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Adaptive Backstepping Control of a Class of Uncertain Nonlinear Systems with Unknown Dead-Zone

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Abstract **In this paper, we consider the same class of systems as in [2], i.e., a class of uncertain dynamic nonlinear systems preceded by unknown dead-zone nonlinearities, in the presence of bounded external disturbances. By using backstepping technique, robust adaptive backstepping control algorithms are developed. Unlike some existing control schemes for systems with dead-zone, the developed backstepping controllers do not require the uncertain parameters within known intervals. Also no knowledge is assumed on the bound of the 'disturbance-like' term, a combination of the external disturbances and a term separated from the dead-zone model. It is shown that the proposed controllers not only can guarantee global stability, but also transient performance.**

Keywords: Adaptive control, backstepping, dead-zone, nonlinear system, robust control.

I. Introduction

Dead-zone, which can severely limit system performances, is one of the most important nonsmooth nonlinearities arisen in actuators, such as valves and DC servo motors and other devices. Therefore the effect of dead-zone should be taken into consideration in the design and analysis of control systems. In most practical motion systems, the dead-zone parameters are poorly known, and thus robust adaptive control techniques may be applied to design controllers. The study of adaptive control for systems with unknown dead-zone at the input was initiated in [3], where an adaptive scheme was proposed with full state measurement. An immediate method for the control of dead-zone is to construct an adaptive dead-zone inverse. This approach was used in [4]-[14], where the output of a dead-zone is measurable. [9] a fuzzy precompensator was proposed in nonlinear industrial motion system and neural networks were employed in [10] to construct a dead-zone precompensator. Such approaches promise to improve the tracking performance of motion system in presence of unknown dead-zones. An alternative approach based on sliding mode control is proposed in [11], [12]. In [13] and [14], an adaptive state feedback controller employees an adaptive inverse for a class of nonlinear systems. In the above mentioned approaches, the term multiplying the control and the uncertain parameters of the system and the dead-zone model must be within a known compact set. In [2], a robust adaptive control scheme is developed for a class of nonlinear systems without using the dead-zone inverse, where the dead-zone slopes in the positive and negative region are the same and the unknown system parameters are inside a known compact set. A saturation function was used to handle the effect from the dead-zone nonlinearity.
In this paper, we develop two simple backstepping adaptive control schemes for the same class of nonlinear systems as in [2], with bounded external disturbances included in our case. Besides showing global stability of the system, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus our scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. In our design, the term multiplying the control and the system parameters are not assumed to be within known intervals. The bound of the 'disturbance-like' term, a combined effect of the deadzone and external disturbances, is not required. To handle such a term, an estimator is used to estimate its bound.
This paper is organized as follows: Section 2 states the problem of this paper and assumptions on the nonlinear systems. Sections 3 presents the adaptive control design based on the backstepping technique and analyzes the stability and performance. Simulation results are presented in Section 4. Finally, Section 5 concludes the paper.

II. Problem Statement

We consider the same class of systems as in [2]. For completeness, the system model is given as follows:

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = b\omega(v) + \bar{d}(t) \qquad (1)$$

where Y_i are known continuous linear or nonlinear functions, $\bar{d}(t)$ denotes bounded external disturbances, parameters a_i are unknown constants and control gain b is unknown bounded constant, v is the control input, $u(v)$ denotes dead-zone nonlinearity described by

$$u(t) = \begin{cases} m(v(t) - b_r) & v(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ m(v(t) - b_l) & v(t) \leq b_l \end{cases} \qquad (2)$$

where $b_r \geq 0, b_l \leq 0$ and $m > 0$ are constants, v is the input and u is the output. A graphical representation of the dead-zone is shown in Figure 1.

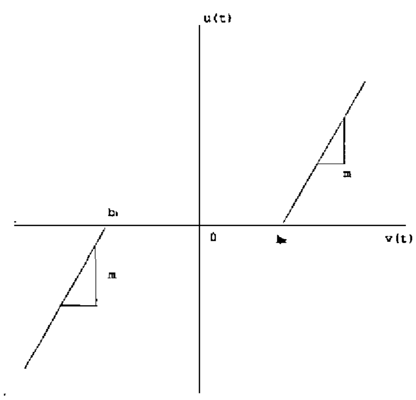

Fig. 1. Dead-zone model

For plant (1) with dead-zone nonlinearity, the $u(t)$ can be expressed as

$$u(t) = mv(t) - d_1(v(t)) \qquad (3)$$

where

$$d_1(v(t)) = \begin{cases} -mb_r & v(t) \geq b_r \\ -mv(t) & b_l < v(t) < b_r \\ -mb_l & v(t) \leq b_l \end{cases} \qquad (4)$$

It is clear that $d(v(t))$ is bounded.
From the structure (3) of model (2), (1) becomes

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = \beta v(t) + d(t) \qquad (5)$$

where $\beta = bm$ and $d(t) = bd_1(v(t)) + \bar{d}(t)$. The effect of $d(t)$ is due to both external disturbances and $bd_1(v(t))$. We call $d(t)$ a 'disturbance-like' term for simplicity of presentation and use D to denote its bound.
Now equation (5) is rewritten in the following form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= -\sum_{i=1}^{r} a_i Y_i(x_1(t), x_2(t), \ldots, x_{(n-1)}(t)) \\ &\quad + \beta v(t) + d(t) \\ &= a^T Y + \beta v(t) + d(t) \end{aligned} \qquad (6)$$

where $x_1 = x, x_2 = \dot{x}, \ldots, x_n = x^{(n-1)}$, $a = [-a_1, -a_2, \ldots, -a_r]^T$ and $Y = [Y_1, Y_2, \ldots, Y_r]^T$.
For the development of control laws, the following assumptions are made.
Assumption 1. The uncertain parameters b and m are such that $\beta > 0$.
Assumption 2. The desired trajectory $y_r(t)$ and its $(n-1)$th order derivatives are known and bounded.
The control objectives are to design backstepping adaptive control laws such that

- The closed loop is globally stable in sense that all the signals in the loop are uniformly ultimately bounded;
- The tracking error $x(t) - y_r(t)$ is adjustable during the transient period by an explicit choice of design parameters and $\lim_{t\to\infty} x(t) - y_r(t) = 0$ or $\lim_{t\to\infty} |x(t) - y_r(t)| \leq \delta_1$ for an arbitrary specified bound δ_1.

Remark 1: Compared with [2], the uncertain parameters β and a_i are not assumed inside known intervals. The bound D for $d(t)$ is not assumed to be known and it will be estimated by our adaptive controllers. Also the control objectives are not only to ensure global stability, but also transient performance.

III. Design of Adaptive Controllers

Before presenting the adaptive control design using the backstepping technique to achieve the desired control objectives, the following change of coordinates is made.

$$z_1 = x_1 - y_r \qquad (7)$$

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \qquad i = 2, 3, \ldots, n \qquad (8)$$

where α_{i-1} is the virtual control at the *ith step* and will be determined in later discussion. In the following, two control schemes are proposed.

A. Control Scheme I

To illustrate the backstepping procedures, only the last step of the design, i.e. *step n* below, is elaborated in details.

- *Step* 1: We start with the equation for the tracking error z_1 obtained from (6) to (8):

$$\dot{z}_1 = z_2 + \alpha_1 \qquad (9)$$

We design the virtual control law α_1 as

$$\alpha_1 = -c_1 z_1 \qquad (10)$$

where c_1 is a positive design parameter. From (9) and (10) we have

$$z_1\dot{z}_1 = -c_1 z_1^2 + z_1 z_2 \tag{11}$$

• *Step i (i = 2, ..., n − 1)*: Choose

$$\alpha_i = -c_i z_i - z_{i-1} + \dot{\alpha}_{i-1}(x_1, \ldots, x_{i-1}, y_r, \ldots, y_r^{(i-1)}) \tag{12}$$

where $c_i, i = 2, \ldots, n-1$ are positive design parameters. From (8) and (12) we obtain

$$z_i\dot{z}_i = -z_{i-1}z_i - c_i z_i^2 + z_i z_{i+1} \tag{13}$$

• *Step n*: From (6) and (8) we obtain

$$\dot{z}_n = \beta v(t) + a^T Y + d(t) - y_r^{(n)} - \dot{\alpha}_{n-1} \tag{14}$$

Then the adaptive control law is designed as follows

$$\begin{aligned} v &= \hat{e}\bar{v} & (15)\\ \bar{v} &= -c_n z_n - z_{n-1} - \hat{a}^T Y - sgn(z_n)\hat{D} \\ &\quad + y_r^{(n)} + \dot{\alpha}_{n-1} & (16)\\ \dot{\hat{e}} &= -\gamma\bar{v}z_n & (17)\\ \dot{\hat{a}} &= \Gamma Y z_n & (18)\\ \dot{\hat{D}} &= \eta|z_n| & (19) \end{aligned}$$

where c_n, γ and η are three positive design parameters, Γ is a positive definite matrix, $\hat{e}$, $\hat{a}$ and $\hat{D}$ are estimates of $e = 1/\beta$, a and D. Let $\tilde{e} = e - \hat{e}$, $\tilde{a} = a - \hat{a}$ and $\tilde{D} = D - \hat{D}$. Note that $\beta v(t)$ in (14) can be expressed as

$$\beta v = \beta\hat{e}\bar{v} = \bar{v} - \beta\tilde{e}\bar{v} \tag{20}$$

From (14),(16) and (20) we obtain

$$\dot{z}_n = -c_n z_n - z_{n-1} + \tilde{a}^T Y - sgn(z_n)\hat{D} + d(t) - \beta\tilde{e}\bar{v} \tag{21}$$

We define Lyapunov function as

$$V = \sum_{i=1}^{n}\frac{1}{2}z_i^2 + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a} + \frac{\beta}{2\gamma}\tilde{e}^2 + \frac{1}{2\eta}\tilde{D}^2 \tag{22}$$

Then the derivative of V along with (6) and (15) to (19) is given by

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n} z_i\dot{z}_i + \tilde{a}^T\Gamma^{-1}\dot{\tilde{a}} + \frac{\beta}{\gamma}\tilde{e}\dot{\tilde{e}} + \frac{1}{\eta}\tilde{D}\dot{\tilde{D}} \\ &\le -\sum_{i=1}^{n} c_i z_i^2 + \tilde{a}^T\Gamma^{-1}(\Gamma Y z_n - \dot{\hat{a}}) \\ &\quad -\frac{\beta}{\gamma}\tilde{e}(\gamma\bar{v}z_n + \dot{\hat{e}}) + \frac{1}{\eta}\tilde{D}(\eta|z_n| - \dot{\hat{D}}) & (23)\\ &= -\sum_{i=1}^{n} c_i z_i^2 & (24) \end{aligned}$$

where we have used (11),(13),(21) and the fact that $z_n d(t) \le |z_n|D$ to obtain (53).

We then have the following stability and performance results based on this scheme.

Theorem 1: Consider the uncertain nonlinear system (1) satisfying Assumptions 1-2. With the application of controller (15) and the parameter update laws (17) to (19), the following statements hold:

• The resulting closed loop system is globally stable.

• The asymptotic tracking is achieved, i.e.,

$$\lim_{t\to\infty}[x(t) - y_r(t)] = 0 \tag{25}$$

• The transient tracking error performance is given by

$$\begin{aligned} \| x(t) - y_r(t) \|_2 &\le \frac{1}{\sqrt{c_1}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) \\ &\quad + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\Big)^{1/2} & (26) \end{aligned}$$

Proof: From (24) we established that V is non increasing. Hence, $z_i, i = 1, \ldots, n$, $\hat{e}$, $\hat{a}$, $\hat{D}$ are bounded. By applying the LaSalle-Yoshizawa theorem in [15] to (24), it further follows that $z_i(t) \to 0, i = 1, \ldots, n$ as $t \to \infty$, which implies that $\lim_{t\to\infty}[x(t) - y_r(t)] = 0$.

From (24) we also have that

$$\begin{aligned} \| z_1 \|_2^2 &= \int_0^{\infty}|z_1(\tau)|^2 d\tau \le \frac{1}{c_1}(V(0) - V(\infty)) \\ &\le \frac{1}{c_1}V(0) & (27) \end{aligned}$$

Thus, by setting $z_i(0) = 0, i = 1, \ldots, n$, we obtain

$$V(0) = \frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2 \tag{28}$$

a decreasing function of γ, η and Γ, independent of c_1. This means that the bound resulting from (27) and (28) is

$$\begin{aligned} \| z_1 \|_2 &\le \frac{1}{\sqrt{c_1}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) \\ &\quad + \frac{\beta}{2\gamma}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\Big)^{1/2} & (29) \end{aligned}$$

$\triangle\triangle\triangle$

Remark 2: From Theorem 1 the following conclusions can be obtained:

• The transient performance depends on the initial estimate errors $\tilde{e}(0)$, $\tilde{a}(0)$, $\tilde{D}(0)$ and the explicit design parameters. The closer the initial estimates $\hat{e}(0)$, $\hat{a}(0)$ and $\hat{D}(0)$ to the true values e, a and D, the better the transient performance.

• The bound for $\| x(t) - y_r(t) \|_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains γ, η and Γ.

• To improve the tracking error performance we can also increase the gain c_1. However, increasing c_1 will influence other performance such as $\| \dot{x} - \dot{y}_r \|_2$ as shown below.

Since $\dot{V} \le 0$, immediately from (22) we know

$$\begin{aligned} V(t) &= \sum_{i=1}^{n}\frac{1}{2}z_i^2 + \frac{1}{2}\tilde{a}^T\Gamma^{-1}\tilde{a} + \frac{\beta}{2\gamma}\tilde{e}^2 + \frac{1}{2\eta}\tilde{D}^2 \\ &\le V(0) & (30) \end{aligned}$$

Then

$$\| z_i \|_\infty \le \sqrt{2V(0)}, \quad i = 1, \ldots, n \tag{31}$$

$$\| \tilde{a} \|_\infty \le \sqrt{\bar{\lambda}(\Gamma)}\sqrt{2V(0)} \tag{32}$$

From equations (7),(8) for $i = 2$ and (10), we get

$$\begin{aligned} \| \dot{x} - \dot{y}_r \|_2 &= \| z_2 - c_1 z_1 \|_2 \\ &\leq \| z_2 \|_2 + c_1 \| z_1 \|_2 \end{aligned} \quad (33)$$

Similar to the proof of (29), we can get $\| \; z_2 \; \|_2 \leq \frac{1}{\sqrt{c_2}} \sqrt{V(0)}$ and thus

$$\| \dot{x} - \dot{y}_r \|_2 \leq (\frac{1}{\sqrt{c_2}} + \sqrt{c_1}) \sqrt{V(0)} \quad (34)$$

From equation (34) we can see that increasing c_1 also increase the error $\| \; \dot{x} - \dot{y}_r \; \|_2$. This suggests to fix the gain c_1 to some acceptable value and adjust the other gains such as $\;$, η and Γ.

B. Control Scheme II

In the previous scheme, a discontinuous function $sgn(z_n)$ is involved in the control and this may cause chattering. To avoid this, we now propose an alternative smooth control scheme.

Firstly we define a function $sg_i(z_i)$ as follows

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|} & |z_i| \quad \delta_i \\ \frac{z_i}{(\delta_i^2 - z_i^2)^{n \;\; i+2} + |z_i|} & |z_i| < \delta_i \end{cases} \quad (35)$$

where $\delta_i (i = 1, \ldots, n)$ is a positive design parameter. It can be shown that $sg_i(z_i)$ is $(n - i + 2)th$ order differentiable. We also design a function $f_i(z_i)$ as

$$f_i(z_i) = \begin{cases} 1 & |z_i| \quad \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \quad (36)$$

Then we can get

$$sg_i(z_i) f_i(z_i) = \begin{cases} 1 & z_i \quad \delta_i \\ 0 & |z_i| < \delta_i \\ -1 & z_i \leq \delta_i \end{cases} \quad (37)$$

To ensure the resultant functions are differentiable, we replace z_i^2 by $(|z_i| - \delta_i)^{n \;\; i+2} f_i$ in the Lyapunov functions for $i = 1, \ldots, n$ in Section 3.1 and we also replace z_i by $(|z_i| - \delta_i)^{n \;\; i+1} sg_i$ in the design procedure as detailed below.

- *Step 1*: we design virtual control law $\;_1$ as

$$_1 = -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) - (\delta_2 + 1) sg_1(z_1) \quad (38)$$

where c_1 is a positive design parameter. We choose Lyapunov function V_1 as

$$V_1 = \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1 \quad (39)$$

Then the derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= (|z_1| - \delta_1)^n f_1 sg_1(z_1) \dot{z}_1 \\ &\leq -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} f_1 \\ &\quad + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 \end{aligned} \quad (40)$$

where (9) and (38) have been used.

- *Step 2*: we design virtual control law $\;_2$ as

$$\begin{aligned} _2 &= -(c_2 + \frac{5}{4})(|z_2| - \delta_2)^{n \;\; 1} sg_2(z_2) \\ &\quad + \dot{}_1 - (\delta_3 + 1) sg_2(z_2) \end{aligned} \quad (41)$$

where c_2 is positive design parameter.We design Lyapunov function V_2 as

$$V_2 = \frac{1}{n}(|z_2| - \delta_2)^n f_2 + V_1 \quad (42)$$

Then the derivative of V_2 is

$$\begin{aligned} \dot{V}_2 &\leq \sum_{i=1}^{2} (|z_i| - \delta_i)^{2(n \;\; i+1)} f_i + M_2 \\ &\quad + (|z_2| - \delta_2)^{n \;\; 1} (|z_3| - \delta_3 - 1) f_2 \end{aligned} \quad (43)$$

where $M_2 = -\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + (|z_1| - \delta_1)^n (|z_2| - \delta_2 - 1) f_1 - (|z_2| - \delta_2)^{2(n \;\; 1)} f_2$. Now we show that $M_2 < 0$. It is clear that $M_2 \leq 0$ for $|z_2| < \delta_2 + 1$. For $|z_2| \quad \delta_2 + 1$

$$\begin{aligned} M_2 &\leq -\frac{1}{4}(|z_1| - \delta_1)^{2n} f_1 + \frac{1}{4}(|z_1| - \delta_1)^{2n} f_1^2 \\ &\quad + (|z_2| - \delta_2 - 1)^2 - (|z_2| - \delta_2)^{2(n \;\; 1)} \\ &< (|z_2| - \delta_2)^2 - (|z_2| - \delta_2)^{2(n \;\; 1)} \\ &= (|z_2| - \delta_2)^2 (1 - (|z_2| - \delta_2)^{2(n \;\; 2)}) \\ &\leq 0 \end{aligned} \quad (44)$$

Then (43) is written as

$$\begin{aligned} \dot{V}_2 &\leq -\sum_{i=1}^{2} c_i (|z_i| - \delta_i)^{2(n \;\; i+1)} f_i \\ &\quad + (|z_2| - \delta_2)^{n \;\; 1} (|z_3| - \delta_3 - 1) f_2 \end{aligned} \quad (45)$$

- *Step i* ($i = 3, \ldots, n - 1$): Choose

$$\begin{aligned} _i &= -(c_i + \frac{5}{4})(|z_i| - \delta_i)^{n \;\; i+1} sg_i(z_i) + \dot{}_{i \;\; 1} \\ &\quad -(\delta_{i+1} + 1) sg_i(z_i) \end{aligned} \quad (46)$$

where c_i is positive design parameter.

- *Step n*: The control law and parameter update laws are designed as follows

$$\begin{aligned} v &= \hat{e} \bar{v} && (47) \\ \bar{v} &= -(c_n + 1)(|z_n| - \delta_n) sg_n(z_n) - \hat{a}^T Y \\ &\quad - sg_n \hat{D} + y_r^{(n)} + \dot{}_{n \;\; 1} && (48) \\ \dot{\hat{e}} &= - \;\; \bar{v}(|z_n| - \delta_n) f_n sg_n(z_n) && (49) \\ \dot{\hat{a}} &= \Gamma Y (|z_n| - \delta_n) f_n sg_n(z_n) && (50) \\ \dot{\hat{D}} &= \eta(|z_n| - \delta_n) f_n && (51) \end{aligned}$$

where c_n, $\;$ and η are three positive design parameters, Γ is a positive definite matrix, $\hat{e}$, $\hat{a}$ and $\hat{D}$ are estimates of $e = 1/\beta$, a and D. We define Lyapunov function as

$$\begin{aligned} V &= \sum_{i=1}^{n} \frac{1}{n - i + 2}(|z_i| - \delta_i)^{n \;\; i+2} f_i + \frac{1}{2}\tilde{a}^T \Gamma^{\;\; 1} \tilde{a} \\ &\quad + \frac{\beta}{2}\tilde{e}^2 + \frac{1}{2\eta}\tilde{D}^2 \end{aligned} \quad (52)$$

Then the derivative of V is given by

$$\begin{aligned}
\dot{V} &= \dot{V}_i + (|z_n| - \delta_n)^2 f_n sg_n(z_n)\dot{z}_n + \tilde{a}^T\Gamma^{-1}\dot{\tilde{a}} \\
&\quad + \frac{\beta}{\,}\tilde{e}\dot{\tilde{e}} + \frac{1}{\eta}\tilde{D}\dot{\tilde{D}} \\
&\le -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i \\
&\quad + \tilde{a}^T\Gamma^{-1}(\Gamma Y(|z_n| - \delta_n) f_n sg_n(z_n) - \dot{\hat{a}}) \\
&\quad - \frac{\beta}{\,}\tilde{e}(\ \bar{v}(|z_n| - \delta_n) f_n sg_n(z_n) + \dot{\hat{e}}) \\
&\quad + \frac{1}{\eta}\tilde{D}\big(\eta(|z_n| - \delta_n) f_n - \dot{\hat{D}}\big) \qquad (53) \\
&= -\sum_{i=1}^{n} c_i(|z_i| - \delta_i)^{2(n-i+1)} f_i \qquad (54)
\end{aligned}$$

where (6),(38),(41) and (47) to (51) have been used.

Theorem 2: Consider the uncertain nonlinear system (1) satisfying Assumptions 1-2. With the application of controller (47) and the parameter update laws (49) to (51), the following statements hold:

- The resulting closed loop system is globally stable.
- The tracking error approaches δ_1 asymptotically, i.e.,

$$\lim_{t\to\infty} |x(t) - y_r(t)| = \delta_1 \qquad (55)$$

- The transient tracking error performance is given by

$$\begin{aligned}
\| x(t) - y_r(t) \|_2 &\le \delta_1 + \frac{1}{c_1^{2n}}\Big(\frac{1}{2}\tilde{a}(0)^T\Gamma^{-1}\tilde{a}(0) \\
&\quad + \frac{\beta}{2}\tilde{e}(0)^2 + \frac{1}{2\eta}\tilde{D}(0)^2\Big)^{1/2n} \qquad (56)
\end{aligned}$$

with $z_i(0) = \delta_i, i = 1, \ldots, n$,

Proof: Based on (54), the results can be shown by following similar steps to that of Theorem1.

△△△

Note that similar remarks made in Remark 2 are also applicable here.

IV. Simulation Studies

In this section, we illustrate the above methodology on the same example system in [2] which is described as:

$$\begin{aligned}
\ddot{x} &= a_1\frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} - a_2(\dot{x}^2 + 2x)sin(\dot{x}) \\
&\quad - 0.5a_3 x sin(3t) + bu(t) \qquad (57)
\end{aligned}$$

where $u(t)$ represents the output of the dead-zone nonlinearity. The actual parameter values are $b = 1$ and $a_1 = a_2 = a_3 = 1$. Without control, i.e. $u(t) = 0$, (57) is unstable. The parameters of the dead-zone ar $b_r = 0.5, b_l = -0.6, m = 1$. The objective is to control the system state x to follow a desired trajectory $y_r(t) = 2.5sin(t)$ as in [2].

In the simulation of Scheme I, the robust adaptive control law (15)-(19) was used, taking $c_1 = 0.8, c_2 = 0.5, \ \ = 0.1, \Gamma = 0.1I_3, \eta = 0.2$. The initial values are chosen as follows: $\hat{e}(0) = 0.25, \hat{a}(0) = [1.5\ 1\ 1]^T, \hat{D}(0) = 2, x(0) = [1\ 1.05]^T$ and $v(0) = 0$. The simulation results presented in the Figure 2 and Figure 3 are system tracking error and input.

In the simulation of Scheme II by using the robust adaptive control law (47)-(51), we choose c_1, $, \eta, \Gamma$ and the initial values to be same as above and $\delta_1 = 0.1$. The simulation results presented in the Figure 4 and Figure 5 are system tracking error and input. Clearly all the results verify our theoretical findings and show the effectiveness of the control schemes.

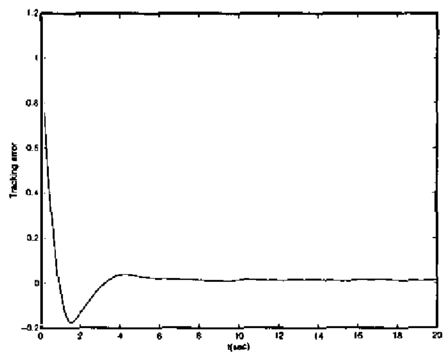

Fig. 2. Tracking error-Scheme I

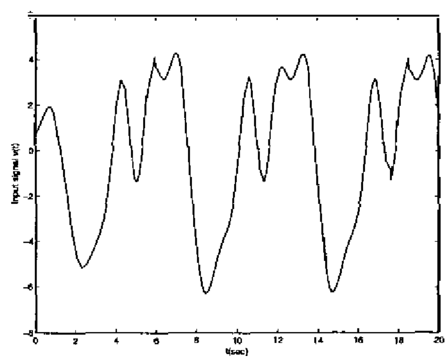

Fig. 3. Control signal $v(t)$-Scheme I

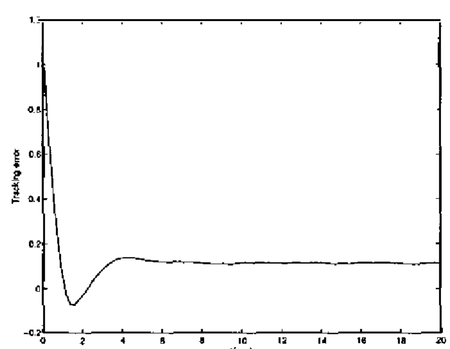

Fig. 4. Tracking error-Scheme II

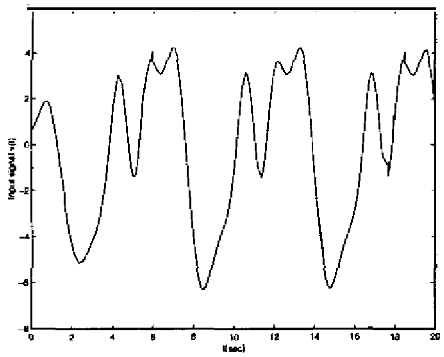

Fig. 5. Control signal $v(t)$-Scheme II

V. Conclusion

This paper presents two backstepping adaptive controller design schemes for a class of uncertain nonlinear SISO system preceded by unknown dead-zone nonlinearities, in the presence of bounded external disturbances. In the first scheme, a sign function is involved and this can ensure perfect tracking. To avoid possible chattering caused by the sign function, we propose an alternative smooth control law and the tracking error is still ensured to approach a prescribed bound in this case. Unlike some existing control schemes, the developed backstepping controls do not require the model parameters within known intervals and the knowledge on the bound of 'disturbance-like' term is not required. Besides showing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters. Simulation results illustrates the effectiveness of our schemes.

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