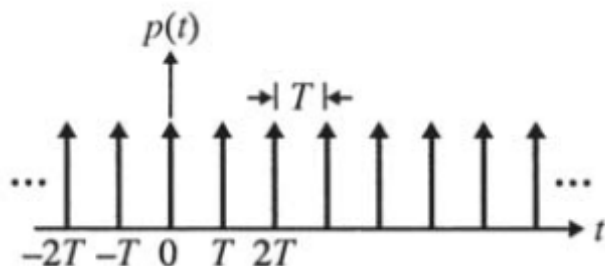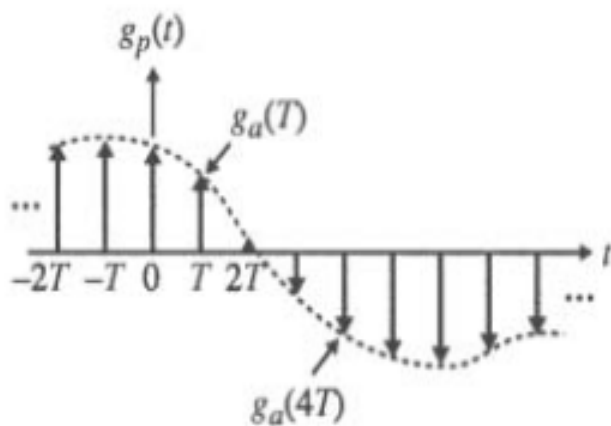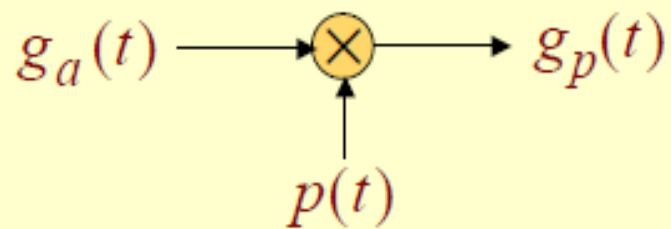# 1. 奈奎斯特采样定律

a.连续时间信号 $g_a(t)$:

b.周期冲激序列 $p(t)$:

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$g_p(t) = g_a(t)p(t) = \sum_{n=-\infty}^{\infty} g_a(nT)\delta(t-nT)$$

b.周期冲激序列p(t):

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$p(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j(2\pi/T)kT} = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j\Omega_T kt}$$
$$\text{where } \Omega_T = 2\pi/T$$

c.采样信号的连续傅氏变换CTFT
G$_P$(jΩ):

$$g_p(t) = g_a(t)p(t) = \sum_{n=-\infty}^{\infty} g_a(nT)\delta(t-nT)$$

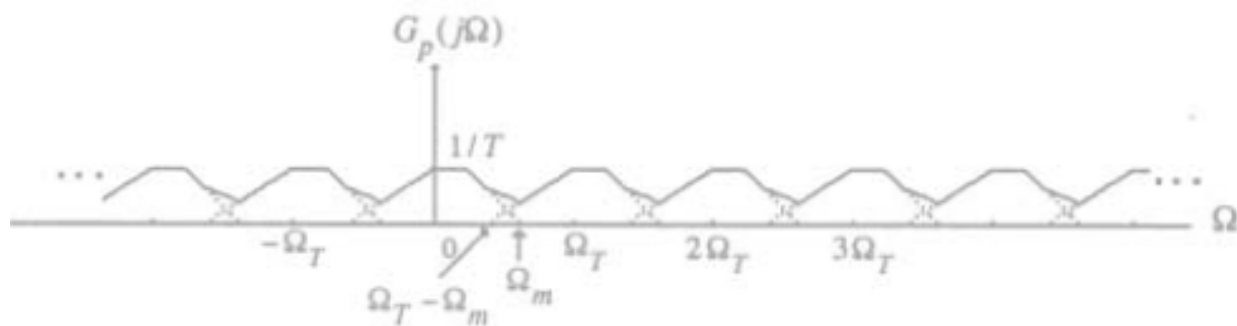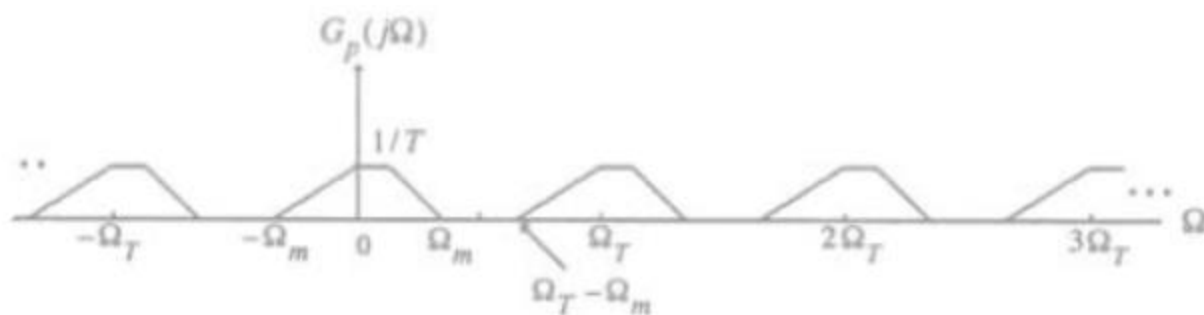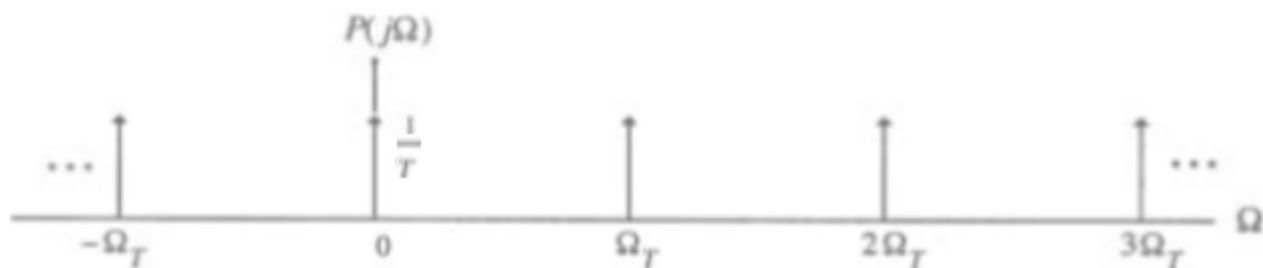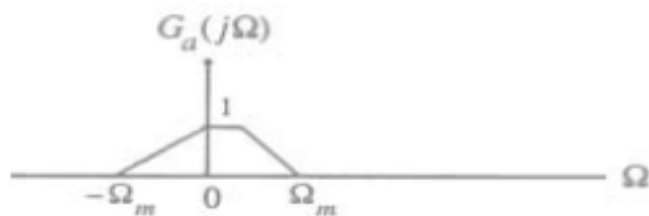$$g_p(t) = \left(\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j\Omega_T kt} \right) \cdot g_a(t)$$

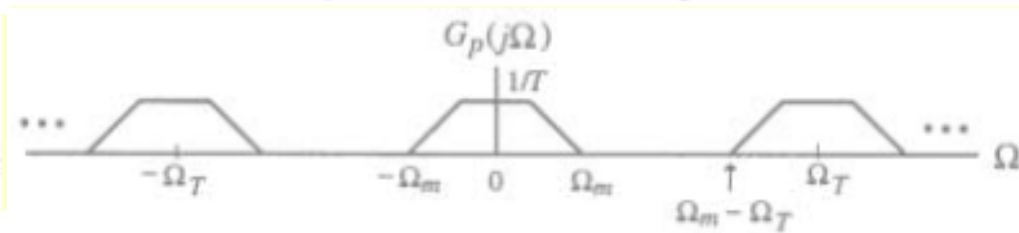$$e^{j\Omega_T kt} g_a(t) \rightarrow G_a\big(j(\Omega - k\Omega_T)\big)$$

$$G_p(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} G_a\big(j(\Omega - k\Omega_T)\big)$$

$$G_p(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty}G_a\left(j(\Omega - k\Omega_T)\right)$$

d.

$g_a(t) \longrightarrow \times \xrightarrow{\ g_p(t)\ } \boxed{H_r(j\Omega)} \longrightarrow \hat{g}_a(t)$

$p(t)$

$H_r(j\Omega)$

T

$\Omega_m < \Omega_c < (\Omega_T - \Omega_m)$

$-\Omega_c \qquad 0 \qquad \Omega_c$

Ω

$G_a(j\Omega)$

1

$-\Omega_m \quad 0 \quad \Omega_m$

Ω

$G_p(j\Omega)$

$1/T$

$\cdots$

$-\Omega_T \qquad -\Omega_m \quad 0 \quad \Omega_m \qquad \qquad \Omega_T \qquad \cdots$

$\Omega_m - \Omega_T$

Ω

$\hat{G}_a(j\Omega)$

1

$-\Omega_m \quad 0 \quad \Omega_m$

Ω
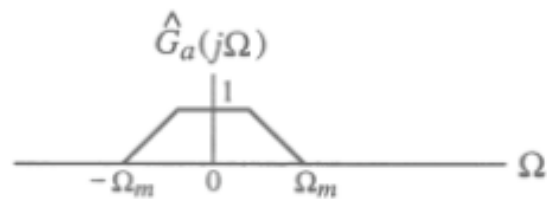
带阻连续时间信号

带宽：$\Delta\Omega = \Omega_H - \Omega_L$

假设最高频率满足条件：

$\Omega_H = M(\Delta\Omega)$
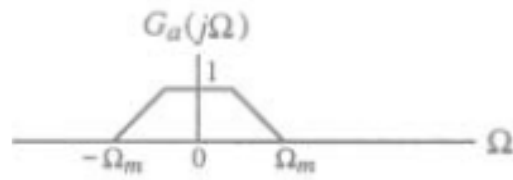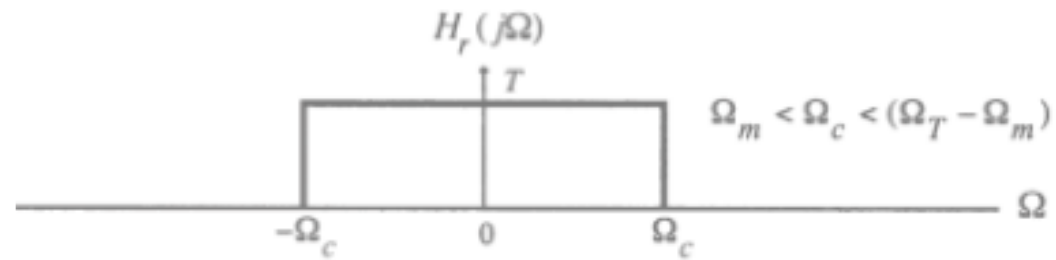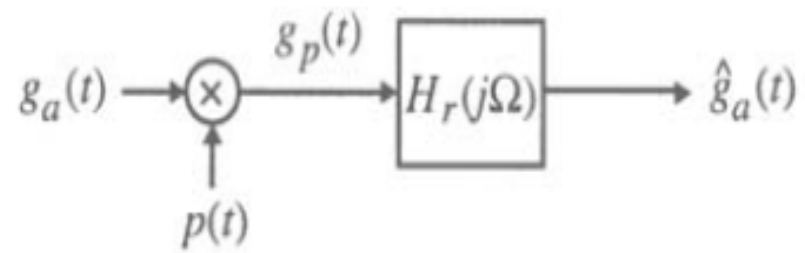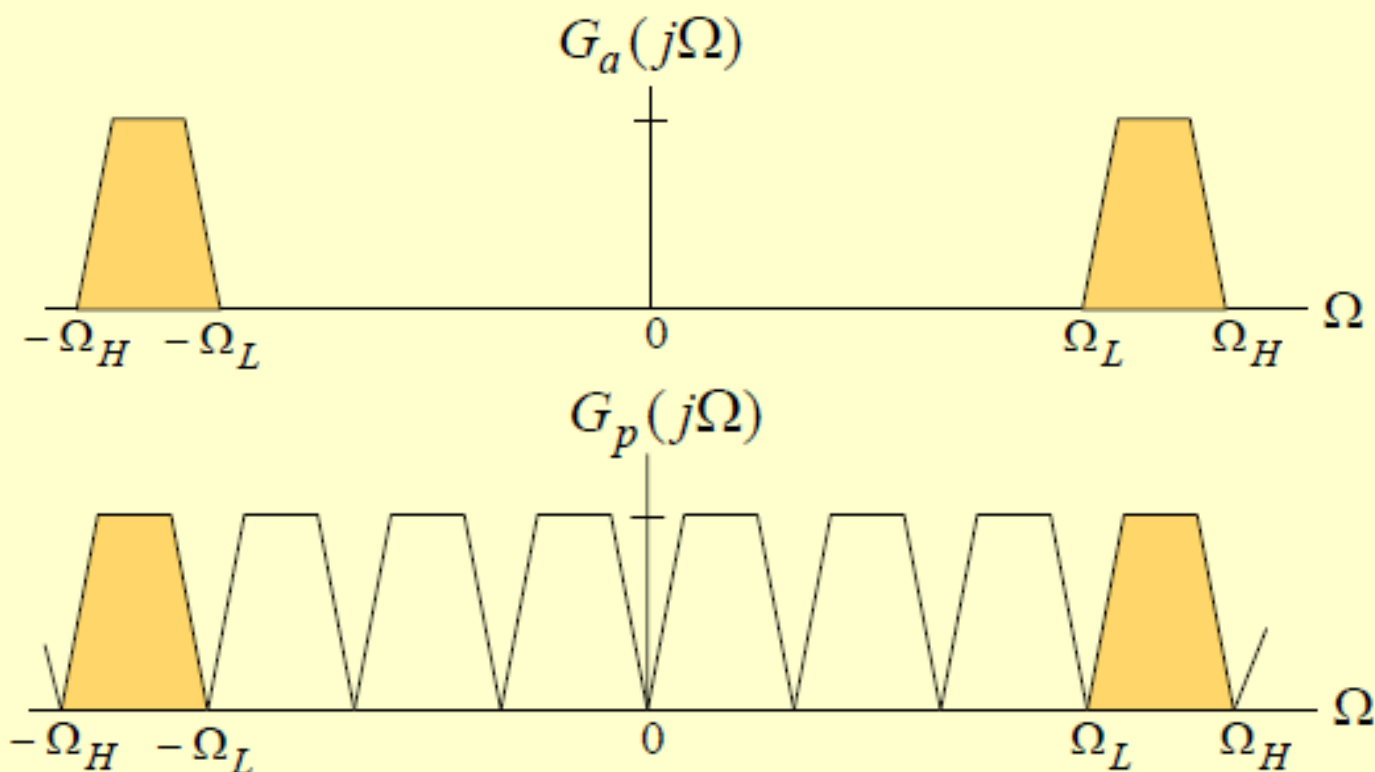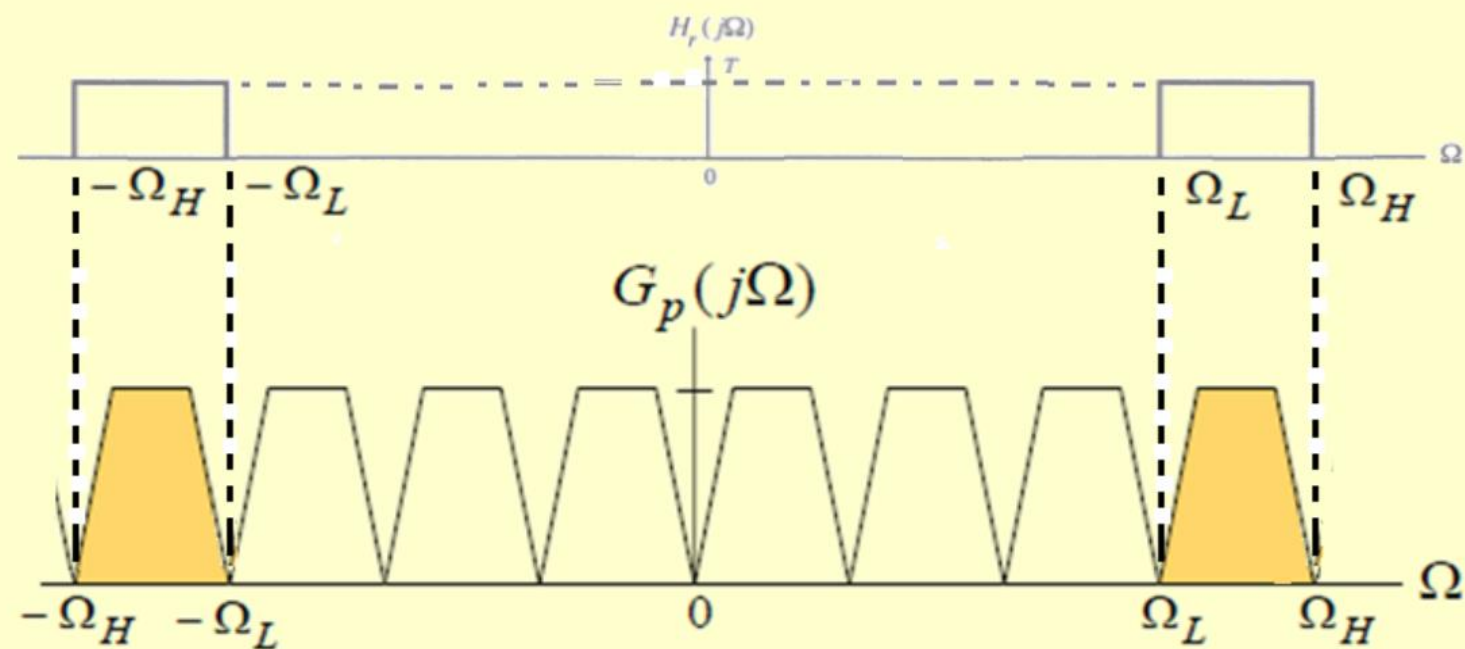
采样角频率为：
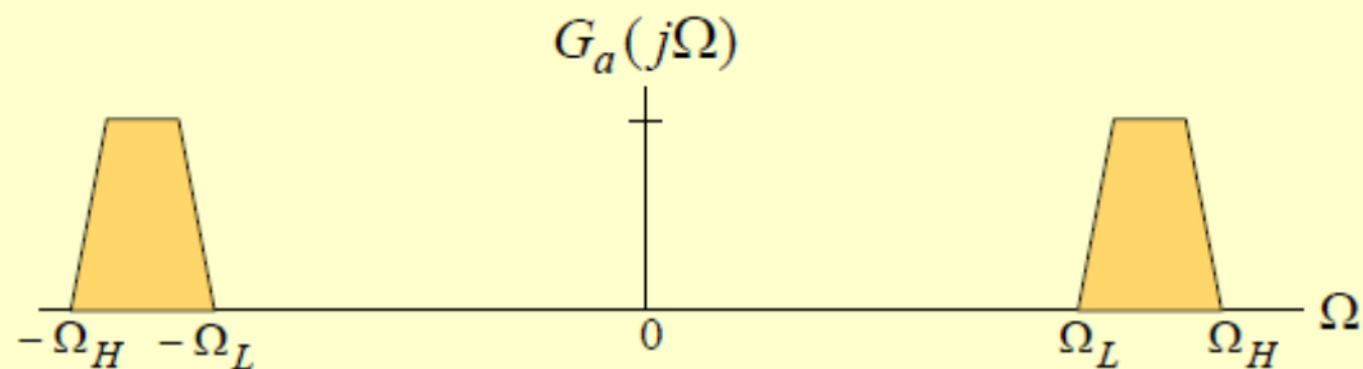
$\Omega_T = 2(\Delta\Omega) = \dfrac{2\Omega_H}{M}$

$$G_p(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} G_a\big(j(\Omega - k\Omega_T)\big)$$

$$G_p(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} G_a\big(j\Omega - j2k(\Delta\Omega)\big)$$
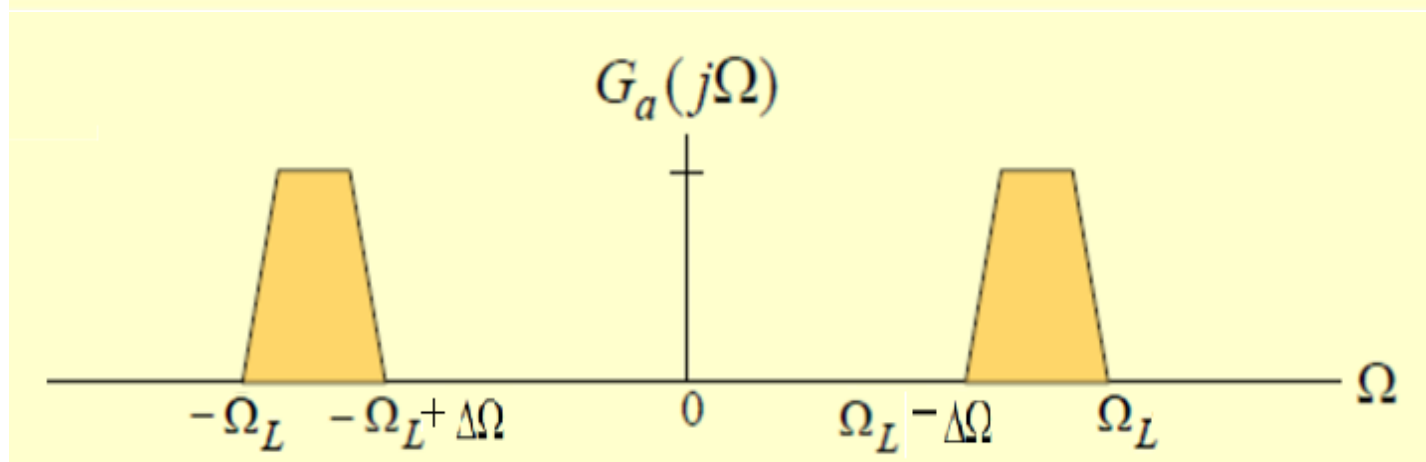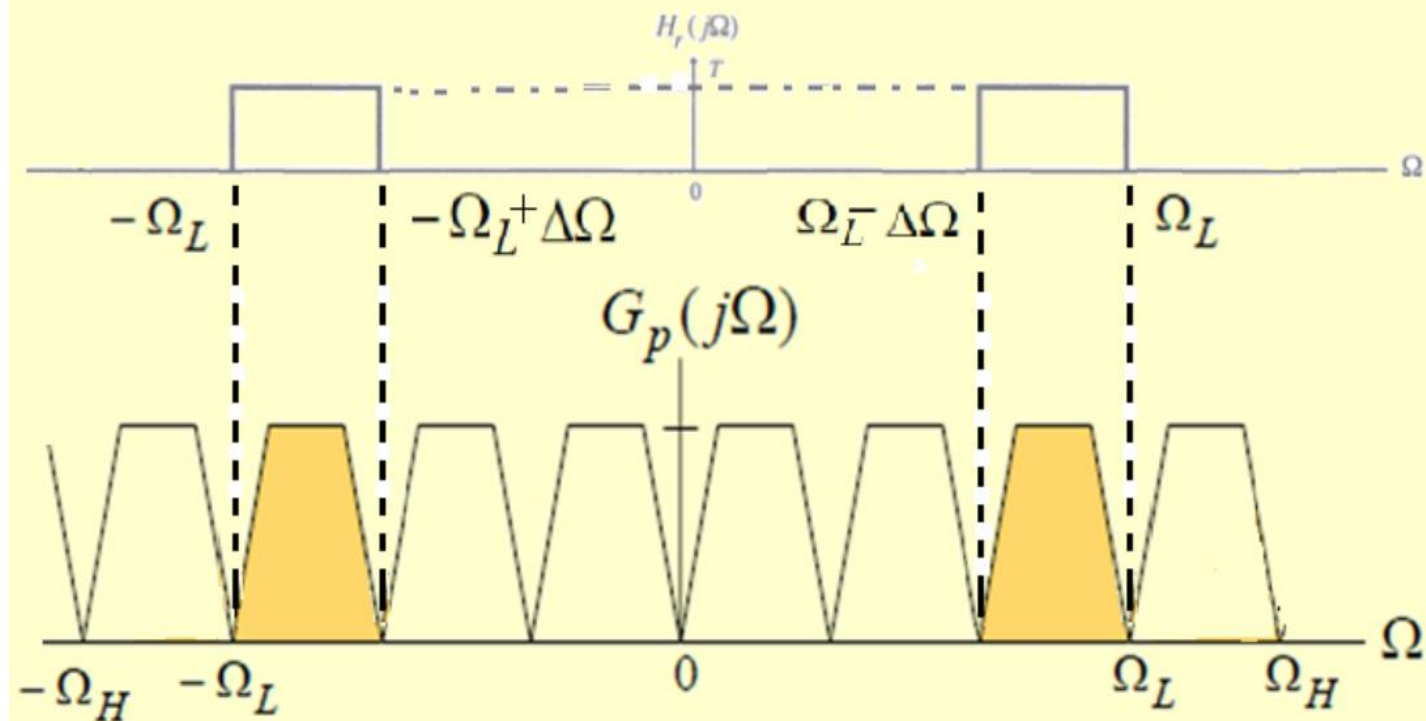
$$H_r(j\Omega) = \begin{cases} T, & \Omega_L \leq |\Omega| \leq \Omega_H \\ 0, & |\Omega| < \Omega_L \text{ or } \Omega_H < |\Omega| \end{cases}$$

获得低频频谱的带通滤波器：

$$H_r(j\Omega) = \begin{cases} T, & \Omega_L - k(\Delta\Omega) \leq |\Omega| \leq \Omega_H - k(\Delta\Omega) \\ & 1 \leq k \leq M-1 \\ 0, & |\Omega| < \Omega_L - k(\Delta\Omega) \\ & \text{or } \Omega_H - k(\Delta\Omega) < |\Omega| \end{cases}$$

$p(t) = \sum\limits_{n=-\infty}^{\infty} \delta(t - nT)$. Since p(t) is periodic function of time t with a period T, it can be

represented as a Fourier series: $p(t) = \sum\limits_{n=-\infty}^{\infty} c_n e^{j(2\pi nt/T)}$, where $c_n = \int\limits_{-T/2}^{T/2} \delta(t) e^{-j(2\pi nt/T)} dt = \dfrac{1}{T}$.

Hence $p(t) = \sum\limits_{n=-\infty}^{\infty} \delta(t - nT) = \dfrac{1}{T} \sum\limits_{n=-\infty}^{\infty} e^{j(2\pi nt/T)}$.