

Chapter 3

Traditional Symmetric-Key Ciphers

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3-1 INTRODUCTION

Figure 3.1 shows the general idea behind a symmetric-key cipher. The original message from Alice to Bob is called plaintext; the message that is sent through the channel is called the ciphertext. To create the ciphertext from the plaintext, Alice uses an encryption algorithm and a shared secret key. To create the plaintext from ciphertext, Bob uses a decryption algorithm and the same secret key.

Topics discussed in this section:

- 3.1.1 Kerckhoff's Principle
- 3.1.2 Cryptanalysis
- 3.1.3 Categories of Traditional Ciphers

3.3

Chapter 3 Objectives To define the terms and the concepts of symmetric key ciphers To emphasize the two categories of traditional ciphers: substitution and transposition ciphers To describe the categories of cryptanalysis used to break the symmetric ciphers To introduce the concepts of the stream ciphers and block ciphers To discuss some very dominant ciphers used in the past, such as the Enigma machine

Figure 3.1 General idea of symmetric-key cipher Alice Plaintext Secure key-exchange channel algorithm Shared secret key Ciphertext Insecure channel

3.1 Continued

If P is the plaintext, C is the ciphertext, and K is the key,

Encryption:
$$C = E_k(P)$$

Decryption: $P = D_k(C)$

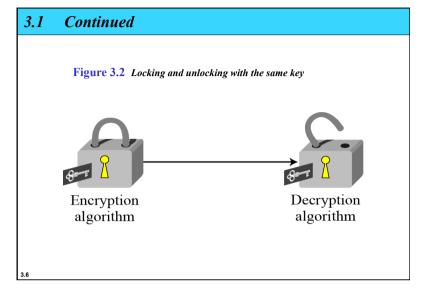
In which,
$$D_k(E_k(x)) = E_k(D_k(x)) = x$$

We assume that Bob creates P_1 ; we prove that $P_1 = P$:

Alice:
$$C = E_k(P)$$

Bob:
$$P_1 = D_k(C) = D_k(E_k(P)) = P$$

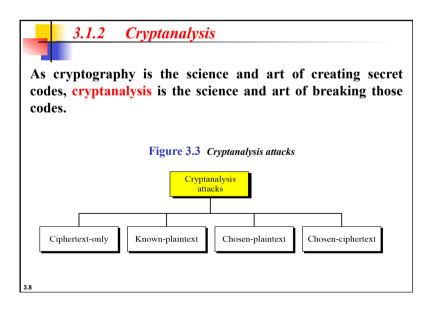
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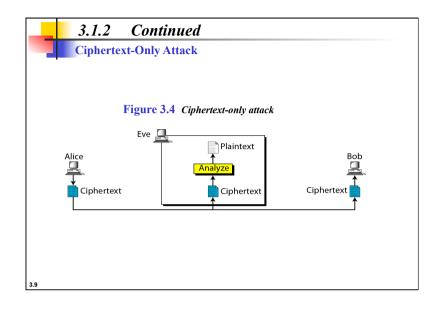


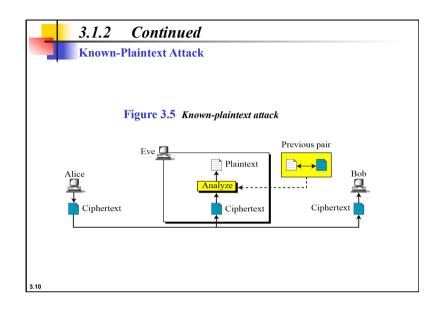
3.1.1 Kerckhoff's Principle

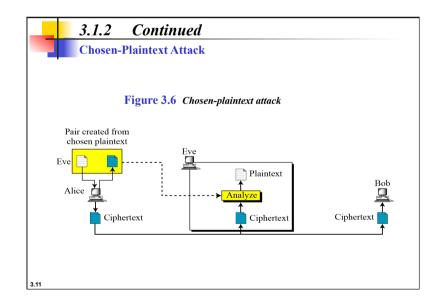
Based on Kerckhoff's principle, one should always assume that the adversary, Eve, knows the encryption/decryption algorithm. The resistance of the cipher to attack must be based only on the secrecy of the key.

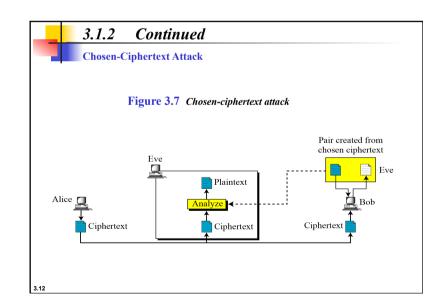
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3-2 SUBSTITUTION CIPHERS

A substitution cipher replaces one symbol with another. Substitution ciphers can be categorized as either monoalphabetic ciphers or polyalphabetic ciphers.

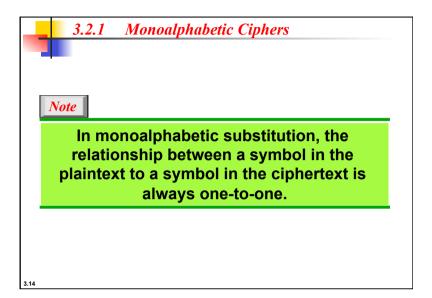
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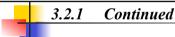
A substitution cipher replaces one symbol with another.

Topics discussed in this section:

- 3.2.1 Monoalphabetic Ciphres
- **3.2.2** Polyalphabetic Ciphers

3.13





Example 3.1

The following shows a plaintext and its corresponding ciphertext. The cipher is probably monoalphabetic because both l's (els) are encrypted as O's.

Plaintext: hello Ciphertext: KHOOR

Example 3.2

The following shows a plaintext and its corresponding ciphertext. The cipher is not monoalphabetic because each l (el) is encrypted by a different character.

Plaintext: hello Ciphertext: KHOOR

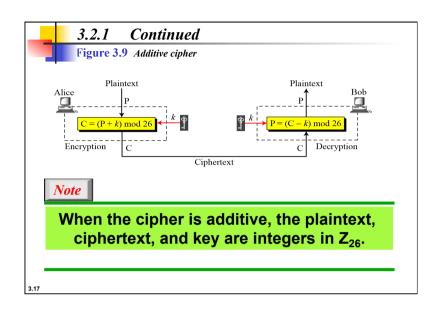
3.1

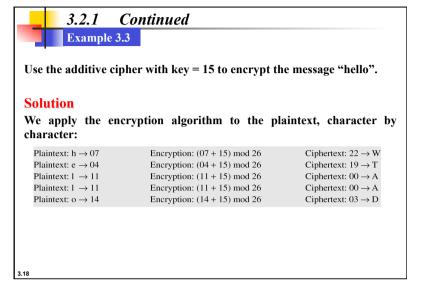


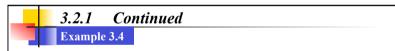
The simplest monoalphabetic cipher is the additive cipher. This cipher is sometimes called a shift cipher and sometimes a Caesar cipher, but the term additive cipher better reveals its mathematical nature.

Figure 3.8 Plaintext and ciphertext in Z₂₆









Use the additive cipher with key = 15 to decrypt the message "WTAAD".

Solution

We apply the decryption algorithm to the plaintext character by character:

Ciphertext: W \rightarrow 22	Decryption: (22 – 15) mod 26	Plaintext: $07 \rightarrow h$
Ciphertext: T \rightarrow 19	Decryption: (19 – 15) mod 26	Plaintext: $04 \rightarrow e$
Ciphertext: A \rightarrow 00	Decryption: $(00-15) \mod 26$	Plaintext: $11 \rightarrow 1$
Ciphertext: A \rightarrow 00	Decryption: $(00-15) \mod 26$	Plaintext: $11 \rightarrow 1$
Ciphertext: D \rightarrow 03	Decryption: (03 – 15) mod 26	Plaintext: $14 \rightarrow 0$

3.19



3.2.1 Continued

Shift Cipher and Caesar Cipher

Historically, additive ciphers are called shift ciphers. Julius Caesar used an additive cipher to communicate with his officers. For this reason, additive ciphers are sometimes referred to as the Caesar cipher. Caesar used a key of 3 for his communications.



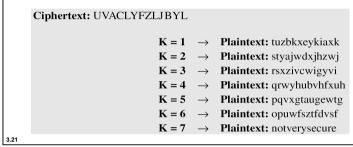
Additive ciphers are sometimes referred to as shift ciphers or Caesar cipher.



Eve has intercepted the ciphertext "UVACLYFZLJBYL". Show how she can use a brute-force attack to break the cipher.

Solution

Eve tries keys from 1 to 7. With a key of 7, the plaintext is "not very secure", which makes sense.





Eve has intercepted the following ciphertext. Using a statistical attack, find the plaintext.

 $\label{eq:continuity} XLILSYWIMWRSAJSVWEPIJSVJSYVQMPPMSRHSPPEVWMXMWASVX-LQSVILY-VVCFIJSVIXLIWIPPIVVIGIMZIWQSVISJJIVW$

Solution

When Eve tabulates the frequency of letters in this ciphertext, she gets: $I=14,\,V=13,\,S=12,$ and so on. The most common character is I with 14 occurrences. This means key = 4.

the house is now for sale for four million dollars it is worth more hurry before the seller receives more offers

. . .

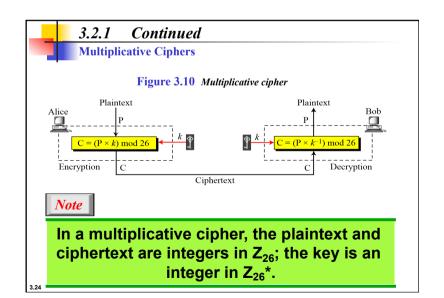
3.2.1 Continued

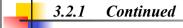
Table 3.1 Frequency of characters in English

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
Е	12.7	Н	6.1	W	2.3	K	0.08
Т	9.1	R	6.0	F	2.2	J	0.02
A	8.2	D	4.3	G	2.0	Q	0.01
О	7.5	L	4.0	Y	2.0	X	0.01
I	7.0	С	2.8	P	1.9	Z	0.01
N	6.7	U	2.8	В	1.5		
S	6.3	M	2.4	V	1.0		

Table 3.2 Frequency of diagrams and trigrams

Digram	TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, NT, HA, ND, OU, EA, NG, AS, OR, TI, IS, ET, IT, AR, TE, SE, HI, OF
Trigram	THE, ING, AND, HER, ERE, ENT, THA, NTH, WAS, ETH, FOR, DTH





Example 3.7

What is the key domain for any multiplicative cipher?

Solution

The key needs to be in Z_{26}^* . This set has only 12 members: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25.

Example 3.8

We use a multiplicative cipher to encrypt the message "hello" with a key of 7. The ciphertext is "XCZZU".

Plaintext: $h \rightarrow 07$ Plaintext: $e \rightarrow 04$	Encryption: $(07 \times 07) \mod 26$ Encryption: $(04 \times 07) \mod 26$	ciphertext: $23 \rightarrow X$ ciphertext: $02 \rightarrow C$
Plaintext: $l \rightarrow 04$ Plaintext: $l \rightarrow 11$	Encryption: $(04 \times 07) \mod 26$	ciphertext: $02 \rightarrow C$ ciphertext: $25 \rightarrow Z$
Plaintext: $1 \rightarrow 11$	Encryption: $(11 \times 07) \mod 26$	ciphertext: $25 \rightarrow Z$
Plaintext: $o \rightarrow 14$	Encryption: $(14 \times 07) \mod 26$	ciphertext: $20 \rightarrow U$

3.25

3.2.1 Continued

Example 3.09

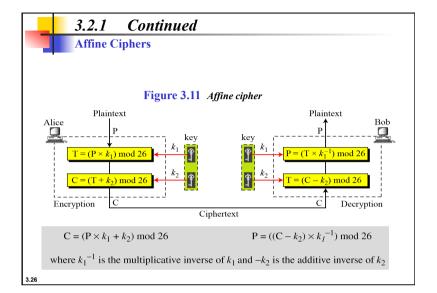
The affine cipher uses a pair of keys in which the first key is from Z_{26}^* and the second is from Z_{26} . The size of the key domain is $26 \times 12 = 312$.

Example 3.10

Use an affine cipher to encrypt the message "hello" with the key pair (7, 2).

P: $h \rightarrow 07$	Encryption: $(07 \times 7 + 2) \mod 26$	$C: 25 \rightarrow Z$
P: $e \rightarrow 04$	Encryption: $(04 \times 7 + 2) \mod 26$	$C: 04 \rightarrow E$
$P: 1 \rightarrow 11$	Encryption: $(11 \times 7 + 2) \mod 26$	$C: 01 \rightarrow B$
$P: 1 \rightarrow 11$	Encryption: $(11 \times 7 + 2) \mod 26$	$C: 01 \rightarrow B$
P: $o \rightarrow 14$	Encryption: $(14 \times 7 + 2) \mod 26$	$C: 22 \rightarrow W$

3.27



3.2.1 Continued

Example 3.11

Use the affine cipher to decrypt the message "ZEBBW" with the key pair (7, 2) in modulus 26.

Solution

$C: Z \rightarrow 25$	Decryption: $((25 - 2) \times 7^{-1}) \mod 26$	$P:07 \rightarrow h$
$C: E \rightarrow 04$	Decryption: $((04 - 2) \times 7^{-1}) \mod 26$	$P:04 \rightarrow e$
$C: B \rightarrow 01$	Decryption: $((01 - 2) \times 7^{-1}) \mod 26$	$P:11 \rightarrow 1$
$C: B \rightarrow 01$	Decryption: $((01 - 2) \times 7^{-1}) \mod 26$	$P:11 \rightarrow 1$
$C: W \rightarrow 22$	Decryption: $((22 - 2) \times 7^{-1}) \mod 26$	$P:14 \rightarrow 0$

Example 3.12

The additive cipher is a special case of an affine cipher in which $k_1=1$. The multiplicative cipher is a special case of affine cipher in which $k_2=0$.



3.2.1 Continued

Monoalphabetic Substitution Cipher

Because additive, multiplicative, and affine ciphers have small key domains, they are very vulnerable to brute-force attack.

A better solution is to create a mapping between each plaintext character and the corresponding ciphertext character. Alice and Bob can agree on a table showing the mapping for each character.

Figure 3.12 An example key for monoalphabetic substitution cipher



3 20



3.2.2 Polyalphabetic Ciphers

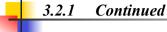
In polyalphabetic substitution, each occurrence of a character may have a different substitute. The relationship between a character in the plaintext to a character in the ciphertext is one-to-many.

Autokey Cipher

$$P = P_1 P_2 P_3 \ ... \qquad \qquad C = C_1 C_2 C_3 ... \qquad \qquad k = (k_1, \, P_1, \, P_2, \, ...)$$

Encryption: $C_i = (P_i + k_i) \mod 26$ Decryption: $P_i = (C_i - k_i) \mod 26$

3.31



Example 3.13

We can use the key in Figure 3.12 to encrypt the message

this message is easy to encrypt but hard to find the key

The ciphertext is

ICFVQRVVNEFVRNVSIYRGAHSLIOJICNHTIYBFGTICRXRS

3.30



3.2.2 Continued

Example 3.14

Assume that Alice and Bob agreed to use an autokey cipher with initial key value $k_1 = 12$. Now Alice wants to send Bob the message "Attack is today". Enciphering is done character by character.

 Plaintext:
 a
 t
 t
 a
 c
 k
 i
 s
 t
 o
 d
 a
 y

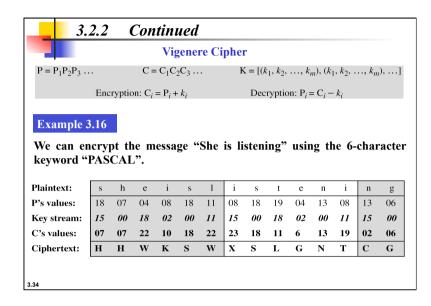
 P's Values:
 00
 19
 19
 00
 02
 10
 08
 18
 19
 14
 03
 00
 22

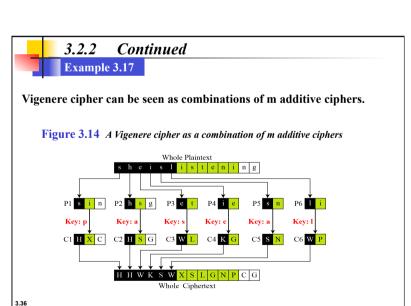
 Key stream:
 12
 00
 19
 19
 00
 02
 10
 08
 18
 19
 14
 03
 00

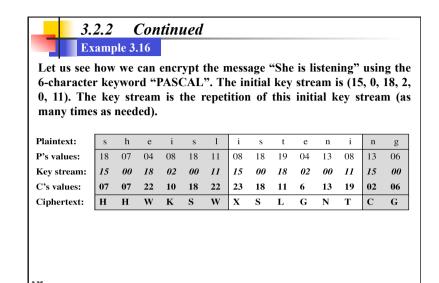
 C's Values:
 12
 19
 12
 19
 02
 12
 18
 00
 11
 7
 17
 03
 24

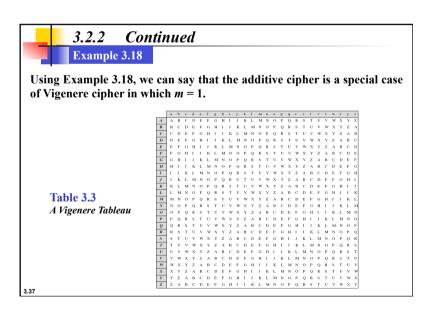
 Ciphertext:
 M
 T
 M
 T
 C
 M
 S
 A
 L
 H
 R
 D
 Y

. ..











3.2.2 Continued

Vigenere Cipher (Crypanalysis)

Example 3.19

Let us assume we have intercepted the following ciphertext:

LIOMWGFEGGDVWGHHCOUCRHRWAGWIOWOLKGZETKKMEVLWPCZVGTH-VTSGXQOVGCSVETQLTJSUMVWVEUVLXEWSLGFZMVVWLGYHCUSWXQH-KVGSHEEVFLCFDGVSUMPHKIRZDMPHHBVWVWJWIXGFWLTSHGJOUEEHH-VUCFVGOWICQLTJSUXGLW

The Kasiski test for repetition of three-character segments yields the results shown in Table 3.4.

String	First Index	Second Index	Difference
JSU	68	168	100
SUM	69	117	48
VWV	72	132	60
MPH	119	127	8



Continued 3.2.2

Example 3.19 (Continued)

The greatest common divisor of differences is 4, which means that the key length is multiple of 4. First try m = 4.

C1: LWGWCRAOKTEPGTQCTJVUEGVGUQGECVPRPVJGTJEUGCJG

P1: jueuapymircneroarhtsthihytrahcieixsthcarrehe

C2: IGGGQHGWGKVCTSOSQSWVWFVYSHSVFSHZHWWFSOHCOQSL

P2: usssctsiswhofeaeceihcetesoecatnpntherhctecex

C3: OFDHURWQZKLZHGVVLUVLSZWHWKHFDUKDHVIWHUHFWLUW

P3: lcaerotnwhiwedssirsiirhketehretltiideatrairt

C4: MEVHCWILEMWVVXGETMEXLMLCXVELGMIMBWXLGEVVITX

P4: iardysehaisrrtcapiafpwtethecarhaesfterectpt

In this case, the plaintext makes sense.

Julius Caesar used a cryptosystem in his wars, which is now referred to as Caesar cipher. It is an additive cipher with the key set to three. Each character in the plaintext is shifted three characters to create ciphertext.



3.2.2 Continued

Example 3.19

Let us assume we have intercepted the following ciphertext:

LIOMWGFEGGDVWGHHCQUCRHRWAGWIOWQLKGZETKKMEVLWPCZVGTH-VTSGXQOVGCSVETOLTJSUMVWVEUVLXEWSLGFZMVVWLGYHCUSWXQH-KVGSHEEVFLCFDGVSUMPHKIRZDMPHHBVWVWJWIXGFWLTSHGJOUEEHH-VUCFVGOWICOLTJSUXGLW

The Kasiski test for repetition of three-character segments yields the results shown in Table 3.4.

String	First Index	Second Index	Difference
JSU	68	168	100
SUM	69	117	48
VWV	72	132	60
MPH	119	127	8



3.2.2 Continued

One-Time Pad

One of the goals of cryptography is perfect secrecy. A study by Shannon has shown that perfect secrecy can be achieved if each plaintext symbol is encrypted with a key randomly chosen from a key domain. This idea is used in a cipher called one-time pad, invented by Vernam.

3-3 TRANSPOSITION CIPHERS

A transposition cipher does not substitute one symbol for another, instead it changes the location of the symbols.

Note

A transposition cipher reorders symbols.

Topics discussed in this section:

- 3.3.1 Keyless Transposition Ciphers
- 3.3.2 Keyed Transposition Ciphers
- 3.3.3 Combining Two Approaches

3.4



3.3.1 Continued

Example 3.23

Alice and Bob can agree on the number of columns and use the second method. Alice writes the same plaintext, row by row, in a table of four columns.

m e e t
 m e a t
 t h e p
 a r k

She then creates the ciphertext "MMTAEEHREAEKTTP".

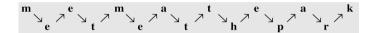
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3.3.1 Keyless Transposition Ciphers

Simple transposition ciphers, which were used in the past, are keyless.

Example 3.22

A good example of a keyless cipher using the first method is the rail fence cipher. The ciphertext is created reading the pattern row by row. For example, to send the message "Meet me at the park" to Bob, Alice writes



She then creates the ciphertext "MEMATEAKETETHPR".

3.4



3.3.1 Continued

Example 3.24

The cipher in Example 3.23 is actually a transposition cipher. The following shows the permutation of each character in the plaintext into the ciphertext based on the positions.

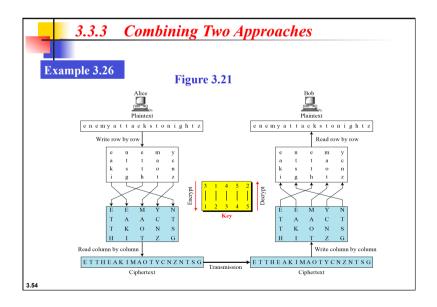
The second character in the plaintext has moved to the fifth position in the ciphertext; the third character has moved to the ninth position; and so on. Although the characters are permuted,

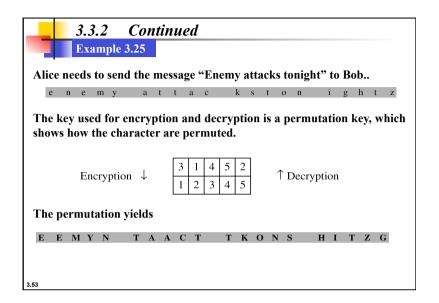
there is a pattern in the permutation: (01, 05, 09, 13), (02, 06, 10, 13), (03, 07, 11, 15), and (08, 12). In each section, the difference between the two adjacent numbers is 4.

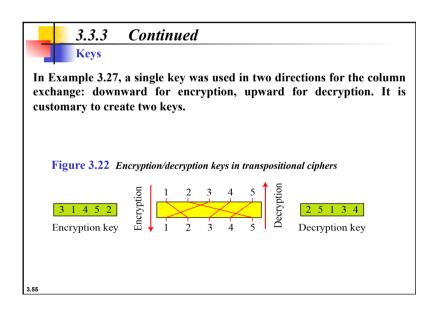


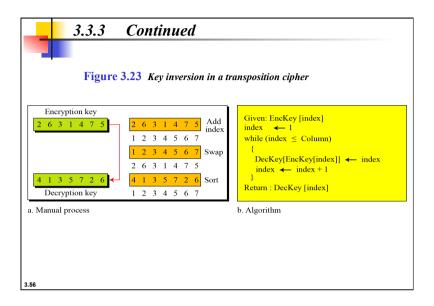
The keyless ciphers permute the characters by using writing plaintext in one way and reading it in another way The permutation is done on the whole plaintext to create the whole ciphertext. Another method is to divide the plaintext into groups of predetermined size, called blocks, and then use a key to permute the characters in each block separately.

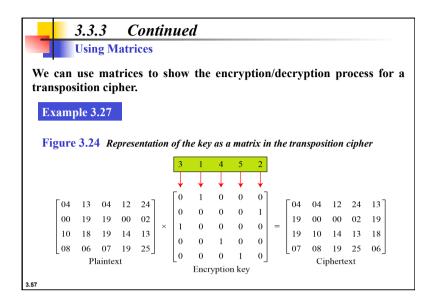
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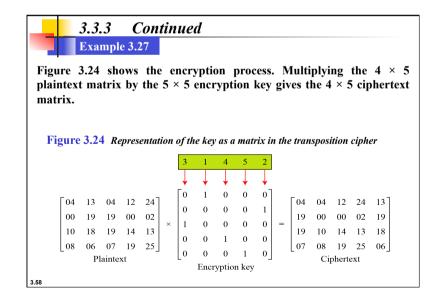


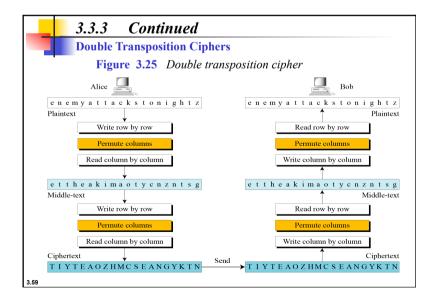












3-4 STREAM AND BLOCK CIPHERS

The literature divides the symmetric ciphers into two broad categories: stream ciphers and block ciphers. Although the definitions are normally applied to modern ciphers, this categorization also applies to traditional ciphers.

Topics discussed in this section:

- 3.4.1 Stream Ciphers
- 3.4.2 Block Ciphers
- 3.4.3 Combination

3 60

3.4.1 Continued

Example 3.30

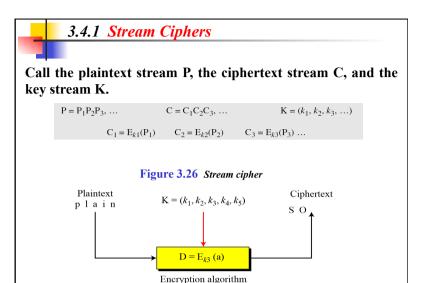
Additive ciphers can be categorized as stream ciphers in which the key stream is the repeated value of the key. In other words, the key stream is considered as a predetermined stream of keys or

K = (k, k, ..., k). In this cipher, however, each character in the ciphertext depends only on the corresponding character in the plaintext, because the key stream is generated independently.

Example 3.31

The monoalphabetic substitution ciphers discussed in this chapter are also stream ciphers. However, each value of the key stream in this case is the mapping of the current plaintext character to the corresponding ciphertext character in the mapping table.

3.62



3.4.1 Continued

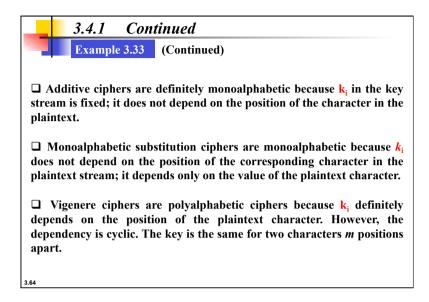
Example 3.32

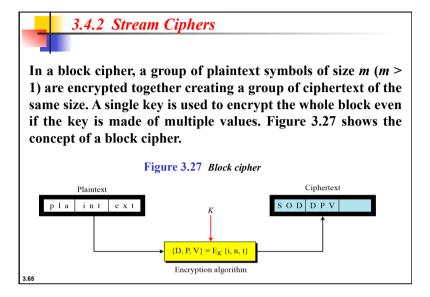
Vigenere ciphers are also stream ciphers according to the definition. In this case, the key stream is a repetition of m values, where m is the size of the keyword. In other words,

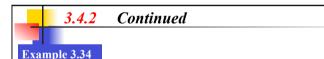
$$K = (k_1, k_2, \dots k_m, k_1, k_2, \dots k_m, \dots)$$

Example 3.33

We can establish a criterion to divide stream ciphers based on their key streams. We can say that a stream cipher is a monoalphabetic cipher if the value of k_i does not depend on the position of the plaintext character in the plaintext stream; otherwise, the cipher is polyalphabetic.







Playfair ciphers are block ciphers. The size of the block is m = 2. Two characters are encrypted together.

Example 3.35

Hill ciphers are block ciphers. A block of plaintext, of size 2 or more is encrypted together using a single key (a matrix). In these ciphers, the value of each character in the ciphertext depends on

all the values of the characters in the plaintext. Although the key is made of $m \times m$ values, it is considered as a single key.

Example 3.36

From the definition of the block cipher, it is clear that every block cipher is a polyalphabetic cipher because each character in a ciphertext block depends on all characters in the plaintext block.



In practice, blocks of plaintext are encrypted individually, but they use a stream of keys to encrypt the whole message block by block. In other words, the cipher is a block cipher when looking at the individual blocks, but it is a stream cipher when looking at the whole message considering each block as a single unit.