Monte Carlo Tree Search for Climate-Economy Models

*An exploration in Progress

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Executive Summary

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- **Approach:** Integration of Monte Carlo Tree Search (MCTS) with climate-economy modeling
- **Key Features:**
  - Nested optimization structure separating climate policy and economic equilibrium
  - Dynamic damage function learning
  - Tractable computation (6 periods)
- **Framework Components:**
  - Two-period general equilibrium model
  - Markov Decision Process for policy. MCTS for finding optimal pathways.
  - Endogenous technological change
  - Deep Uncertainty about Climate Damages
- **Next Steps:** Evaluation and full working proof of concept

Conceptual Road Map

The model follows a nested structure:

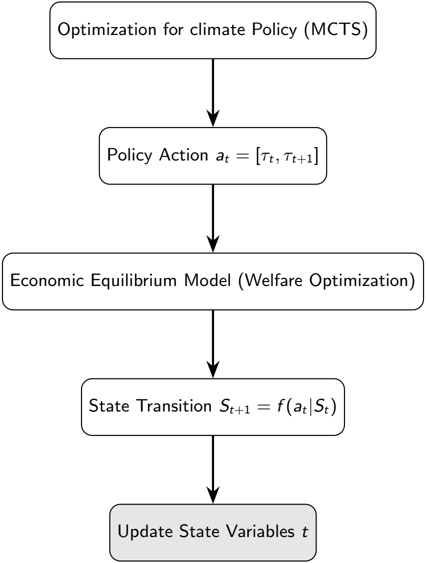
- **Outer Loop**: Climate Policy Optimization
- **Inner Loop**: Two-Period General Equilibrium (GE) Model

This structure supports a separation between climate policy and equilibrium, allowing for independent optimization without compounding computational costs.

GE is solved for 2 periods, the current period and a future period based on expectations. Each state is chained together. In the end there is a longer time horizon. Feasible to compute up to 6 time-periods on my laptop.

Nested Loop Structure of the Model

Outer Loop: Policy Optimization



Motivation

Current Challenges in Climate-Economy Modeling

- Computational complexity in joint optimization of policy and equilibrium
- Limitations due to perfect foresight assumptions and static technology-emissions links
- Need for improved uncertainty handling and credible policy analysis

Why Now?

- Advances in Monte Carlo Tree Search (MCTS) and parallel computing
- Growing demand for dynamic policy analysis and technology transition modeling

Markov Decision Process Formalization

The system evolves as a Markov Decision Process (MDP) where:

$$S_t = f(a_t | S_{t-1})$$

- S_t: State vector at time t containing:
  - Economic variables from GE model (Y_t, C_t, etc.)
  - Cumulative emissions $E_t = \sum_{i=0}^{t} \eta_i Y_i$
  - Current damage function parameters θ_t
- a_t: Policy action vector $[\tau_t, \tau_{t+1}]$
- $f(\cdot)$: Transition function (Two-Period GE model)

Policy Maker's Problem

Objective Function

The policy maker maximizes expected social welfare:

$$\max_{a_t} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(u(S_t) - D(E_t, \theta_t) Y_t\right)\right]$$

- β: Social discount factor
- $u(S_t)$: Social utility function
- $D(E_t, \theta_t)$: Damage function
- Y_t: Aggregate output
- E_t: Cumulative emissions
- θ_t: Vector of damage function parameters

Damage Function Learning

The damage function uncertainty evolves as:

$$\theta_t = g(E_t, \theta_{t-1}, \epsilon_t)$$

- $g(\cdot)$: Represents the learning process
- ϵ_t: New information received in period t
- Uncertainty band narrows as $t \to \infty$
- Distribution shape depends on E_t

Policy Action Space

$a_t \in A$ where A is the discrete action space:

$$A = \{\tau_t \pm \{5, 10\} \text{ basis points}\}$$

Reason: Action space has to be discrete and small for search to work. Modelled like monetary policy adjustments used by central banks.

Agent Expectations

Agents in the GE model form expectations of future carbon taxes:

$$\mathbb{E}_t[\tau_{t+1}] = h(a_t, \Omega_t)$$

- Ω_t: Information set at time t containing:
  - Policy maker's announced rates $[\tau_t, \tau_{t+1}]$
  - Policy maker's objective function
  - Current state S_t
- $h(\cdot)$: Expectation formation function
  - Incorporates credibility of policy maker
  - Rational expectations using all available information
  - Policy deviations come from new information about damages, or technological shocks

Time Structure

1. Period t begins with state S_t
2. Policy maker observes S_t and chooses $a_t = [\tau_t, \tau_{t+1}]$
3. Agents observe a_t and form expectations
4. GE model solves for equilibrium
5. Damages realized, new information ϵ_t received
6. System transitions to S_{t+1}

Monte Carlo Tree Search (MCTS): Why Use It?

Traditional Challenges

- Perfect foresight assumption
- Curse of Dimensionality with Model Complexity
- Limited uncertainty handling. Requires handmade scenarios

MCTS Solutions

- Can handle Bayesian Updating
- Managing vast search space: asymptotically optimal
- Robust to modeling exogenous shocks. Can test multiple scenarios at once.

MCTS Implementation

Core Algorithm Steps

1. **Selection:** Choose promising policy paths
2. **Expansion:** Add new policy scenarios
3. **Simulation:** Run GE model for outcomes
4. **Backpropagation:** Update policy values

Key Advantages

- No pre-commitment to policy paths
- Incorporates new damage information
- Handles deep uncertainty

The GE Model Climate Extensions: Overview

- Joint Distribution of Productivity and Carbon Intensity
  - Firms select capital varieties with both productivity (A_t) and carbon intensity (η_t).
  - Dynamic Joint Distribution: The joint distribution of productivity (A_t) and carbon intensity (η_t) evolves over time, driven by an exogenous technological innovation rate.
  - Trade-Offs: Joint distribution with correlation (ρ) between A_t and η_t introduces a trade-off.
  - Objective: Firms face an emissions tax that affects their choice, balancing productivity against carbon intensity.

The GE Model Climate Extensions: Overview

- Emission Tax on Output
  - An emission tax ($\tau \eta_t Y_t$) is imposed on firms based on net emissions.
  - Incentive to Reduce Emissions: Higher carbon intensity increases tax burden, impacting profits.
  - Policy Relevance: Allows analysis of how emission taxes influence firm technology choices and market dynamics.

The GE Model: Labor Market and Adjustment Costs

- Labor Market Adjustment Costs:
  - Worker skills (θ) inversely relate to adjustment costs when firms change carbon intensity.
  - Adjustment Cost Function: Increasing on change in carbon intensity, with higher skill workers affected less and lower skill workers facing higher costs.

$$\int_{\underline{\theta}}^{\overline{\theta}} \phi(\Delta\eta_t)\psi(\theta)dF(\theta)$$

  - Impact: Large swings in carbon intensity increases labor frictions, affecting labor supply.
- Government and Market Mechanisms:
  - Emission tax revenue ($T_t = \tau\eta_t Y_t$) is rebated as lump-sum transfers to households.

Opportunities for Extension

- **Economic Complexity**: Heterogeneous Agents, Sectors
- **Policy Analysis**: Credibility dynamics, learning effects, regional heterogeneity
- **Research Applications**: Policy uncertainty impact, technology adoption, distributional effects

Challenges

Technical

- Equilibrium convergence, parallelization, Bugs

Practical

- Deadline for Thesis. No time for calibration/validation.

Next Steps

Thesis

- Complete core implementation, sensitivity analysis

Future

- Calibrate with Climate Models
- Compute optimal climate Policy

Long Term

- Very hypothetical
- Heterogeneous agents, full uncertainty quantification, applied policy cases

Discussion & Questions

Appendix: Firm Optimization

Firm's Objective: Profit Maximization

- Firms choose capital (K_t) and labor (L_t) to maximize profits.
- Profit Function:

$$\Pi_t = Y_t - w_t L_t - r_t K_t$$

  where:
  - Y_t: Output produced by the firm at time t.
  - w_t: Wage rate paid for labor.
  - r_t: Rental rate of capital.

Appendix: Firm Optimization

Production Function

- Cobb-Douglas Production Function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

where:

  - A_t: Total factor productivity at time t.
  - α: Capital share in output, with $\alpha \in (0, 1)$.

First-Order Conditions for Profit Maximization

- Labor Demand:

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \Rightarrow w_t = (1-\alpha)\frac{Y_t}{L_t}$$

- Capital Demand:

$$\frac{\partial \Pi_t}{\partial K_t} = 0 \Rightarrow r_t = \alpha \frac{Y_t}{K_t}$$

Appendix: Household Optimization

CRRA : Utility Function for maximization Reason: Extensible for economic insecurity. (Original thesis research question)

- Households choose consumption (C_t) and labor supply (L_t) to maximize lifetime utility.
- Utility Function:

$$U = \sum_{t=0}^{T} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\nu}}{1+\nu} \right)$$

  where:
  - β: Discount factor, $\beta \in (0, 1)$.
  - σ: Coefficient of relative risk aversion.
  - χ: Weight on disutility of labor.
  - ν: Inverse of the Frisch elasticity of labor supply.

Appendix: Household Optimization

Budget Constraint

- Period t Budget Constraint:

$$C_t + S_t = w_t L_t + (1 + r_t) S_{t-1}$$

where:

  - S_t: Savings in period t, which earn a return r_t.

First-Order Conditions for Utility Maximization

- Labor Supply:

$$\chi L_t^{\nu} = C_t^{-\sigma} w_t$$

- Consumption-Savings Decision (Euler Equation):

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{\sigma} = 1 + r_{t+1}$$