

I. Vector and Matrix Algebra

- a. Operations on vectors: +, -, scaling
- b. Vector norms: L_1 and L_2 norm
- c. Cosine angle between two vectors, orthogonal vectors, orthonormal vectors
- d. Scalar/inner/dot product between two vectors
- e. Operations on matrices: matrix-vector multiplication, matrix-matrix multiplication, transposition
- f. Identity matrix, diagonal matrix, inverse matrix
- g. Eigenvalues and eigenvectors of a matrix

II. Calculus & Optimization

- a. Derivative, partial derivative, matrix calculus
- b. Gradient
- c. Hessian Matrix
- d. Least Squares

III. Probability and Statistics

- a. Definition of the probability function
- b. Discrete and continuous random variables
- c. Probability Density Function (PDF)
- d. Continuous and Discrete Variable Distributions: Uniform, Gaussian, Binomial, Bernoulli; their parameters (mean and variance)
- e. Expectation, variance, and their properties
- f. Joint probability, marginal, conditional probability, chain rule
- g. Independence and conditional independence
- h. Bayes rule/theorem
- i. Covariance, covariance matrix
- j. Maximum likelihood estimation (MLE)
- k. P-values, confidence intervals (CI), and Hypothesis testing

IV. Data Analysis, Data Mining, Machine Learning

- a. Linear regression: optimization function, ordinary least squares, underlying assumptions, equation, statistically significant coefficients
- b. Supervised vs unsupervised learning
- c. Classification and clustering: Naïve Bayes classifier, k-means clustering
- d. Performance metrics: contingency tables, accuracy, precision, recall, specificity, sensitivity, F1-measure
- e. Cross-validation: k-fold, leave-one-out
- f. Similarity, dissimilarity and distance measure: cosine, Euclidean, Jaccard
- g. Correlation: Pearson

V. Graphs and Graph Theory

- a. Definition of a graph
- b. Types of graphs: directed, undirected, labeled, unlabeled, attributed, bipartite, weighted, acyclic
- c. Paths, cycles
- d. Cliques
- e. Adjacency matrix