Backstepping Control: An Introduction

1 Introduction

In this lecture, we explore a powerful control technique known as **backstepping**. This technique is particularly useful for designing controllers for nonlinear systems and offers several advantages:

- It is systematic once the initial step is overcome.
- Applicable to a broad class of nonlinear systems.
- Capable of handling both matched and unmatched uncertainties.

The primary drawback of backstepping is the complexity of the final control law. This lecture outlines the step-by-step procedure for designing a backstepping controller.

2 System Description

We begin with a nonlinear system described by the following state equations:

$$\dot{\eta} = f(\eta) + g(\eta)z, \quad \dot{z} = u$$

where:

- η and z are state variables.
- $f(\eta)$ and $g(\eta)$ are smooth nonlinear functions.

The goal is to design a control input u such that the equilibrium point at the origin is asymptotically stable.

3 Backstepping Procedure

The backstepping design process is outlined below:

3.1 Step 1: Subsystem Control

1. Consider the subsystem:

$$\dot{\eta} = f(\eta) + g(\eta)z$$

2. Treat z as an input and define a control law:

$$z = F(\eta)$$

such that the closed-loop system:

$$\dot{\eta} = f(\eta) + g(\eta)F(\eta)$$

is asymptotically stable. Find a Lyapunov function $V(\eta)$ for this subsystem satisfying:

$$\dot{V}(\eta) = \frac{\partial V}{\partial \eta}\dot{\eta} \leq -W(\eta)$$

where $W(\eta)$ is a positive definite function.

3.2 Step 2: Overall System Control

1. Define a state transformation:

$$\zeta = z - F(\eta)$$

2. Rewrite the system in terms of the new state variables:

$$\dot{\eta} = f(\eta) + g(\eta)F(\eta) + g(\eta)\zeta, \quad \dot{\zeta} = u - \dot{F}(\eta)$$

3. Define a composite Lyapunov function:

$$V_c(\eta, \zeta) = V(\eta) + \frac{1}{2}\zeta^2$$

4. Compute the derivative of V_c along system trajectories:

$$\dot{V}_c = \frac{\partial V}{\partial \eta}\dot{\eta} + \zeta\dot{\zeta}$$

Substitute $\dot{\eta}$ and $\dot{\zeta}$ to derive an expression for $\dot{V}_c$.

5. Choose u to ensure $\dot{V}_c$ is negative definite. For example:

$$u = \dot{F}(\eta) - k\zeta$$

where $k > 0$ is a design parameter.

4 Summary of Results

The backstepping design procedure can be summarized as follows:

1. Design a control law for the subsystem:

$$z = F(\eta)$$

such that the subsystem is asymptotically stable.

2. Define a composite Lyapunov function:

$$V_c(\eta, \zeta) = V(\eta) + \frac{1}{2}\zeta^2$$

3. Derive the control law for the overall system:

$$u = \dot{F}(\eta) - k\zeta$$

5 Example

To demonstrate the backstepping approach, consider a simple nonlinear system. Follow the outlined steps to derive the control law and verify stability using a Lyapunov function.

6 Conclusion

Backstepping is a systematic method for designing controllers for nonlinear systems, capable of addressing matched and unmatched uncertainties. While the complexity of the final control law can be a challenge, its systematic nature makes it a robust choice for many applications.