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Involutive distribution

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The geometric interpretation of a completely-integrable differential system on an n-dimensional differentiable manifold M^n of class C^k, $k \geq 3$. A p-dimensional distribution (or a differential system of dimension p) of class C^r, $1 \leq r < k$, on M^n is a function associating to each point $x \in M^n$ a p-dimensional linear subspace $D(x)$ of the tangent space $T_x(M^n)$ such that x has a neighbourhood U with p C^r vector fields $X_1, \dots, X_p$ on it for which the vectors $X_1(y), \dots, X_p(y)$ form a basis of the space $D(y)$ at each point $y \in U$. The distribution D is said to be involutive if for all points $y \in U$,

$$[X_i, X_j](y) \in D(y), \ \ 1 \leq i, j \leq p.$$

This condition can also be stated in terms of differential forms. The distribution D is characterized by the fact that

$$D(y) = \{X \in T_y(M^n) : \omega^\alpha(y)(X) = 0\}, \ p < \alpha \leq n,$$

where $\omega^{p+1}, \dots, \omega^n$ are 1-forms of class C^r, linearly independent at each point $x \in U$; in other words, D is locally equivalent to the system of differential equations $\omega^\alpha = 0$. Then D is an involutive distribution if there exist 1-forms ω_β^α on U such that

$$d\omega^\alpha = \sum_{\beta=p+1}^{n} \omega^\beta \wedge \omega_\beta^\alpha,$$

that is, the exterior differentials $d\omega^\alpha$ belong to the ideal generated by the forms ω^β.

A distribution D of class C^r on M^n is involutive if and only if (as a differential system) it is an integrable system (Frobenius' theorem).

References

[1] C. Chevalley, "Theory of Lie groups" , **1** , Princeton Univ. Press (1946)

[2] R. Narasimhan, "Analysis on real and complex manifolds" , North-Holland & Masson (1968) (Translated from French)