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Exact and Approximate Modeling of Linear Systems

A Behavioral Approach

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Preface

The behavioral approach, put forward in the three part paper by J. C. Willems [Wil87], includes a rigorous framework for deriving mathematical models, a field called system identification. By the mid 80's there was a well developed stochastic theory for linear time-invariant system identification—the prediction error approach of L. Ljung—which has numerous "success stories". Nevertheless, the rationale for using the stochastic framework, the question of what is meant by an optimal (approximate) model, and even more basically what is meant by a mathematical model remained to some extent unclear.

A synergy of the classical stochastic framework (linear system driven by white noise) and a key result of [Wil87] that shows how a state sequence of the system can be obtained directly from observed data led to the very successful subspace identification methods [VD96]. Now the subspace methods together with the prediction error methods are the classical approaches for system identification.

Another follow-up of [Wil87] is the global total least squares approach due to Roorda and Heij. In a remarkable paper [RH95], Roorda and Heij address an approximate identification problem truly in the behavioral framework, i.e., in a representation-free setting. Their results lead to practical algorithms that are similar in structure to the prediction error methods: double minimization problems, of which the inner minimization is a smoothing problem and the outer minimization is a nonlinear least squares problem. Unfortunately, the global total least squares method has gained little attention in the system identification community and the algorithms of [RH95, Roo95] did not find their way to robust numerical implementation and consequently to practical applications.

The aim of this book is to present and popularize the behavioral approach to mathematical modeling among theoreticians and practitioners. The framework we adopt applies to static as well as dynamic and to linear as well as nonlinear problems. In the linear static case, the approximate modeling problem considered specializes to the total least squares method, which is classically viewed as a generalization of the least squares method to fitting problems $Ax \approx b$, in which there are errors in both the vector b and the matrix A. In the quadratic static case, the behavioral approach leads to the orthogonal regression method for fitting data to ellipses. In the first part of the book we examine static approximation problems: weighted and structured total least squares problems and estimation of bilinear and quadratic models, and in the second part of the book we examine dynamic approximation problems: exact and approximate system identification. The exact identification problem falls in the field of subspace identification and the approximate identification problem is the global total least squares problem of Roorda and Heij.

Most of the problems in the book are presented in a deterministic setting, although one can give a stochastic interpretation to the methods derived. The appropriate stochastic model for this aim is the errors-in-variables model, where all observed variables are assumed inexact due to measurement errors added on “true data” generated by a “true model”. The assumption of the existence of a true model and the additional stochastic ones about the measurement errors, however, are rarely verifiable in practice.

Except for the chapters on estimation of bilinear and quadratic models, we consider total least squares-type problems. The unifying framework for approximate modeling put forward in the book is called *misfit approach*. In philosophy it differs essentially from the classical approach, called *latency approach*, where the model is augmented with unobserved latent variables. A topic of current research is to clarify how the misfit and latency approaches compare and complement each other.

We do not treat in the book advanced topics like statistical and numerical robustness of the methods and algorithms. On the one hand, these topics are currently less developed in the misfit setting than in the latency setting and, on the other hand, they go beyond the scope of a short monograph. Our hope is that robustness as well as recursivity, further applications, and connections with other methods will be explored and presented elsewhere in the literature.

The prerequisites for reading the book are modest. We assume an undergraduate level linear algebra and systems theory knowledge. Familiarity with system identification is helpful but is not necessary. Sections with more specialized or technical material are marked with $*$. They can be skipped without loss of continuity on a first reading.

This book is accompanied by a software implementation of the described algorithms. The software is callable from MATLAB® and most of it is written in MATLAB code. This allows readers who have access to and knowledge of MATLAB to try out the examples, modify the simulation setting, and apply the methods on their own data.

The book is based on the first author’s Ph.D. thesis at the Department of Electrical Engineering of the Katholieke Universiteit Leuven, Belgium. This work would be impossible without the help of sponsoring organizations and individuals. We acknowledge the financial support received from the Research Council of K.U. Leuven and the Belgian Programme on Interuniversity Attraction Poles, projects IUAP IV–02 (1996–2001) and IUAP V–22 (2002–2006). The work presented in the first part of the book is done in collaboration with Alexander Kukush from the National Taras Shevchenko University, Kiev, Ukraine, and the work presented in the second part is done in collaboration with Paolo Rapisarda from the University of Maastricht, The Netherlands. We would like to thank Diana Sima and Rik Pintelon for useful discussions and proofreading the drafts of the manuscript.

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