UNIVERSITY OF TORONTO
Faculty of Arts and Science

Midterm 1, Version 1
CSC165H1S

Date: Thursday February 9, 6:10-7:00pm
Duration: 50 minutes
Instructor(s): David Liu, Toniann Pitassi

No Aids Allowed

Name:

Student Number:

Please read the following guidelines carefully!

- **Please write your name on both the front and back of this exam**.
- This examination has **4** questions. There are a total of **8 pages, DOUBLE-SIDED**.
- Answer questions clearly and completely, with justifications unless explicitly asked not to.
- Unless stated otherwise, your formulas can use *only* the propositional connectives and quantifiers we have seen in class, arithmetic operators (like $+$, $\times$, and exponentiation), comparison operators (like $=$ and $>$), and the divisibility and $Prime$ predicates. You may not define your own sets or predicates unless asked to do so.
- All formulas must have negations applied directly to propositional variables or predicates (e.g., $\neg Prime(n)$). You do *not* need to show your work for computing negations.
- In your proofs, you may always use definitions of predicates. You may *not* use any external facts about rates of growth, divisibility, primes, or greatest common divisor unless you prove them, or they are given to you in the question.
- You may **not** use induction for your proofs on this midterm.

Take a deep breath.

This is your chance to show us

How much you've learned.

We **WANT** to give you the credit

That you've earned.

A number does not define you.

Good luck!

1. **[6 marks] Statements in logic.**

   (a) **[3 marks]** Write the truth table for the following formula. No rough work is required.

   $$\big((p \Leftrightarrow q) \land r\big) \Rightarrow \neg p$$

   Hint: use vacuous truth to quickly find some rows where the formula is true.

   (b) **[3 marks]** Consider the pair of statements:

   (1) $\forall n \in \mathbb{N},\ P(n) \land Q(n)$ (2) $\forall n \in \mathbb{N},\ P(n) \Rightarrow Q(n)$

   Define the predicates P and Q with domain $\mathbb{N}$ so that one of these statements is true and the other one false. Note that you're only defining the predicates *once*: the two statements must use the same definitions for P and Q.

   Also, briefly explain which statement is true and which one is false, and why; no formal proofs necessary.

2. **[7 marks] Translating statements**.

A **power of two** is a natural number that can be written in the form 2^n, where n is some natural number. A **Mersenne prime** is a natural number that is prime and one less than some power of two. For example, $2^2 - 1 = 3$ and $2^5 - 1 = 31$ are Mersenne primes.

Express each of the following statements using predicate logic. No justification is required.

Note: please review the instructions on the midterm's front page for our expectations in this question. In particular, you may *not* define any helper predicates or sets.

(a) 165165 is not a Mersenne prime.

Hint: it may be easier to think about how to express "165165 *is* a Mersenne prime" first.

(b) There are infinitely many Mersenne primes.

3. **[6 marks] Proofs (inequalities).** Consider the following statement: "There exists a positive real number y such that for every positive real number x, $x^2 + 165 < 2y$."

   (a) **[1 mark]** Translate the above statement into predicate logic. Use the symbol $\mathbb{R}^+$ to denote the set of positive real numbers.

   (b) **[1 mark]** Write the negation of this statement, fully-simplified so that all negation symbols are applied directly to predicates. You can simplify $\neg(a < b)$ to $a \geq b$.

   (c) **[4 marks]** Disprove the original statement by proving its negation. In your proof, any chains of calculations must follow a top-down order; don't start with the inequality you're trying to prove! Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be looked at if your proof is not completely correct.

Discussion.

Proof.

4. **[5 marks] Proofs (number theory).** Consider the following statement: "Every integer that is divisible by 100 is divisible by 2 and by 5."

   (a) **[1 mark]** Translate the above statement into predicate logic.

   (b) **[4 marks]** Prove the above statement using the definition of divisibility:

   $$x \mid y : \quad \exists k \in \mathbb{Z},\ y = kx$$

   Do not use any external facts about divisibility.
   Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be looked at if your proof is not completely correct.

Discussion.

Proof.

Use this page for rough work. If you want work on this page to be marked, please indicate this clearly *at the location of the original question.*

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Name:

Question	Grade	Out of
Q1		6
Q2		7
Q3		6
Q4		5
Total		24