
  3321
  3322
  3323
  3324
  3325
  3326
  3327
  3328 as w = 2n + 1.
  3329
  3330
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    Perform Adaptive Fractal Analysis (AFA) to compute the Hurst exponent.

Parameters:
    signal (array-like): time-series data to analyze.
    n_values (iterable): Sequence of `n` values to define window sizes
    Returns:
        float: Estimated Hurst exponent (H).
    """
# Define window sizes as w = 2n + 1
window_sizes = [2 * n + 1 for n in n_values]
fluctuations = []

for window_size in window_sizes:
    segment_variances = []
    for start in range(0, len(signal) - window_size + 1, window_size):
        # Extract the window
        window = signal[start:start + window_size]
        # Fit a second-order polynomial (quadratic fit) and compute residual
        x = np.arange(len(window))
        p = np.polyfit(x, window, deg=2) # Degree 2 polynomial
        residual = window - np.polyval(p, x)

```

```

3359         # Compute variance of the residuals
3360         variance = np.var(residual)
3361         segment_variances.append(variance)
3362
3363
3364
3365
3366         # Compute average variance for this window size
3367         fluctuations.append(np.mean(segment_variances))
3368
3369
3370
3371         # Fit the scaling law: log(fluctuations) vs. log(window_sizes)
3372         log_window_sizes = np.log(window_sizes)
3373         log_fluctuations = np.log(fluctuations)
3374
3375         slope, intercept = np.polyfit(log_window_sizes, log_fluctuations, deg=1)
3376
3377
3378
3379         # The slope corresponds to the Hurst exponent
3380
3381     return slope
3382
3383
3384 Endogenous human brain dynamics recover slowly following
3385
3386 cognitive effort - Barnes et al. (2009) [55]
3387
3388 1) Does performance of a cognitively effortful task significantly change fractal
3389 scaling properties of fMRI time-series compared to their values before task per-
3390 formance? 2) If so, can we relate the extent of task-related perturbation to the
3391 difficulty of the task? This result supports the model that endogenous low fre-
3392 quency oscillatory dynamics are relevant to the brain's response to exogenous
3393 stimulation. Moreover, it suggests that large-scale neurocognitive systems mea-
3394 sured using fMRI, like the heart and other physiological systems subjected to
3395 external demands for enhanced performance, can take a considerable period of
3396 time to return to a stable baseline state.
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```

<b>Notes:</b>	3405
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	3407
• maximum likelihood in the wavelet domain	3408
	3409
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<b>Wavelets and functional magnetic resonance imaging of the human brain - Bullmore et al. (2004) [56]</b>	3411
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	3414
We provide a brief formal introduction to key properties of the DWT and review the growing literature on its application to fMRI. We focus on three applications in particular: (i) wavelet coefficient resampling or “wavestrapping” of 1-D time-series, 2- to 3-D spatial maps and 4-D spatiotemporal processes; (ii) wavelet-based estimators for signal and noise parameters of time-series regression models assuming the errors are fractional Gaussian noise (fGn); and (iii) wavelet shrinkage in frequentist and Bayesian frameworks to support multiresolution hypothesis testing on spatially extended statistic maps.	3415
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<b>Notes:</b>	3427
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• This paper suggests that motion correction translates many fBm signals to fGn... however, it is not clear where this data comes from.	3430
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<b>Fractal-Based Analysis of fMRI BOLD Signal During Naturalistic Viewing Conditions - Campbell et al. (2021) [57]</b>	3435
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We performed fractal analysis on Human Connectome Project 7T fMRI data ( $n = 72$ , 41 females, mean age $29.46 \pm 3.76$ years) to compare H across movie-watching and rest. Results: In contrast to previous work using conventional tasks, we found higher H values for movie relative to rest (mean difference = 0.014; $p = 5.279 \times 10^{-7}$ ; 95% CI [0.009, 0.019]). H was significantly higher in movie than rest in the visual, somatomotor and dorsal attention networks, but was significantly lower during movie in the frontoparietal and default networks. We found no cross-condition differences in test-retest reliability of	3439
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3451 H. Finally, we found that H of movie-derived stimulus properties (e.g., luminance changes)  
3452 were fractal whereas H of head motion estimates were non-fractal.  
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3455 **Notes:**  
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3461 **Scale-free brain dynamics under physical and psychological distress:**  
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3463 **Pre-treatment effects in women diagnosed with breast cancer -**  
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3465 **Churchill et al. (2015) [58]**  
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3467 In a BOLD functional magnetic resonance imaging study, we scanned three groups during a  
3468 working memory task: women scheduled to receive chemotherapy or radiotherapy and aged-  
3469 matched controls. Surprisingly, patients' BOLD signal exhibited greater H with increasing  
3470 intensity of anticipated treatment. However, an analysis of H and functional connectivity  
3471 against self-reported measures of psychological distress (Worry, Anxiety, Depression) and  
3472 physical distress (Fatigue, Sleep problems) revealed significant interactions. The modula-  
3473 tion of (Worry, Anxiety) versus (Fatigue, Sleep Problems, Depression) showed the strongest  
3474 effect, where higher worry and lower fatigue was related to reduced H in regions involved  
3475 in visuospatial search, attention, and memory processing. This is also linked to decreased  
3476 functional connectivity in these brain regions.  
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3480 **Notes:**  
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3484 **The suppression of scale-free fMRI brain dynamics across three**  
3485 **different sources of effort: Aging, task novelty and task difficulty -**  
3486

3487 **Churchill et al. (2016) [37]**  
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3489 Decreases in the Hurst exponent (H), which quantifies scale-free signal, was related to three  
3490 different sources of cognitive effort/task engagement: 1) task difficulty, 2) task novelty,  
3491 and 3) aging effects. These results were consistently observed across multiple datasets  
3492

and task paradigms. We also demonstrated that estimates of H are robust across a range of time-window sizes. H was also compared to alternative metrics of BOLD variability (SDBOLD) and global connectivity (Gconn), with effort-related decreases in H producing similar decreases in SDBOLD and Gconn.

**Notes:**

**Interplay between functional connectivity and scale-free dynamics in intrinsic fMRI networks - Ciuciu et al. (2014) [35]**

We applied this framework to fMRI data acquired from healthy young adults at rest and performing a visual detection task. First, we found that scale-invariance existed beyond univariate dynamics, being present also in bivariate cross-temporal dynamics. Second, we observed that frequencies within the scale-free range do not contribute evenly to interregional connectivity, with a systematically stronger contribution of the lowest frequencies, both at rest and during task. Third, in addition to a decrease of the Hurst exponent and inter-regional correlations, task performance modified cross-temporal dynamics, inducing a larger contribution of the highest frequencies within the scale-free range to global correlation. Lastly, we found that across individuals, a weaker task modulation of the frequency contribution to inter-regional connectivity was associated with better task performance manifesting as shorter and less variable reaction times. These findings bring together two related fields that have hitherto been studied separately – resting-state networks and scale-free dynamics, and show that scale-free dynamics of human brain activity manifest in cross-regional interactions as well.

**Notes:**

3543 **Temporal fractal analysis of the rs-BOLD signal identifies brain**  
3544  
3545 **abnormalities in autism spectrum disorder - Dona et al. (2017) [43]**  
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3547 "It is important to mention here that fractal dimension estimation based on adisper-  
3548 sional analysis is quite robust with respect to uncorrelated noise and does not require  
3549 preprocessing"  
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3552  
3553 **Notes:**  
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3556 • ASD = reduced FD = increased H;  
3557 • rare study to properly define fGn vs fBm first?  
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3561 **Fractal analysis of brain blood oxygenation level dependent (BOLD)**  
3562 **signals from children with mild traumatic brain injury (mTBI) -**  
3563  
3564 **Dona et al. (2017) [47]**  
3565  
3566 **Notes:**  
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3569 • children with mTBI; mTBI = reduced FD = increased H  
3570  
3571 • rare study to properly define fGn vs fBm first?  
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3574 **Hurst Exponent Analysis of Resting-State fMRI Signal Complexity**  
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3576 **across the Adult Lifespan - Dong et al. (2018) [33]**  
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3578 Region-wise and voxel-wise analyses were performed to investigate the effects of age, gen-  
3579 der, and their interaction on complexity. In region-wise analysis, we found that the healthy  
3580 aging is accompanied by a loss of complexity in frontal and parietal lobe and increased  
3581 complexity in insula, limbic, and temporal lobe. Meanwhile, differences in HE between gen-  
3582 ders were found to be significant in parietal lobe ( $p = 0.04$ , corrected). However, there was  
3583 no interaction between gender and age. In voxel-wise analysis, the significant complexity  
3584 decrease with aging was found in frontal and parietal lobe, and complexity increase was  
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found in insula, limbic lobe, occipital lobe, and temporal lobe with aging. Meanwhile, differences in HE between genders were found to be significant in frontal, parietal, and limbic lobe. Furthermore, we found age and sex interaction in right parahippocampal gyrus ( $p = 0.04$ , corrected). Our findings reveal HE variations of the rs-fMRI signal across the human adult lifespan and show that HE may serve as a new parameter to assess healthy aging process. 3589  
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**Notes:**

- They state that increase in age = decrease in complexity

**Pitfalls in fractal time-series analysis: fMRI BOLD as an exemplary case - Eke et al. (2012) [59]**

**Wavelet-Generalized Least Squares: A New BLU Estimator of Linear Regression Models with 1/f Errors - Fadili & Bullmore (2002) [60]**

**Not in one metric: Neuroticism modulates different resting state metrics within distinctive brain regions - Gentili et al. (2017) [61]**

Metrics more related to the measurement of regional intrinsic brain activity (fALFF, ALFF and REHO), or that provide a parsimonious index of integrated and segregated brain activity (HE), were more broadly modulated in regions related to emotions and their regulation. Metrics related to connectivity were modulated across a wider network of areas. Overall, these results show that neuroticism affects distinct aspects of brain resting state activity.

3635 **Notes:**

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3638 • “parsimonious index of integrated and segregated brain activity (HE)”

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3640 • HE was inversely correlated to neuroticism

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3643 **Proneness to social anxiety modulates neural complexity in the**

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3645 **absence of exposure: A resting state fMRI study using Hurst**

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3647 **exponent - Gentili et al. (2015) [62]**

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3649 Results from fALFF were highly consistent with those obtained using LSAS and BFNE  
3650 to predict HE. Overall our data indicate that spontaneous brain activity is influenced by  
3651 the degree of social anxiety, on a continuum and in the absence of social stimuli. These  
3652 findings suggest that social anxiety is a trait characteristic that shapes brain activity and  
3653 predisposes to different reactions in social contexts.

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3656 **Notes:**

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3660 • “A recent article (Rubin et al., 2013) analyzes the robustness of different algorithms

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3662 with respect to possible fMRI artifacts and time-series lengths. In particular, the  
3663 relevance of preprocessing steps as motion correction, detrending and filtering were  
3664 evaluated both on simulated and real fMRI data, while other preprocessing steps  
3665 like segmentation were not evaluated, although they may have an impact on”

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3667 • “The HE of fMRI time-series is generally higher in gray matter than in white matter  
3668 (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with  
3669 cholinergic transmission enhancement (Wink et al., 2006)”

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3671 • “As pointed out by Maxim (Maxim et al., 2005), fMRI noise, after these pre-  
3672 processing steps, can be described as fGn.”

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• “The HE of fMRI time-series is generally higher in gray matter than in white matter (Maxim et al., 2005), augments in the hippocampus with aging, and decreases with cholinergic transmission enhancement (Wink et al., 2006).”	3681 3682 3683 3684 3685 3686 3687 3688 3689 3690 3691 3692 3693 3694 3695 3696 3697 3698 3699 3700 3701 3702 3703 3704 3705 3706 3707 3708 3709 3710 3711 3712 3713 3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
<b>Real-time fractal signal processing in the time domain. - Hartmann et al. (2013) [63]</b>	3697 3698 3699 3700 3701 3702 3703 3704 3705 3706 3707 3708 3709 3710 3711 3712 3713 3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
Here we introduce real-time variants of the Detrended Fluctuation Analysis (DFA) and the closely related Signal Summation Conversion (SSC) methods, which are suitable to estimate the fractal exponent in one pass.	3701 3702 3703 3704 3705 3706 3707 3708 3709 3710 3711 3712 3713 3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
<b>Altered fractal dynamics of gait: Reduced stride-interval correlations with aging and Huntington’s disease. - Hausdoff et al. (1997) [64]</b>	3707 3708 3709 3710 3711 3712 3713 3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
<b>Notes:</b>	3711 3712 3713 3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
• Gait... not fMRI	3714 3715 3716 3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
<b>Scale-Free Properties of the Functional Magnetic Resonance Imaging Signal during Rest and Task - He (2011) [36]</b>	3717 3718 3719 3720 3721 3722 3723 3724 3725 3726
its power-law exponent differentiates between brain networks and correlates with fMRI signal variance and brain glucose metabolism. Importantly, in parallel to brain electrical field potentials, the variance and power-law exponent of the fMRI signal decrease during	3721 3722 3723 3724 3725 3726

task activation, suggesting that the signal contains more long-range memory during rest and conversely is more efficient at online information processing during task. The scale-free properties of the fMRI signal and brain electrical field potentials bespeak their respective stationarity and nonstationarity. This suggests that neurovascular coupling mechanism is likely to contain a transformation from nonstationarity to stationarity.

The fMRI signal time course from each ROI was extracted for each subject and fMRI run. The normalized or non-normalized power spectrum of the fMRI signal was computed using the Bartlett smoothing procedure of deriving the power spectral function from the lagged autocorrelation or auto-covariance function, respectively (Jenkins and Watts, 1998). A Tukey window of 20 fMRI frame width was applied for additional smoothing. The power spectra were then averaged across runs and subjects and across homologous ROIs, resulting in an average power spectrum for each of 21 brain regions (Fig. 2A). Finally, to obtain the power-law exponent , the <0.1 Hz range of each average power spectrum was fit with a power-law function:  $P(f) \sim 1/f$  using a least-squares fit. Using the low-frequency range to fit the power-law exponent avoids aliasing artifact in higher-frequency range (we used TR of 2.16 s, hence Nyquist limit is 0.23 Hz) and yields reliable measurement of the scale-free distribution (Eke et al., 2002).

The DFA method has the particular advantage of being applicable to both stationary and nonstationary data. To analyze our fMRI data, window lengths of 5, 10, 19, 38, and 95 fMRI frames were chosen so that the number of frames in each run (190 after discarding the first four frames) is an integer multiple of the window length.

<b>Fractal characterization of complexity in dynamic signals:</b>	3773
Application to cerebral hemodynamics - Herman (2009) [12]	3774
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<b>Identification of brain activity from fMRI data: Comparison of three fractal scaling analyses. - Hu (2006) [65]</b>	3777
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<b>A shift to randomness of brain oscillations in people with autism.</b>	3781
Lai (2010) [44]	3782
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Complex fractal scaling of fMRI time-series was found in both groups but globally there was a significant shift to randomness in the ASC (mean $H = .758$ , $SD = .045$ ) compared with neurotypical volunteers (mean $H = .788$ , $SD = .047$ ).	3785
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<b>Extraversion is encoded by scale-free dynamics of default mode network. Lei (2013) [66]</b>	3791
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<b>Fractional Gaussian noise, functional MRI and Alzheimer's disease.</b>	3795
Maxim (2005) [41]	3796
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we adopted the Davies-Harte algorithm, which is both exact and fast, to generate the fGn simulations used here. For each value of $H = 0.1, \dots, 0.9$ , we simulated 1000 realizations of fGn with 512 time-points in each series; we set $\omega^2 = 1$ for all simulations.	3799
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<b>NOTES:</b>	3805
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• This paper has the figure showing signal goes from fBm to fGn with proper motion regression	3808
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<b>Decomposing multifractal crossovers. Nagy (2017) [67]</b>	3813
The NIRS and fMRI-BOLD low-frequency fluctuations were dominated by a multifractal component over an underlying biologically relevant random noise, thus forming a bimodal	3814
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3819 signal. The crossover between the EEG signal components was found at the boundary  
3820 between the and bands, suggesting an independent generator for the multifractal  
3821 rhythm. The robust implementation of the SFD method should be regarded as essential  
3822 in the seamless processing of large volumes of bimodal fMRI-BOLD imaging data for the  
3823 topology of multifractal metrics free of the masking effect of the underlying random noise.  
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3828 **Optimizing complexity measures for fMRI data: Algorithm,**

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3830 **artifact, and sensitivity. Rubin (2013) [68]**

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3832 Power-spectrum, Higuchi's fractal dimension, and generalized Hurst exponent based esti-  
3833 mates were most successful by all criteria; the poorest-performing measures were wavelet,  
3834 detrended fluctuation analysis, aggregated variance, and rescaled range. Our results clearly  
3835 demonstrate that decisions regarding choice of algorithm, signal processing, time-series  
3836 length, and scanner have a significant impact on the reliability and sensitivity of com-  
3837 plexity estimates. operating on the edge of chaos, complex systems position themselves  
3838 for optimal responsivity to inputs, as well as ability to maintain homeostatic regulation.  
3839 Daubechies wavelet based computations (Hdb) *have long computation times, are not sen-*  
3840 *sitive to spikes, and show poor sensitivity to activation, tissue type, and emotional content;*  
3841 *for these Daubechies wavelet based estimates the overall performance increases with the*  
3842 *wavelet order up to a point (Hdb8), and then deteriorates. HRS and HAV, performed poorly*  
3843 *across the board. In terms of image contrast, overlap with activation, and group differences,*  
3844 *HDFA performed poorly as well, with HDFA-S outperforming HDFA and HDFA-L, sug-*  
3845 *gesting that the bulk of useful information is found at shorter lags. The most consistently*  
3846 *successful measures were the powerspectrum based measures HFFT and HpWelch, with*  
3847 *the latter slightly outperforming the former while taking much longer to compute Second,*  
3848 *it appears that detrending, regressing out the global mean, and excluding low frequencies*  
3849 *improves agreement between complexity and activation. 300-600 time-points Finally, the*  
3850 *best measures to use are either the power-spectrum based ones (HFFT or HpWelch) on a*  
3851 *restricted frequency range (above ,0.01 Hz),*

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**Mutual information identifies spurious Hurst phenomena in resting state EEG and fMRI data. von Wegner (2018) [69]**

In these processes, which do not have long-range memory by construction, a spurious Hurst phenomenon occurs due to slow relaxation times and heteroscedasticity (time-varying conditional variance). In summary, we find that mutual information correctly distinguishes long-range from short-range dependence in the theoretical and experimental cases discussed. Our results also suggest that the stationary fGn process is not sufficient to describe neural data, which seem to belong to a more general class of stochastic processes, in which multiscale variance effects produce Hurst phenomena without long-range dependence. In our experimental data, the Hurst phenomenon and long-range memory appear as different system properties that should be estimated and interpreted independently.

3911 **References**

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<b>Appendix</b>	3957
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<b>Python code</b>	3959
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<b>Sample python code for testing power-law scaling</b>	3962
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<b>print("HELLO")</b>	3964
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