Advanced Backstepping Control for Nonlinear Systems

1 Introduction

In this lecture, we extend the concept of backstepping control to a more general class of nonlinear systems. While previous discussions focused on systems with two states, this approach can handle systems with more than two states, described by a chain of integrators. The procedure remains systematic, albeit with increasing complexity in the final control law.

2 System Description

The system under consideration is described by the following state equations:

$$\begin{aligned}
\dot{x}_1 &= f(x_1) + g(x_1)x_2, \\
\dot{x}_2 &= x_3, \\
&\vdots \\
\dot{x}_{n-1} &= x_n, \\
\dot{x}_n &= u.
\end{aligned}$$

Here:

- $x_1, x_2, \ldots, x_n$ are the state variables.
- u is the control input.

The goal is to design a control input u such that the equilibrium point at the origin is asymptotically stable.

3 Design Procedure

The backstepping procedure for this system is as follows:

3.1 Step 1: Subsystem Control

1. Consider the subsystem:
$$\dot{x}_1 = f(x_1) + g(x_1)x_2.$$

2. Treat x_2 as an input and design a control law:

$$x_2 = \phi_1(x_1)$$

such that:

$$\dot{x}_1 = f(x_1) + g(x_1)\phi_1(x_1)$$

is asymptotically stable.

3. Define a Lyapunov function:

$$V_1(x_1) = \frac{1}{2}x_1^2,$$

and verify that:

$$\dot{V}_1 = x_1\dot{x}_1 < 0.$$

3.2 Step 2: State Transformation and Composite Lyapunov Function

1. Define the state transformation:

$$z_2 = x_2 - \phi_1(x_1).$$

2. Rewrite the system in terms of the new state variables:

$$\dot{x}_1 = f(x_1) + g(x_1)\phi_1(x_1) + g(x_1)z_2, \quad \dot{z}_2 = x_3 - \dot{\phi}_1(x_1).$$

3. Define a composite Lyapunov function:

$$V_2(x_1, z_2) = V_1(x_1) + \frac{1}{2}z_2^2.$$

3.3 Step 3: Recursive Steps

For each subsequent state x_k ($k \geq 3$):

1. Treat x_{k+1} as an input and define:

$$z_k = x_k - \phi_{k-1}(x_1, \ldots, x_{k-1}).$$

2. Rewrite the system in terms of the new state variables:

$$\dot{z}_k = x_{k+1} - \dot{\phi}_{k-1}.$$

3. Define a Lyapunov function:

$$V_k = V_{k-1} + \frac{1}{2}z_k^2.$$

3.4 Step n: Final Control Law

1. For the final state x_n, define:

$$z_n = x_n - \phi_{n-1}(x_1, \ldots, x_{n-1}).$$

2. Rewrite the system and define:

$$V_n = V_{n-1} + \frac{1}{2} z_n^2.$$

3. Compute the control law:

$$u = \phi_{n-1}(x_1, \ldots, x_{n-1}) - k_n z_n,$$

where $k_n > 0$ is a design parameter.

4 Simulation Results

The proposed control law was implemented in MATLAB Simulink for a system with three states. The simulation results show that all state trajectories converge to the equilibrium point at the origin. The control law ensures stability while allowing for tuning of the overshoot and transient response through the parameter k.

5 Conclusion

Backstepping control provides a systematic approach to handling nonlinear systems with a chain of integrators. Although the final control law becomes increasingly complex as the number of states increases, the methodology remains consistent. This lecture demonstrates its application to a general class of systems, paving the way for extensions to even more complex scenarios.