MRI Reconstruction Using Noise2Noise

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Abstract

In this work, we use basic statistical reasoning to show: it is possible for neural network to learn to restore corrupted data by only looking at corrupted data. In pratice, we show Noise2Noise can restore corrupted MRI scans which are generated by different downsampling methods.

1. Introduction

Image denoising and image reconstruction from corrupted or incomplete measurement are very important subfields in computer vision and computional imaging. In recent years, Using deep neural network to map corrupted observations to the unobserved clean data (Noise2Clean) have achieved a lot of improvements in this field. This is actually training a regression model, e.g., a convolution neural network (CNN), with a large number of pairs $(\hat{x}_i, y_i)$ of corrutped inputs $\hat{x}_i$ and clean targets y_i and minimizing the empirical risk

$$\underset{\theta}{\operatorname{argmin}} \sum_i L(f_\theta(\hat{x}_i), y_i), \tag{1}$$

where f_θ is parametric family of mappings, and L is loss function.

While these mapping methods use clean data (non-noise data) as training target. But sometimes it is difficult or even impossible to acquire clean data, which sets a higher requirement for the reconstruction method.

Noise2Noise[1] is a method to handle this problem. It is a state of the art method for image denoising and reconstruction. It can learn to restore corrutped data by only looking at noise data. This happens by training a regression model with a lot of pairs of corrupted inputs and corrupted targets.

Magnetic Resonance Imaging (MRI) is an essential medical imaging tool. It is actually a method to produce volumetric images of biological tissues by sampling the Fourier transform (k-space) of the signal[1] . For MRI, the data acquiaition process is inherently slow, which may be imprecision because of the motion of patients, and may be harmful for patients, too. So, modern MRI techniques rely heavily on compressed sensing[2], which can reconstruct the image from under-sampled datas in Fourier domain.

In this work, we apply Noise2Noise to reconstruct MRI from undersampled data. We also compare the reconstruct result of Noise2Noise with the result of Noise2Clean to show: Noise2Noise method have similar effect as Noise2Clean method.

2. Theoretical Background

Noise2Clean method is training with a lot of pairs $(\hat{x}_i, y_i)$ and minimizing the empirical risk

$$\underset{\theta}{\operatorname{argmin}} \sum_i L(f_\theta(\hat{x}_i), y_i), \tag{2}$$

where $\hat{x}_i$ is corrupted input, y_i is clean target, θ is the parameters of the model we are looking for, and L is loss function.

Similarly, Noise2Noise method is training with a lot of pairs $(\hat{x}_i, \hat{y}_i)$ and minimizing the empirical risk

$$\underset{\theta}{\operatorname{argmin}} \sum_i L(f_\theta(\hat{x}_i), \hat{y}_i), \tag{3}$$

where both $\hat{x}_i$ and $\hat{y}_i$ are corrupted targets, conditioned on the unobserved clean target y_i such that $E\{\hat{y}_i|\hat{x}_i\} = y_i$.

If we use $L2$ loss $L(x, y) = (x - y)^2$, for each corrupted input $\hat{x}_i$, the Noise2Clean method is trying to find θ to satisfy

$$f_\theta(\hat{x}_i) = E\{y_i\} \tag{4}$$
$$= y_i \tag{5}$$

while Noise2Noise method is trying to find θ to satisfy

$$f_\theta(\hat{x}_i) = E\{\hat{y}_i|\hat{x}_i\} \tag{6}$$
$$= y_i \tag{7}$$

The network can, in theory, minimize this loss by solving the point estimatin problem separately for each input sample[1]. So, in theory, if we use same model, Noise2Noise and Noise2Clean method will find the same θ for the model which means they will have the same effect.

3. Experiment and Results

3.1. Dataset

To train the CNN, we first build our dataset from the IXI brain scan MRI dataset.[1] The IXI dataset includes brain scans from 581 subjects, and each subjects have 150 brain scans image. The image resolution is 256×256.

We randomly chose 5000 images from 500 subjects to build the training set, and for validation set we randomly chose 1000 image from the other 81 subjects. We corrupt images in the validation set to generate a specific validation set for each corrupt method. And we normalize our images to represent them in range $[-0.5, 0.5]$.

3.2. U-net

In this work, the deep neural network we chose is U-net[3]. U-net can learn spatial information in high level layers, can learn sematic information in low level layers. In the process of upsampling, U-net can combinate spatial information and sematic information to restroe the corrupted images.

In the experiments, we use the Mean Square Error (MSE) as the loss funciton, and we use the Peak Singal to Noise Ratio (PSNR) as the measurment.

3.3. Corrupt and Feed Online

For training with Noise2Noise method, we corrupt the images from training set and feed them online. Every time we feed data to Unet, we choose a clean image from training set, and then corrupt it to generate two differnt corrupted images, one for input image, the other for target image. In this way, even for the same clean image, U-net will be fed different noise input and target data in different iteration.

3.4. Additive Gaussian Noise

In this section, we use additive gaussain noise to corrupt the clean image. The mean of the noise is 0. Every time we corrupt a clean image, we randomly choose a value from $[0, \alpha]$ as the standard deviation of the noise. In this experiment, we set α to 0.2.

The PSNR of the validation set is $23.88dB$. When trained using the standard way with clean targets, U-net can achieve $33.62dB$. When trained with noise targets, which is actually Noise2Noise method, U-net can achieve $33.60dB$. Noise2Noise only gives about $0.02dB$ worse results. Furthermore, as shown in Figure 1, the training converges as quickly as Noise2Clean method. Figure 2 shows some restored images using Noise2Noise and Noise2Clean method.

3.5. Noise2Noise Downsampling

The Noise2Noise downsampling is a downsampling method in Fourier domain. The pipeline of Noise2Noise downsampling is shown in Figure 3. In Fourier domain, each individual frequency has a probability $p(u, v) = e^{-\lambda\sqrt{u^2+v^2}}$ of being selected for acquisition. Here, we chose λ to save about 10% of the spectrum.

[1]http://brain-development.org/ixi-dataset/ $\rightarrow$ T1 images.

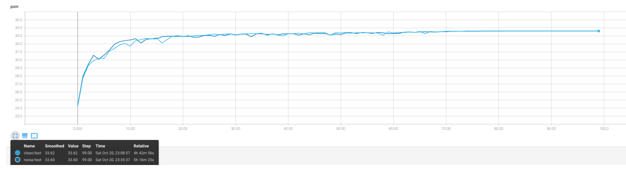

Figure 1. PSNR Curve with Additive Gaussian Noise

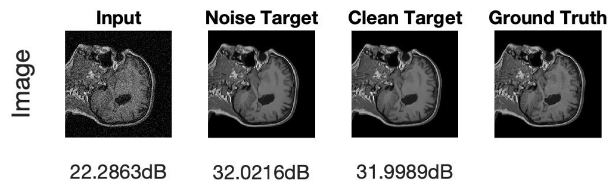

Figure 2. Example Results from Additive Gaussian Noise

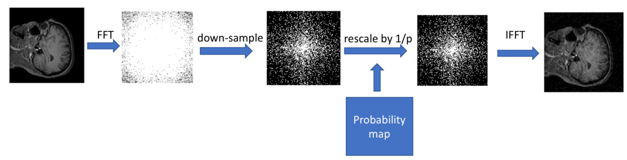

Figure 3. Noise2Noise Downsampling Pipeline

According to the probability $p(u, v)$, we get the value of pixel in probability map by computing the probability of beind seclected of the corresponding pixel in spectrum. After downsampling in Fourier domain, we rescaled the downsampled spectrum map by the probability map to make sure the expectation of the downsamped spectrum is equal to the spectrum of the clean image. And then, we use Inverse Fast Fourier Transform (IFFT) to reconstruct the corrupted image.

The PSNR of the validation set is $19.28dB$. When trained with clean targets, Noise2Clean method achieve $28.24dB$. When trained with noise targets, Noise2Noise method achieve $28.07dB$. Figure 4 shows the PSNR result of Noise2Clean and Noise2Noise in validation set and Figure 5 shows some restored images using Noise2Noise and Noise2Clean method. The first row of Figure 5 is the reconstructed image, the second row is the spectrum of the corresponding image in the first row.

3.6. Star Downsampling

The Star downsampling is another downsampling method in Fourier domain. The pipeline of Star downsampling is shown in Figure 6. In Fourier domain, we chose n

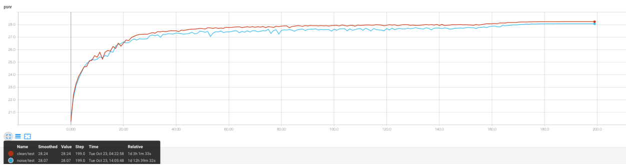

Figure 4. PSNR with Noise2Noise Downsampling

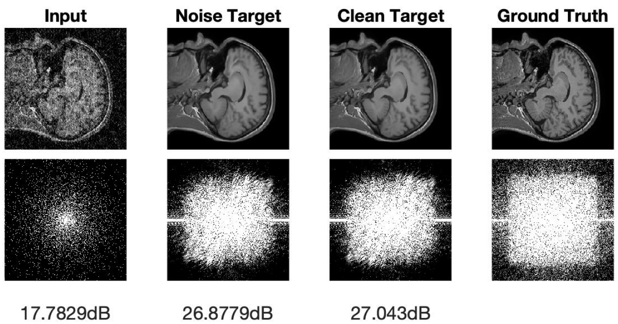

Figure 5. Example Results from Noise2Noise Downsampling

lines which all go though the center of the spectrum, and pixels on the lines will be selected for acquisition. Here, we chose 60 lines which save about 20.9% of the spectrum.

For computing the probability map of star downsampling, we set the center of the spectrum to be origin, and then compute the slope of all pixels in the first quadrant. We compute the angel between the adjacent lines which are decided by the sloped we computed, and assign a value according to the computed angle to the pixels which are on the lines. Then, we use systematic property to compute the value of other pixels. Through these processes, probability map P_1 for one line downsampling will be known. Probability map P_n for n lines downsampling can be computed by $1-(1-P_1)^n$.

As the same as in Noise2Noise downsampling, after star downsampling in Fourier domain, we rescaled the downsampled spectrum map and then use IFFT to reconstruct the corrupted image.

The PSNR of the validation set is $21.83dB$. When trained with clean target, Noise2Clean method achieve $29.03dB$. When trained with noise targets, Noise2Noise method achieve $28.88dB$. Figure 7 shows the PSNR result of Noise2Clean and Noise2Noise in validation set and Figure 8 shows some restored images using Noise2Noise and Noise2Clean method.

4. Discussion

We have shown that Noise2Noise method is able to reconstruct MRI image without observing clean data, without an explicit statistical characterization of the corruption, at

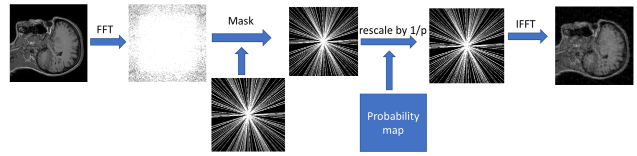

Figure 6. Star Downsampling Pipeline

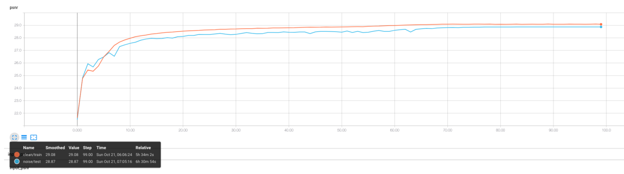

Figure 7. PSNR with Star Downsampling

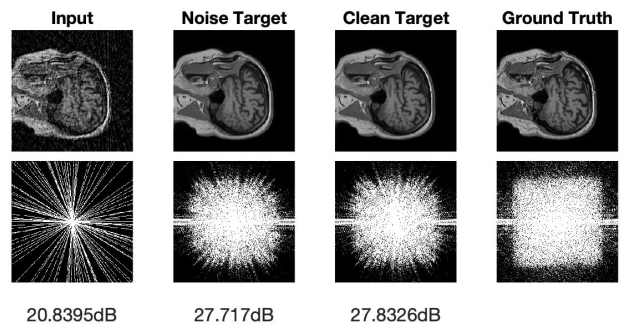

Figure 8. Example Results from Star Downsampling

performance levels equal or close to Noise2Clean method.

References

[1] J. Lehtinen, J. Munkberg, J. Hasselgren, S. Laine, T. Karras, M. Aittala, and T. Aila. Noise2Noise: Learning image restoration without clean data. In *Proceedings of the 35th International Conference on Machine Learning*, 2018.

[2] M. Lustig, D. L. Donoho, J. M. Santos, and J. M. Pauly. Compressed sensing mri. *IEEE signal processing magazine*, 25(2):72–82, 2008.

[3] O. Ronneberger, P. Fischer, and T. Brox. U-net: Convolutional networks for biomedical image segmentation. In *International Conference on Medical image computing and computer-assisted intervention*, pages 234–241. Springer, 2015.