UNIVERSITY OF TORONTO

Faculty of Arts and Science

Midterm 1, Version 2
CSC165H1S

Date: Friday February 10, 12:10-1:00pm
Duration: 50 minutes
Instructor(s): David Liu, Toniann Pitassi

No Aids Allowed

Name:

Student Number:

Please read the following guidelines carefully!

- **Please write your name on both the front and back of this exam**.
- This examination has **4** questions. There are a total of **8 pages, DOUBLE-SIDED**.
- Answer questions clearly and completely, with justifications unless explicitly asked not to.
- Unless stated otherwise, your formulas can use *only* the propositional connectives and quantifiers we have seen in class, arithmetic operators (like $+$, $\times$, and exponentiation), comparison operators (like $=$ and $>$), and the divisibility and $Prime$ predicates. You may not define your own sets or predicates unless asked to do so.
- All formulas must have negations applied directly to propositional variables or predicates (e.g., $\neg Prime(n)$). You do *not* need to show your work for computing negations.
- In your proofs, you may always use definitions of predicates. You may *not* use any external facts about rates of growth, divisibility, primes, or greatest common divisor unless you prove them, or they are given to you in the question.
- You may **not** use induction for your proofs on this midterm.

Take a deep breath.

This is your chance to show us
How much you've learned.
We **WANT** to give you the credit
That you've earned.
A number does not define you.

Good luck!

1. **[6 marks] Statements in logic.**

   (a) [**3 marks**] Write the truth table for the following formula. No rough work is required.

   $$\big((p \Leftrightarrow q) \land r\big) \Rightarrow \neg r$$

   Hint: use vacuous truth to quickly find some rows where the formula is true.

   (b) [**3 marks**] Consider the pair of statements:

   (1) $\forall n \in \mathbb{N},\ P(n) \Rightarrow Q(n)$ (2) $\forall n \in \mathbb{N},\ P(n) \Leftrightarrow Q(n)$

   Define the predicates P and Q with domain $\mathbb{N}$ so that one of these statements is true and the other one false. Note that you're only defining the predicates *once*: the two statements must use the same definitions for P and Q.

   Also, briefly explain which statement is true and which one is false, and why; no formal proofs necessary.

2. **[7 marks] Translating statements**.

   A **semiprime** is a natural number that can be written as the product of two (possibly equal) prime numbers. For example, $6 = 2 \cdot 3$ and $49 = 7 \cdot 7$ are semiprimes.

   Express each of the following statements using predicate logic. No justification is required.

   Note: please review the instructions on the midterm's front page for our expectations in this question. In particular, you may *not* define any helper predicates or sets.

   (a) There is a semiprime greater than 165.

   (b) There are no semiprimes.
   Hint: it may be easier to first express the negation of this statement.

3. **[6 marks] Proofs (inequalities).** Consider the following statement: "For every real number x greater than or equal to 3, $x^3 - x^2 - 10 > 2x$."

   (a) [**1 mark**] Translate the above statement into predicate logic.

   (b) [**5 marks**] Prove the above statement. In your proof, any chains of calculations must follow a top-down order; don't start with the inequality you're trying to prove!
   Hint: try starting with the expression $x^3 - x^2 - 2x$ and factor.
   Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be graded if your proof is not completely correct.

Discussion.

Proof.

4. **[5 marks] Proofs (number theory).** Consider the following statement: "There exists a positive integer x such that for every integer y, if $y \neq 0$ then $x \nmid x + y$."

   (a) **[1 mark]** Translate the above statement into predicate logic. Use $\mathbb{Z}^+$ to represent the set of positive integers, and $\mathbb{Z}$ to represent the set of integers.

   (b) **[1 mark]** Write the negation of this statement, fully-simplified so that all negation symbols are applied directly to predicates.

   (c) **[3 marks]** Disprove the original statement by proving its negation, using the definition of divisibility:

$$x \mid y : \quad \exists k \in \mathbb{Z},\ y = kx$$

   Do not use any external facts about divisibility.

   Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be looked at if your proof is not completely correct.

Discussion.

Proof.

Use this page for rough work. If you want work on this page to be marked, please indicate this clearly *at the location of the original question.*

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Name:

Question	Grade	Out of
Q1		6
Q2		7
Q3		6
Q4		5
Total		24