MFE 409: Financial Risk Management
Problem set 2

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due 4/14 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and submit it on BruinLearn before Monday April 14 at midnight.

1 Expected shortfall

1. Derive a formula for the expected shortfall if gains are normally distributed $\mathcal{N}\left(\mu, \sigma^2\right)$.

2. The expected shortfall can also be defined as the average of the VaR for all confidence level above c:

$$ES = \frac{1}{1-c}\int_c^1 VaR_\alpha d\alpha$$

   Prove that this definition is equivalent to the one we have seen in class. *Hint: You can use integration by part and a change of variable.*

2 Decomposing the VaR of a portfolio

The attached python-code contains a function called "draw_returns" with one argument N. Calling the function returns N random draws for the returns of 3 assets, A, B, and C. Please take this function as given and do not modify it (assume it is a black box).

1. Assume you have a portfolio of \$3m in asset A, \$4m in asset B and \$3m in asset C. What is the VaR of your portfolio? What is the CVaR and DVaR for each of the assets? Check that the sum of CVaRs coincides with VaR. Which asset is responsible for the most risk of the portfolio?

2. When you approximate the derivatives involved in DVaR and CVaR, vary the size of the position change you use. What do you observe when you change the value? Report a graph that shows the effect of different size of the step for the derivative. Explain what is happening.

3. You change your portfolio to $3m in asset A, $5m in asset B and $2m in asset C. What are the CVaR and DVaR for asset C?

3 Managing a Currency Trading Desk

Deutsche Bank (DB) is a German bank that manages its book in EUR. Consider 2 desks in DB, one is long 150 million USD and the other is short 50 million GBP. The exchange rates are 1 USD = 0.93 EUR and 1 GBP = 1.17 EUR. The daily volatilities for changes in USD/EUR and GBP/EUR are 0.40% and 0.30%, respectively and means of 1 basis point and 2 basis points. The correlation between them is 0.7. For risk calculations, assume that the returns have mean zero and are normally distributed.

1. What is the 99% 1-day VaR for each desk?
2. What is the 99% 1-day VaR for the combined portfolio?

Optional, more challenging:

3. Consider an arbitrary portfolio with positions x_1 and x_2 in two assets. If you increase your position in x_1 by a small amount Δx_1, by how much do you need to change your position in asset 2 to keep your VaR constant? What is the effect of these changes on your expected profits? Obtain mathematical expressions as function of (some but not necessarily all of) Δx_1, VaR, DVaR, CVaR, and expected returns.

4. How would you change the allocation of DB's trading desk? Give a quantitative argument.