MFE 409: Financial Risk Management
Problem set 5

Valentin Haddad

due 5/12 before midnight

You should work with your assigned group but should write up your answer for question 1 individually. Give the name of your group members in your writeup and **post your answer on BruinLearn** before Monday May 12 at midnight. For the Case Study, only one member of your group needs to submit.

1 VaR for option on two underlying

1. We are interested in managing the risk of an option on two stocks with prices $S_{1,t}$ and $S_{2,t}$. Assume a short rate r and:

$$\begin{aligned} \frac{dS_{1,t}}{S_{1,t}} &= \mu_1 dt + \sigma_1 dW_{1,t} \\ \frac{dS_{1,t}}{S_{1,t}} &= \mu_2 dt + \sigma_2 dW_{2,t} \\ corr(dW_{1,t}, dW_{2,t}) &= \rho dt \end{aligned}$$

   where all the parameters are in daily units. Call $M(S_{1,t}, S_{2,t})$ the price of the option. *Derive a formula for the 99% 1-day VaR for the option using the delta approach.*

2. Consider the case of a European option on the minimum of the two stock price, with maturity T and final payoff:

$$M_T = \max\left(\min\left(S_{1,T}, S_{2.T}\right) - K, 0\right)$$

   From Stulz (1982) (attached paper), the price of the option when the time to maturity is τ is:

$$\begin{aligned} M(S_1, S_2) = {} & S_1 \mathcal{N}_2\left(\gamma_1 + \sigma_1\sqrt{\tau}, \left(\ln\left(S_2/S_1\right) - \frac{1}{2}\sigma^2\sqrt{\tau}\right) / \left(\sigma\sqrt{\tau}\right), \left(\rho\sigma_2 - \sigma_1\right)/\sigma\right) \\ & + S_2 \mathcal{N}_2\left(\gamma_2 + \sigma_2\sqrt{\tau}, \left(\ln\left(S_1/S_2\right) - \frac{1}{2}\sigma^2\sqrt{\tau}\right) / \left(\sigma\sqrt{\tau}\right), \left(\rho\sigma_1 - \sigma_2\right)/\sigma\right) \\ & - K e^{-r\tau} \mathcal{N}_2\left(\gamma_1, \gamma_2, \rho\right) \end{aligned}$$

where

$$\gamma_1 = \frac{\ln(S_1/K) + \left(r - \frac{1}{2}\sigma_1^2\right)\tau}{\sigma_1\sqrt{\tau}}$$
$$\gamma_2 = \frac{\ln(S_2/K) + \left(r - \frac{1}{2}\sigma_2^2\right)\tau}{\sigma_2\sqrt{\tau}}$$
$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

and $\mathcal{N}_2(\alpha, \beta, \theta)$ is the bivariate cumulative standard normal distribution with upper limits α and β and correlation θ. That is, if X_1 and $X_2$are standard normal with correlation θ:

$$\mathcal{N}_2(\alpha, \beta, \theta) = P(X_1 \leq \alpha, X_2 \leq \beta)$$

Assume: $r = 0.02\%, \sigma_1 = \sigma_2 = 1.5\%, \rho = 0.35, \mu_1 = \mu_2 = 0.025\%$, where again all parameters are in daily units. Further assume that at date 0, we have $T = 6$months, and that $S_{1,0} = 99$, $S_{2,0} = 101$ and $K = 100$. *Compute the price of the option at date* 0.

3. *Compute the 99% 1-day VaR for the option using the formula you have derived in question 1.*

4. *Compute the 99% 1-day VaR for the option using simulations. Compare to the results of the previous question and explain the intuition behind this result.*

5. **Optional:** *Derive a formula for the 99%-VaR for the option using the delta-gamma approach. Implement the formula and compare your results to the other two approaches. Explain the intuition for this result.*

6. *Now assume you are a trader in the real world and you do not know for sure that the model for the underlying is correct. What other types of risks would you worry about? If you had to worry about just one more risk, what would it be? Explain (quantitatively) how you get to this conclusion.*

2 Case Study: Implementing Quantitative Risk Management and VaR in a Chinese Investment Bank

You should buy the case study material using this link: `https://hbsp.harvard.edu/import/1283027`.

1. Explain the objectives and priorities of each player: Jasper Wang, Jianguo Lu, and Charles Pan. What is motivating the different players? What tensions existed among their different objectives?

2. Why does Jasper choose to make the VaR model the first step towards rationalizing the trading function? What is the appeal of the VaR model generally?

3. Why do Jianguo and the traders resist the VaR model? Do you think their pattern of resistance to risk management is unique to China, or might it be found elsewhere too?

4. Using the spreadsheet provided, run backtests of the VaR predictions against actual daily gains or losses for both the S&P 500 index and the Shanghai Composite index.

   (a) Starting with a lookback period of three months, observe the number of exceptions in all years for both the Shanghai and S&P indexes. How do they compare?

   (b) Try different lookback periods (say, 3, 6 and 9 months) to see if the length of the period changes your conclusions.

   (c) Given that Jasper's VaR model assumes a 95% confidence level, how well does the backtest validate the model?

5. How might Jasper use the backtest results to bolster his case for introducing the VaR model?

6. How successful do you think Jasper will be in his attempt to implement Western risk management practices? What advice would you give to someone in a role similar to his?

7. What is the current regulation environment of risk followed by Chinese banks and how has it evolved since the crisis? *(Find information beyond the case study material)*