HW 6 for MEM 636

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Fall 2024; due November 19 at $11:59$pm

Problem 1

Compute the ranks of the distribution Δ_1 and Δ_2 and decide if they are involutive:

(a) The domain is $D_1 = \mathbb{R}^3$.

$$\Delta_1(x) = span\{f_1(x), f_2(x)\},\ f_1(x) = \begin{pmatrix} 1 \\ 0 \\ x_1 + x_2 \end{pmatrix},\ f_2(x) = \begin{pmatrix} x_1 \\ 0 \\ x_2 \end{pmatrix}. \tag{1}$$

(b) The domain is $D_2 = \{x \in \mathbb{R}^3 \| x_1^2 + x_2^2 \neq 0\}$.

$$\Delta_2(x) = span\{f_1(x), f_2(x)\},\ f_1(x) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ f_2(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{2}$$

Problem 2

(a) Consider the following linear system:

$$\dot{x_1} = x_1 + x_2 + u, \qquad \dot{x_2} = x_1 + 2x_2, \qquad y = x_1 \tag{3}$$

Build a state feedback controller of the form $u = -Kx + k_f r$ for this system. Explain step-by-step how you built the state feedback to track the reference signal $r = 1$.

(b) Consider the following linear system:

$$\dot{x_1} = x_1 + x_2 + u, \qquad \dot{x_2} = 2x_2 \tag{4}$$

Can this system be stabilized via state feedback? Is it controllable?