MFE 409: Financial Risk Management
Problem set 7

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Due 5/26 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and post it on BruinLearn before May 26 at midnight.

1 Bootstrapping a CDS curve

1. Recover the hazard rate curve from the slide "Bootstrapping Default Probabilities from CDS" (slide 17) of the notes.
2. Use this hazard rate curve to price a 7-year bond on the same company which pays 3% coupon every 6 month and has face value \$100.

2 Historical vs bond-implied hazard rates

Explain the patterns you see in the table on the slide "Comparing Hazard Rates" (slide 19) of the notes.

3 *Optional:* Dynamic credit model

Consider 8 categories: AAA, AA, A BBB, BB, B, CCC and default. We are interested in constructing a stochastic dynamic model of rating and default in continuous time. For this we will use the information in slide "Rating Transitions" (slide 7) of the notes.

1. Let us call $P(t)$ the 8×8 matrix of transition probability after time t. This means that $P_{ij}(t)$ is the probability of being in category j at date t if the firm is in category i at date 0.
   (a) What is $P(0)$?
   (b) What is $P(1)$?

2. Just like we defined the hazard rate has the instantaneous probability of default, we can consider instantaneous transition probability λ_{ij} such that $\lambda_{ij}dt$ is the probability of going from rating i to rating j during an interval dt if $i \neq j$. When $i = j$, we define λ_{ii} as the opposite of the intensity of leaving state i: $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$. We can put all these in a matrix Λ. Express Λ as a function of P and its first derivative.

3. Assuming that Λ is constant over time, derive an expression relating $P(1)$ and Λ.

4. Compute Λ for the values of slide "Rating Transitions" (slide 7) of the notes.

5. Use this matrix Λ to compute the probabilities of default at horizon 1, 2, 3, 4, 5, 7, and 10 years given each initial rating.

6. Compare your results to the slide "Historical Default Probabilities" (slide 6) of the notes. What can explain the similarities and differences?

7. Use this model to price a 6-year bond on a BBB company which pays 3% coupon every 6 month and has face value \$100. Assume that the risk-free interest rate is 0% and recovery is 60%

8. Compute the 3, 5, and 10-year CDS spreads for the same company.