Rešitve nalog: Jordanova kanonična forma

1 Jordanova kanonična forma

1.1. (a) $J = \begin{bmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{bmatrix}$, $P = \begin{bmatrix} -1 & 5 & 1 \\ -1 & 6 & 0 \\ 1 & -7 & 0 \end{bmatrix}$, $m_A(\lambda) = \lambda^3$

(b) $J = \begin{bmatrix} -2 & 1 & & & \\ & -2 & & & \\ & & -2 & & \\ & & & -1 & \\ & & & & -1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & -1 \end{bmatrix}$, $m_A(\lambda) = (\lambda+2)^2(\lambda+1)$

(c) $J = \begin{bmatrix} 1 & 1 & & & & \\ & 1 & & & & \\ & & 2 & 1 & & \\ & & & 2 & 1 & \\ & & & & 2 & \end{bmatrix}$, $P = \begin{bmatrix} 1 & 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $m_A(\lambda) = (\lambda-1)^2(\lambda-2)^3$

(d) $J = \begin{bmatrix} 1 & 1 & & & & \\ & 1 & 1 & & & \\ & & 1 & & & \\ & & & 1 & 1 & \\ & & & & 1 & \end{bmatrix}$, $P = \begin{bmatrix} 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, $m_A(\lambda) = (\lambda-1)^3$

(e) $J = \begin{bmatrix} 1 & 1 & & & & & \\ & 1 & 1 & & & & \\ & & 1 & & & & \\ & & & 1 & 1 & & \\ & & & & 1 & & \\ & & & & & 0 & \end{bmatrix}$, $P = \begin{bmatrix} -2 & 0 & 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -6 & 1 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 \end{bmatrix}$, $m_A(\lambda) = (\lambda-1)^3\lambda$

1.2. $a = 1$, $J = \begin{bmatrix} 0 & & \\ & 1 & 1 \\ & & 1 \end{bmatrix}$, $P = \begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

1.3. $J = \begin{bmatrix} 1 & 1 & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & 3 & 1 & \\ & & & & & 3 & \\ & & & & & & 3 \end{bmatrix}$

1.4. $J = \begin{bmatrix} 0 & 1 & & & & & & & & \\ & 0 & 1 & & & & & & & \\ & & 0 & 1 & & & & & & \\ & & & 0 & & & & & & \\ & & & & 0 & 1 & & & & \\ & & & & & 0 & 1 & & & \\ & & & & & & 0 & & & \\ & & & & & & & 0 & 1 & \\ & & & & & & & & 0 & 1 \\ & & & & & & & & & 0 \end{bmatrix}$, $\dim \ker A = 4$

2 Funkcije matrik

2.1. $J = \begin{bmatrix} 1 & 1 & \\ & 1 & \\ & & 2 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $e^A = \begin{bmatrix} e & e & -e \\ 0 & 2e - e^2 & 2\left(e^2 - e\right) \\ 0 & e - e^2 & 2e^2 - e \end{bmatrix}$

2.2. $J = \begin{bmatrix} -1 & 1 & & & \\ & -1 & & & \\ & & & 1 & 1 \\ & & & & 1 \end{bmatrix}$, $P = \begin{bmatrix} 0 & -\frac{1}{2} & -1 & -\frac{1}{2} \\ 0 & 0 & 0 & 2 \\ 0 & \frac{1}{2} & 3 & \frac{9}{2} \\ 1 & 0 & 1 & 0 \end{bmatrix}$, $f(A) = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{\pi}{2\sqrt{2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{3\pi}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{3\pi}{\sqrt{2}} & -\sqrt{2}\pi & \frac{\pi}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$