Exercise sheet: Auto-diff

The following exercises have different levels of difficulty indicated by (*), (**), (***). An exercise with (*) is a simple exercise requiring less time or effort to solve compared to an exercise with (***), which is a more complex exercise.

Let $\mathbf{f}$ be a vector-valued function that maps from $\mathbb{R}^3$ to $\mathbb{R}^2$,

$$
y_1 = f_1(x_1, x_2, x_3) = x_1 x_3 + \log(x_2 + x_1) \times e^{-x_3}
$$
$$
y_2 = f_2(x_1, x_2, x_3) = e^{-x_2} + \cos(x_1 x_3).
$$

1. (*) Compute the Jacobian using manual differentiation and evaluate the Jacobian at the point ($x_1 = 3, x_2 = 5, x_3 = 1$)

2. (*) Compute the Jacobian at the same point that in the previous point, but using finite difference approximation.

3. (*) Draw the computational graph.

4. (**) Compute the Jacobian using AD in forward mode. Write the expressions for all the intermediate variables $\dot{v}_i$ in the forward tangent trace.

5. (**) Compute the Jacobian using AD in reverse mode. Write the expressions for all the adjoints $\bar{v}_i$ in the reverse derivative trace.