

-

- [Log in](#)

 

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Navigation

- [Main page](#)
- [Pages A-Z](#)
- [StatProb Collection](#)
- [Recent changes](#)
- [Current events](#)
- [Random page](#)
- [Help](#)
- [Project talk](#)
- [Request account](#)

Tools

- [What links here](#)
- [Related changes](#)
- [Special pages](#)
- [Printable version](#)
- [Permanent link](#)
- [Page information](#)

Namespaces

- [Page](#)
- [Discussion](#)

Variants

Views

- [View](#)
- [View source](#)
- [History](#)

Actions

Involutive distribution

From Encyclopedia of Mathematics

Jump to: [navigation](#), [search](#)

The geometric interpretation of a completely-integrable differential system on an n -dimensional differentiable

manifold M^n of class C^k , $k \geq 3$. A p -dimensional distribution (or a differential system of dimension p) of class C^r , $1 \leq r < k$, on M^n is a function associating to each point $x \in M^n$ a p -dimensional linear subspace $D(x)$ of the tangent space $T_x(M^n)$ such that x has a neighbourhood U with p C^r vector fields X_1, \dots, X_p on it for which the vectors $X_1(y), \dots, X_p(y)$ form a basis of the space $D(y)$ at each point $y \in U$. The distribution D is said to be involutive if for all points $y \in U$,

$$[X_i, X_j](y) \in D(y), \quad 1 \leq i, j \leq p.$$

This condition can also be stated in terms of differential forms. The distribution D is characterized by the fact that

$$D(y) = \{X \in T_y(M^n) : \omega^\alpha(y)(X) = 0\}, \quad p < \alpha \leq n,$$

where $\omega^{p+1}, \dots, \omega^n$ are 1-forms of class C^r , linearly independent at each point $x \in U$; in other words, D is locally equivalent to the system of differential equations $\omega^\alpha = 0$. Then D is an involutive distribution if there exist 1-forms ω_β^α on U such that

$$d\omega^\alpha = \sum_{\beta=p+1}^n \omega^\beta \wedge \omega_\beta^\alpha,$$

that is, the exterior differentials $d\omega^\alpha$ belong to the ideal generated by the forms ω^β .

A distribution D of class C^r on M^n is involutive if and only if (as a differential system) it is an [integrable system](#) (Frobenius' theorem).

References

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[Categories](#):

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- [TeX done](#)
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