MFE 409 Lecture 5
Risk for Options

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Lecture Objectives

Risk management for option trading

- What are the risks of option strategies?
- How to quantify these risks?

Trading Derivatives and Risk Management

- Two broad levels of risk management inside financial institutions
  - Trader level: (hard) risk limits
    - ⋆ Often expressed in terms of Greeks
  - Institution level: aggregate positions and construct broad measures of risk
    - ⋆ Often around VaR

Delta

- Delta (Δ) of a portfolio: change in portfolio price in response to a change in underlying price

$$\Delta = \frac{\partial P}{\partial S}$$

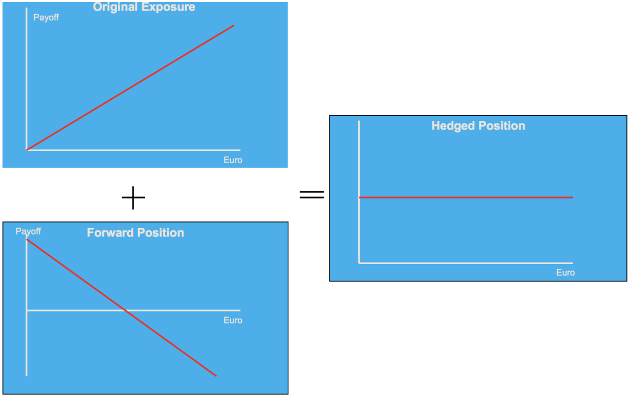
- Buying $-\Delta$ of the underlying protects the portfolio against local changes in underlying price
- Can also hedge with another option

Linear Products

- If the value of the portfolio is linear in the price of the underlying, delta-hedging eliminates all risk
- Examples: forwards, futures, fixed promises in foreign currency, ...
- Static hedging works perfectly: "hedge and forget"

Example

- A U.S. company has a receivable of EUR 10mil in one year.
- One-year forward exchange rate $F = 1.436\text{USD/EUR}$



Nonlinear Products

- If portfolio payoff nonlinear, static delta-hedging does not protect against larger shocks
- But ...

  - Continuous delta-hedging eliminates all risk
  - By no arbitrage, can be used to find option prices

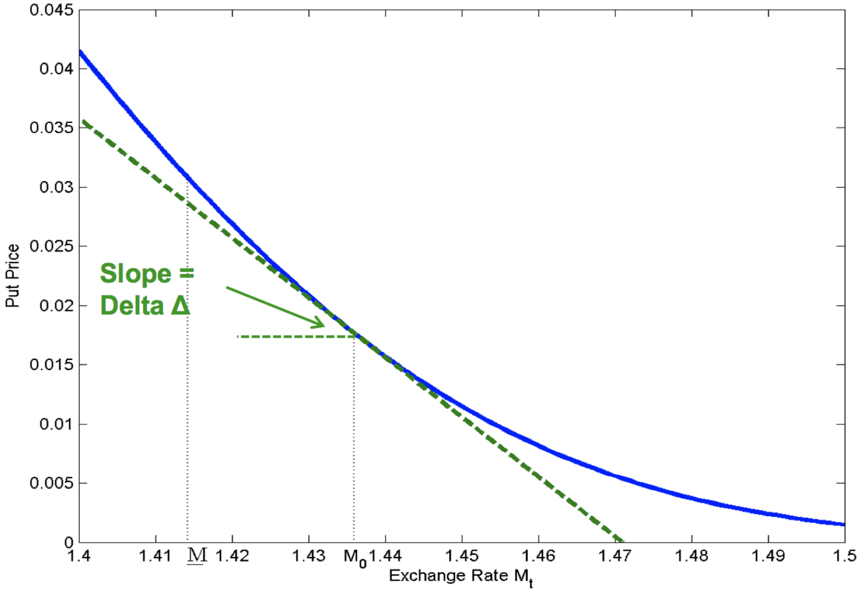
VaR for Options: Delta Approach

- Portfolio:
  - Long EUR10m, EUR/USD $= M_t = 1.436$, volatility of EUR/USD 0.65%)
  - Short 10m puts to sell euros in 6 months, $\Delta = -0.5044$
- 99% VaR?
  - Put price $p_t = G(M_t)$, where
  - Approximately: $p_{t+1} - p_t \approx G'(M_t) \times (M_{t+1} - M_t) = \Delta \times (M_{t+1} - M_t)$
  - Portfolio gain:

$$\begin{aligned} V_{t+1} - V_t &= 10\text{m} \times (M_{t+1} - M_t) - 10\text{m} \times (p_{t+1} - p_t) \\ &\approx 10\text{m} \times (1 - \Delta) \times (M_{t+1} - M_t) \\ &\approx \$14.36\text{m} \times (1 - \Delta) \times R_{M,t} \end{aligned}$$

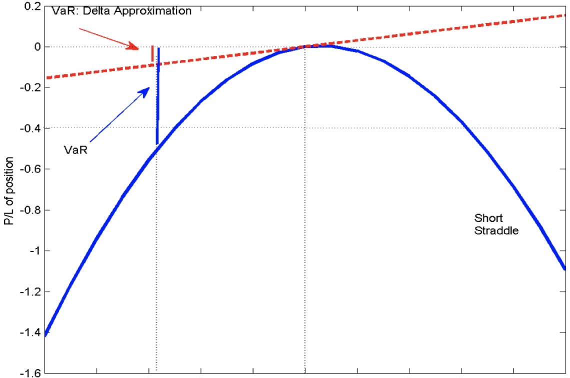
  - 99% 1-day VaR $= 1.5044 \times \$217,204 = \$326,762$

Put Price: Delta Approximation



When The Delta Approach Goes Wrong

- Nick Leeson, 1995 Barings Bank 
- Short puts and Calls with the same strike price on Nikkei Index



Gamma

- Gamma (Γ): second derivative of portfolio price with respect to the price of the underlying asset

$$\Gamma = \frac{\partial^2 P}{\partial S^2}$$

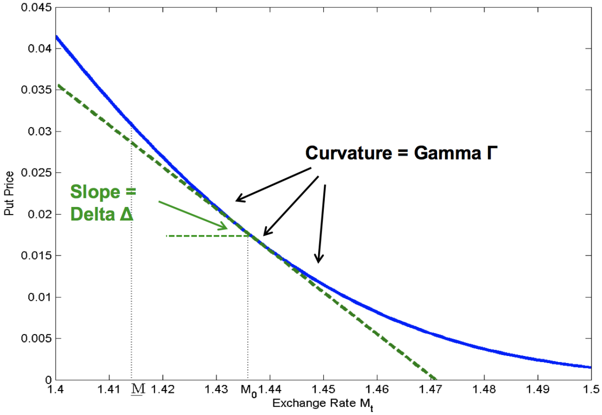
- Also rate of change of Delta with respect to the price of the underlying asset:

$$\Gamma = \frac{\partial \Delta}{\partial S}$$

  - Delta-Gamma hedging does better with less frequent readjustments

Put Price: Delta Gamma Approximation

$$P(S) \approx P(S_0) + \underbrace{P'(S_0)}_{\Delta}(S - S_0) + \frac{1}{2}\underbrace{P''(S_0)}_{\Gamma}(S - S_0)^2$$



VaR for Options: Delta-Gamma Approach

- Assume change in underlying price $S_{t+1} - S_t \sim \mathcal{N}(\mu_S, \sigma_S^2)$
- Change in portfolio value:

$$P_{t+1} - P_t = \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2$$

- Can compute moments of $R_S = S_{t+1} - S_t$:

$$\mathbb{E}[R_s] = \mu_S$$

$$\mathbb{E}[R_s^2] = \sigma_S^2 + \mu_S^2$$

$$\mathbb{E}[R_s^3] = \mu_S^3 + 3\mu_S\sigma_S^2$$

$$\mathbb{E}[R_s^4] = \mu_S^4 + 6\mu_S^2\sigma_S^2 + 3\sigma^4$$

VaR for Options: Delta-Gamma Approach

- Obtain mean and variance of $P_{t+1} - P_t$:

$$\begin{aligned}
\mathbb{E}[P_{t+1} - P_t] &= \Delta \mathbb{E}[R_s] + \frac{1}{2}\Gamma \mathbb{E}[R_s^2] \\
&= \Delta \mu_S + \frac{1}{2}\Gamma(\mu_S^2 + \sigma_S^2) \\
\text{var}[P_{t+1} - P_t] &= \Delta^2 \text{var}[R_s] + \frac{1}{4}\Gamma^2 \text{var}[R_s^2] + \Delta\Gamma \text{cov}[R_s, R_s^2] \\
&= \Delta^2 \sigma_S^2 + \frac{1}{2}\Gamma^2 \sigma_S^2(2\mu_S^2 + \sigma_S^2) + 2\Delta\Gamma \mu_S \sigma_S^2
\end{aligned}$$

- Plug in the estimator for the normal distribution:

$$\text{VaR}(c) = -\mathbb{E}[P_{t+1} - P_t] + z(c)\sqrt{\text{var}[P_{t+1} - P_t]}$$

- Can also deal with portfolio of options with more than one risk

Cornish-Fisher Expansion

- With this approach, we could also compute any moments of the portfolio: skewness, kurtosis, ...
- How to incorporate into VaR calculation?
- **Cornish-Fisher expansion**: asymptotic expansion for the quantile of a distribution
  - Skewness: $\xi_P = \mathbb{E}[(R_P - \mu_P)^3]/\sigma_P^3$
  - Quantile $1 - c$:

$$\mu_P + \left(z(1-c) + \frac{1}{6} \left(z(1-c)^2 - 1 \right) \xi_P \right) \sigma_P$$

  - Can also include kurtosis and higher moments

Vega

- Vega (ν): derivative of option value with respect to the volatility of the underlying asset

$$\nu = \frac{\partial P}{\partial \sigma}$$

- Under the assumptions of Black-Scholes, there is no risk of change in volatility ... but in practice volatility can move

- We can add changes in volatility to our previous calculations:

$$P_{t+1} - P_t = \Delta \times (S_{t+1} - S_t) + \frac{1}{2}\Gamma \times (S_{t+1} - S_t)^2 + \nu(\sigma_{t+1} - \sigma_t) + \ldots$$

Other Greeks

- Theta (Θ): change of the value of the portfolio due to passage of time:

$$\Theta = \frac{\partial P}{\partial t}$$

  - Often ignored for risk management (same as means)

- Rho: change of the value of the portfolio due to a parallel shift in all interest rates in a particular country

$$\text{Rho} = \frac{\partial P}{\partial r}$$

  - Particularly relevant for interest rate and exchange rate products

In practice

- Traders must be delta neutral at least once a day
- Traders must keep Gamma and Vega within limits set by risk management
  - Adjust whenever the opportunity arises
- Delta can be adjusted by trading the underlying
- Gamma and Vega need trading of other options

Takeaways

- When trading options, identify the key risks and hedge them
- Think one step ahead and about potential large shocks: gamma-hedging
- For risk management: crucial to take into account the non-linearity of option contracts