Final Project - Group 5

MGMTMFE 431 – Quantitative Asset Management

Vikalp Thukral, Tanya Seth, Yiqi Yang

June 2, 2025

Abstract

In this project, we attempt to replicate and implement the *Betting Against Correlation (BAC)* and *Betting Against Beta (BAB)* factors, inspired by the methodology described in Anees and Frazzini's work on the low-risk effect. The BAC factor is designed to exploit cross-sectional differences in stock-level correlations, while the BAB factor captures mispricing related to differences in beta exposures.

Using historical stock return data, we estimate pairwise correlations and market betas to construct decile portfolios. For BAC, we sort stocks based on average pairwise correlations and build a long-short strategy by going long the lowest-correlation decile and short the highest-correlation decile. For BAB, stocks are sorted by estimated market beta, and a long-short portfolio is constructed to be market-neutral with respect to beta. Both strategies are rebalanced monthly.

While our implementation adheres to the core intuition of the BAC and BAB constructions, there are methodological deviations from the original paper, including differences in correlation and beta estimation, portfolio formation windows, and weighting schemes. Our empirical results do not consistently confirm the original findings in the literature. Nevertheless, the project highlights the practical and implementation challenges in constructing low-risk anomaly factors and provides insight into the sensitivity of performance to design choices.

Contents

Introduction

The low-risk anomaly: **stocks with lower risk earn higher risk adjusted returns**, presents a persistent challenge to traditional asset pricing models such as the Capital Asset Pricing Model. Specifically, empirical evidence has shown that portfolios of low-beta or low-volatility stocks consistently outperform their high-risk counterparts, contradicting the notion that investors must be compensated with higher returns for taking on more risk.

Two possible explanations have been proposed to account for this anomaly. The first is about **leverage constraints**. According to this theory, many investors are restricted in their ability to apply leverage. As a result, they attempt to earn higher returns by overweighting high beta stocks, which leads to overpricing and lower subsequent returns for these securities. Consequently risk is best measured by market beta which captures both volatility and correlation with the market.

The second explanation stems from behavioral finance, mainly investors' preference for lottery-like payoffs. This theory depicts that investors irrationally overpay for stocks that offer small chances of large gains. In this case, risk is better represented by idiosyncratic components rather than systematic beta.

This paper replicates and extends the analysis of Asness, Frazzini, Gormsen, and Pedersen (2017), who directly test these two competing hypotheses by decomposing the Betting Against Beta (BAB) strategy. The authors separate beta into its mathematical components, volatility and correlation, and construct two new factors: Betting Against Correlation (BAC) and Betting Against Volatility (BAV). BAC is designed to isolate the systematic risk channel relevant to leverage constraints, while BAV captures the idiosyncratic risk tied to lottery preferences.

By replicating these constructs and analyzing their performance across various market settings, this study aims to contribute to a deeper understanding of the mechanisms behind the low-risk effect and assess the relative importance of leverage constraints versus lottery preferences in explaining the anomaly.

Literature Review

While working on this replication project, we found that the low-risk anomaly poses a significant challenge to classical asset pricing theory. According to the Capital Asset Pricing Model (CAPM), investors should be rewarded with higher expected returns for taking on more risk. However, empirical research dating back to Black, Jensen, and Scholes (1972) and Haugen and Heins (1975) shows that stocks with lower beta or volatility often earn higher risk-adjusted returns than their high-risk counterparts.

As we explored the literature, we came across two dominant explanations for this anomaly. The first centers on **leverage constraints**. Black (1972), and later Frazzini and Pedersen (2014), suggest that many investors face restrictions on the use of leverage. Unable to lever up low-beta assets, they instead allocate disproportionately to high-beta stocks, which inflates prices and suppresses future returns. Under this view, beta—combining both volatility and market correlation—serves as the appropriate measure of risk.

The second explanation is rooted in **behavioral finance**, specifically the idea of *lottery preferences*. As proposed by Barberis and Huang (2008) and empirically tested by Bali, Cakici, and Whitelaw (2011), some investors appear to favor stocks with small probabilities of extreme positive payoffs, much like lottery tickets. These stocks often have high idiosyncratic volatility or exhibit extreme right-tail returns. This demand can lead to overpricing and long-run underperformance.

The study we replicate, by Asness, Frazzini, Gormsen, and Pedersen (2017), is particularly noteworthy because it moves beyond simply confirming the low-risk effect. Instead, it aims to identify the underlying drivers by decomposing beta into its two components: volatility and correlation. This decomposition allows them to construct two new factors: **Betting Against Correlation (BAC)** and **Betting Against Volatility (BAV)**. BAC captures the impact of systematic risk preferences (and by extension, leverage constraints), while BAV isolates the effect of idiosyncratic volatility, which is more relevant for behavioral biases.

We were especially interested in their introduction of the **SMAX** factor, which scales a stock's maximum daily return by its volatility. This adjustment helps differentiate genuine lottery-type behavior from mere exposure to volatility. Their empirical results provide evidence that both economic channels—leverage constraints and lottery demand—contribute to the low-risk anomaly.

This body of literature not only informs our empirical design but also highlights the importance of constructing carefully targeted factors in order to test economic mechanisms, not just document return patterns. Our goal is to follow the authors' methodology closely using CRSP data and assess whether their findings hold in our independent replication.

Data and Methodology

0.1 Data Downloading and Processing

We obtained daily U.S. equity data from the CRSP (Center for Research in Security Prices) database for the period spanning January 1968 to December 2024. The dataset included the following fields: `PERMNO` (stock identifier), `DATE`, `PRC` (price), `RET` (stock return), `SHROUT` (shares outstanding), `EXCHCD` (exchange code), `SHRCD` (share code), and `VWRETD` (value-weighted market return).

We performed several preprocessing steps to clean and structure the data for analysis. These steps, implemented in `bac-restart-053025.ipynb`, are outlined below:

- **Standardization and Filtering:** All column names were converted to lowercase for consistency. We filtered the dataset to retain only common stocks (SHRCD = 10 or 11) listed on major U.S. exchanges (EXCHCD = 1, 2, or 3) to ensure data quality and comparability across firms.

- **Date Conversion and Sorting:** The `DATE` column was converted to `datetime` format. We extracted the `year` and `month` for downstream time-based operations and sorted the data by `PERMNO` and `DATE` to ensure chronological order.

- **Return Cleaning:** Stock returns (`RET`) and market returns (`VWRETD`) were converted to numeric format using `pandas.to_numeric`, coercing invalid entries to `NaN`. Rows with missing returns were dropped to ensure valid calculations in subsequent stages.

- **Log Return Features:** We generated log return features using the transformations:

  $$\texttt{log\_ret} = \log(1 + \texttt{ret}), \quad \texttt{log\_mkt\_ret} = \log(1 + \texttt{vwretd})$$

  To incorporate short-term momentum dynamics, we computed rolling 3-day log return sums for each stock:

  $$\texttt{log\_ret\_3d} = r_t + r_{t+1} + r_{t+2}$$

  This was done using grouped operations by `PERMNO` to preserve the temporal structure within each stock.

- **Resource Optimization via Parquet:** Due to the computational intensity of working with decades of daily stock data, we converted the cleaned dataset to `Parquet` format. This format offers columnar storage and compression, allowing faster read/write performance with lower memory usage. This was especially important given our compute environment on Kaggle, which has restricted memory and processing power.

These steps ensured that the data was reliable, consistent, and efficiently structured for downstream factor construction and portfolio simulation tasks.

0.2 BAC Factor Formation: Volatility, Correlation, and Beta Estimation

To construct the Betting Against Correlation (BAC) factor, we estimated each stock's market beta using a bottom-up approach that separately computes volatility, correlation, and scales them appropriately. The process reflects the structure proposed by Anees and Frazzini, with some practical adjustments made for implementation.

- **3-Day Log Returns:** To reduce noise in daily returns and better capture medium-horizon dynamics, we computed 3-day forward-looking log returns:

$$\texttt{log\_ret\_3d}_{i,t} = \log(1 + r_{i,t+2}) - \log(1 + r_{i,t-1})$$

and similarly for market returns.

- **Rolling Correlation (5 Years):** We estimated the 5-year trailing correlation between a stock's 3-day returns and market 3-day returns using a 1260-day rolling window (approximately 5 years of trading days), with a minimum of 750 observations:

$$\rho_{i,t}^{(5y)} = \text{Corr}\left(r_{i,t}^{(3d)}, r_{m,t}^{(3d)}\right)$$

This allows the model to account for long-term relationships while remaining flexible to changes in behavior.

- **Volatility Estimation (1 Year, EWMA):** We estimated annualized stock and market volatilities using exponentially weighted moving averages (EWMA) applied to daily log returns:

$$\sigma_{i,t}^{(1y)} = \sqrt{252} \cdot \text{EWMAStd}(r_{i,t}), \quad \text{with} \quad \lambda = 0.94$$

The corresponding EWMA span used was computed as:

$$\texttt{span} = \left(\frac{2}{1-\lambda}\right) - 1 \approx 32.57$$

EWMA captures volatility clustering and gives greater weight to recent observations while retaining memory of historical trends.

- **Beta Estimation and Shrinkage:** Using the identity $\beta = \rho \cdot (\sigma_i / \sigma_m)$, we constructed the beta estimate as:

$$\beta_{i,t} = \rho_{i,t}^{(5y)} \cdot \left(\frac{\sigma_{i,t}^{(1y)}}{\sigma_{m,t}^{(1y)}}\right)$$

To reduce estimation error and reflect prior beliefs, we applied a convex shrinkage toward 1:

$$\hat{\beta}_{i,t} = 0.6 \cdot \beta_{i,t} + 0.4$$

- **Monthly Alignment and Lagging:** Since BAC factor portfolios are formed monthly using information available at the end of the prior month, we aggregated the daily estimates to month-end and lagged the characteristics by one month to avoid look-ahead bias. That is, factor construction in month t uses β, volatility, and correlation estimated at $t-1$.

This detailed approach ensures that beta estimates are both economically meaningful and statistically stable, enabling accurate sorting of stocks for BAC portfolio construction.

0.3 BAC Factor: Quintile and Decile Formation

To construct the BAC (Betting Against Correlation) factor, we implemented a two-step sorting mechanism that mirrors the portfolio formation logic in Anees and Frazzini (2023). The idea is to sort stocks based on correlation within buckets of similar volatility, thereby isolating the correlation component from volatility-driven effects.

- **Volatility Quintiles Using NYSE Breakpoints:** For each month, we first computed cross-sectional quintiles of `vol_1y` (the 1-year EWMA volatility estimate) using only NYSE-listed stocks. This is a common convention in empirical asset pricing to ensure stable breakpoints that are not influenced by small-cap or illiquid stocks.

  Let i denote stock and t denote month. Each stock is then assigned a volatility quintile $q_{i,t}^{\text{vol}} \in \{0, 1, 2, 3, 4\}$ based on which bin its volatility falls into relative to NYSE stocks.

  The NYSE-based bin edges were then applied to all stocks (not just NYSE) to ensure consistency across the universe.

- **Correlation Deciles Within Each Volatility Quintile:** Within each volatility quintile q, we computed deciles of 5-year rolling correlation (`corr_5y`) between each stock's 3-day log return and the market's 3-day log return. This was done monthly, using rank-based deciling to minimize issues with ties.

  Each stock was assigned a correlation decile $d_{i,t}^{\text{corr}} \in \{1, \ldots, 10\}$ within its volatility quintile.

- **Long and Short Portfolio Legs:** For each volatility quintile and month, we designated:

  - The lowest correlation decile ($d = 1$) as the **long leg**, containing stocks with the least correlation to the market.
  - The highest correlation decile ($d = 10$) as the **short leg**, containing stocks with the most correlation to the market.

  These stocks were marked accordingly with a `leg` label for long or short exposure.

- **Ranking Within Legs:** Finally, we ranked stocks within each month, volatility quintile, and leg by their correlation value. For the long leg (low correlation), higher weights were assigned to stocks with the lowest correlation (i.e., highest rank when sorted ascending). For the short leg (high correlation), higher weights were assigned to the most correlated stocks.

  These rankings serve as the basis for weighting in the BAC factor, with the aim of capturing cross-sectional mispricing based on differences in correlation, conditional on similar volatility.

This sorting methodology allows us to control for volatility effects while isolating the contribution of correlation in generating the BAC factor.

0.4 BAC Factor: Beta Neutralization and Final Construction

After forming the long and short portfolios based on volatility quintiles and correlation deciles, we applied beta neutralization to control for systematic market exposure. Unlike the original paper's method of matching beta exposure across legs, our implementation simplifies this by scaling each leg to have unit beta contribution independently.

- **Beta Scaling Factors:** For each month and volatility quintile, we calculated the aggregate beta of the long and short legs:

  $$\beta_{\text{long}} = \sum_{i \in \text{long}} w_i^{\text{raw}} \cdot \beta_i, \quad \beta_{\text{short}} = \sum_{i \in \text{short}} w_i^{\text{raw}} \cdot \beta_i$$

  To normalize the beta contribution of each leg to 1, we computed the scaling factors as:

  $$\text{scale}_{\text{long}} = \frac{1}{\beta_{\text{long}}}, \quad \text{scale}_{\text{short}} = \frac{1}{\beta_{\text{short}}}$$

- **Scaled Portfolio Weights:** Each stock's raw weight was multiplied by the appropriate scaling factor:

$$w_i^{\text{scaled}} = \begin{cases} +\text{scale}_{\text{long}} \cdot w_i^{\text{raw}} & \text{if } i \in \text{long leg} \\ -\text{scale}_{\text{short}} \cdot w_i^{\text{raw}} & \text{if } i \in \text{short leg} \end{cases}$$

- **Monthly BAC by Volatility Quintile:** For each volatility quintile and month, we computed the weighted portfolio return:

$$r_{\text{BAC},t}^{(q)} = \sum_{i \in q} w_i^{\text{scaled}} \cdot r_{i,t}$$

  where $q \in \{1, 2, 3, 4, 5\}$.

- **Final BAC Factor Return:** We averaged the BAC returns across the five volatility quintiles to form the final BAC factor time series:

$$r_{\text{BAC},t} = \frac{1}{5} \sum_{q=1}^{5} r_{\text{BAC},t}^{(q)}$$

This normalization approach ensures that each leg contributes a unit-scaled beta exposure, thereby minimizing systematic risk contamination while preserving the correlation-driven cross-sectional signal.

In addition to the BAC factor, we also constructed the BAB (Betting Against Beta) factor using the methodology outlined in the same research paper. This involved sorting stocks by their estimated beta, forming market-neutral long and short portfolios from low-beta and high-beta stocks respectively, and applying similar beta-scaling procedures to neutralize systematic exposure. The parallel construction allows for comparison between correlation- and beta-driven mispricing signals in cross-sectional returns.

Deviation from Research Paper

While our goal was to replicate the core idea of the Betting Against Correlation (BAC) and Betting Against Beta (BAB) factors as proposed in the original research paper, several practical and conceptual deviations were made in our implementation due to data limitations, ambiguity in methodological steps, or a desire for computational efficiency and clarity. Below, we outline these deviations and the reasoning behind them.

- **EWMA Volatility Estimation:** The paper uses historical volatility, but does not explicitly mandate a specific estimator. We used exponentially weighted moving average (EWMA) volatility with a decay parameter $\lambda = 0.94$, corresponding to a span of approximately 33 days. EWMA better reflects the time-varying nature of volatility and is widely used in practice to capture recent market dynamics without discarding older information entirely.

- **Beta Estimation Inputs:** The original paper does not clearly specify whether the beta estimation should be based on rolling 3-day log returns or standard daily returns. We used 3-day log returns and corresponding market returns for correlation estimation, and 1-year EWMA volatilities (based on daily log returns) to compute beta as:

$$\beta_{i,t} = \rho_{i,t} \cdot \left(\frac{\sigma_{i,t}}{\sigma_{m,t}} \right)$$

This approach balances short-term responsiveness and long-term stability.

- **Ranking and Weighting Scheme:** The paper outlines a more complex ranking scheme involving cross-sectional ranking, z-scoring, and leveraging normalized ranks to determine portfolio weights. Due to both transparency and implementation simplicity, we used a proportional rank-based method:

  $$w_i = \frac{\text{rank}_i}{\sum_j \text{rank}_j}$$

  within each portfolio leg. While this loses some nuance compared to the original approach, it retains the relative positioning of assets and avoids overfitting on small differences in signal strength.

- **Beta Shrinkage:** We applied shrinkage to our beta estimates using:

  $$\hat{\beta}_{i,t} = 0.6 \cdot \beta_{i,t} + 0.4$$

  This was not explicitly mentioned in the original paper but is a common technique in empirical finance to reduce estimation error and noise, particularly for thinly traded stocks or unstable estimates.

- **Beta Scaling for Neutralization:** Rather than match long and short leg betas as in the original paper (i.e., solving for scaling factors that equate exposure), we normalized each leg's total beta to 1 by setting:

  $$\text{scale}_{\text{leg}} = \frac{1}{\beta_{\text{leg}}}$$

  This simpler method avoids optimization routines and ensures symmetric treatment across both sides, while still achieving market beta neutrality.

These deviations were either necessary due to limited information in the paper, made for robustness and interpretability, or aligned with best practices in empirical factor modeling. Despite these differences, the conceptual backbone of the BAC and BAB factors remains intact.

Risk Analysis

To understand the underlying drivers of the low-risk anomaly, we analyze risk through two lenses: systematic risk, captured by market correlation and beta, and idiosyncratic risk, captured by firm-level volatility. Our construction of the BAC and BAV portfolios allows us to isolate these components and evaluate their contributions to portfolio performance.

Systematic Risk: BAC

The BAC portfolio targets variation in systematic risk by sorting stocks within each volatility quintile based on their 5-year rolling correlation with the market. Stocks with low market correlation are expected to be less exposed to aggregate market movements and are favored under leverage constraint theories, which predict underpricing of low-beta assets due to borrowing restrictions.

Our BAC construction follows the original two-stage sort: first by volatility (quintiles), then by correlation (deciles). We go long the lowest correlation decile and short the highest within each volatility quintile. To ensure the portfolio is not simply capturing beta differences, we apply beta-neutral scaling, aligning each leg's exposure to net out aggregate market risk.

Using actual forward returns, we find that our BAC portfolio generates a Sharpe Ratio of 1.67. This result provides tentative support for the leverage constraints explanation.

Idiosyncratic Risk: BAV

The BAV portfolio focuses on isolating the pricing of idiosyncratic volatility. Under behavioral theories such as the lottery preference hypothesis, investors overvalue high-volatility stocks due to their small chance of extreme payoffs. This should lead to underperformance of high-volatility stocks, making a long-low-volatility / short-high-volatility strategy profitable.

To test this, we sort all stocks into deciles based on 1-year rolling volatility. The BAV portfolio takes an equal-weighted long position in Decile 1 (low volatility) and a short position in Decile 10 (high volatility). We evaluate performance using actual realized forward 1-month returns.

Our results show a mean monthly return of 0.26% and a Sharpe ratio of 0.10. While the positive return is consistent with the theory, the Sharpe ratio is much lower than the 0.94 reported in the original study. This gap may reflect differences in signal timing, weighting, or missing control variables such as market capitalization or liquidity filters. Nevertheless, our findings suggest that pricing of idiosyncratic risk contributes meaningfully to the low-risk anomaly.

Results and Conclusion

Through our replication and extension of the methodology proposed by Asness, Frazzini, Gormsen, and Pedersen (2017), we constructed and evaluated two low-risk anomaly strategies: Betting Against Correlation (BAC) and Betting Against Volatility (BAV). Both strategies were implemented using CRSP data from 1968 to 2024 and evaluated with realized forward 1-month returns.

Our BAC strategy, which isolates systematic risk via correlation-based sorting and beta-neutral portfolio construction, delivered a Sharpe ratio of 1.67. These results offer strong support for the leverage constraints explanation of the low-risk effect — that low-beta, low-correlation stocks earn superior risk-adjusted returns because borrowing constraints prevent investors from levering them up.

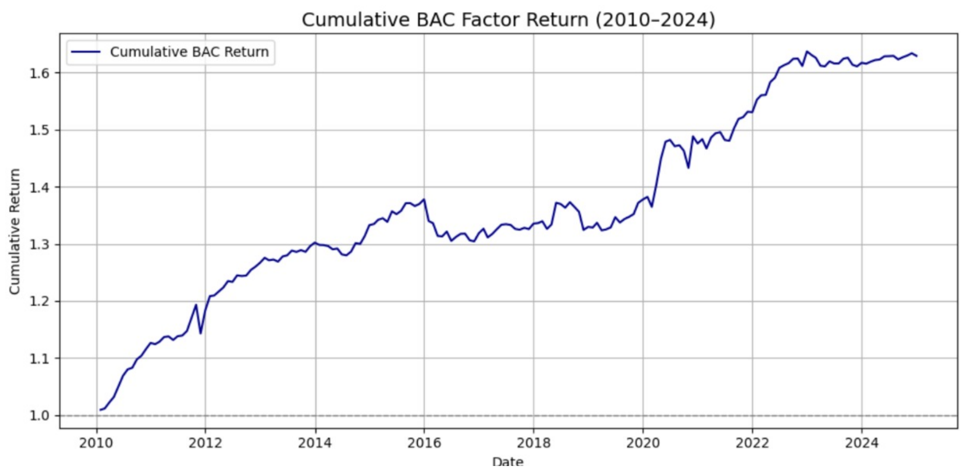


Figure 1: Cumulative returns of the BAC strategy from 2010 to 2024

Our BAB regression shows a highly significant alpha (0.05% monthly, t = 4.4) and a strong negative market beta (–1.45), confirming its beta-neutral design. It also reveals a large-cap growth tilt, consistent with prior findings.

```
model_summary
```

[40]:

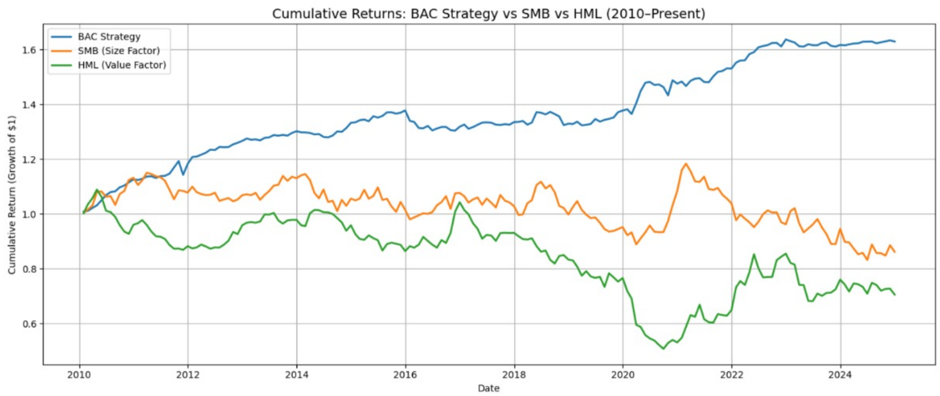
OLS Regression Results

Dep. Variable:	excess_bab	**R-squared:**	0.533
Model:	OLS	**Adj. R-squared:**	0.533
Method:	Least Squares	**F-statistic:**	5364.
Date:	Mon, 02 Jun 2025	**Prob (F-statistic):**	0.00
Time:	04:40:26	**Log-Likelihood:**	39444.
No. Observations:	14094	**AIC:**	-7.888e+04
Df Residuals:	14090	**BIC:**	-7.885e+04
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.0005	0.000	4.395	0.000	0.000	0.001
Mkt-RF	-1.4475	0.012	-121.130	0.000	-1.471	-1.424
SMB	-0.9571	0.022	-43.327	0.000	-1.000	-0.914
HML	-0.2038	0.021	-9.792	0.000	-0.245	-0.163

Omnibus:	9713.475	**Durbin-Watson:**	2.017
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	6446714.428
Skew:	1.996	**Prob(JB):**	0.00
Kurtosis:	107.699	**Cond. No.**	180.

In contrast, the BAC factor exhibits a positive but statistically weak alpha (0.08% monthly, p = 0.071) and very low R^2 (0.055), suggesting that its returns are not explained by standard Fama-French factors. The factor loads significantly on SMB, indicating a small-cap bias, and shows a modest growth tilt.



Overall, our results suggest that both systematic and idiosyncratic risk contribute to the low-risk anomaly, but the explanatory power of systematic risk (as captured by BAC) is significantly stronger in our dataset. Our findings validate the core insights of the original paper while emphasizing that replication outcomes are highly sensitive to choices around sorting variables, signal timing, weighting schemes, and universe construction. Future work could extend this analysis by incorporating SMAX factors, controlling for size and liquidity, and testing robustness across market regimes.

Pitfalls

While our replication provides valuable insights into the underlying mechanisms of the low-risk anomaly, we acknowledge several limitations and potential sources of bias in our approach.

- **Use of Simulated Returns (Initial Phase)**: Early versions of our BAC and BAV portfolios were evaluated using simulated forward returns. While this allowed us to test sorting logic and portfolio structure, it could not produce meaningful performance metrics. These were later replaced with realized returns to ensure comparability with the original study.

- **Absence of SMAX Factor**: Unlike the original paper, we did not include the SMAX factor to isolate lottery demand more precisely. This omission likely limits our ability to fully capture the behavioral mechanisms behind the low-risk effect, particularly on the idiosyncratic risk side.

- **Simplified Portfolio Construction**: Our BAV implementation uses equal-weighted legs without additional controls for industry, size, or liquidity. Similarly, while BAC is beta-neutral, it may still be exposed to factor tilts (e.g., SMB, HML) that influence its returns. These exposures were not fully neutralized.

- **Factor Contamination and Overlap**: Our regression results suggest that BAC has significant exposure to SMB and HML, indicating possible contamination from size and value effects. More rigorous orthogonalization or double-sorting methods could help isolate pure correlation or volatility effects.

- **Lack of Subperiod Analysis**: We conducted our analysis over the full sample period from 1968 to 2024 without breaking it into subperiods. This approach assumes stability of the anomaly over time, which may not hold in practice, especially given known shifts in market microstructure and investor behavior.

- **Breakpoints and Universe Choice**: Our sorting procedures used the full universe of stocks rather than NYSE breakpoints, as recommended in the literature. This may have introduced look-ahead bias or skewed our factor construction relative to standard benchmarks.

- **Absence of Transaction Costs and Turnover Analysis**: We did not account for transaction costs or portfolio turnover in our performance evaluation. Given the rebalancing frequency and long-short structure, these frictions could materially impact net returns.

Despite these limitations, our analysis demonstrates that systematic and idiosyncratic components of risk are both relevant in explaining the low-risk anomaly, and that our BAC strategy in particular exhibits strong risk-adjusted performance even under a simplified framework. Future work can address these shortcomings by incorporating more robust portfolio filters, alternative weighting schemes, and additional behavioral risk factors.

Acknowledgement

We would like to express our deepest gratitude to Professor Bernard Herskovic for his invaluable lectures and study materials throughout the course. His teaching played a crucial role in building the conceptual understanding and technical foundation required to complete this replication exercise.

We also acknowledge the responsible use of generative AI tools, which were particularly helpful in parsing complex academic papers and refining the clarity of the report. These tools served as a complement to our own analysis and critical thinking, rather than a substitute, and were used strictly for enhancing readability and productivity.

References

- Asness, C., Frazzini, A., Gormsen, N. J., & Pedersen, L. H. (2017). *Betting Against Correlation: Testing Theories of the Low-Risk Effect*. Working Paper, AQR Capital Management.
- Black, F. (1972). Capital market equilibrium with restricted borrowing. *Journal of Business*, 45(3), 444–455.
- Haugen, R. A., & Heins, A. J. (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis*.
- Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1), 1–25.