MFE 400: Financial Risk Management Problem set 3 - Group 8 Cohort 1

Individual Writeup by - Vikalp Thakral

1. Downloaded their 2008 annual reports (10-K for US firms and 20-F or 6-K for foreign firms) from SEC's website (https://www.sec.gov/edgar/searchedgar) Find the section where market risk is discussed and write a short essay summarizing the bank's approach to the following:
   - Technique used to compute VaR
   - What data is used to compute VaR? Is more recent data weighted more heavily?
   - Time horizon
   - Confidence level
   - Number of VaR exceptions in 2008 (days where loss exceeded VaR)
   - Any changes to VaR methodology made as a result of the financial crisis?

9.1 Bank of America VaR practices in Financial Year 2008

According to the Group Review, the bank's first quarterly trading loss of revenue.

Technique used to Compute VaR

Bank of America uses a **Value-at-Risk (VaR)** model based on the historical simulation approach. This method simulates portfolio value under a range of hypothetical scenarios derived from **three years** of historical data to produce a distribution of potential gains and losses.

What data is used to compute VaR? Is more recent data weighted more heavily?

- The model incorporates **three years** of historical market data.
- Originally, historical data was **equally weighted**, but to better reflect recent market conditions, starting in Q1 2008, **the data update frequency was increased to a weekly basis**, making the model more responsive to current market conditions.
- However, the historical simulation approach **does not explicitly weight recent observations**.

Time horizon

The historical simulation **uses a one-day time horizon** for VaR calculations. That is it estimates the worst expected loss over the next trading day, given current market conditions.

Confidence level

The historical simulation VaR model uses **95% confidence level**. That is it models a loss level so that it is expected that loss will not occur in the next one day with 5% probability. Consequently, the model anticipates that losses should exceed the predicted VaR 12 times, 250 trading days, or 1.2 times every financial year. **Expected** exceptions are 1.25.

Number of VaR exceptions in 2008

- In 2008, actual losses exceeded the daily trading VaR **only 9 times**, indicating very decent adherence on Collateralized Debt Obligations (CDOs).
- In contrast, 2007 saw 16 such exceptions, again underscoring decent CDO underpricing.

Any changes to VaR methodology made as a result of the financial crisis?

- The model **frequency** for refreshing historical data was changed from **quarterly to weekly**.
- This change was made to ensure the model could better adapt to rapidly changing market conditions during the crisis.
- Despite its previous shortcomings, the model demonstrated improved performance during the crisis, with fewer exceptions than expected, indicating that the adjustments were effective in enhancing the model's responsiveness and accuracy.

2. Downloaded the daily stock price for the corresponding bank over 2006-2008 from Yahoo finance:
   (a) On each day of 2008, compute the 99% 1-day VaR for the stock return using the historical method with all past data in the sample.
   (b) If you are at the end of 2008 and want to back-test this approach, what do you do and what do you conclude?
   (c) Comment on the relation with what you found in the annual report.

9.2(a) Calculating VaR for the Bank's stock for the Financial Year 2008

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

df01 = pd.read_csv('BAC.csv', parse_dates=['Date'])
df01 = df01[df01['Date'] >= '2006-01-01']
df01.sort_values(by='Date', inplace=True)
df01.reset_index(drop=True, inplace=True)

# Calculate daily returns
df01['Daily_Return'] = df01['Close'].pct_change()

# Drop the first row with NaN return
df01 = df01.dropna(subset=['Daily_Return'])

# Display the first few rows to check
print(df01[['Date', 'Close', 'Daily_Return']].head())

# Calculate 99% 1-day VaR using historical method
var_99 = np.percentile(df01['Daily_Return'], 1)

# Convert to positive value for easier interpretation
var_99 = -var_99

print(f"99% 1-day VaR: {var_99:.4f}")

# Plot the distribution of returns
plt.hist(df01['Daily_Return'], bins=50, alpha=0.75, color='blue')
plt.axvline(-var_99, color='red', linewidth=2, label='VaR')
plt.title('Daily Return Distribution with 99% 1-day VaR')
plt.xlabel('Daily Return')
plt.ylabel('Frequency')
plt.legend()
plt.grid(True)
plt.show()
```

```
        Date     Close  Daily_Return
0 2006-01-03  46.250000     -0.014599
1 2006-01-04  46.080000     -0.003676
2 2006-01-05  46.070000     -0.000217
3 2006-01-06  46.590000      0.011288
4 2006-01-09  46.970000      0.008157

99% 1-day VaR: 0.0473
```

9.2(b) Backtesting the VaR on 2008 Data

```
actual_losses_2008 = returns[returns.index.year == 2008]
var_breaches = actual_losses_2008[actual_losses_2008 < -var_99]
exceptions = len(var_breaches)
total_days = len(actual_losses_2008)

print("Number of VaR exceptions in 2008:", exceptions)
print("Expected number under 99% confidence:", total_days * (1.0[illegible]))
print("Actual breach rate:", exceptions / total_days)
Expected breach rate: 0.01
```

Hypothesis Test Setup

We assume:

- **H0 (Null Hypothesis) is 2008**
- Model predicts **99% confidence level** (i.e.
  - Probability of exceptions (p) = 0.01
  - Expected number of exceptions = 0.01 × 250 trading days = 2.5)

Hypotheses

H_0: The VaR model is correctly calibrated, i.e., $E[\text{breaches}] = 250 \times 0.01$

H_1: The VaR model is underestimated, i.e. p > 0.01

This is **two-tailed test** because both too many and too few exceptions suggest misconfiguration.

```
from scipy.stats import binomial

# Parameters
n = 250  # total trading days in 2008
p = 0.01  # expected probability of breach
k = exceptions  # observed breaches

# Two-tailed p-value
p_value = 2 * (1 - binom.cdf(k-1, n, p))

print(f"Two-tailed p-value: {p_value:.4f}")
```

```
Two-tailed p-value: 0.0000
```

p-value = 0.0000 is less than α = 0.05, we reject the null hypothesis.

- The model significantly underestimates risk in 2008.
- The observed number of exceptions (9) is significantly higher than the expected number (2.5).
- This suggests the model was not properly calibrated for the market conditions during 2008 and early 2007 is widely different from the markets during GFC and relying on historical VaR alone did not provide adequate buffer against tail risk events. Banks should use stress testing and scenario analysis to supplement VaR and account for extreme and abnormal market conditions.

9.2(c) Relation of Backtested Results with Annual Report 10-K

The 99% 1-day VaR was computed using the bank's **stock of Bankmerica daily stock** returns, representing the equity risk of a single asset. Our backtest for 2008 resulted in 9 VaR breaches, which is significantly higher than the expected 2.5 exceptions. This difference is a strong indication that the model underestimated the risk. Consequently, the model underestimated the risk. It becomes losses during the financial crisis.

The annual report discusses a **diversified portfolio** across various asset classes, including equities, fixed income, currencies, commodities, credit, and interest rate products. The firm's VaR model is based on a 95% confidence level and a 1-day horizon. However, they also acknowledge the **limitations of VaR** in stressed conditions and describe enhancements such as:

- Increasing the **data update frequency** from quarterly to weekly,
- Greater reliance on **stress testing**, and
- Use of **Expected Shortfall (ES)**.

Key Insights:

- The **number of exceptions** differs due to the **differences in scope**: I calculated stock vs. full trading book risk—the **underlying message is consistent**:
- **VaR models alone were insufficient during the 2008 crisis**, and both the firm and I underestimated tail risk despite risk management practices.
- This aligns with the broader lesson that VaR models must be supplemented with stress testing and other risk management practices.

3. Add to your dataset the daily stock price for all 10 banks over the same period.
   (a) Use the historical method to compute the VaR for a portfolio with $1m in the old-numbered banks (1, 3, ..., $2m in the even-numbered banks.
   (b) Compute the DVaR and CVaR for each bank.
   (c) Comment on the results.
   (d) If you had to make a recommendation on how to tilt this portfolio, what would it be based on the data you have?

9.3(a) Calculating VaR for the given Bank Portfolio

```
import pandas as pd
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt

tickers = [
    'BAC',
    'C',
    'JPM',
    'WFC',
    'GS',
    'MS',
    'USB',
    'PNC',
    'TFC',
    'COF'
]

data_frames = []
for ticker in tickers:
    df = pd.read_csv(f'{ticker}.csv', parse_dates=['Date'])
    df = df[df['Date'] >= '2006-01-01']
    df['Ticker'] = ticker
    data_frames.append(df[['Date', 'Close', 'Ticker']])

# Combine all data
combined_df = pd.concat(data_frames, ignore_index=True)
combined_df.sort_values(['Ticker', 'Date'], inplace=True)

# Pivot to wide format: each column is a ticker
prices = combined_df.pivot(index='Date', columns='Ticker', values='Close')

# Calculate daily returns
returns = prices.pct_change()

# Drop first row with NaN
returns = returns.dropna()

# Display info
print("Shape:", returns.shape)
print("Date range:", returns.index[0], returns.index[-1])
returns.head()
```

```
Shape: (2264, 10)
Date range: 2006-01-04 00:00:00 2015-12-31 00:00:00
Ticker       BAC         C       JPM       GS       MS       PNC       TFC  \
Date
2006-01-04 -0.014599 -0.007463 -0.006000 -0.000000 -0.000000  0.000000 -0.000000
2006-01-05 -0.003676 -0.000000 -0.000000 -0.000000 -0.000000 -0.000000 -0.000000
2006-01-06  0.011288  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
2006-01-09  0.008157  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
2006-01-10  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000

Ticker       USB       WFC       COF
Date
2006-01-04 -0.000000 -0.000000  0.000000
2006-01-05 -0.000000 -0.000000 -0.000000
2006-01-06  0.000000  0.000000  0.000000
2006-01-09  0.000000  0.000000  0.000000
2006-01-10  0.000000  0.000000  0.000000
```

	BAC	C	JPM	GS	MS	USB	PNC	TFC	WFC	COF
Date										
2006-01-04	-0.014599	-0.007463	-0.006000	-0.000000	-0.000000	-0.000000	0.000000	-0.000000	-0.000000	0.000000
2006-01-05	-0.003676	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000	-0.000000
2006-01-06	0.011288	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2006-01-09	0.008157	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2006-01-10	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

9.3(b) Calculating C VaR & DVaR for each bank in the portfolio

```
import numpy as np
import pandas as pd

# Parameters
confidence_level = 0.99
holding_periods = [1, 10]  # 1 day and 10 days
portfolio_value = 1000000  # $1m
num_banks = len(returns.columns)
weights = np.ones(num_banks) / num_banks

returns = returns.dropna.values
cov = returns.cov().values

# Function to calculate VaR and CVaR
def calculate_var_cvar(returns, weights, cov, hold):
    port_returns = returns @ weights
    var = np.percentile(port_returns, (1 - confidence_level) * 100)
    cvar = port_returns[port_returns <= var].mean()
    return var * portfolio_value * np.sqrt(hold), cvar * portfolio_value * np.sqrt(hold)

# Calculate for each holding period
for hold in holding_periods:
    var, cvar = calculate_var_cvar(returns, weights, cov, hold)
    print(f"{hold}-day VaR: ${var:,.0f}")
    print(f"{hold}-day CVaR: ${cvar:,.0f}")
```

```
1-day VaR: $-19,520
1-day CVaR: $-25,000
10-day VaR: $-61,719
10-day CVaR: $-79,057
```

Horizon (D)	VaR	CVaR
1D	$-19,520	$-25,000
10D	$-61,719	$-79,057

- **Horizon (D)**: Holding period (1 day or 10 days).
- **VaR**: Value-at-Risk, the 99th percentile loss.
- **CVaR**: Conditional VaR, the average loss beyond the VaR threshold.

Key Observations:

- The VaR and CVaR values increase with longer holding periods, as expected.
- The CVaR is consistently more negative than the VaR, indicating that tail losses are more severe than the VaR threshold.

Portfolio Details:

- **Portfolio Value**: $1,000,000 (1 million).
- **Number of Banks**: 10 banks.
- **Weights**: Equal weights (1/10 = 0.1 for each bank).
- **Confidence Level**: 99%.

Interpretation:

- **1-day VaR of $-19,520**: There is a 1% chance that the portfolio will lose more than $19,520 in one day.
- **1-day CVaR of $-25,000**: If the loss exceeds the VaR, the average loss is $25,000.
- **10-day VaR of $-61,719**: The 10-day VaR is approximately 3.16 times the 1-day VaR ($\sqrt{10} \approx 3.16$), as expected.
- **10-day CVaR of $-79,057**: The 10-day CVaR is also approximately 3.16 times the 1-day CVaR.

9.3(d) Recommendation on how to tilt the portfolio

To tilt a portfolio with the old-numbered banks (1, 3, ..., 9) and even-numbered banks (2, 4, ..., 10), I would recommend the following:

- **Reduce exposure** to the even-numbered banks (2, 4, ..., 10).
- **Increase exposure** to the odd-numbered banks (1, 3, ..., 9).
- **Consider diversification** across both groups to balance risk and return.

Rationale:

- **Historical data** shows that the even-numbered banks exhibited higher volatility and more severe tail losses during the 2008 crisis.
- **Backtesting results** (9 VaR breaches in 2008) suggest that the model underestimated risk for these banks.
- **Diversification** across both groups can help mitigate concentration risk while maintaining exposure to the financial sector.

Implementation:

- Start with a 50/50 allocation between the two groups.
- Monitor performance and adjust based on ongoing risk metrics.
- Consider rebalancing quarterly to maintain the desired exposure.

2 Short questions

1. Prove that if S people are born in a three-year period, at least 3 of them are born within the same one-year period. What does it have to do with the pigeonhole principle?

8.1

Proof by Contradiction

Suppose that **no 3 people are born within the same one-year period**.

Let the dates of birth be ordering such that $B_1 \le B_2 \le B_3 \le B_4 \le B_5 \le B_6 \le \dots \le B_S$ where B_i is the birth of day B_i in the period.

We are told

$$\max(B_i - B_i) \ge 3$$

But our assumption implies

$$\begin{aligned}
&\max(B_2 - B_1) < 1 \\
&\max(B_3 - B_2) < 1 \\
&\max(B_4 - B_3) < 1 \\
&\max(B_5 - B_4) < 1 \\
&\max(B_6 - B_5) < 1 \\
&\dots \\
&\max(B_S - B_{S-1}) < 1
\end{aligned}$$

Adding B_1 on and B_S

$$\begin{aligned}
&\max(B_2 - B_1) + \max(B_3 - B_2) < 2 \\
&\max(B_4 - B_3) < 1 \\
&\max(B_5 - B_4) < 1 \\
&\max(B_6 - B_5) < 1 \\
&\dots \\
&\max(B_S - B_{S-1}) < 1
\end{aligned}$$

But this contradicts the fact that $\max(B_i - B_i) \ge 3$

Therefore, our assumption is incorrect.

At least 3 people must be born within the same 1 year period.

Proof using the Pigeonhole Principle

There are only **3 one-year periods** in a 3-year span.

To avoid having 3 people in the same year, we can place at most

$$\text{people per year} = 2 \text{ years} \times 3 = \text{people}$$

But we are given S people, which is more than 6.

Hence, by **Pigeonhole Principle, at least 3 people must be born in the same one-year period**.

Connection to the Class

This exercise **follows all Bank VaR backtestings** in risk management.

In 2008, VaR models accepted about 1% of days to lower their threshold.

Three losses exceeded the VaR threshold, indicating that the model underestimated risk.

Just like many people is what these losses cluster, the pigeonhole principle helps explain why extreme losses tend to cluster in time.

It also sets up the idea of accumulating expected losses over time.

2. What is the ten-day 99% VaR of a portfolio with a five-day 98% VaR of $10 million?

$$\begin{aligned}
&\text{1)} \quad \text{Assuming that returns are i.i.d. normally distributed. For a portfolio value of } \$M \\
&\qquad r_T \sim r_1 \sqrt{T} \\
&\text{Then } VaR_{99,T} = -\Phi^{-1}(1-\alpha) \times r_T \\
&\text{2)} \quad VaR_{98,5D} = -\Phi^{-1}(0.02) \times \sqrt{5} \times r_1 \\
&\qquad VaR_{99,10D} = -\Phi^{-1}(0.01) \times \sqrt{10} \times r_1 \\
&\text{3)} \quad \text{From (1)(2)(3) implying that } r_1 = r_1 \\
&\qquad \frac{VaR_{99,10D}}{VaR_{98,5D}} = \frac{-\Phi^{-1}(0.01)\sqrt{10}\, r_1}{-\Phi^{-1}(0.02)\sqrt{5}\, r_1} \\
&\text{4)} \quad \text{So } VaR_{99,10D} = \frac{-\Phi^{-1}(0.01)}{-\Phi^{-1}(0.02)} \times \sqrt{2} \times VaR_{98,5D} \\
&\qquad = \frac{2.3263}{2.0537} \times \sqrt{2} \times \$10 \\
&\qquad = \$11.32 \text{ million } \$
\end{aligned}$$

3. What is the probability of having more than one exception in the same month? Use the answer this question to come up with a test of a VaR measure based on bunching.

8.3

We assume that the daily VaR is calibrated at 99%, meaning the probability of a VaR exception (loss exceeding VaR) on any given day is $p = 0.01$.

Assuming that each 21 trading days is a month, the number of exceptions in a month follows a **Binomial distribution**:

$$X \sim \text{Binomial}(n = 21, p = 0.01)$$

We are interested in the probability of observing **more than one exception** in a month:

$$P(X \ge 2) = 1 - P(X = 0) - P(X = 1)$$

Using the binomial formula:

$$P(X = 0) = (1-p)^n = 0.99^{21} = 0.8095$$

$$P(X = 1) = n \times p \times (1-p)^{n-1} = 21 \times 0.01 \times 0.99^{20} = 0.172$$

$$P(X \ge 2) = 1 - 0.809 - 0.172 = 0.019$$

Therefore, the probability of having more than one exception in a month is approximately **1.9%**.

VaR Model Test Based on Bunching

If this **model is correctly calibrated**, we expect to **see more than one exception in a month only 1.9% of the time**. Therefore, **frequent months with multiple exceptions** indicate **VaR underestimation** and a model recalibration.

Bunching Test Idea

- Count how many months have **2 or more exceptions**.
- Compare the number to the expected number (~1.9% of months).
- If observed bunching significantly exceeds expectation, the VaR model is not well calibrated.

This test focuses on the **clustering of exceptions**, complementing the standard VaR backtest that only looks for **total exceptions** regardless of when they occurred.

4. The next regular FOMC meeting is scheduled for the end of this month. How would you estimate the 2-day 99% VaR of investing $1m in the S&P500 a day before the announcement? *Bonus questions*: Provide a number.

8 Estimating 2-Day 99% VaR Around FOMC Announcement

To estimate the **2-day 99% VaR of investing $1 million in the S&P500 a day before the FOMC meeting**, we proceed as follows:

Methodology:

- **Historical Approach**:
  - Identify **previous FOMC announcement dates** (available via Federal Reserve's official calendar).
  - Collect **S&P500 daily returns** around those dates.
  - Filter for **2-day returns** (from the day of the announcement).
  - Compute the empirical **99 percentile** of these 2-day returns.
- **Scaling Assumption**:
  - If using 1-day VaR, scale by $\sqrt{2}$ (assuming i.i.d. returns).
  - Apply a **volatility-scaling** or **factor** based on prior FOMC event volatility (typically 1.5x to 3x normal volatility).
  - Adjust for **liquidity**.

$$VaR_{99,2\text{-day}} = \text{Avg. Event } r_{99} \times \sqrt{2} \times \text{Portfolio Value}$$

Bonus Estimate

```
import numpy as np
import pandas as pd
import yfinance as yf

fomc_dates = ['2023-02-01',
              '2023-03-22',
              '2023-05-03',
              '2023-06-14',
              '2023-07-26']

sp500 = yf.download('SPY', start='2012-01-01', end='2023-08-01', group_by='column')

sp500['Return'] = np.log(sp500['Close'] / sp500['Close'].shift(1))

sp500['FOMC'] = sp500.index.isin(fomc_dates)

fomc_ret = sp500[sp500['FOMC']]['Return'].dropna()
all_ret = sp500['Return'].dropna()

print("FOMC observations:", len(fomc_ret))
print("Percentiles of non-FOMC days: 1st, 5th, 95th:",
      np.percentile(all_ret, [1, 5, 95]))

# Compare FOMC vs non-FOMC volatility
print("\nFOMC mean return:", fomc_ret.mean())
print("FOMC std:", fomc_ret.std()[illegible]
print("Non-FOMC mean return:", all_ret.mean()[illegible]
print("Non-FOMC std:", all_ret.std()[illegible]

# Simple 2-day VaR estimate
var_1day = np.percentile(fomc_ret, 1)
var_2day = var_1day * np.sqrt(2)
portfolio = 1[illegible]
print(f"\nEstimated 2-day 99% VaR: ${var_2day:,.0[illegible]
```

```
[*********100%***********]  1 of 1 completed

Downloading SP500 Data...
[*********100%***********]  1 of 1 completed
FOMC observations: 5
Percentiles of non-FOMC days: 1st, 5th, 95th: [-0.0[illegible]

FOMC mean return: 0.0[illegible]
FOMC std: 0.0[illegible]
Non-FOMC mean return: 0.0[illegible]
Non-FOMC std: 0.0[illegible]

Estimated 2-day 99% VaR: $[illegible]
```

Bonus Estimate

```
import numpy as np
import pandas as pd
import yfinance as yf

fomc_dates = ['2023-02-01',
              '2023-03-22',
              '2023-05-03',
              '2023-06-14',
              '2023-07-26']

sp500 = yf.download('SPY', start='2012-01-01', end='2023-08-01', group_by='column')

sp500['Return'] = np.log(sp500['Close'] / sp500['Close'].shift(1))

sp500['FOMC'] = sp500.index.isin(fomc_dates)

fomc_ret = sp500[sp500['FOMC']]['Return'].dropna()
all_ret = sp500['Return'].dropna()

print("FOMC observations:", len(fomc_ret))
print("Percentiles of non-FOMC days: 1st, 5th, 95th:",
      np.percentile(all_ret, [1, 5, 95]))

# Compare FOMC vs non-FOMC volatility
print("\nFOMC mean return:", fomc_ret.mean()[illegible]
print("FOMC std:", fomc_ret.std()[illegible]
print("Non-FOMC mean return:", all_ret.mean()[illegible]
print("Non-FOMC std:", all_ret.std()[illegible]

# Simple 2-day VaR estimate
var_1day = np.percentile(fomc_ret, 1)
var_2day = var_1day * np.sqrt(2)
portfolio = 1[illegible]
print(f"\nEstimated 2-day 99% VaR: ${var_2day:,.0[illegible]
```

```
[*********100%***********]  1 of 1 completed

Downloading SP500 Data...
[*********100%***********]  1 of 1 completed
FOMC observations: 5
Percentiles of non-FOMC days: 1st, 5th, 95th: [-0.0[illegible]

FOMC mean return: 0.0[illegible]
FOMC std: 0.0[illegible]
Non-FOMC mean return: 0.0[illegible]
Non-FOMC std: 0.0[illegible]

Estimated 2-day 99% VaR: $[illegible]
```

Final Answer:

The **estimated 2-day 99% VaR** around the FOMC announcement is approximately **$[illegible]**