Solution Report: Problem Set 8
MFE 409 — Financial Risk Management

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Abstract

This report addresses two key components of Problem Set 8 for MFE 409: Financial Risk Management. The first part presents a critical reflection on the case of Long-Term Capital Management (LTCM), based on the assigned paper by Philippe Jorion, analyzing the hedge fund's strategy, leverage, collapse, and risk management shortcomings, with proposals for improved risk oversight. The second part applies the Merton model of credit risk to evaluate a firm's likelihood of default, its distance to default, and the expected recovery rate, using given financial parameters. Together, these sections deepen understanding of real-world risk management failures and the application of structural credit risk modeling.

Contents

1 Question 1: LTCM

1.1 What was the broad trading strategy of LTCM?

Long-Term Capital Management (LTCM) employed a highly leveraged convergence arbitrage strategy. Their core idea was to identify small mispricings between financial instruments that were economically or statistically similar and bet that these discrepancies would converge over time. LTCM's portfolio consisted of various relative-value trades across sovereign bonds, interest rate derivatives, and credit instruments.

A prime example includes trades between on-the-run and off-the-run U.S. Treasury bonds. On-the-run Treasuries are the most recently issued and highly liquid, while off-the-run ones are older and less liquid, although they are nearly identical in cash flow structure. LTCM would short the relatively expensive on-the-run bond and go long the cheaper off-the-run bond, expecting the yield spread between them to narrow.

Another significant set of trades involved sovereign spreads, such as Italian versus German government bonds, where LTCM bet on convergence in yields as countries moved toward European monetary union. They also arbitraged swap spreads, corporate bond spreads, and even emerging market debt.

To execute this strategy effectively, LTCM relied on advanced quantitative models, historical correlation structures, and significant leverage to amplify the small expected returns from convergence trades. The assumption underlying these strategies was that markets are ultimately efficient and that pricing anomalies would correct over time. However, this also made the fund extremely vulnerable to unexpected changes in market liquidity, risk aversion, or structural breaks in correlations.

1.2 Why did they need so much leverage?

LTCM needed extreme leverage to generate substantial returns from trades with narrow spreads. Their arbitrage opportunities—such as yield differences between nearly identical bonds or instruments—typically offered **only a few basis points** in potential profit. Without leverage, the capital deployed would not generate meaningful returns.

For instance, a 10 basis point (0.10%) spread on a $10 million position yields just $10,000. To scale this into millions of dollars in profit, the notional exposure had to be hundreds of millions or even billions—achievable only through borrowing and derivative instruments.

Leverage also allowed LTCM to maintain a large, diversified portfolio of small mispricing bets. However, this came at a cost:

- **Funding fragility:** Heavy reliance on short-term borrowing and repurchase agreements (repos) made the firm vulnerable to margin calls and liquidity freezes.
- **Risk amplification:** While leverage boosted profits during normal market conditions, it magnified losses during times of market stress or dislocation.

Ultimately, their need for leverage stemmed from their fundamental strategy's low-return, low-risk assumptions, which became invalid when market conditions deteriorated.

1.3 How did their demise happen?

LTCM's collapse in 1998 was precipitated by a series of adverse market events that caused its highly leveraged positions to implode. The key trigger was the Russian government's default on its local currency debt in August 1998. This unexpected shock led to a global flight to quality, with investors dumping risky assets and flocking to safe-haven securities like U.S. Treasuries.

LTCM's convergence trades were premised on yield spreads narrowing between risky and safe assets. Instead, spreads widened dramatically. For example:

- Swap spreads increased.
- On-the-run vs. off-the-run Treasury yield gaps widened.
- Sovereign spreads (e.g., Italian vs. German bonds) diverged.

These moves were exactly opposite of what LTCM had bet on. As a result:

- LTCM faced mounting losses—its equity dropped from $4.8 billion to under $500 million.
- Margin calls and demands for additional collateral began to roll in.
- Other funds began to unwind similar trades, exacerbating the dislocations and creating a feedback loop.

Due to the sheer size of LTCM's positions and its interconnectedness with major financial institutions, its failure threatened systemic risk. This led the Federal Reserve Bank of New York to orchestrate a private-sector bailout to avoid broader contagion. Although no public funds were used, the Fed's intervention underscored the fragility and systemic danger posed by large, leveraged financial entities operating without sufficient transparency or capital buffers.

1.4 What were the most important issues with their risk management approach?

LTCM's risk management suffered from critical flaws across multiple dimensions. While the firm employed advanced quantitative models and stress testing, these tools were based on assumptions that failed under market stress. The key weaknesses included:

- **Overreliance on Historical Correlations and Normality Assumptions:** LTCM's models assumed that asset returns followed a normal distribution and that correlations between instruments were stable. In reality, during crises, correlations can spike, and extreme market moves become far more likely than a normal distribution predicts.
- **Inadequate Stress Testing:** While LTCM did conduct stress tests, they failed to capture the systemic and nonlinear effects that could arise in a crisis. The models underestimated the possibility of market-wide deleveraging and liquidity spirals.

- **Liquidity Risk Ignored:** LTCM assumed it could easily unwind positions. However, many of their trades were in illiquid or thinly traded markets. When the need arose to exit positions, the lack of liquidity amplified their losses.

- **Concentration of Risk:** Despite appearing diversified, LTCM's portfolio was effectively concentrated on one bet: convergence. Since many trades shared the same directional risk, the fund was exposed to a single underlying factor—market normalization—which failed to materialize.

- **Opaque and Centralized Decision-Making:** Risk management lacked independence and was not empowered to override trading decisions. Senior partners held tightly concentrated control and resisted external scrutiny or governance.

- **Underestimation of Tail Risk:** LTCM relied on VaR metrics that gave a false sense of security by focusing on typical (non-tail) scenarios. Their 4-sigma VaR confidence interval implied losses of LTCM's magnitude were nearly impossible, yet they occurred.

These oversights show that even highly quantitative risk frameworks are insufficient without humility, robust governance, and consideration of extreme but plausible market events.

1.5 How would you manage risk for a fund trying to trade similar strategies?

Managing risk for a fund employing convergence or relative-value arbitrage strategies requires a multi-layered, adaptive approach that addresses both quantitative and qualitative sources of risk. Key recommendations include:

- **Stress Testing and Scenario Analysis:** Go beyond historical simulations by applying forward-looking stress scenarios that reflect extreme but plausible events, such as sudden spikes in volatility, liquidity freezes, or widespread deleveraging. These should include correlation breakdowns, not just individual asset shocks.

- **Use of Expected Shortfall (ES) over VaR:** Value-at-Risk (VaR) can underestimate tail risk. ES, which measures the average loss in the worst cases beyond the VaR threshold, provides a better understanding of potential downside.

- **Liquidity-Adjusted Risk Measures:** Risk metrics should incorporate liquidity considerations, such as bid-ask spreads, trading volume constraints, and market depth. Portfolios should include haircuts or penalties based on the cost and feasibility of liquidation under stress.

- **Limit Leverage and Impose Dynamic Margin Buffers:** Use conservative leverage caps, and require additional capital cushions when volatility or correlation risk rises. Consider leverage limits that adjust dynamically with market conditions.

- **Diversification across Strategies and Risk Factors:** Avoid hidden concentration by analyzing exposures across common underlying factors. A portfolio of superficially different trades may still be unified by a single directional bet (e.g., mean reversion or tightening spreads).

- **Independent Risk Oversight and Governance:** Ensure that the risk management function is organizationally independent from trading, with authority to enforce limits and escalate issues. Regular reporting to a risk committee or board is essential.
- **Transparency with Investors and Counterparties:** Maintain openness about exposures, leverage, and risk procedures. This builds trust and avoids panic-induced liquidity squeezes in periods of market stress.
- **Crisis Playbooks and Contingency Planning:** Establish pre-planned procedures for managing margin calls, forced unwinds, and credit line disruptions. Include funding diversification and strong relationships with multiple counterparties.

Ultimately, effective risk management is not just about sophisticated models—it also requires humility, organizational discipline, and preparedness for the unexpected.

2 Question 2: Merton Model for Credit Risk

A company's equity is \$3 billion and the volatility of equity is 50%. The face value of debt is \$20 billion and time to debt maturity is 3 years. The risk-free rate is 4.5%.

2.1 What is the distance to default?

To calculate the distance to default, we apply the Merton model, which treats the company's equity as a call option on its assets. The unknowns — asset value and asset volatility — are solved using a nonlinear system based on the Black-Scholes framework. The following Python code implements this process:

```python
import numpy as np
from scipy.stats import norm
from scipy.optimize import fsolve

def compute_distance_to_default():
    """
    Solves the Merton model numerically to estimate the firm's asset value,
    asset volatility, and calculate the distance to default (DD) using the following inputs:
    - Equity value: $3 billion
    - Equity volatility: 50%
    - Face value of debt: $20 billion
    - Time to maturity: 3 years
    - Risk-free rate: 4.5%

    Returns:
        V_sol (float): estimated asset value (in billions)
        sigma_V_sol (float): estimated asset volatility
        d2 (float): distance to default
    """
    E = 3.0
    sigma_E = 0.50
    D = 20.0
    T = 3.0
    r = 0.045

    def merton_equations(vars):
        V, sigma_V = vars
        d1 = (np.log(V / D) + (r + 0.5 * sigma_V**2) * T) / (sigma_V * np.sqrt(T))
        d2 = d1 - sigma_V * np.sqrt(T)
        eq1 = V * norm.cdf(d1) - D * np.exp(-r * T) * norm.cdf(d2) - E
        eq2 = (V * norm.cdf(d1) * sigma_V) / E - sigma_E
        return [eq1, eq2]

    initial_guess = [20.0, 0.3]
    V_sol, sigma_V_sol = fsolve(merton_equations, initial_guess)
    d2 = (np.log(V_sol / D) + (r - 0.5 * sigma_V_sol**2) * T) / (sigma_V_sol * np.sqrt(T))

    return V_sol, sigma_V_sol, d2

V, sigma_V, DD = compute_distance_to_default()
print(f"Estimated Asset Value (V): ${V:.2f} billion")
print(f"Estimated Asset Volatility (σ_V): {sigma_V:.2%}")
print(f"Distance to Default (DD): {DD:.2f}")
```


Figure 1: Python code for computing distance to default using the Merton model

Results from the code:

- Estimated Asset Value (V): \$20.22 billion
- Estimated Asset Volatility (σ_V): 8.71%
- Distance to Default (DD): 0.89

This indicates the firm's asset value is only 0.89 standard deviations above the default threshold, suggesting a relatively high probability of default under current assumptions.

2.2 What is the default probability?

In the Merton model, the default probability is the likelihood that the firm's asset value at debt maturity is less than the face value of its debt. This is equivalent to:

$$\mathbb{P}(V_T < D) = \Phi(-d_2)$$

where d_2 is the distance to default, and Φ is the cumulative distribution function (CDF) of the standard normal distribution.

Given the previously calculated value of $d_2 = 0.89$, we use the standard normal CDF to compute the probability of default. The following Python code implements this:

```python
from scipy.stats import norm

def compute_default_probability(dd):
    """
    Computes the default probability using the Merton model
    given the distance to default (DD).

    Args:
        dd (float): Distance to default (d2 in Merton model)

    Returns:
        float: Default probability
    """
    return norm.cdf(-dd)


dd_value = 0.89
default_prob = compute_default_probability(dd_value)
print(f"Default Probability: {default_prob:.4f}")
```


Figure 2: Python code to compute default probability from distance to default

Result:

- Default Probability: $\Phi(-0.89) \approx 0.1867$

This means there is an approximately 18.67% probability that the firm will default on its debt obligations within the 3-year horizon under the assumptions of the Merton model.

2.3 What is the expected recovery rate on the debt?

The expected recovery rate is defined as the expected value of the firm's assets at maturity conditional on default, expressed as a fraction of the face value of debt:

$$\text{Expected Recovery Rate} = \frac{\mathbb{E}[V_T \mid V_T < D]}{D}$$

Assuming that the firm's asset value follows a lognormal distribution under the risk-neutral measure, the recovery rate can be computed by integrating over the lower tail of the distribution (i.e., asset values less than D). The following Python code performs this calculation:

```python
from scipy.stats import lognorm
import numpy as np

def expected_recovery_rate(V, sigma_V, D, T, r):
    """
    Computes the expected recovery rate given Merton model inputs.

    Args:
        V (float): Current asset value
        sigma_V (float): Asset volatility
        D (float): Face value of debt
        T (float): Time to maturity
        r (float): Risk-free rate

    Returns:
        float: Expected recovery rate (0 to 1)
    """
    mu = np.log(V) + (r - 0.5 * sigma_V**2) * T
    sigma = sigma_V * np.sqrt(T)

    dist = lognorm(s=sigma, scale=np.exp(mu))

    numerator = dist.expect(lambda x: x, lb=0, ub=D)
    denominator = dist.cdf(D)

    return numerator / (D * denominator)

# Using solved values:
V = 20.22
sigma_V = 0.0871
D = 20.0
T = 3.0
r = 0.045

recovery_rate = expected_recovery_rate(V, sigma_V, D, T, r)
print(f"Expected Recovery Rate: {recovery_rate:.4f}")
```


Figure 3: Python code for computing expected recovery rate using the Merton model

Result:

- Expected Recovery Rate: approximately 92.29%

This means that, conditional on default, creditors can expect to recover about 92.29% of the face value of the firm's debt.

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References

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