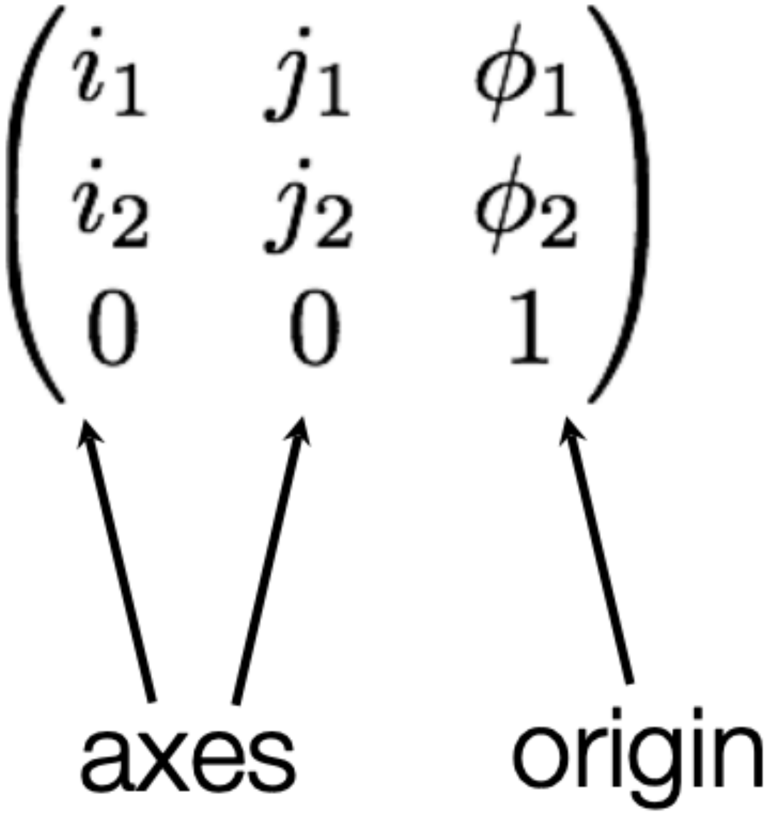
COMP9415 Review

2D Transformations

- **Affine transformations**
  - **Translation**
    - Translation is the process of moving an object in space
    - $\begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \phi_1 \\ 0 & 1 & \phi_2 \\ 0 & 0 & 1 \end{pmatrix}$
  - **Rotation**
    - Rotate objects around the origin
    - $\begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$
  - **Scaling**
    - Scale along both axes.
    - $\begin{pmatrix} s_x p_1 \\ s_y p_2 \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$
  - **Shear**
    - Shear is the unwanted child of affine transformations.
    - Horizontal
      - $\begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$
    - Vertical
      - $\begin{pmatrix} q_1 \\ q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ 1 \end{pmatrix}$
- **Transformation pipeline**
  - $P_{camera} \overset{view}{\leftarrow} P_{world} \overset{model}{\leftarrow} P_{local}$
  - The model transform transforms points in the local coordinate system to the world coordinate system
  - The view transform transforms points in the world coordinate system to the camera's coordinate system
  - ***Matrix***

    - $P_{world} = Trans(Rot(Scale(P_{camera})))$
    - The view matrix: $P_{camera} = Scale^{-1}(Rot^{-1}(Trans^{-1}(P_{camera})))$
- **Decomposing**
  - $Translation = (\phi_1, \phi_2, 1)^T$
  - $Rotation = atan2(i_2, i_1)$
  - $Scale = |i|$
  - **Perpendicular** (垂直) : $i \cdot j = 0$



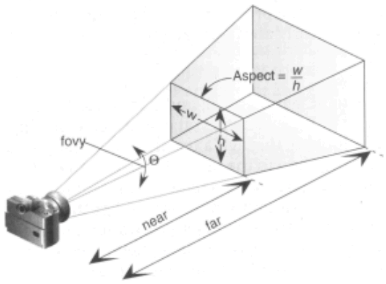
- **Camera**
  - In addition to the transformation properties inherited from SceneObject the Camera has an **aspect ratio**.

3D Transformations

- **TransTranslation**
  - $M_T = \begin{pmatrix} 1 & 0 & 0 & \phi_1 \\ 0 & 1 & 0 & \phi_2 \\ 0 & 0 & 1 & \phi_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- **Scale**

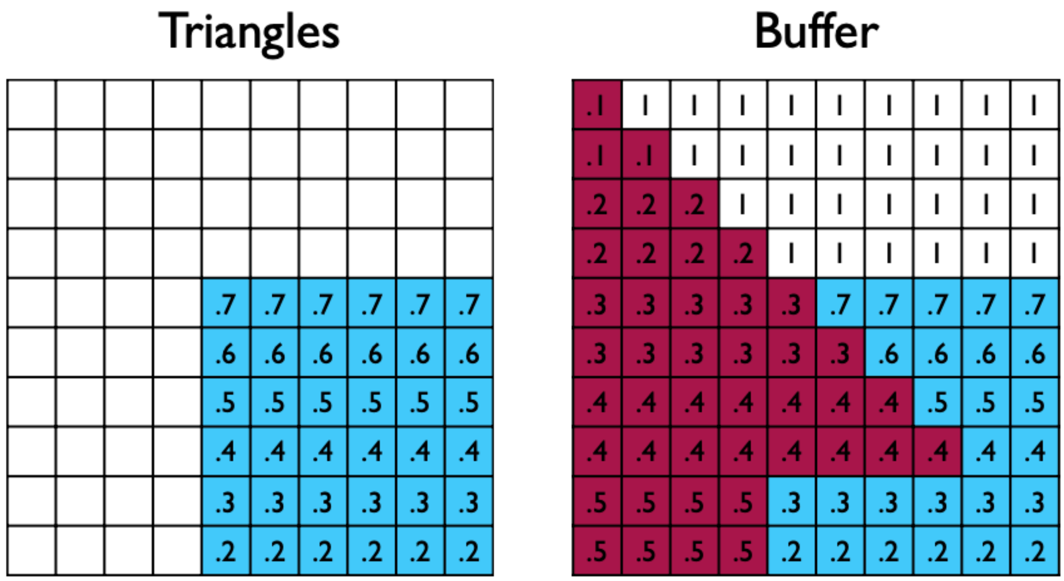
- $M_T = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

- **Shear**
  - $M_T = \begin{pmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- **Rotate**
  - Right Hand Rule: For any axis, if the right thumb points in the positive direction of the axis the right fingers curl in the direction of rotation
  - ***Rotate X***
    - $M_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
  - ***Rotate Y***
    - $M_y = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
  - ***Rotate Z***
    - $M_z = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- **Projection**
  - Projection happens after the model and view transformations have been applied, so all points are in camera coordinates.
  - Points with negative z values in camera coordinates are in front of the camera.
  - **Canonical View Volume (CVV)**
- **Perspective**
  - $M_{perspective} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{pmatrix}$
  - This matrix is **not affine**.
  -

Situation	Matrix
	$\mathbf{M_P} = \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix}$

Draw Algorithm

- Painter's algorithm
  - Sort geometric primitives by depth
  - Draw in order from back to front
  - ***Problem***
    - Intersect ploygon
- BSP Tree
  - One possible solution is to use Binary Space Partitioning trees (BSP trees)
  - They recursively divide the world into polygons that are behind or in front of other polygons and split polygons when necessary.
  - Then it is easy to traverse and draw polygons in a front to back order
  - Building the tree is slow and it needs to be rebuilt every time the geometry changes.
  - Best for rendering static geometry where tree can just be loaded in.
- Depth Buffer
  - Initially the depth buffer is initialised to 1 (maximum depth in window coords).
  - Each polygon is drawn fragment by fragment.
  - If it is closer, we update the pixel in the colour buffer and update the buffer value to the new pseudodepth. If not we discard it.

- 

Limitations

- **Ambient light**
  - $I_{ambient} = I_a \rho_a$
    - I_a is the ambient light intensity
    - ρ_a is the ambient reflection coefficient

      in the range (0,1) (usually ρ_a = ρ_d)
  - Lighting with just diffuse and specular lights gives very stark shadows.
  - In reality shadows are not completely black.
  - It is too computationally expensive to model this in detail.
  - Light is coming from all directions, reflected off other objects, not just from 'sources'
- **Diffuse light (Lambert's Cosine Law)**
  - $I_d = I_s \rho_d (\hat{s} \cdot \widehat{m})$
    - I_s is the source intensity
    - ρ_d is the diffuse reflection coefficient in [0,1]
  - Both vectors are normalised
  - When the angle is 0 degrees the cosine is 1
    - All the reflected light back
  - When the angle is 90 degrees
    - None of the light is reflected back
  - When the angle is > 90 degrees
    - cos gives us a negative value! This is not what we want.
- **Specular light (Phong model)**
  - $\hat{r} = -s + 2(s \cdot \widehat{m})\widehat{m}$

- $I_{sp} = max(0, I_s \rho_{sp} (\hat{r} \cdot \hat{v})^f)$
    - ρ_{sp} is the specular reflection coefficient in the range [0,1]
    - f is the phong exponent, typically in the range [1,128]
    - Larger values of the Phong exponent *f* make cos(Φ)*f* smaller, produce less scattering, creating more mirror-like surfaces.
- **Blinn Phong Model**
  - $I_{sp} = max(0, I_s \rho_{sp} (\widehat{h} \cdot \widehat{m})^f)$
  - halfway vector $\widehat{h} = \frac{\hat{s}+\hat{v}}{2}$
  - Phong/Blinn Phong model only reflects light sources, not the environment.
  - It is good for adding bright highlights but cannot create a true mirror.

- **Total intensity for the vertex**
  - $I = I_{ambient} + I_d + I_{sp}$
  - $I = I_a \rho_a + max(0, I_s \rho_d (\hat{r} \cdot \hat{v})) + max(0, I_s \rho_{sp} (\hat{r} \cdot \hat{v})^f)$
- **Spotlights**
  - A spotlight has a direction and a cutoff angle
  - Spotlights are also attenuated, so the brightness falls off as you move away from the centre.
  - $I = I_s (cos(\beta))^\varepsilon$
    - ε is the attenuation factor

Shading

- **Flat shading**
  - Calculated for each face
  - The simplest option is to shade the entire face the same colour
  - ***Pro***
    - Diffuse illumination
    - For flat surfaces with distant light sources
    - Non-realistic/retro rendering
    - The fastest shading option
  - ***Con***
    - close light sources
    - specular shading
    - curved surfaces
- **Gouraud shading**
  - Calculated for each vertex and interpolated for every fragment
  - Illumination is calculated at each of these vertices.
  - Gouraud shading is a simple smooth shading model.
  - We calculate fragment colours by bilinear interpolation on neighbouring vertices.
  - ***Pro***

        - curved surfaces
        - close light sources
        - diffuse shading
    - ***con***
        - more expensive than flat shading
        - handles specular highlights poorly
- **Phong shading**
    - Calculated for every fragment
    - designed to handle specular lighting better than Gouraud.
    - It also handles diffuse better as well.
    - illumination values are calculated per fragment rather than per vertex
    - ***Pro***
        - Handles specular lighting well
        - Improves diffuse shading
        - More physically accurate
    - ***Con***
        - Slower than Gouraud as normals and illumination values have to be calculated per pixel rather than per vertex.
        - In the old days this was a BIG issue. Not so much any more.

	Where	Pro	Con
Flat	Face	Flat surfaces, a retro blocky look	Curved surfaces, specular highlights
Gouraud	Vertex	Curved surfaces, diffuse shading	Specular highlights
Phong	Fragment	Diffuse and specular shading	Old hardware

Color

- **Alpha blending**
    - $\begin{pmatrix} r \\ g \\ b \end{pmatrix} \leftarrow a \begin{pmatrix} r_{image} \\ g_{image} \\ b_{image} \end{pmatrix} + (1-a) \begin{pmatrix} r \\ g \\ b \end{pmatrix}$
    - ***Example:***
        - If the pixel on the screen is currently green, and we draw over it with a red pixel, with alpha = 0.25

- $p = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ $p_{image} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0.25 \end{pmatrix}$
- $p = 0.25 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0.25 \end{pmatrix} + 0.75 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.75 \\ 0 \\ 0.8125 \end{pmatrix}$

- **Con**
  - Alpha blending depends on the order that pixels are drawn.
  - You need to draw transparent polygons after the polygons behind them.
  - Must be Back-to-front order

Curves

- **de Casteljau Algorithm** (Bézier curves)
  - Linear interpolation
    - $P(t) = (1-t)P_0 + tP_1$
  - Quadratic interpolation
    - $P(t) = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$
  - Cubic interpolation
    - $P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$
- **Bézier curves**
  - $\sum_{k=0}^{m} B_k^m(t)P_k$
    - ***m*** is the degree of the curve
    - P_0**...**P_m are the control points
  - The coefficient functions are called Bernstein polynomials.
    - $B_k^m(t) = \begin{pmatrix} m \\ k \end{pmatrix} t^k(1-t)^{m-k}$
    - $\begin{pmatrix} m \\ k \end{pmatrix} = \frac{m!}{k!(m-k)!}$ is binomial function
  - ***example***
    - m = 3
    - $B_0^3(t) = (1-t)^3$
    - $B_1^3(t) = 3t(1-t)^2$
    - $B_2^3(t) = 3t^2(1-t)$
    - $B_3^3(t) = t^3$
    - $P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$
- **Tangents**
  - Tangent vector
    - $\frac{dP(t)}{dt} = \sum_{k=0}^{m} \frac{dB_k^m(t)}{dt} P_k$

- $\frac{dP(t)}{dt} = \sum_{k=0}^{m-1} \frac{dB_k^{m-1}(t)}{dt}(P_{k+1} - P_k)$

- **L-System** (Lindenmayer System)
  - Can give us realistic plants and trees
  - L-system is a formal grammar
  - Symbols
    - A, B, +, -
    - A→B - A - B
    - B→A + B + A
  - Use a **LIFO** stack to save and restore global state like position and heading

Textures

- Usage
  - 1. Load or creating textures
    2. Passing the texture to a shader
    3. Mapping texture co-ordinates to vertices
- Texture WRAP
  - GL.GL_REPEAT (default)
  - GL.GL_MIRRORED_REPEAT
  - GL.GL_CLAMP_TO_EDGE

    ```
    gl.glTexParameteri( GL.GL_TEXTURE_2D, GL.GL_TEXTURE_WRAP_S,
    GL.GL_REPEAT);
    gl.glTexParameteri( GL.GL_TEXTURE_2D, GL.GL_TEXTURE_WRAP_T,
    GL.GL_REPEAT);
    ```

- **Textures and shading**
  - The simplest approach is to replace illumination calculations with a texture look-up.
    - $I(P) = T(s(P), t(P))$
    - This produces objects which are not affected by lights or color.
  - A more common solution is to use the texture to modulate the ambient and diffuse reflection coefficients.
    - $I(P) = T(s,t)[I_a\rho_a + I_d\rho_d(\hat{s}\cdot\widehat{m})] + I_s\rho_s(\hat{r}\cdot\hat{v})^f$
    - We usually leave the specular term unaffected because it is unusual for the material colour to affect specular reflections.
- **Magnification** (zoom in)
  - Nearest Texel
    - Find the nearest texel
    - image
  - Bilinear Filtering
    - Find the nearest four texels and use bilinear interpolation over them
- **Minification** (zoom out)

    - ***Aliasing***
        - It occurs when samples are taken from an image at a lower resolution than repeating detail in the image.
    - ***Filtering***
        - One screen pixel overlaps multiple texels but is taking its value from only one of those texels.
        - A better approach is to average the texels that contribute to that pixel.
        - Doing this on the fly is expensive.
- **MIP mapping**
    - Starting with a 512x512 texture we compute and store 256x256, 128x128, 64x64, 32x32, 16x16, 8x8, 4x4, 2x2 and 1x1 versions.
    - This takes total memory = 4/3 original.

    ```
    gl.glGenerateMipmap(GL.GL_TEXTURE_2D);
    ```

    - The simplest approach is to use the next smallest mipmap for the required resolution.
- **Trilinear filtering**
    - A more costly approach is trilinear filtering
        - Use bilinear filtering to compute pixel values based on the next highest and the next lowest mipmap resolutions.
        - Interpolate between these values depending on the desired resolution.
- **Aniso Filtering**
    - If a polygon is on an oblique angle away from the camera, then minification may occur much more strongly in one dimension than the other.
    - Anisotropic filtering is filtering which treats the two axes independently.
- **RIP Mapping**
    - RIP mapping is an extension of MIP mapping which down-samples each axis and is a better approach to anisotropic filtering
        - 256x256 image has copies at: 256x128, 256x64, 256x32, 256x16, ...,   128x256, 128x128, 128x64, ... 64x256, 64x128, etc.
    - ***Con***
        - Does not handle diagonal anisotropy.
        - More memory required for RIP maps (4 times as much).
        - Not implemented in OpenGL
- **Multi-texturing**
    - Have to pass two different textures to the shader.
    - two different sets of texture coordinates
- **Animated textures**
    - Animated textures can be achieved by loading multiple textures and using a different one on each frame.
- **Rendering to a texture**
    - A common trick is to set up a camera in a scene, render the scene into an offscreen

buffer, then copy the image into a texture to use as part of another scene.

- **Reflection mapping (cube mapping)**
  - Doing this in general is expensive, but we can make a reasonable approximation with textures
    - Generate a cube that encloses the reflective object.
    - Place a camera at the centre of the cube and render the outside world onto the faces of the cube.
    - Use this image to texture the object
  - To apply the reflection-mapped texture to the object we need to calculate appropriate texture coordinates.
  - We do this by tracing a ray from the camera, reflecting it off the object and then calculating where it intersects the cube.
  - ***Pros***
    - Produces reasonably convincing polished metal surfaces and mirrors
  - ***Cons***
    - Expensive: Requires 6 additional render passes per object
    - Angles to near objects are wrong.
    - Does not handle self-reflections or recursive reflections.
- **Shadow buffering**
  - keep a shadow buffer for each light source.
  - The shadow buffer is like the depth buffer, it records the distance from the light source to the closest object in each direction.
    - Render the scene from each light's viewpoint capturing only z-info in shadow (depth) buffer (color buffer turned off)
    - Render the scene from camera's point of view, using the previously captured shadow buffers to modulate the fragments
  - When rendering a point P
    - Project the point into the light's clip space.
    - Calculate the index (i,j) for P in the shadow buffer
    - Calculate the pseudodepth d relative to the light source
    - If shadow[i,j] < d then P is in the shadow
  - ***Pro***
    - Provides realistic shadows
    - No knowledge or processing of the scene geometry is required
  - ***Cons***
    - More computation
    - Shadow quality is limited by precision of shadow buffer. This may cause some aliasing artefacts.
    - Shadow edges are hard.
    - The scene geometry must be rendered once per light in order to generate the shadow map for a spotlight, and more times for an omnidirectional point light.
- **Light Mapping**

- If our light sources and large portions of the geometry are static then we can precompute the lighting equations and store the results in textures called light maps.
    - This process is known as baked lighting.
    - ***Pro***
        - Sophisticated lighting effects can be computed at compile time, where speed is less of an issue.
    - ***Con***
        - Memory and loading times for many individual light maps.
        - Not suitable for dynamic lights or moving objects.
        - Potential aliasing effects depending on the resolution of the light maps.
- **Normal mapping**
    - Phong shader we are assuming that the surface of the polygon is smoothly curved.
    - One solution would be to increase the number of polygons to represent all the deformities, but this is computationally unfeasible for most applications.
    - Instead we use textures called normal maps to simulate minor perturbations in the surface normal.
    - Rather than arrays of colours, normal maps can be considered as arrays of ***vectors***. These vectors are added to the interpolated normals to give the appearance of roughness.
    - ***Pro***
        - Provide the illusion of surface texture
    - ***Cons***
        - Does not affect silhouette
        - Does not affect occlusion calculation

Rasterisation

- Rasterisation is the process of converting lines and polygons represented by their vertices into fragments.
- Fragments are like pixels but include color, depth, texture coordinate. They may also never make it to the screen due to hidden surface removal or culling.
- This operation needs to be accurate and efficient.

- **Bresenham's algorithm**
    - The key idea is that calculations are doneincrementally, based on the values for the previous pixel.
    - assume to begin with that the line is in the first octant.
    - For each x we work out which pixel we set next
        - The next pixel with the same y value if the line passes below the midpoint between

the two pixels

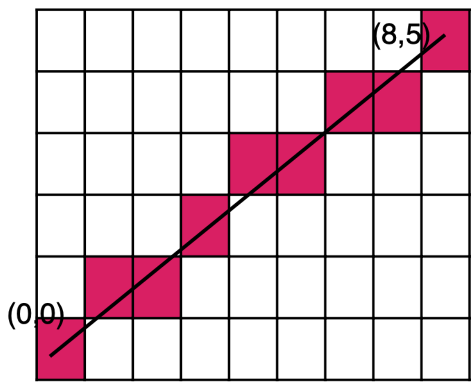
- Or the next pixel with an increased y value if the line passes above the midpoint between the two pixels

```
int y = y0;
for (int x = x0; x <= x1; x++) {
    setPixel(x,y);
    M = (x + 1, y + 1/2)
    if (M is below the line)
        y++
}
```

```
int y = y0;
int w = x1 - x0;
int h = y1 - y0;
int F = 2 * h - w;

for (int x = x0; x <= x1; x++) {
  drawPixel(x,y);
  if (F < 0)
    F += 2*h;
  else {
    F += 2*(h-w); y++;
  }
}
```

- Example



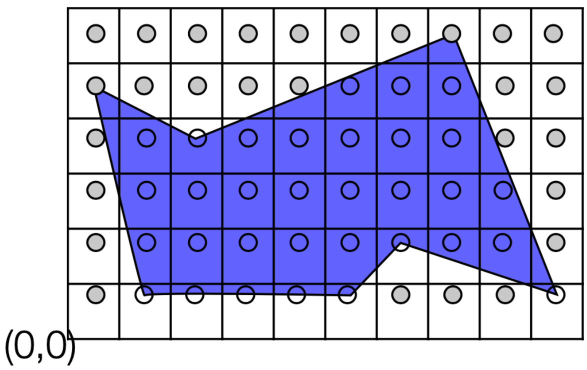
w = 8
h = 5
2 * (h - w) = -6
2 * h = 10

x	y	F
0	0	2
1	1	-4
2	1	6
3	2	0
4	3	-6
5	3	4
6	4	-2
7	4	8
8	5	2

- **Polygon filling**
  - Shared edges
    - We adopt a rule: The edge pixels belong to the rightmost and/ or upper polygon;

i.e. do not draw rightmost or uppermost edge pixels

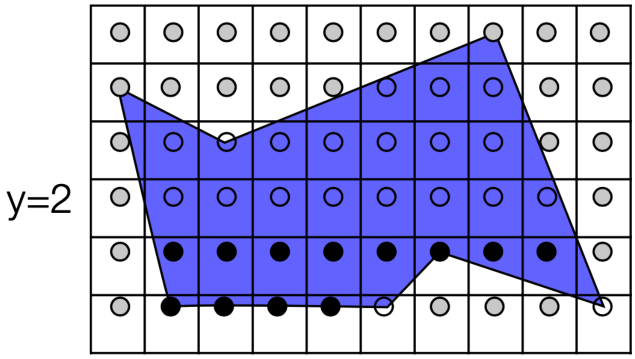
- **Scanline algorithm**
  - Testing every pixel is very inefficient.
  - We only need to check where the result changes value, i.e. when we cross an edge
  - We proceed row by row:
    - Calculate intersections incrementally.
    - Sort by x value.
    - Fill runs of pixels between intersections.
  - ***Edge table***
    - The edge table is a lookup table indexed on the y-value of the lower vertex of the edge.
    - Horizontal edges are not added
    - We store the the x-value of the lower vertex, the increment (inverse gradient) of the edge and the y-value of the upper vertex.



Edge table

y in	x	inc	y out
0	1	-0.25	4
0	5	1	1
0	9	-3	1
0	9	-0.4	5
3	2	-2	4
3	2	2.5	5

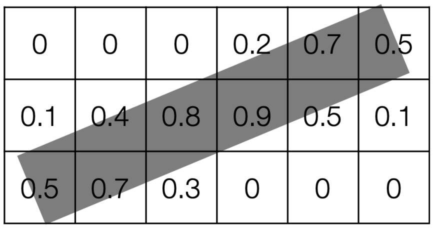
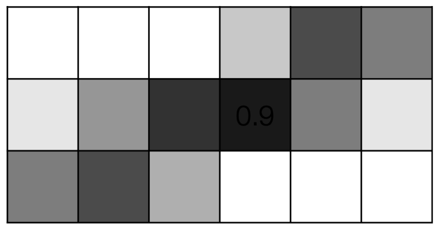
  - ***Active Edge List***
    - On the left edge, round up to the nearest integer, with round(n) = n if n is an integer.
    - On the right edge, round down to the nearest integer, but with round(n) = n-1 if n is an integer.



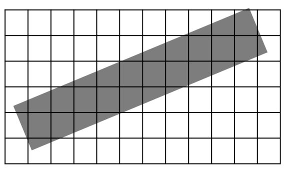
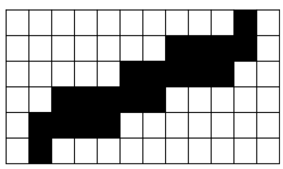
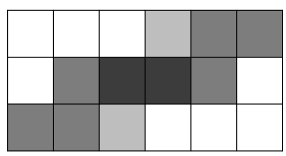
Active edge list

x	inc	y out
0.5	-0.25	4
8.2	-0.4	5

- **Antialiasing**
  - ***Prefiltering***
    - Prefiltering is computing exact pixel values geometrically rather than by sampling.

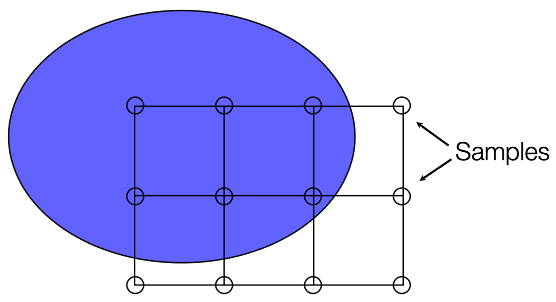
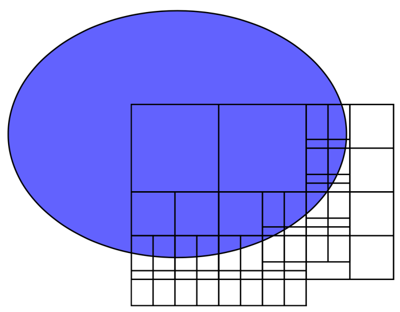
Input	Result
	

    - ***Prefiltering is most accurate but requires more computation.***
  - ***Postfiltering***
    - ***Postfiltering can be faster. Accuracy depends on how many samples are taken per pixel. More samples means larger memory usage.***
    - Postfiltering is taking samples at a higher resolution (supersampling) and then averaging.
    - Draw the line at a higher resolution and average (supersampling).

Higher resolution	Sample	Output
		

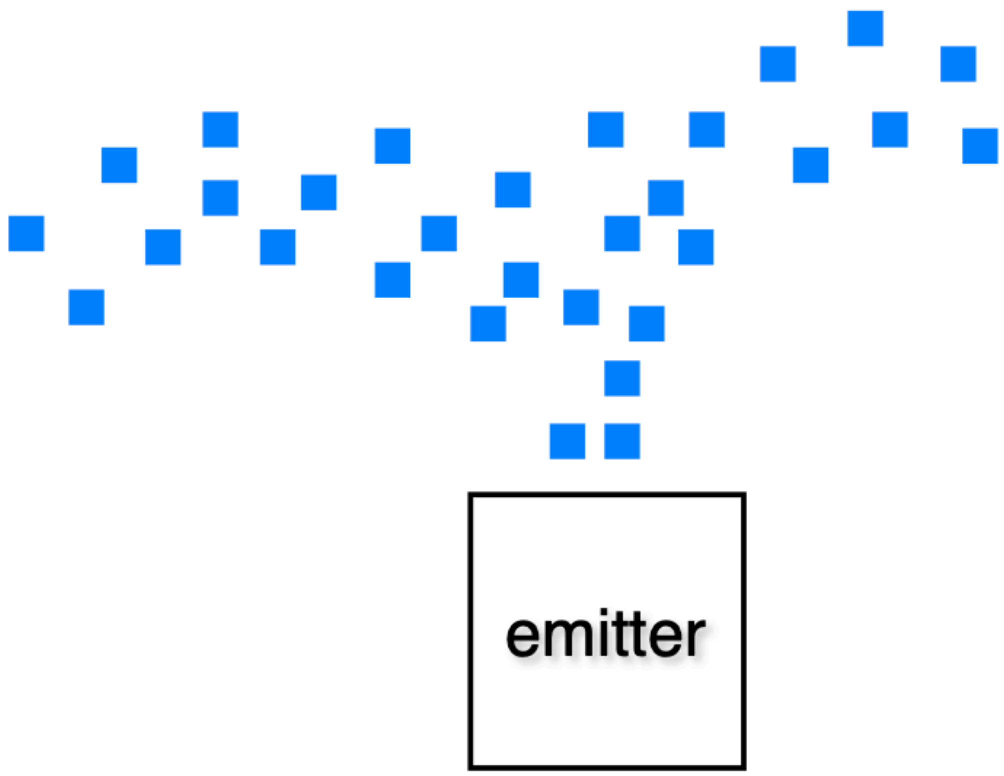
  - ***Weighted postfiltering***
    - It is common to apply weights to the samples to favour values in the center of the pixel.
  - ***Stochastic sampling*** (Random)

        - Taking supersamples in a grid still tends to produce noticeably regular aliasing effects.
        - Adding small amounts of jitter to the sampled points makes aliasing effects appear as visual noise.
    - ***Adaptive Sampling***
        - Supersampling in large areas of uniform colour is wasteful.
        - Supersampling is most useful in areas of major colour change.
        - Solution: Sample recursively, at finer levels of detail in areas with more colour variance.

Input	Output
	

Particle systems (粒子系统)

- Some visual phenomena are best modelled as collections of small particles.
- Particles are usually represented as small textured quads or point sprites – single vertices with an image attached.
- They are billboarded, i.e transformed so that they are always face towards the camera.
- Particles are created by an emitter object and evolve over time, usually changing position, size, colour.



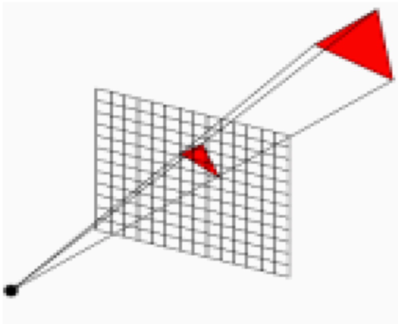
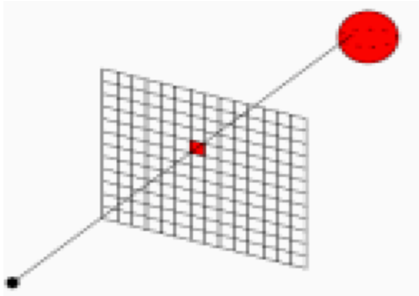
Global Lighting

- The lighting equation we looked at earlier only handled direct lighting from sources:
    - $I = I_a\rho_a + \sum_{l\varepsilon lights} I_l(\rho_d(\widehat{S_1}\cdot\widehat{m}) + \rho_{sp}(\widehat{r_1}\cdot\hat{v})^f)$
    - We added an ambient fudge term to account for all other light in the scene.
    - Without this term, surfaces not facing a light source are black.
    - Methods that take this kind of multi-bounce lighting into account are called global lighting methods.

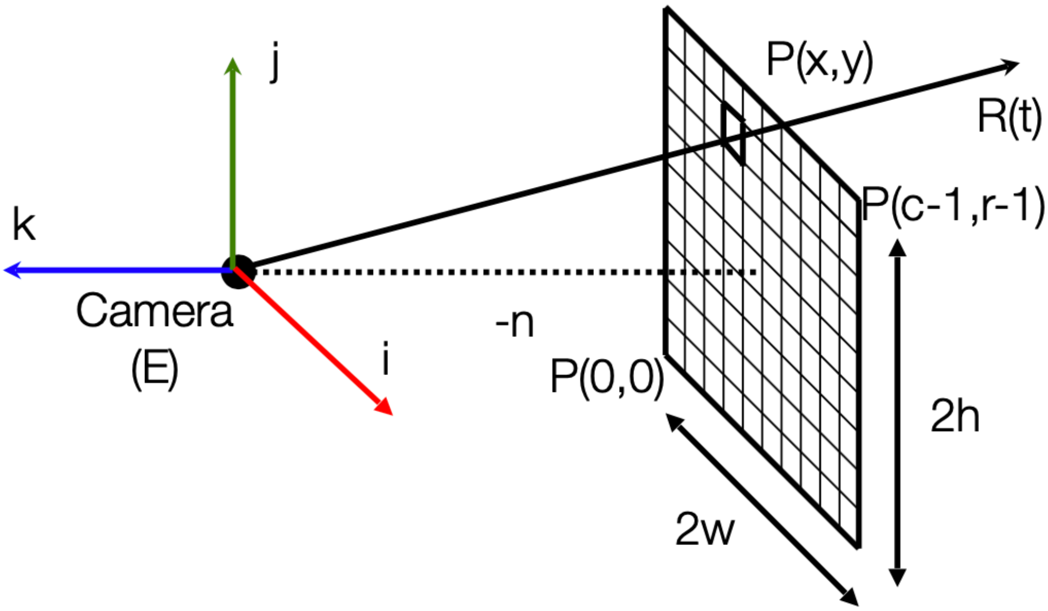
Raytracing (Global lighting)

- Raytracing models specular reflection and refraction.
- Both methods are computationally expensive and are rarely suitable for real-time rendering.
- Ray tracing is a different approach to rendering than the pipeline we have seen so far.
- **Projective Methods vs RayTracing**
    - ***Projective Method***
        - For each **object:** Find and update each pixel it influences
    - ***Ray Tracing***
        - For each **pixel:** Find each object that influences it and update accordingly
    - Same
        - shading models
        - calculation of intersections,
    - Difference

- projection and hidden surface removal come for 'free' in ray tracing

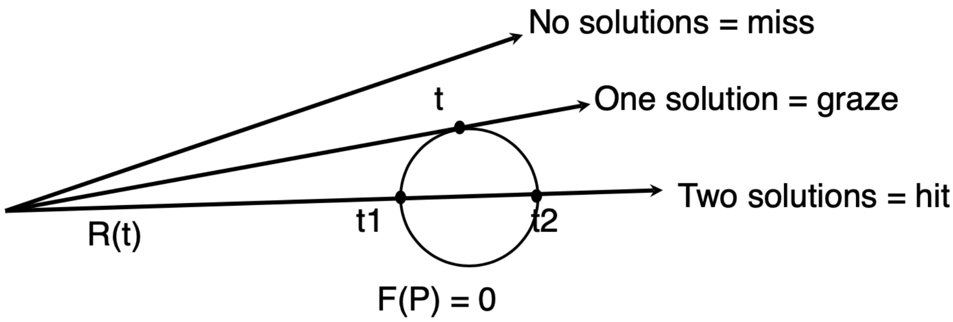
Projective	Ray Tracing
	 Same

- Location of pixel



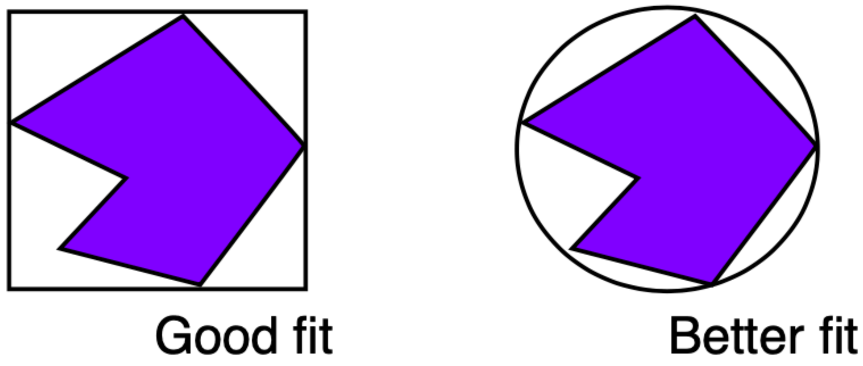
- $pixelWidth = \frac{2w}{c}$
- $pixelHeight = \frac{2h}{r}$
- $i_c = -w + x(\frac{2w}{c}) = w(\frac{2x}{c} - 1)$
- $j_r = h(\frac{2y}{r} - 1)$
- The point P(x,y) of pixel (x,y) is given by:
  - $P(x, y) = E + w(\frac{2x}{c} - 1)i + h(\frac{2y}{r} - 1)k - nk$
- A ray from the camera through P(x,y) is given by:
  - $R(t) = E + t(P(x, y) - E) = E + tv$
  - $v = w(\frac{2x}{c} - 1)i + h(\frac{2y}{r} - 1)k - nk$

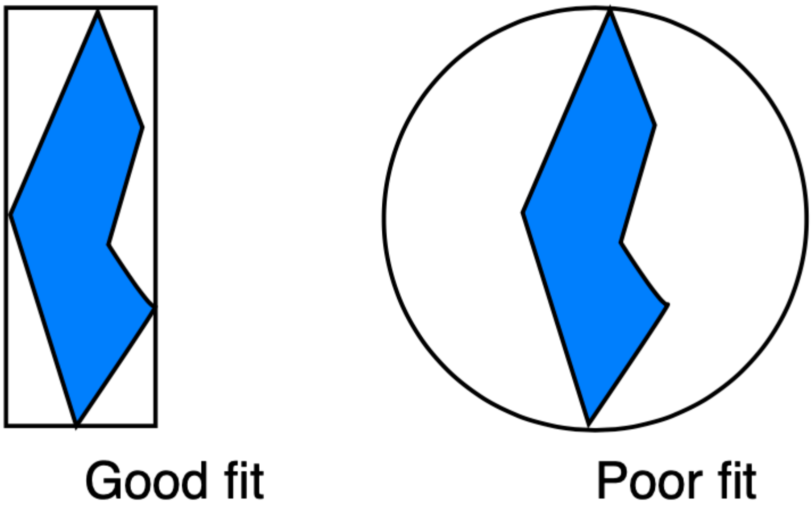
        - t = 0, we get E (Eye/Camera)
        - t = 1, we get P(x,y) – the point on the near plane
        - t > 1 point in the world
        - t < 0 point behind the camera – not on ray
    - ***Generic Sphere***

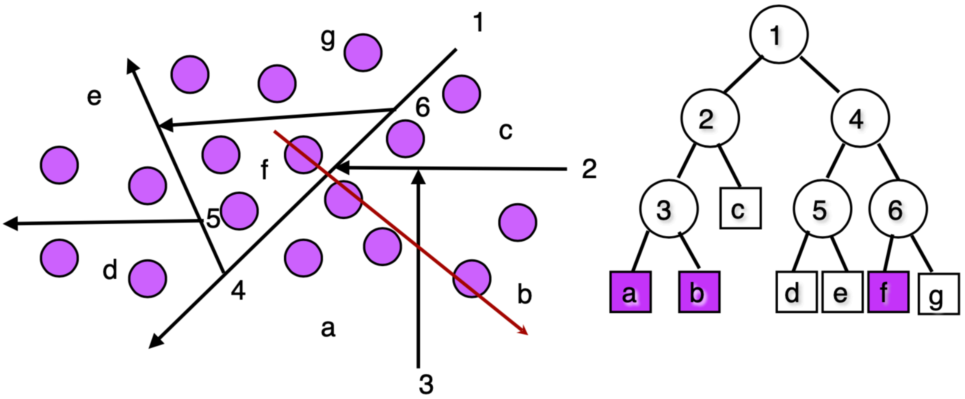


$$t_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

    - ***Shadows***
        - At each hit point we cast a new ray towards each light source. These rays are called shadow feelers.
    - ***Extents*** (大小)
        - Extents are bounding boxes or spheres which enclose an object
        - To compute a box extent for a mesh we simply take the min and max x, y and z coordinates over all the points.
        - To compute a sphere extent we find the centroid of all the vertices by averaging their coordinates. This is the centre of the sphere.



- 

- ***Projection extents***
  - A projection extent of an object is a bounding box which encloses all the pixels which would be in the image of the object (ignoring occlusions).
- ***Binary Space Partitioning (BSP)***
  - Another approach to optimisation is to build a Binary Space Partitioning (BSP) tree dividing the world into cells, where each cell contains a small number of objects.
  - 

- **Raytracing Can't Do**
  - Basic recursive raytracing cannot do:
    - Light bouncing off a shiny surface like a mirror and illuminating a diffuse surface

        - Light bouncing off one diffuse surface to illuminate others
        - Light transmitting then diffusing internally
    - Also a problem for rough specular reflection
        - Fuzzy reflections in rough shiny objects
- **Realtime ray-tracing (RTX)**
    - Works by arranging objects in a bounding volume hierarchy (BVH)
    - Specialised hardware offers fast traversal of these hierarchies to find ray intersections.

Radiosity (Global lighting)

- Radiosity models diffuse reflection
- Radiosity is a global illumination technique which performs indirect diffuse lighting.
- Direct lighting techniques only take into account light coming directly from a source.
- Raytracing takes into account specular reflections of other objects.
- Radiosity takes into account diffuse reflections of everything else in the scene.
- **Finite elements**
    - We divide the scene up into small patches.
    - We then calculate the energy transfer from each patch to every other patch.
    - ***Energy transfer***
        - The basic equation for energy transfer is:
        - Light output = Light emitted + ρ * Light input
        - Where ρ is the diffuse reflection coefficient.
        - $B_i = E_i + \rho_i \sum_j B_j F_{ij}$
            - B_i is the radiosity of patch i
            - E_i is the energy emitted by patch i
            - ρ_i is the reflectivity of patch i
            - $F_{ij}$is a form factor which encodes what fraction of light from patch j reaches patch i.

Color

- **RGB**
    - Red, Green, Blue
    - Colour is expressed as the addition of red, green and blue components.
- **CMYK**
    - Cyan, Magenta,Yellow, Black
    - CMY is a subtractive colour model, typically used in describing printed media.
- **HSV**

    - Hue, Saturation,Value (Brightness)
    - HSV (aka HSB) is an attempt to describe colours in terms that have more perceptual meaning
    - H represents the hue as an angle from 0° (red) to 360° (red)
    - S represents the saturation from 0 (grey) to 1 (full colour)
    - V represents the value/brightness form 0 (black) to 1 (bright colour).
- **HSL**
    - Hue, Saturation, Lightness
    - HSL (aka HLS) replaces the brightness parameter with a (perhaps) more intuitive lightness value.
    - H represents the hue as an angle from 0° (red) to 360° (red)
    - S represents the saturation from 0 (grey) to 1 (full colour)
    - L represents the lightness form 0 (black) to 1 (white).