评分

一、(1) 互信息量：对两个离散的事件集X和Y，事件y_j的出现给出关于事件x_i的信息量，定义为 $I(x_i;y_j)=\log\frac{p(x_i/y_j)}{p(x_i)}$

(2) 平均互信息量：互信息量$I(x_i;y_j)$在集合X和Y上的概率加权平均值称为 $I(X;Y)=\sum_X\sum_Y I(x_i;y_j)$

$=\sum_X\sum_Y p(xy)\log\frac{p(x/y)}{p(x)}$

(3). 性质：① 非负性：即 $I(X;Y)\geq 0$

② 互易性(对称性)：$I(X;Y)=I(Y;X)$

③ 极值性：$I(X;Y)\leq H(X)$

$I(X;Y)\leq H(Y)$

④ 与各种熵之间的关系：

① $I(X;Y)=H(X)-H(X/Y)$

② $I(X;Y)=H(Y)-H(Y/X)$

$I(X;Y)=H(X)+H(Y)-H(X,Y)$.

⑤ 凸函数性：$I(X;Y)$是关于$p(x)$与$p(y/x)$的凸函数.

二、证明：∵ $H(X)=-\sum_X p(x)\log p(x)$.

$H(Y/X)=-\sum_X\sum_Y p(xy)\log p(y/x)$

$H(Y)=-\sum_Y p(y)\log p(y)$

又∵ $\sum_Y p(y)=1$. $\sum_X p(x)=1$. $\sum_X p(x|y)=1$. $\sum_X p(xy)=p(y)$

∴ $H(Y/X)=-\sum_X\sum_Y p(xy)\log p(y/x)$

$\leq-\sum_X\sum_Y p(xy)\log\left[\sum_X p(x)\cdot p(y/x)\right]$

$= -\sum_x \sum_y p(xy) \log \left(\sum_x p(xy) \right)$

$= -\sum_x \sum_y p(y) \cdot p(x/y) \log p(y)$

$= -\sum_y \left(\sum_x p(x/y) \right) \cdot p(y) \log p(y)$

$= -\sum_y p(y) \cdot \log p(y) = H(Y)$

即 $H(Y/X) \leq H(Y)$.

三、证明：$H(S) = -\sum_{i=1}^{q} p(s_i) \log p(s_i)$

$\bar{L} = \sum_{i=1}^{q} p(s_i) \cdot l_i$

(1) 先证 $H(S) - \bar{L} \cdot \log r \leq 0$.

$\because H(S) - \bar{L} \log r = -\sum_{i=1}^{q} p(s_i) \log p(s_i) - \sum_{i=1}^{q} p(s_i) \cdot l_i$

$= -\sum_{i=1}^{q} p(s_i) \log p(s_i) + \sum_{i=1}^{q} p(s_i) \log r^{-l_i} = \sum_{i=1}^{q} p(s_i) \log \frac{r^{-l_i}}{p(s_i)}$

由詹森不等式可得

$H(S) - \bar{L} \log r \leq \log \left(\sum_{i=1}^{q} p(s_i) \cdot \frac{r^{-l_i}}{p(s_i)} \right) = \log \sum_{i=1}^{q} r^{-l_i}$

又 $\because$ 存在唯一可译码的充要条件为 $\sum_{i=1}^{q} r^{-l_i} \leq 1$.

$\therefore$ 可得 $H(S) - \bar{L} \log r \leq \log 1 = 0$

即 $H(S) - \bar{L} \log r \leq 0$. 可得 $\bar{L} \geq \frac{H(S)}{\log r}$

(2). ~~再~~证 $\bar{L} < 1 + \frac{H(S)}{\log r}$

$)=p(y)$. $\because l_i \geq -\frac{\log p(s_i)}{\log r}$ $\therefore p(s_i) \geq r^{-l_i}$

即 $\sum_{i=1}^{q} r^{-l_i} \leq \sum_{i=1}^{q} p(s_i) = 1$.

而 $l_i < \frac{-\log p(s_i)}{\log r} + 1$. 从而有 $\sum_{i=1}^{q} p(s_i) \cdot l_i < \frac{-\sum_{i=1}^{q} p(s_i) \log p(s_i)}{\log r} + 1$

$\therefore$ 有 $\bar{L} < \frac{H(S)}{\log r} + 1$. 综上所述 $\frac{H(S)}{\log r} \leq \bar{L} < \frac{H(S)}{\log r} + 1$.

评分

四. 解: (1) $H(S) = -0.6\log 0.6 - 0.4\cdot\log 0.4 = 0.97$.

对于 $S_1 \to 0$. $S_2 \to 1$. 可得 $\bar{L}_1 = 1$.

$\eta = \frac{H(S)}{\bar{L}_1} = 0.971$. $R = 0.971$

(2)

S^2	$s_1 s_1$	$s_1 s_2$	$s_2 s_1$	$s_2 s_2$
P	0.36	0.24	0.24	0.16
C	0	10	110	111
l_i	1	2	3	3

$\therefore \bar{L}_2 = 0.36\times 1 + 0.24\times 2 + 0.24\times 3 + 0.16\times 3 = 2.06$

$\therefore \bar{L} = \frac{\bar{L}_2}{2} = 1.03$

$\therefore \eta = \frac{H(S)}{\bar{L}} = 0.942$. $R = 0.942$.

五. 解: (1)

X	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

Y	0	1
P	1/2	1/2

Z	0	1
P	7/8	1/8

(2)

X \ Z	0	1
0	1/2	0
1	3/8	1/8

Y \ Z	0	1
0	1/2	0
1	3/8	1/8

$\therefore H(X) = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2} = 1$.

$H(Z) = -\frac{7}{8}\log\frac{7}{8} - \frac{1}{8}\log\frac{1}{8} = 0.543$.

$H(X,Z) = -\frac{1}{2}\log\frac{1}{2} - \frac{3}{8}\log\frac{3}{8} - \frac{1}{8}\log\frac{1}{8} = 1.41$.

$H(X,Y) = -\frac{1}{8}\times 2\cdot\log\frac{1}{8} - \frac{3}{8}\times 2\times\log\frac{3}{8} = 1.82$.

$H(Y/X) = H(X,Y) - H(X) = 0.82$.

$H(Z/X) = H(X,Z) - H(X) = 0.41$

$I(X;Z) = H(X) + H(Z) - H(X,Z) = 0.133$.

七.

97. 六、解：(1) 信道矩阵为 $P_1=\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6}\\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3}\end{bmatrix}$

$L=4$，属于对称信道.

$\therefore C=\log_2 4+2\times\frac{1}{3}\times\log_2\frac{1}{3}+2\times\frac{1}{6}\log_2\frac{1}{6}=0.081$

(2). 信道矩阵 $P_2=\begin{bmatrix}\frac{1}{2} & \frac{1}{3} & \frac{1}{6}\\ 1/6 & 1/2 & 1/3\\ 1/3 & 1/6 & 1/2\end{bmatrix}$

$L=3$. 属于对称信道

$C=\log_2 3-H(\frac{1}{2},\frac{1}{3},\frac{1}{6})$

$=\log_2 3+(\frac{1}{2}\log_2\frac{1}{2}+\frac{1}{3}\log_2\frac{1}{3}+\frac{1}{6}\log_2\frac{1}{6})$

$=0.126$.

七、解：(1)

X	0	1
P	3/4	1/4

(2)

X\Y	0	1
0	1/2	1/4
1	1/12	1/6

Y	0	1
P	7/12	5/12

$\therefore H(X)=-\frac{3}{4}\log\frac{3}{4}-\frac{1}{4}\log\frac{1}{4}=0.811$

$H(X,Y)=-\frac{1}{2}\log\frac{1}{2}-\frac{1}{4}\log\frac{1}{4}-\frac{1}{12}\log\frac{1}{12}-\frac{1}{6}\log\frac{1}{6}=1.729$

$H(Y)=-\frac{7}{12}\times\log\frac{7}{12}-\frac{5}{12}\log\frac{5}{12}=0.98$

$\therefore H(X/Y)=H(X,Y)-H(Y)=0.749$

$H(Y/X)=H(X,Y)-H(X)=0.918$

$I(X;Y)=H(X)+H(Y)-H(X,Y)=0.082$.

(2). 当输入为等概率分布 即

X	0	1
P	1/2	1/2

时

可得 $C=I(X;Y)=0.082$.

评分

八、解：$H(S)=H(0.32,0.22,0.18,0.16,$

(1) 香农编码

s_i	$p(s_i)$	累加$p(s_i)$	$-\log p(s_i)$	l_i	C
s_1	0.32	0	1.64	2	00
s_2	0.22	0.32	2.18	3	010
s_3	0.18	0.54	2.48	3	100
s_4	0.16	0.72	2.64	3	101
s_5	0.08	0.88	3.64	4	1100
s_6	0.04	0.96	4.64	5	11110

$\bar{L}=2\times0.32+3\times0.22+3\times0.18+3\times0.16+4\times0.08+5\times0$

$=2.84.$

$\therefore R=\frac{H(S)}{\bar{L}}=0.83.$

(2) 费诺编码

s_i	$p(s_i)$	第1次分组	第2次分组	第3次分	第4次分	C	l_i
~~0.32~~	0.32	0	0			00	2
~~0.22~~	0.22		1			01	2
~~0.18~~	0.18		0			10	2
~~0.16~~	0.16	1		0		110	3
~~0.08~~	0.08		1		0	1110	4
~~0.04~~	0.04			1	1	1111	4

$, 0.08, 0.04) = 2.35$

$$\bar{L} = 2\times0.32+2\times0.22+2\times0.18+3\times0.16+4\times0.08+4\times0.04$$
$$= 2.32$$

$$\bar{R} = \frac{H(S)}{\bar{L}} = 0.987.$$

(3) 应是最好的霍夫曼编码.

$\times 0.04$

S_i	$p(S_i)$	编码过程	C	l_i
S_1	0.32	0.32 → 0.32 → 0.4 → 0.6 (0)	00	2
S_2	0.22	0.22 → 0.28 → 0.32 (0) → 0.4 (1)	10	2
S_3	0.18	0.18 → 0.22 (0) → 0.28 (1)	11	2
S_4	0.16	0.16 (0) → 0.18 (1)	010	3
S_5	0.08 (0)	0.12 (1)	~~011~~ 0110	~~3~~ 4
S_6	0.04 (1)		0111	4

$$\bar{L} = 2\times0.32+2\times0.22+2\times0.18+3\times0.16+4\times0.08+4\times0.04 = 2.3$$

$$\therefore \bar{R} = \frac{H(S)}{\bar{L}} = 0.987.$$

(4) 应最好三元霍夫曼编码

S_i	$p(S_i)$	编码过程	C	l_i
S_1	0.32	0.32 → 0.44 (0)	1	1
S_2	0.22	0.22 → 0.32 (1)	2	1
S_3	0.18	0.18 (0) → 0.22 (2)	00	2
S_4	0.16	0.16 (1)	01	2
S_5	0.08 (0)	0.12 (2)	020	3
S_6	0.04 (1)		021	3
S_7	0 (2)			

$$\bar{L} = 1\times0.32+1\times0.22+2\times0.18+2\times0.16+3\times0.08+3\times0.04 = 1.58$$

$$\therefore \bar{R} = \frac{H(S)}{\bar{L}} = 1.487$$