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1. find $\cos(0.15) = 0.7317$, $\begin{cases} x: & 0.698 & 0.733 & 0.768 & 0.803 \\ f(x): & 0.7661 & 0.7432 & 0.7193 & 0.6946 \end{cases}$, $f(x) = \cos(x)$,

(a) Degree 1: $P_1 = f(x_0)\frac{x-x_1}{x_0-x_1} + f(x_1)\frac{x-x_0}{x_1-x_0} = f(x_0)L_0(x) + f(x_1)L_1(x)$, 取 $\begin{cases} x_0 = 0.733 \\ x_1 = 0.768 \end{cases}$

$L_0(x) = \frac{x-0.768}{0.733-0.768} = \frac{x-0.768}{-0.035}$, $L_1(x) = \frac{x-0.733}{0.768-0.733} = \frac{x-0.733}{0.035}$

$f'(x) = -\sin(x)$ $f''(x) = -\cos x$
$f'''(x) = \sin(x)$ $f^{(4)}(x) = \cos x$

$P_1(0.15) = 0.7432 \cdot \frac{0.15-0.768}{-0.035} + 0.7193 \cdot \frac{0.15-0.733}{0.035} = 0.73159$ (計算機直接按) $\varepsilon_r = 0.73159 - 0.7317 = -1.1\times10^{-4}$ #

$|R_1(0.15)| \le \frac{|f^{(2)}(\xi)|}{2!} \cdot |0.15-0.733| \cdot |0.15-0.768| = \left|\max_{[x_0,x_1]} f^{(2)}(\xi)\right| \times \frac{1}{2} \times 0.017 \times 0.618 = |f^{(2)}(0.733)| \times 1.53\times10^{-4} = 1.137\times10^{-4}$ #

(b) Degree 2: $P_2(0.15) = \left(\frac{0.15-0.733}{0.698-0.733}\right)\left(\frac{0.15-0.768}{0.698-0.768}\right)(0.7661) + \left(\frac{0.15-0.698}{0.733-0.698}\right)\left(\frac{0.15-0.768}{0.733-0.768}\right)(0.7432) + \left(\frac{0.15-0.698}{0.768-0.698}\right)\left(\frac{0.15-0.733}{0.768-0.733}\right)(0.7193)$

$P_2(0.15) = 0.7661 \cdot \left(\frac{0.017\times-0.018}{(-0.035)(-0.070)}\right) + 0.7432\left(\frac{0.052\times-0.018}{0.035\times-0.035}\right) + 0.7193\left(\frac{-0.053\times0.017}{0.07\times0.035}\right) = 0.7317116$ (計算機) #

$\varepsilon_r \approx 0.7317116 - 0.7317 = 1.6\times10^{-5}$ $|R_2(0.15)| = \frac{|f^{(3)}(\xi)|}{3!} \cdot |(0.15-0.698)(0.15-0.733)(0.15-0.768)| = 1.84\times10^{-6}$ #

$\sin(0.768)$

(c) Degree 3:

$P_3(0.15) = 0.7661 \cdot L_0(x) + 0.7432 L_1(x) + 0.7193 L_2(x) + 0.6946 L_3(x)\Big|_{x=0.15}$

$L_0(0.15) = \frac{(0.15-0.733)(0.15-0.768)(0.15-0.803)}{(0.698-0.733)(0.698-0.768)(0.698-0.803)} = -0.063$ $P_3(0.15) = 0.7317104$ # $\varepsilon_r = 4\times10^{-6}$

$L_1(0.15) = \frac{(0.15-0.698)(0.15-0.768)(0.15-0.803)}{(0.733-0.698)(0.733-0.768)(0.733-0.803)} = 0.5185$ $|R_3(0.15)| = \frac{|f^{(4)}(\xi)|}{24} \times 0.052\times0.017\times0.018\times0.053$

$L_2(0.15) = \frac{(0.15-0.698)(0.15-0.733)(0.15-0.803)}{(0.768-0.698)(0.768-0.733)(0.768-0.803)} = 0.5463$ $= 2.19\times10^{-8}$ #

$L_3(0.15) = \frac{(0.15-0.698)(0.15-0.733)(0.15-0.768)}{(0.803-0.698)(0.803-0.733)(0.803-0.768)} = -0.0618$

(d) Degree 4: $P_4(x)$ & $R_4(x)$ don't exist since there are only 4 point

2.

x	0.3	0.4	0.5	0.6
e^{-x}	0.74818	0.67032	0.60631	0.54882

Iterated inverse interpolation find $x - e^{-x} = 0$

$\Rightarrow x = e^{-x} \Rightarrow y = x \Rightarrow \{(e^{-x_i}, x_i)\}$ ∵ $0.5 = 0.606$, $0.6 = 0.5488$ ∴ $0.54 < x^* < 0.6$

然 y 插出 $P(y)$：

y	猜 0.55	猜 0.56	猜 0.57	猜 0.562	猜 0.563	猜 0.565
$P(y)$	0.59784	0.579839	0.5621	0.576173	0.574496	0.570951

用牛顿插值法 (Lagrange 不同) ⇒

y	猜 0.566	猜 0.567	猜 0.5671	猜 0.56715
$P(y)$	0.56918	0.567148	0.567142	0.567154
		$\varepsilon_r \approx 10^{-4}$		$\varepsilon \approx 10^{-6}$

∵ 插值方程唯一

$x = 0.56715$ 時逼近 $x = e^{-x}$ #

3.

T	0	3	5	8	13
D	0	240	375	630	990
V	15	11	80	14	12

x ⇒ $f(x)$ ⇒ $f'(x)$

$\frac{f[x_1]-f[x_0]}{x_1-x_0} = \frac{630-375}{8-5} = \frac{245}{3}$ $\quad \frac{f[x_4]-f[x_3]}{13-8} = 14$

$\frac{\frac{245}{3}-80}{8-5} = \frac{5}{9}$ $\quad \frac{14-\frac{245}{3}}{8-5} = \frac{-23}{9}$ $\quad \frac{12-14}{13-8} = \frac{-2}{5}$

$\frac{-\frac{23}{9}-\frac{5}{9}}{8-5} = \frac{-28}{27}$ $\quad \frac{\frac{23}{72}+\frac{28}{72}}{13-5} = \frac{293}{1728}$ $\quad \frac{\frac{-2}{25}-\frac{23}{72}}{13-5} = \frac{719}{-14400}$

i	x_i	DD_0	DD_1	DD_2		
0	5	375				
1	5	375	80			
2	8	630	$\frac{245}{3}$	$\frac{5}{9}$		
3	8	630	14	$\frac{-23}{9}$	$\frac{-28}{27}$	
4	13	990	14	0	$\frac{23}{72}$	$\frac{293}{1728}$
5	13	990	12	$\frac{-2}{5}$	$\frac{-2}{25}$	$\frac{-719}{14400}$

-0.0274

$P_4(t) = 375 + 80(t-5) + \frac{5}{9}(t-5)^2 + \frac{-28}{27}(t-5)^2(t-8) +$

$+ \frac{293}{1728}(t-5)^2(t-8)^2 + -0.0274(t-5)^2(t-8)^2(t-13)$

$P_4(10) = 375 + 80\times5 + \frac{5}{9}\times5^2 + \frac{-28}{27}\times5^2\times2 + \frac{293}{1728}\times5^2\times2^2 + -0.0274\times5^2\times2^2\times-3 = 162.213$ #

$P_4'(t) = 80 + \frac{10}{9}(t-5) + \frac{-28}{27}[2(t-5)(t-8)+(t-5)^2] + \frac{293}{1728}[2(t-5)(t-8)^2+2(t-5)^2(t-8)] + (-0.0274)[2(t-5)(t-8)^2(t-13)+2(t-5)^2(t-8)(t-13)+(t-5)^2(t-8)^2]$

$P_4'(10) = 80 + \frac{10}{9}\times5 + \frac{-28}{27}[2(5)(2)+(5)^2] + \frac{293}{1728}[2(5)(2)^2+2(5)^2(2)] + (-0.0274)[2(5)(2)^2(-3)+2(5)^2(2)(-3)+5^2\times2^2]$

$= 80 + \frac{50}{9} + \frac{-28}{27}\times45 + \frac{293}{1728}\times140 + (-0.0274)\times-320 \approx 71.3953$ #

55 mile/h $= 55\times5280\div3600 = 80.67$ ft/s

$P_4'(t) = 80.67$ 根據計算機 $t = 5.0485$ or 1.0155 or 12.0434 ± 0.631，取 $t_{min} = 5.0485$

(c)

$V_{max} = P_4''(0)$ 太難算，since $t = 5.0485$ 第一次到 80.67 ft/s $\quad 5 \le t \le 8$

$P_4'\left(\frac{8+5}{2}\right) = P_4'(6.5) = 83.613$ $\quad P_4'\left(\frac{5+6.5}{2}\right) = P_4'(5.75) = 85.536$ $\quad P_4'\left(\frac{5.75+6.5}{2}\right) = P'(6.125) = 85.323$

$P_4'\left(\frac{6.125+5.75}{2}\right) = P_4'(5.9375) = 85.6178$ $\quad P_4'\left(\frac{5.75+5.9375}{2}\right) = P'(5.84375) = 85.618$ $\quad P_4'\left(\frac{5.84375+5.9375}{2}\right) = 85.635$

Ans:
(a) 162.213 ft
(b) 71.3953 ft/s
(b) 5.0485 s
(c) 85.635 ft/s

Welcome | newton_NumericalMethodHW2.2.py | lag_NumericalMethodHW2.2.py

lag_NumericalMethodHW2.2.py > ...

```python
# 拉格朗日插值法計算機

# 給定的數據點 (y_i, x_i)
y_data = [0.740818, 0.670320, 0.606531, 0.548812]  # y 值
x_data = [0.3, 0.4, 0.5, 0.6]                      # x 值

# 計算拉格朗日插值多項式 P(y)
def lagrange_interpolation(y_data, x_data, y_val):
    n = len(y_data)
    result = 0.0

    # 計算 P(y) = sum(x_i * L_i(y))
    for i in range(n):
        term = x_data[i]  # 從 x_i 開始
        for j in range(n):
            if j != i:
                term *= (y_val - y_data[j]) / (y_data[i] - y_data[j])
        result += term

    return result

# 主程式
def main():

    y_val = float(input("請輸入 y 值："))

    p_y = lagrange_interpolation(y_data, x_data, y_val)

    # 輸出結果
    print(f"P({y_val}) = {p_y:.6f}")

    # 檢查 |y - P(y)| 是否小於 10^-4
    diff = abs(y_val - p_y)
    if diff < 1e-4:
        print(f"提示：輸入值 y = {y_val:.6f} 與輸出值 P(y) = {p_y:.6f} 的差值 {diff:.6f} 小於 10^-4")

if __name__ == "__main__":
    main()
```

```
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.55
P(0.55) = 0.597840
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.56
P(0.56) = 0.579837
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
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PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.57
P(0.57) = 0.562141
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.562
P(0.562) = 0.576273
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.563
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.565
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.566& C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
  File "c:\Users\gunda\Downloads\cpptest\lag_NumericalMethodHW2.2.py", line 38, in <module>
    main()
    y_val = float(input("請輸入 y 值："))
            ^^^^^^^^^^^^^^^^^^^^^^^^^^^^
ValueError: could not convert string to float: '0.566& C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py'
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.566
P(0.566) = 0.569183
P(0.566) = 0.569183
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.567
P(0.567) = 0.567418
請輸入 y 值：0.567
P(0.567) = 0.567418
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.5671
P(0.5671) = 0.567242
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
P(0.567) = 0.567418
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.5671
P(0.5671) = 0.567242
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.5671
P(0.5671) = 0.567242
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
P(0.5671) = 0.567242
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/lag_NumericalMethodHW2.2.py
請輸入 y 值：0.56715
請輸入 y 值：0.56715
P(0.56715) = 0.567154
P(0.56715) = 0.567154
提示：輸入值 y = 0.567150 與輸出值 P(y) = 0.567154 的差值 0.000004 小於 10^-4
PS C:\Users\gunda\Downloads\cpptest>
```

```python
# 給定的數據點 (y_i, x_i)
y_data = [0.740818, 0.670320, 0.606531, 0.548812]  # y 值
x_data = [0.3, 0.4, 0.5, 0.6]                      # x 值

# 計算牛頓插值的差商表
def compute_divided_differences(y_data, x_data):
    n = len(y_data)
    # 初始化差商表
    f = [[0.0] * n for _ in range(n)]

    # 零階差商
    for i in range(n):
        f[i][0] = x_data[i]

    # 計算差商
    for j in range(1, n):
        for i in range(n - j):
            f[i][j] = (f[i + 1][j - 1] - f[i][j - 1]) / (y_data[i + j] - y_data[i])

    return f

# 使用牛頓插值計算 P(y)
def newton_interpolation(y_data, x_data, y_val):
    n = len(y_data)
    f = compute_divided_differences(y_data, x_data)

    # 牛頓插值公式
    result = f[0][0]  # 零階差商
    term = 1.0
    for i in range(1, n):
        term *= (y_val - y_data[i - 1])
        result += f[0][i] * term
    return result


def main():

    y_val = float(input("請輸入 y 值："))

    p_y = newton_interpolation(y_data, x_data, y_val)

    print(f"P({y_val}) = {p_y:.6f}")

    diff = abs(y_val - p_y)

if __name__ == "__main__":
    main()
```

```
loads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.55
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.56
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.57
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.562
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.563
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.565
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.566
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.567
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.5671
PS C:\Users\gunda\Downloads\cpptest> & C:/ProgramData/anaconda3/python.exe c:/Users/gunda/Downloads/cpptest/newton_NumericalMethodHW2.2.py
請輸入 y 值：0.56715
P(0.56715) = 0.567154
PS C:\Users\gunda\Downloads\cpptest>
```