Aufgabe: Rang und Defekt einer Matrix ablesen

$$A_1 = \begin{pmatrix} 1 & 2 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 & 6 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix} \quad \Rightarrow \quad \operatorname{rang}(A_1) = 4\,, \quad \operatorname{def}(A_1) = \; = 1\,,$$

$$A_2 = \begin{pmatrix} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \Rightarrow \quad \operatorname{rang}(A_2) = 3\,, \quad \operatorname{def}(A_2) = \; = 2\,,$$

$$A_3 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 5 & 6 \end{pmatrix} \quad \Rightarrow \quad \operatorname{rang}(A_3) = 2\,, \quad \operatorname{def}(A_3) = \; = 3\,,$$

$$A_4 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix} \quad \Rightarrow \quad \operatorname{rang}(A_4) = 3\,, \quad \operatorname{def}(A_4) = \; = 2\,.$$

Aufgabe: Rang und Defekt einer Matrix bestimmen

$$A_1 = \begin{pmatrix} 2 & 4 & 0 & 4 & 8 \\ 1 & 3 & 1 & 4 & 10 \\ -2 & -2 & 1 & 1 & 7 \\ 0 & -1 & 0 & 1 & -1 \end{pmatrix} \quad \Leftrightarrow \quad \tilde{A}_1 = \begin{pmatrix} 1 & 2 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 & 6 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 1 & 2 & -1 & 1 & 1 \\ 3 & 6 & -3 & 4 & 5 \\ 1 & 4 & 3 & 1 & 3 \\ -1 & -1 & 3 & 2 & 6 \end{pmatrix} \quad \Leftrightarrow \quad \tilde{A}_2 = \begin{pmatrix} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ -4 & 2 & 3 & 3 & 4 \end{pmatrix} \quad \Leftrightarrow \quad \tilde{A}_3 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 5 & 6 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ -4 & 2 & 3 & 3 & 4 \\ 0 & 0 & 1 & 5 & 9 \end{pmatrix} \quad \Leftrightarrow \quad \tilde{A}_4 = \begin{pmatrix} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

Aufgabe: Lineare Gleichungssysteme lösen

$$A_1\boldsymbol{x} = \boldsymbol{b}, \quad (A_1, \boldsymbol{b}) = \left(\begin{array}{cccc|c} 2 & 4 & 0 & 4 & 8 \\ 1 & 3 & 1 & 4 & 10 \\ -2 & -2 & 1 & 1 & 7 \\ 0 & -1 & 0 & 1 & -1 \end{array}\right) \quad \Leftrightarrow \quad \left(\tilde{A}_1, \tilde{\boldsymbol{b}}\right) = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 & 6 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

$$A_2\boldsymbol{x} = \boldsymbol{b}, \quad (A_2, \boldsymbol{b}) = \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 3 & 6 & -3 & 4 & 5 \\ 1 & 4 & 3 & 1 & 3 \\ -1 & -1 & 3 & 2 & 6 \end{array}\right) \quad \Leftrightarrow \quad \left(\tilde{A}_2, \tilde{\boldsymbol{b}}\right) = \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$A_3\boldsymbol{x} = \boldsymbol{b}, \quad (A_3, \boldsymbol{b}) = \left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 1 \\ -4 & 2 & 3 & 3 & 4 \end{array}\right) \quad \Leftrightarrow \quad \left(\tilde{A}_3, \tilde{\boldsymbol{b}}\right) = \left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 5 & 6 \end{array}\right)$$

$$A_4\boldsymbol{x} = \boldsymbol{b}, \quad (A_4, \boldsymbol{b}) = \left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 1 \\ -4 & 2 & 3 & 3 & 4 \\ 0 & 0 & 1 & 5 & 9 \end{array}\right) \quad \Leftrightarrow \quad \left(\tilde{A}_4, \tilde{\boldsymbol{b}}\right) = \left(\begin{array}{cccc|c} 2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right)$$

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\begin{aligned}
A_1\boldsymbol{x} &= \boldsymbol{b} \quad \Rightarrow \mathbb{L}_1 = \left\{ \begin{pmatrix} -6 \\ 3 \\ -1 \\ 2 \end{pmatrix} \right\} \\
A_2\boldsymbol{x} &= \boldsymbol{b} \quad \Rightarrow \mathbb{L}_2 = \left\{ \boldsymbol{x} \in \mathbb{R}^4 : \quad \boldsymbol{x} = \begin{pmatrix} -3 \\ 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ 1 \\ 0 \end{pmatrix},\ \lambda \in \mathbb{R} \right\} \\
A_3\boldsymbol{x} &= \boldsymbol{b} \quad \Rightarrow \mathbb{L}_3 = \left\{ \boldsymbol{x} \in \mathbb{R}^4 : \quad \boldsymbol{x} = \begin{pmatrix} 7/2 \\ 0 \\ 6 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ 0 \\ -5 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1/2 \\ 1 \\ 0 \\ 0 \end{pmatrix},\ \lambda_1, \lambda_2 \in \mathbb{R} \right\} \\
A_4\boldsymbol{x} &= \boldsymbol{b} \quad \Rightarrow \mathbb{L}_4 = \emptyset
\end{aligned}
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