Programming Assignment VI
Segment Tree

Due Date: 2023/12/05 (50) points

1 Description of Assignment

Let m and n be two large integers. Given an array of n integers $x_1, x_2, \ldots, x_n$. Design and implement efficient data structure and algorithms for a set of m queries. Each query takes input l_j and r_j and output $\max_{l_j \le i \le r_j} \{x_i\}$, for $j = 1, 2, \ldots, m$.

Note that each query can also be answered by finding the maximum elements in that interval without any additional data structure. However, when m and n are very large, the straightforward method may take a long time.

On the other hand, a table A of $n \times n$ can be constructed so that $A[l, r]$ contains $\max_{l \le i \le r}\{x_i\}$. By using table A, each query can be answered in $O(1)$ time. However, computing table A needs $O(n^2)$ time and the table takes $O(n^2)$ spaces.

There are two tasks in this assignment. First design a data structure with size $O(n)$ and an algorithm to construct the data structure in linear time. Then, design an $O(\log n)$ time algorithm to process each query using the data structure.

2 Input

The first part of the input is n and $x_1, x_2, \ldots, x_n$. The second part of the input is the m queries. Each query is specified by l_j and r_j, $j = 1, 2, \ldots, m$. Both parts of input data, except m and n, can be generated randomly.

3 Output

For each query l_j and r_j print out the value of the maximum element m_j in $\{x_{l_j}, x_{l_j+1}, \ldots, x_{r_j}\}$.

Compare the performance of your program with the straightforward method. The straightforward method should also be used to verify whether the answer of your program is correct or not.

Notes

The format of the report of the assignment should be close to the format of a research technical report. Incluse at least the following in your report.

1. Title and Author.
   This section should include assignment number, *your name*, *student number* and *email address* on the *first* page of your report.

2. Statement of the problem.
   A "formal" description of the problem in this assignment. In addition to the basic requirements specified in the assignment, highlight other functions or features that you have implemented.

3. Main results.
   This section should include at least the following items.

   (a) Description of the design of your program.

   (b) Describe the data structures used in the program to improve the efficiency of the program. These data structures should be implemented by you and appear in the first part of your program.

   (c) List of your program with comments.

      i. If your program is very long, list only the main parts of the program here and the entire program in appendix.

      ii. Additional comments can be added manually to explain the design of the program.

   (d) Outputs of the compilation and the executions of your program.

4. Conclusions
   Give a brief summary of what you did, and interesting thing you learned from this assignment.

Additional notes:

1. Turn in your report on or before due day.

2. The output of the program execution should indicate the correctness of your program. In other words, a set of comprehensive (but not necessarily exhaustive) annotated test data for the problem should be provided to show that your program is indeed correct. This can be done by carefully selecting a set of test data.

3. Print or write the report on A4 papers. Bind them together in the upper left corner.