EVE: An Implementation of an Efficient Homomorphic Encryption Scheme on Integer Vectors

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Introduction to Homomorphic Encryption

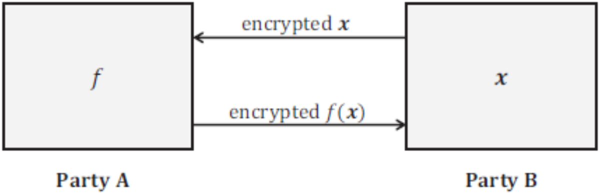


Figure: Most common usage of homomorphic encryption schemes

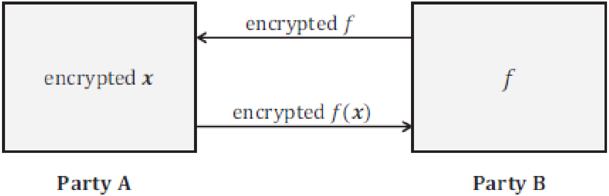


Figure: EVE, the homomorphic encryption scheme considered in our project

Overview of EVE: Supported Operations

Fundamental Operations:

- Encrypt: $\mathbf{c} = E_S(\mathbf{x})$
- Decrypt: $\mathbf{x} = D_S(\mathbf{c})$
- Switching secret keys from S to S'

Supported Operations on Integer Vectors:

- Addition of two vectors: $\mathbf{x}_1 + \mathbf{x}_2$
- Linear transformation: $G\mathbf{x}$
- Weighted inner product of two vectors: $\mathbf{x}_1^T H \mathbf{x}_2$

Can compose these operations to support arbitrary integer polynomials

Details of Fundamental Operations

Encrypt $\mathbf{x}$ with secret key S:

- Choose $\mathbf{c}$ such that $S\mathbf{c} = w\mathbf{x} + \mathbf{e}$

Decrypt $\mathbf{c}$ with S:

- $\mathbf{x} = \left\lceil \frac{S\mathbf{c}}{w} \right\rfloor$

Key switching from S to $S' = [I, T]$:

- Want $S'\mathbf{c}' = S\mathbf{c}$
- Want key-switch matrix M such that $S'M = S + E$
- $M = \begin{pmatrix} -TA + S + E \\ A \end{pmatrix}$ for random matrix A, random noise matrix E
- Then $\mathbf{c}' = M\mathbf{c}$

Details of the Three Supported Operations

Addition: $\mathbf{x}' = \mathbf{x}_1 + \mathbf{x}_2$:

- Let $\mathbf{c}' = \mathbf{c}_1 + \mathbf{c}_2$
- So $S\mathbf{c}' = w(\mathbf{x}_1 + \mathbf{x}_2) + (\mathbf{e}_1 + \mathbf{e}_2)$

Linear Transformation: $\mathbf{x}' = G\mathbf{x}$

- Note $GS\mathbf{c} = wG\mathbf{x} + G\mathbf{e}$
- Hence $E_S(\mathbf{x}) = E_{GS}(G\mathbf{x})$
- So $\mathbf{c}' = \mathbf{c}$ and decrypt with $S' = GS$

Details of the Three Supported Operations (con'd)

Weighted Inner Product: $\mathbf{x}' = \mathbf{x}_1^T H \mathbf{x}_2$

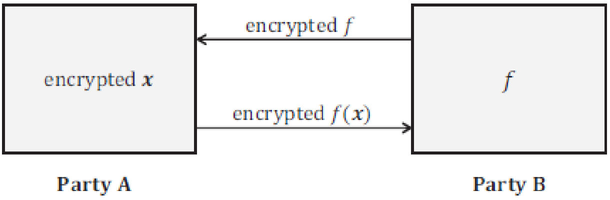
- Note $(S\mathbf{c}_1)^T H (S\mathbf{c}_2) = w^2(\mathbf{x}_1^T H \mathbf{x}_2) + \mathbf{e}''$
- Note $\mathbf{x}_1^T M \mathbf{x}_2 = \text{vec}(\mathbf{x}_1 M \mathbf{x}_2^T) \cdot \text{vec}(M)$ and choose $M = S^T H S$
- Let $S' = \text{vec}(S^T H S)$
- Let $\mathbf{c}' = \left\lceil \frac{\text{vec}(\mathbf{c}_1 \mathbf{c}_2^T)}{w} \right\rfloor$
- So $S'\mathbf{c}' = w(\mathbf{x}_1^T H \mathbf{x}_2) + \mathbf{e}'$

Secrecy in the Scheme

- Clients (Party B) calculate key-switch matrices based on operation f
- Server (Party A) performs the operation without knowing f
- Relies on key-switch matrices being indistinguishable from random

Learning with Error Problem (LWE)

Given S and M, solving $S'M = S + E$ to find S' is hard



Applications

- In each application, we have:
  - Server with data (can be encrypted with S, known to the client)
  - Client that wants to learn a function of the data
- Effective when server has a lot of data, and results are small

Applications

- Search
  - Server has (encrypted) feature vectors for our data
  - Client wants to score each item to rank them
- Classification
  - Can run any polynomial classifier on the server's data (e.g. naive bayes, SVMs with polynomial kernels)
- Feature extraction
  - We can generalize classification to give a low-dimensional representation of data vectors, which will conserve bandwidth over simply querying all the files.

Demo

- We implemented the scheme, and two applications:
  - Private search on encrypted data
    (using TF-IDF relevance on common words)
  - Spam classification (using a naive-bayes model)
- Server has encrypted word counts of 3500 common words for 200 Enron emails
- Server can't learn our queries, or even distinguish between each type

Live Demo!

Conclusions

- In our demo, scheme is slow due mainly to our lack of optimization.
- No overhead in addition
- Multiplicative overhead in linear transformations and inner products equal to the number of bits involved.
  - (Note as given, inner products is slow, but we can combine this with a linear transformation step to reduce the work.)
- This scheme compares well to fully homomorphic encryption by limiting scope of computation.
  - Benchmarks give that HElib achieves 500 multiplications per second for small integers, orders of magnitude worse than our slowdown.

Questions?

Summary of EVE (Any questions?)

Operations:

- Switching secret keys from S to S': $S'M = S + E$
- Addition of two vectors: $S(\mathbf{c}_1 + \mathbf{c}_2) \approx w(\mathbf{x}_1 + \mathbf{x}_2)$
- Linear transformation: $GS\mathbf{c} = wG\mathbf{x}$, $GS \to S'$
- Weighted inner product of two vectors: $\mathbf{c}_1^T S^T H S \mathbf{c}_2 \approx w^2 \mathbf{x}_1^T H \mathbf{x}_2$

Applications:

- Anonymous search
- Classification with hidden models
- Computing hidden polynomials of encrypted data