Assignment 8

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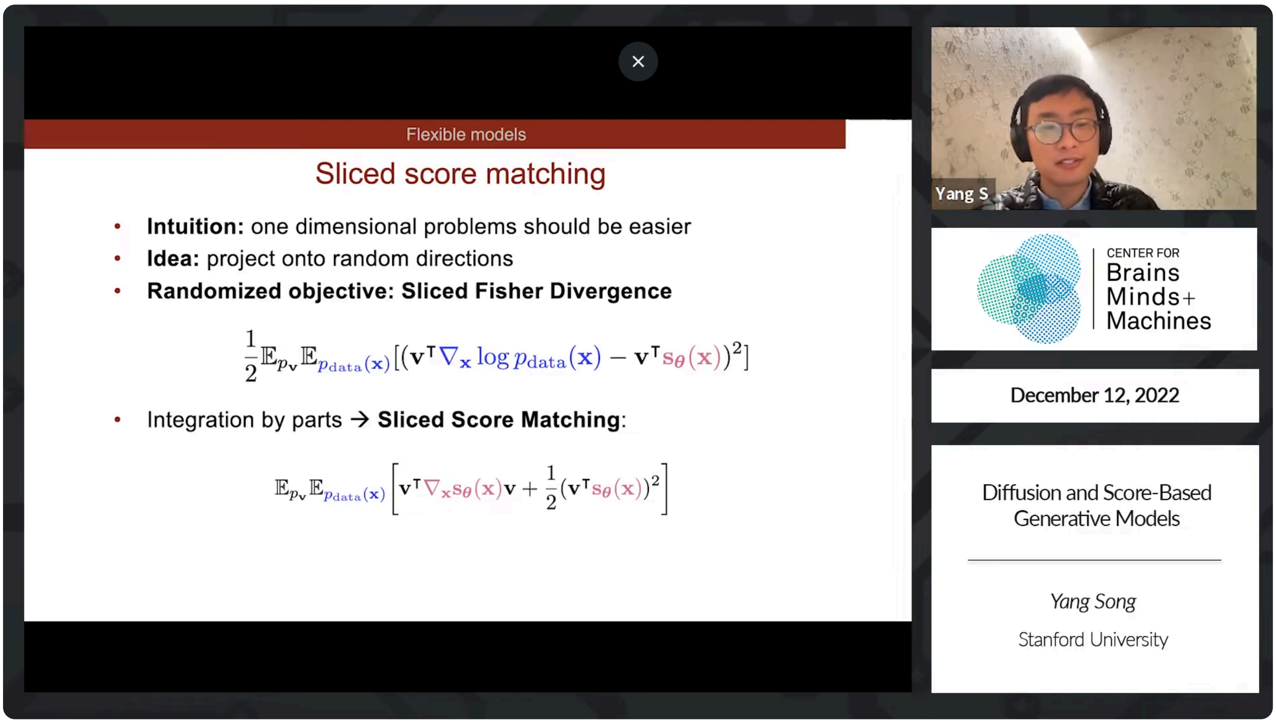
Written Assignment

Q1.



By sliced Fisher Divergence:

$$\mathbb{E}_{x\sim p(x)}\mathbb{E}_{v\sim p(v)}[(v^T\nabla_x log\, p(x) - v^T S(x;\theta))^2]$$

First,

$$\mathbb{E}_{x\sim p(x)}[(v^T S(x;\theta))(v^T\nabla_x log\, p(x))] = \int (v^T S(x;\theta))(v^T\nabla_x log\, p(x))p(x)\, dx$$

And,

$$\nabla_x log\, p(x) = \nabla_x(ln\, p(x)) = \frac{1}{p(x)}\nabla_x p(x)$$

we replace $\nabla_x log\, p(x)$ with $\frac{\nabla_x p(x)}{p(x)}$.
Then we have:

$$\mathbb{E}_{x\sim p(x)}[(v^T S(x;\theta))(v^T \nabla_x log\, p(x))] = \int (v^T S(x;\theta))(v^T \nabla_x p(x))\, dx$$

The following is integration by part:

$$\int (v^T S(x;\theta))(v^T \nabla_x p(x))\, dx = -\int p(x)(v^T \nabla_x (v^T S(x;\theta)))dx$$

So, we will have:

$$\begin{aligned} &\mathbb{E}_{x\sim p(x)}\mathbb{E}_{v\sim p(v)}[(v^T \nabla_x log\, p(x) - v^T S(x;\theta))^2] \\ &= \mathbb{E}_{x\sim p(x)}\mathbb{E}_{v\sim p(v)}[(v^T S(x;\theta))^2 - 2(v^T S(x;\theta))(v^T \nabla_x log\, p(x)) + (v^T \nabla_x log\, p(x))^2] \\ &= \mathbb{E}_{x\sim p(x)}\mathbb{E}_{v\sim p(v)}[\|v^T S(x;\theta)\|^2 + 2v^T \nabla_x (v^T S(x;\theta))] \end{aligned}$$

Since the last term $(v^T \nabla_x log\, p(x))^2$ is constant, we omit it while optimizing. Then we have this result.

Q2.

A stochastic differential equation is an ODE contaminated through a stochastic term. And it is of the form:

$$dX_t = f(X_t, t) + G(X_t, t)dW_t$$

where $f(X_t, t)$ is the drift(deterministic trend) and $G(X_t, t)dW_t$ is the diffusion term driven by Brownian motion W_t.

The main method of solution is to find the PDF as a function of time using the equivalent Fokker-Planck equation(FPE). Since FPE is a deterministic PDE. It tells how the PDF evolves in time.

Q3.

I cannot understand what is p mean in:

$$\frac{dp}{dt} = \frac{\sigma^2}{2} P_{xx}$$

Is it the probability at time t?

I don't know how to implement Fokker-Planck equation to solve SDE. What is the relationship between them?

And can we use FPE to solve the SDEs with different kinds of different PDFs?