CHAPTER 1

Signals and Systems

Content

1.1 Definition and Classification of Signals

- Signals are functions with one or more independent variables that typically carry some type of information.

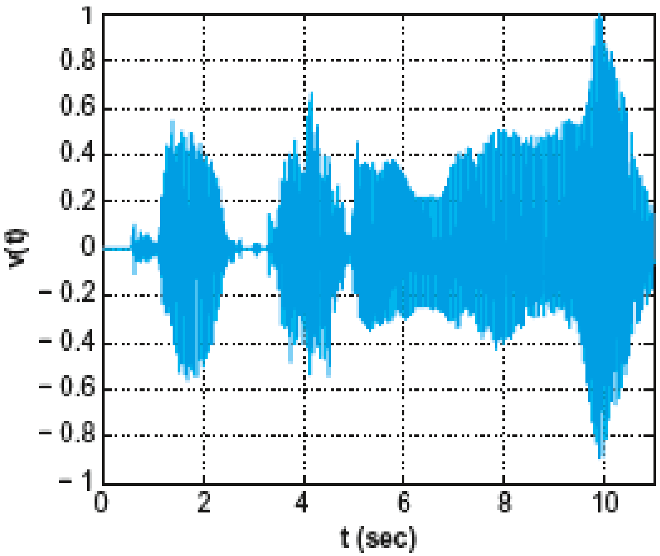
The smoke of wolves' dung burnt at border posts in ancient China to signal alarm

1.1 Definition and Classification of Signals

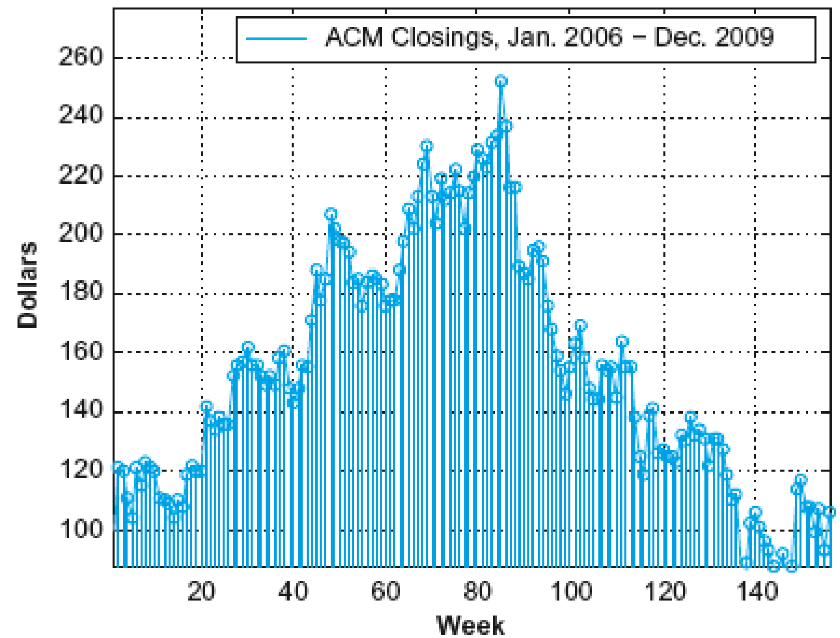
1.1.1 Continuous-time and Discrete-time Signals

（连续时间和离散时间信号）

- Continuous-time signals' independent variable is continuous : $x(t) = e^{-t}$
- Discrete-time signals are defined only at discrete times : $x[n] = 2n$



A segment of the voice signal



Weekly closings of ACM stock for 160 weeks in 2006-2009

Representing Signals Graphically

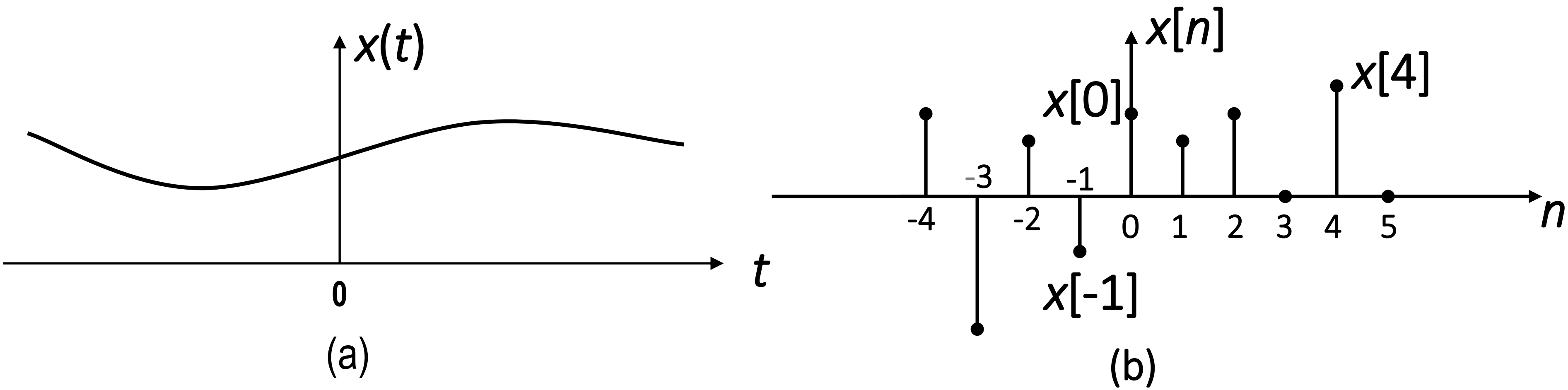


Figure 1.1 Graphical representations of
(a) continuous-time and (b) discrete-time signals

1.1.2 Periodic and Non-periodic Signals

（周期和非周期信号）

- If there is a positive value of T such that

$$x(t) = x(t + T)$$

for all values of t.

- If there is a positive integer of N such that

$$x[n] = x[n + N]$$

for all values of n.

We say that $x(t)$ ($x[n]$) is *periodic with period T(N)*.

Example 1.1 Determine the fundamental period of the signal

$$x(t) = 2cos(10\pi t + 1) - sin(4\pi t - 1).$$

Sol: Let $x_1(t)$=2cos(10πt+1), $x_2(t)$= sin(4πt-1)

T_1=1/5, T_2=1/2

What about the fundamental period T_x of $x(t)$?

Since both T_1 and T_2 are rational, they have the lowest common multiple (最小公倍数) , and LCM of T_1 and T_2 is 1, thus

T_x=1

If T_1 or T_2 is irrational, $x(t)$ is non-periodic. Because you can not find the LCM of T_1 and T_2. For example, $y(t) = 2cos(t+1)-sin(\pi t-1)$ is non-periodic.

However, if let $\pi \approx 3.14$, $y(t)$ is periodic. And since $\omega_1:\omega_2$=1:3.14 =50:157 =$m_1:m_2$, the fundamental period $T = m_1 T_1 = m_2 T_2$. Thus for $y(t)$,

T_y=50×2π=100π=314 or T_y=157×2=314 .

1.1.3 Determinate and Random Signals

（确定信号和随机信号）

- A determinate signal — $x(t) = e^{-t}\cos(\pi t)$
- A random signal — cannot find a function to represent it

1.1.4 Energy and Power Signals
（能量信号和功率信号）

The *instantaneous power* is

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

The *total energy* over time interval (t_1,t_2) is

$$\int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

The *average power* in this interval is

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

1.1 Definition and Classification of Signals

Generally, the total energy over the time interval $t_1 \le t \le t_2$ in a continuous-time signal: $\int_{t_1}^{t_2} |x(t)|^2 \, dt$

Similarly, the total energy in a discrete-time signal *x*[*n*] over the time interval $n_1 \le n \le n_2$: $\sum_{n_1}^{n_2} |x[n]|^2$

The total energy over an infinite interval is,

in continuous time,

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

in discrete time,

$$E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{-\infty}^{\infty} |x[n]|^2$$

The time-average power over an infinite interval:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

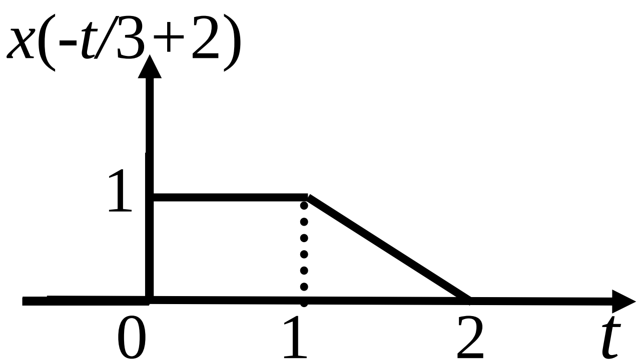
in continuous time and discrete time, respectively. With these definitions, we can identify three important classes of signals:

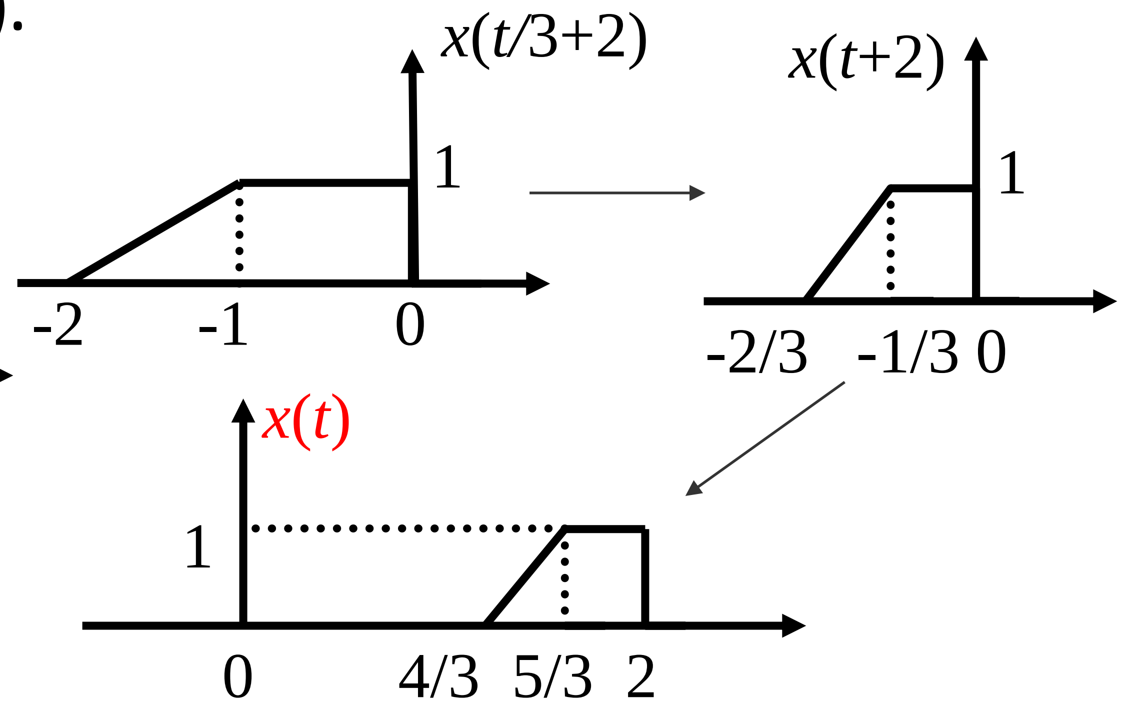
- Energy signals (Energy finite signals)
- Power signals (Power finite signals)
- Signals with neither finite total energy nor finite average power

1.2 Transformations of the Independent Variable of Signals

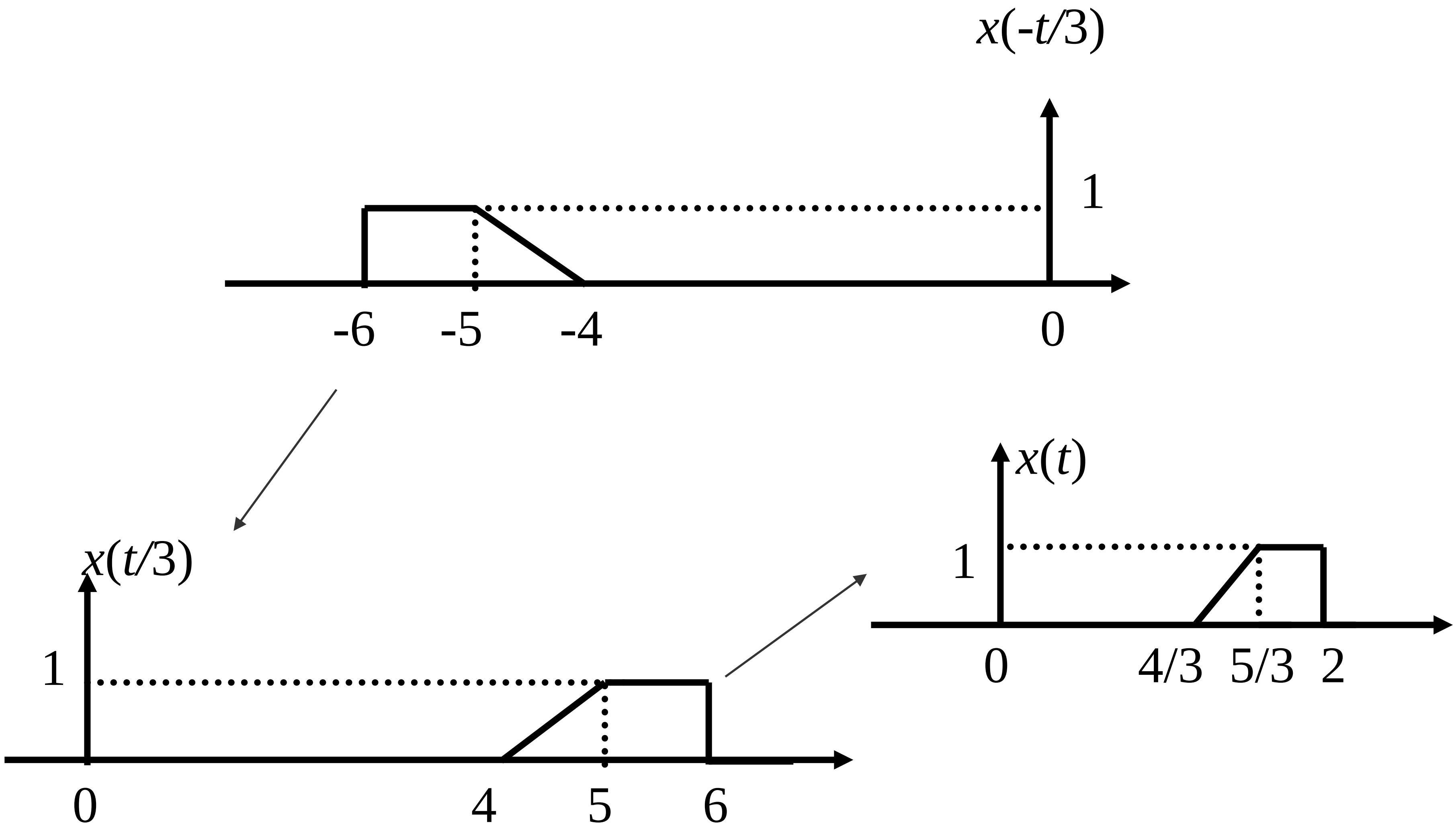
- Time shift (时移): $x(t-t_0)$ or $x[n-n_0]$ (n_0 is an integer)
- Time reversal (反褶) : $x(-t)$ or $x[-n]$
- Time scaling (尺度变换): $x(at)$ or $x[an]$ (The factor a is real)

Example 1.2 Signal $x\left(-\frac{t}{3}+2\right)$ is shown in the following figure, draw the graph of $x(t)$.





1.2 Transformations of the Independent Variable of Signals



- Even and Odd Signals, Even-odd Decomposition of a Signal

$$x(t) = x_e(t) + x_o(t)$$

even part of $x(t)$ *odd part* of $x(t)$

Since $x(-t) = x_e(-t) + x_o(-t) = x_e(t) - x_o(t)$

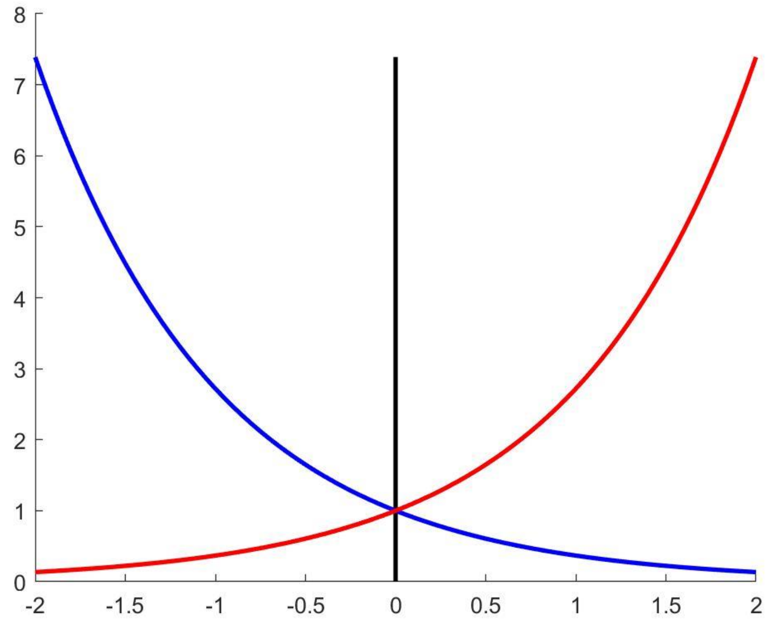
$$x_e(t) = Ev\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x_o(t) = Od\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$

1.3.1 Continuous-time Complex Exponential （复指数函数） and Sinusoidal Signals（正弦信号）

General form of *complex exponential signals*: $x(t) = Ce^{\alpha t}$

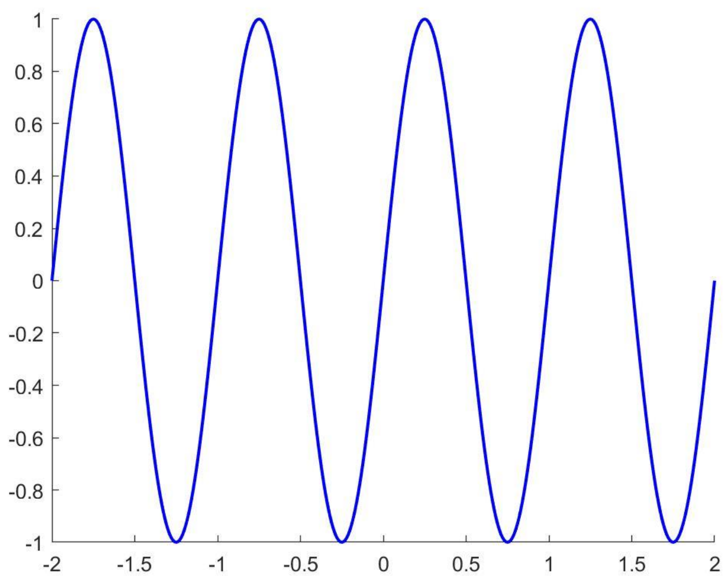
- Real Exponential Signals: Both C and α real



1.3.1 Continuous-time Complex Exponential （复指数函数） and Sinusoidal Signals（正弦信号）

General form of *complex exponential signals*: $x(t) = Ce^{\alpha t}$

- Periodic Complex Exponential: C real and α purely imaginary
  periodic complex exponential $x(t) = Ce^{j\omega_0 t}$



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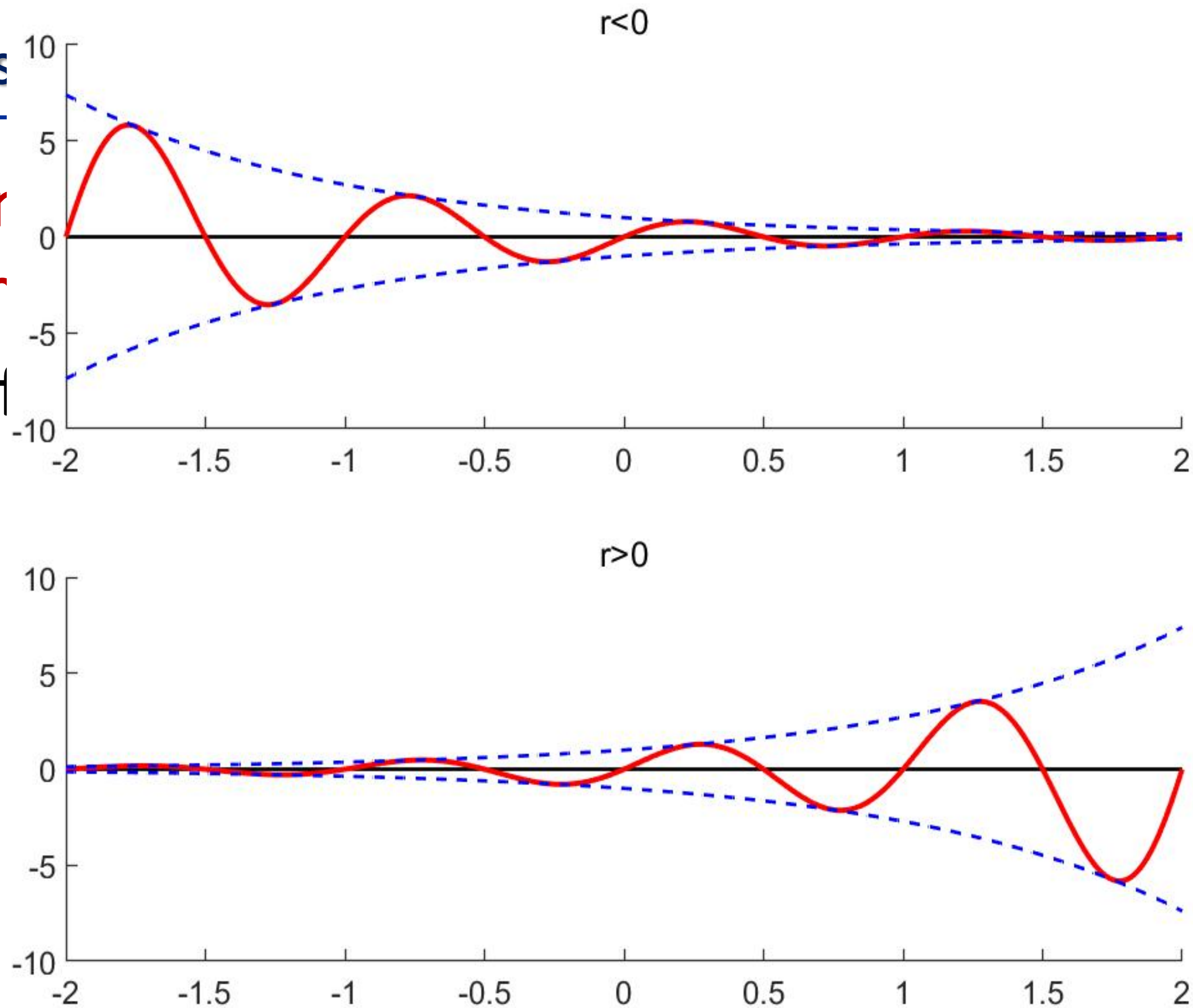
指数函数）

General f $= Ce^{\alpha t}$



- General Complex Exponential Signals: Both *C* and *α* are complex numbers

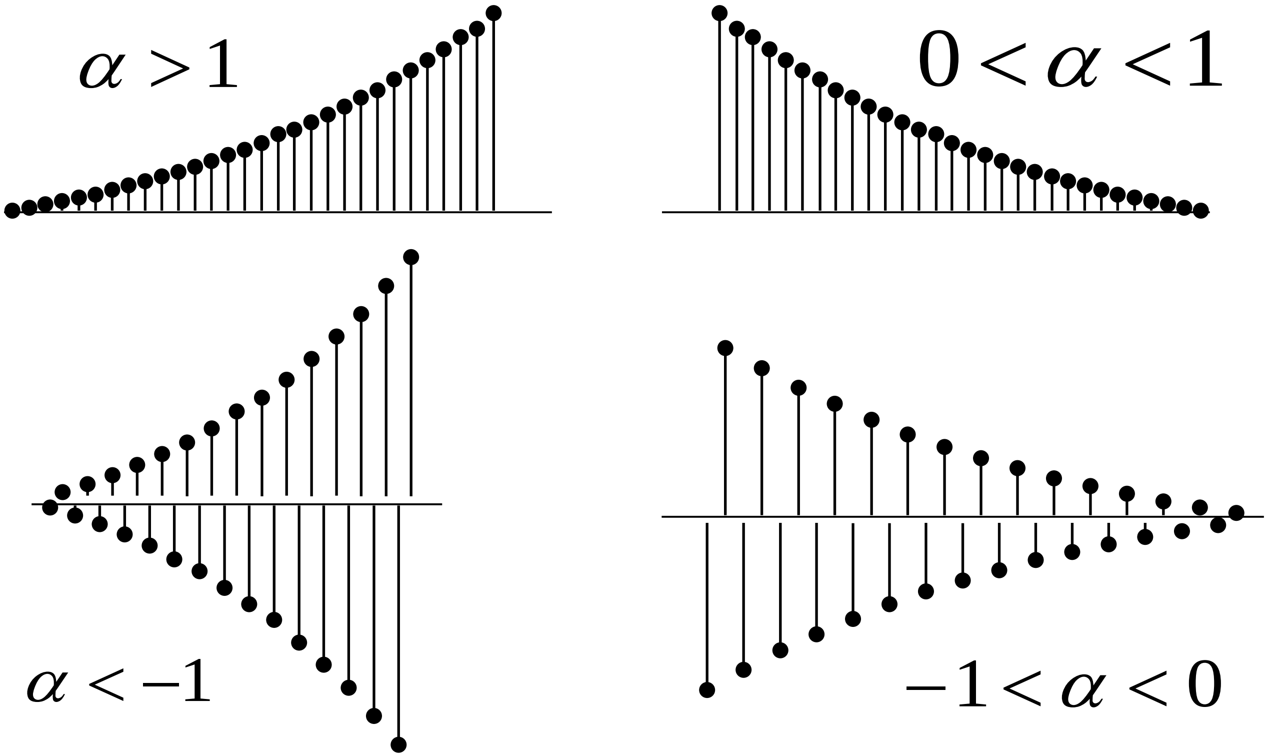
$$C = |C|e^{j\theta}, a = r + j\omega_0$$

$$Ce^{at} = |C|e^{rt}\cos(\omega_0 t + \theta) + j|C|e^{rt}\sin(\omega_0 t + \theta)$$

1.3.2 Discrete-Time Complex Exponential and Sinusoidal signals(*sequences*)（序列）：

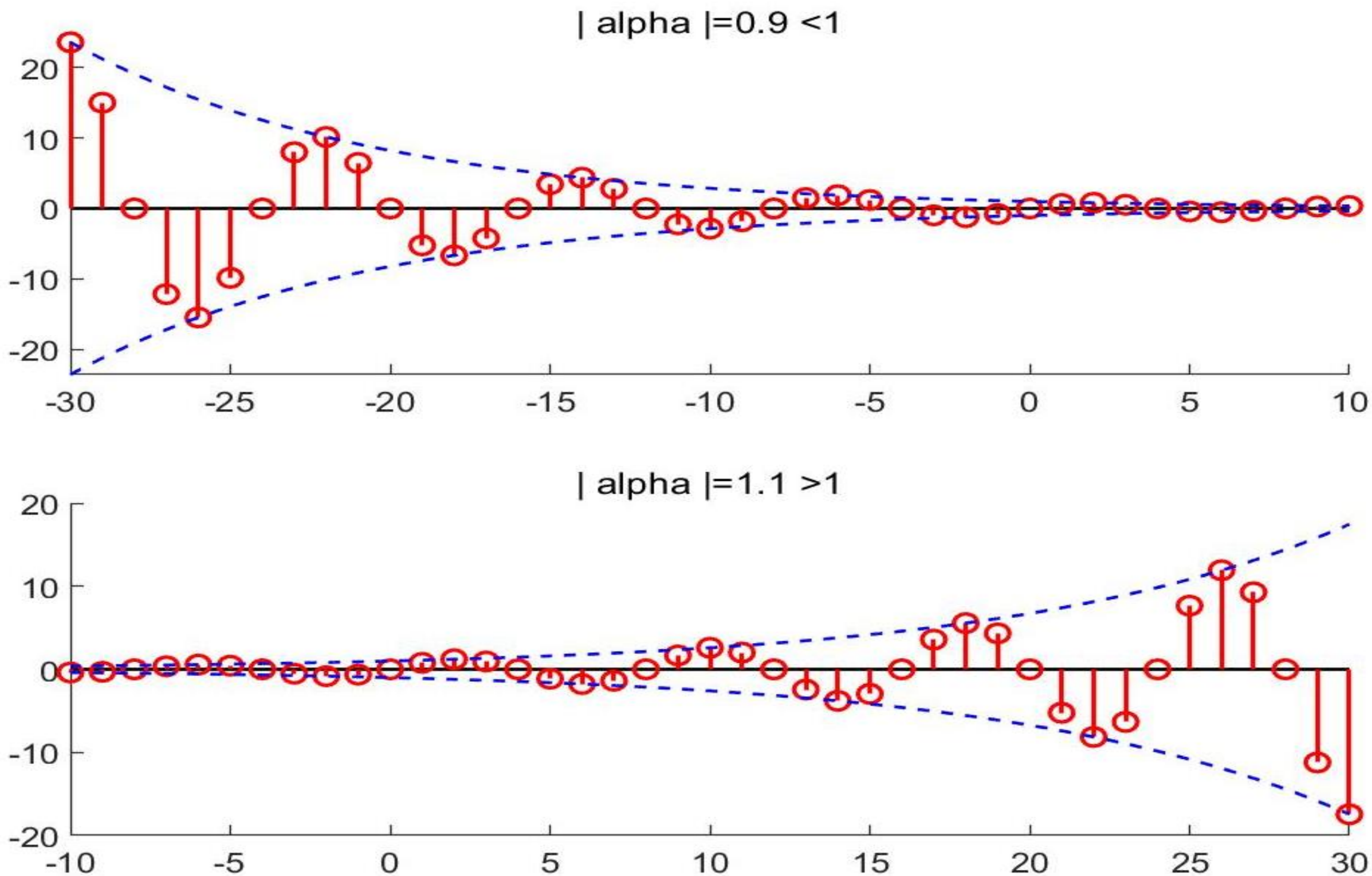
$$x[n] = C\alpha^n$$

- Real Exponential Signals: Both C and α are real



- General Complex Exponential Signals: both C and α are complex numbers

$$C\alpha^{n} = |C||\alpha|^{n}\cos(\omega_0 n+\theta) + j|C||\alpha|^{n}\sin(\omega_0 n+\theta)$$



1.3.3 Periodicity Property of Discrete-Time Complex Exponentials（离散正弦序列的周期性）

A discrete-time sinusoidal signal

$$x[n]=\sin[\omega_0 n]=\sin[\omega_0 n+2\pi m]=\sin\left[\omega_0\left(n+\frac{2\pi m}{\omega_0}\right)\right]$$

Three cases for the periodicity of the sequence:

- $\frac{2\pi}{\omega_0}$ is an integer, the fundamental period $N=\frac{2\pi}{\omega_0}$
- $\frac{2\pi}{\omega_0}=\frac{N}{M}$ is a rational number, the fundamental period

$$N=\frac{2\pi}{\omega_0}M$$

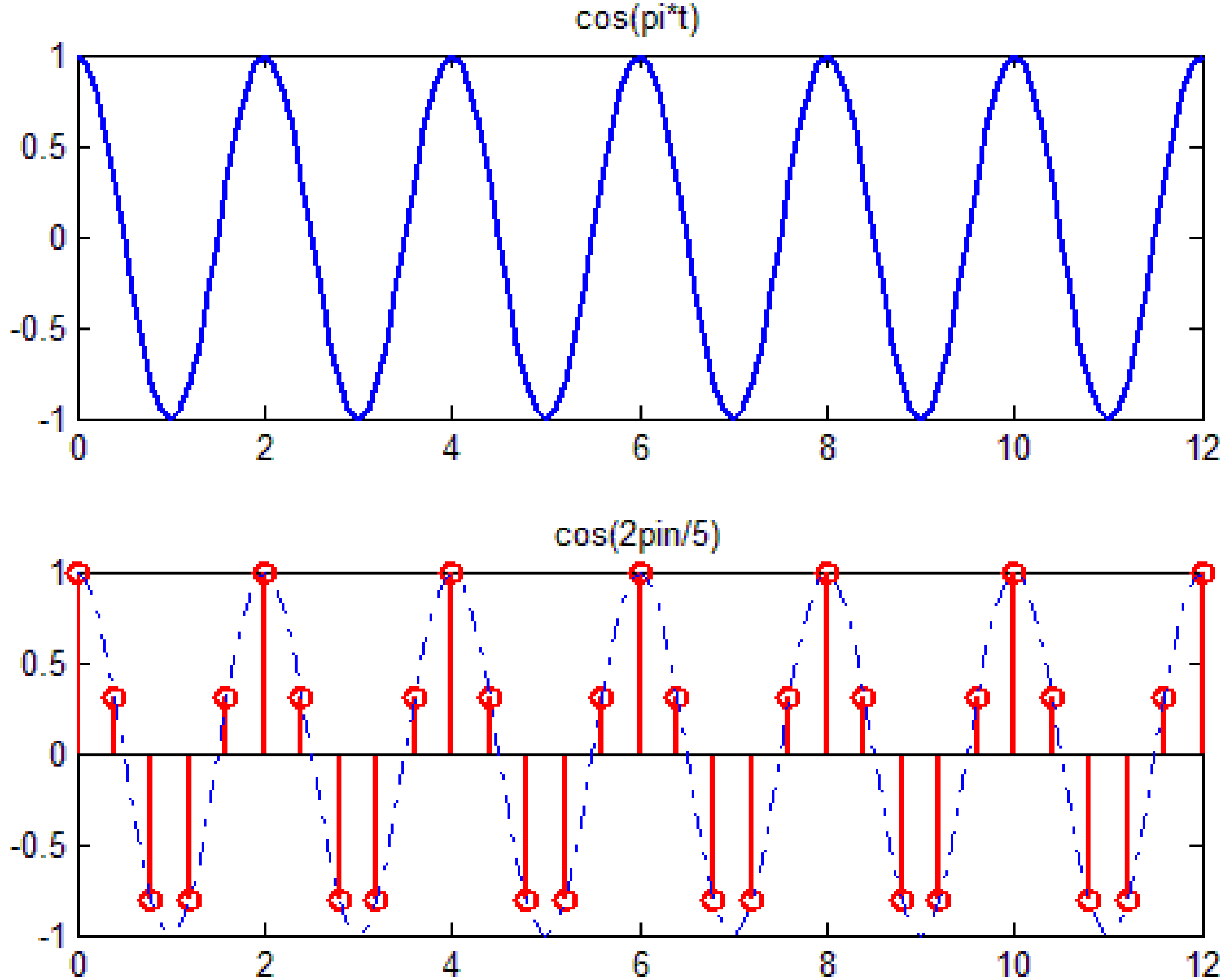
- $\frac{2\pi}{\omega_0}$ is an irrational number, the sequence is not periodic.

Sampling cos(πt) with Different Periods Leading to Different Results

Sampling cos(πt) with period $T_s = 0.4$ s produces

$$\cos\left(\pi \cdot 0.4n\right) = \cos\left(\frac{2\pi}{5}n\right)$$

$N = 5$

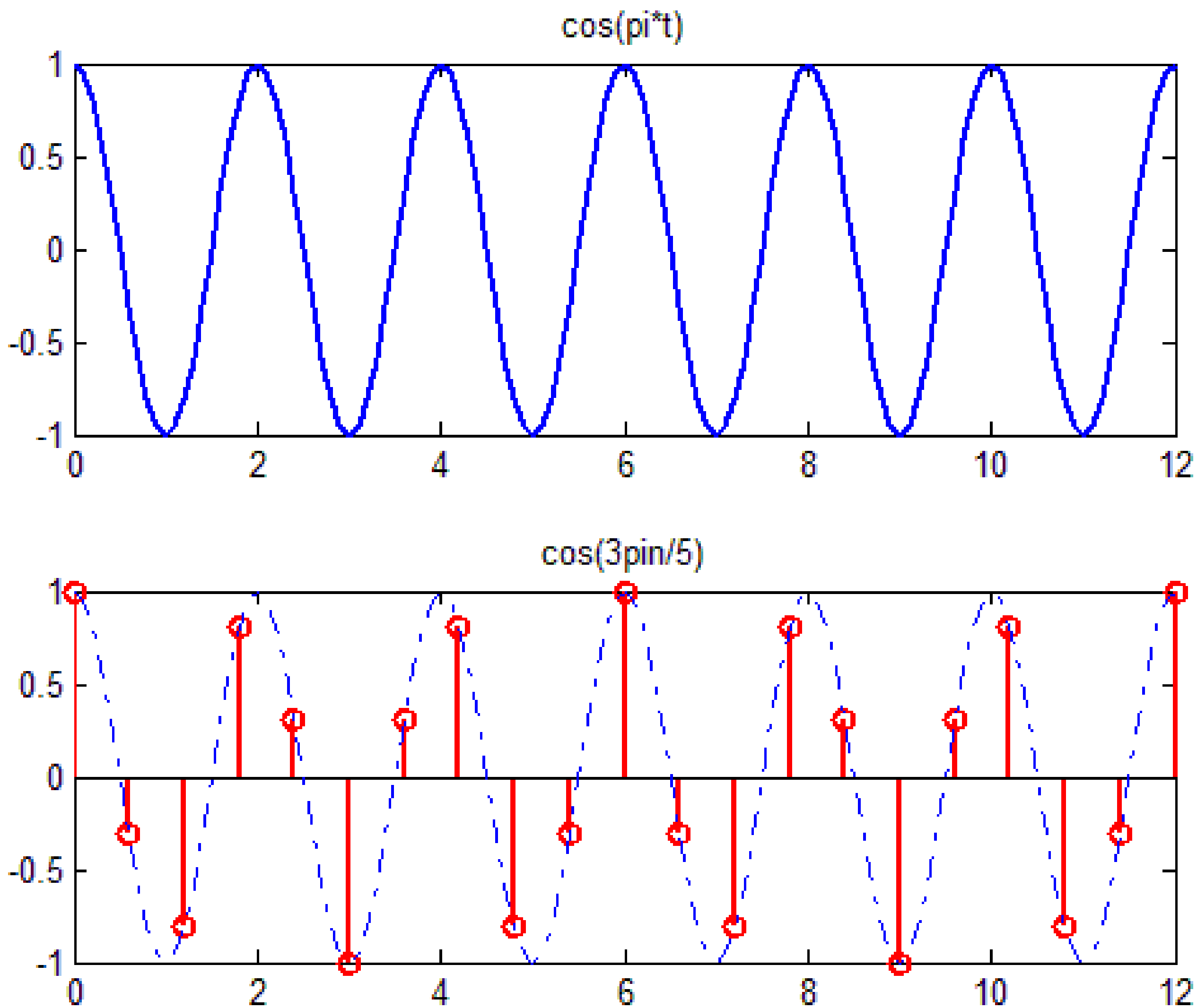


1.3 Representation of Signals Using Basic Functions

Sampling cos(πt) with period $T_s = 0.6$ s produces

$$\cos\left(\pi \cdot 0.6n\right) = \cos\left(\frac{3\pi}{5}n\right)$$

$N = 10$



1.3 Representation of Signals Using Basic Functions

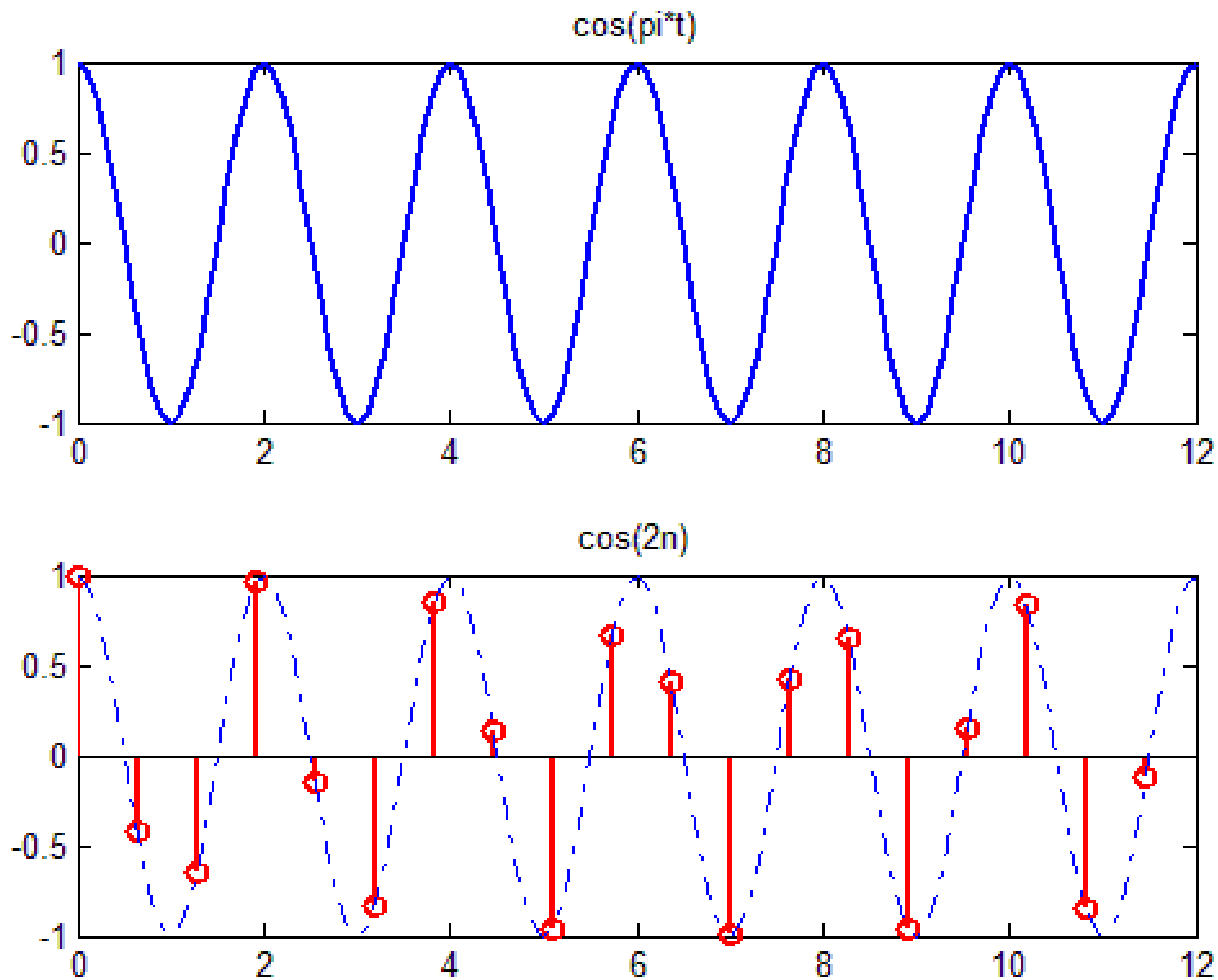
Sampling $\cos(\pi t)$ with

period $T_s = 2/\pi$ s

produces

$$\cos\left(\pi \cdot \frac{2}{\pi} n\right) = \cos(2n)$$

Nonperiodic !



1.3.4 The Discrete-Time Unit Impulse and Unit Step Sequences （单位脉冲和单位阶跃序列）

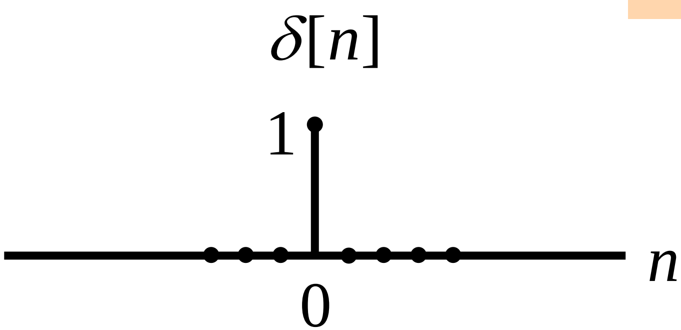
unit impulse (or unit sample):

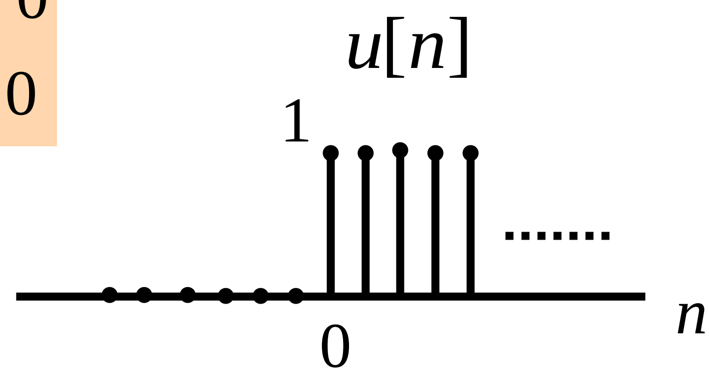
单位脉冲（单位样本）

$$\delta[n]=\begin{cases}0, & n\neq 0\\ 1, & n=0\end{cases}$$

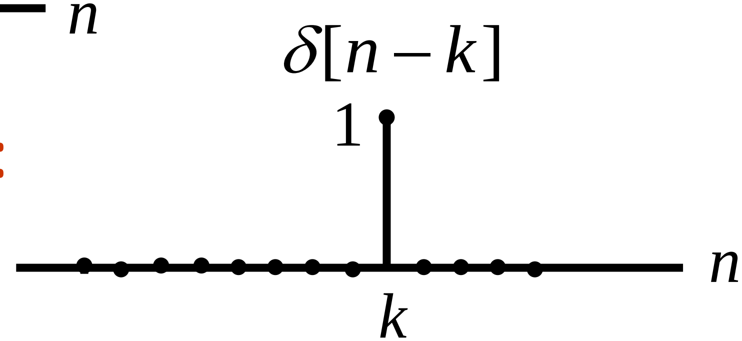
unit step:

$$u[n]=\begin{cases}0, & n<0\\ 1, & n\geq 0\end{cases}$$





shifted unit sample:



1.3 Representation of Signals Using Basic Functions

- Relationship between unit sample and unit step
  - ✓ unit sample is the *first difference*（一阶差分）of the unit step

$$\delta[n]=u[n]-u[n-1]$$

  - ✓ unit step is the *running sum*（连续求和）of the unit sample

$$u[n]=\sum_{m=-\infty}^{n}\delta[m] \qquad u[n]=\sum_{k=0}^{\infty}\delta[n-k]$$

- *sampling property*（抽样性质）*of unit sample:*

$$x[n]\delta[n]=x[0]\delta[n] \qquad x[n]\delta[n-k]=x[k]\delta[n-k]$$

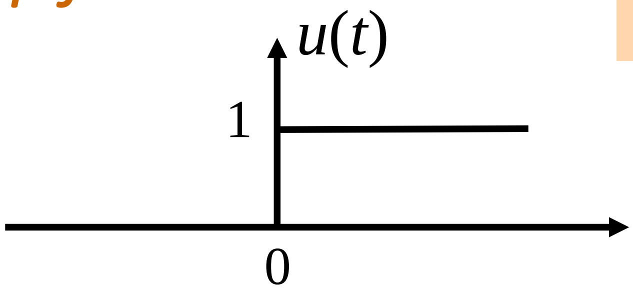
- Representing an arbitrary discrete-time signal in terms of shifted unit sample as

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\delta[n-k]$$

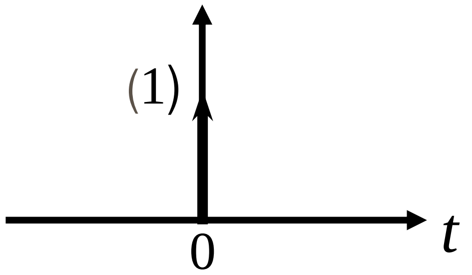
1.3.5 The Continuous-Time Unit Step and Unit Impulse Functions

unit step function:

$$u(t)=\begin{cases}0, & t<0\\ 1, & t>0\end{cases}$$



unit impulse function: $\delta(t)$



- Relationship between unit impulse and unit step

$$u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau \qquad \delta(t)=\frac{du(t)}{dt}$$

Definition of unit impulse

- ✓ Dirac definition:

$$\begin{cases} \int_{-\infty}^{\infty} \delta(t)\,dt = 1 \\ \delta(t) = 0,\ t \neq 0 \end{cases}$$

- ✓ Limit definition:

Gaussian function

$$\delta(t) = \lim_{\Delta\to 0}\frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right] = \lim_{\Delta\to 0}\delta_{\Delta}(t) = \lim_{a\to\infty}\frac{\sin(at)}{\pi t} = \lim_{a\to\infty} a\,e^{-\pi(at)^2}$$

Rectangular function　　Sampling function

- ✓ Generalized function（广义函数）definition:

$$\int_{-\infty}^{\infty} \delta(t)\varphi(t)dt = \varphi(0)$$

1.3 Representation of Signals Using Basic Functions

- Property of unit impulse

✓ Sampling property

$$x(t)\delta(t) = x(0)\delta(t) \qquad \int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0)$$

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0) \qquad \int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

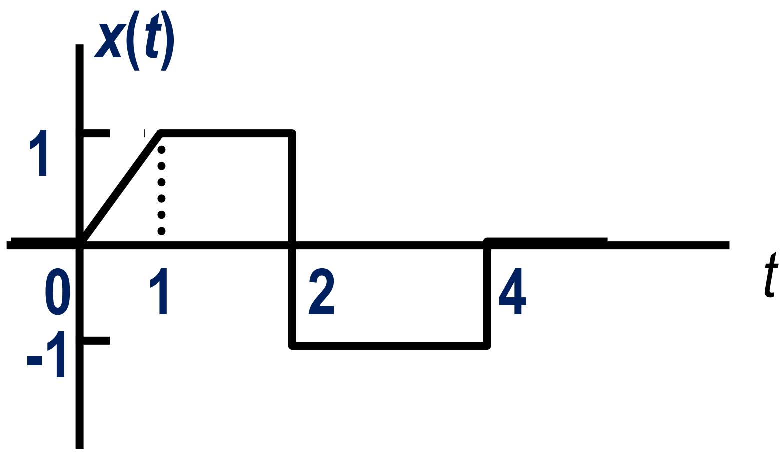
✓ δ(t) is a even function

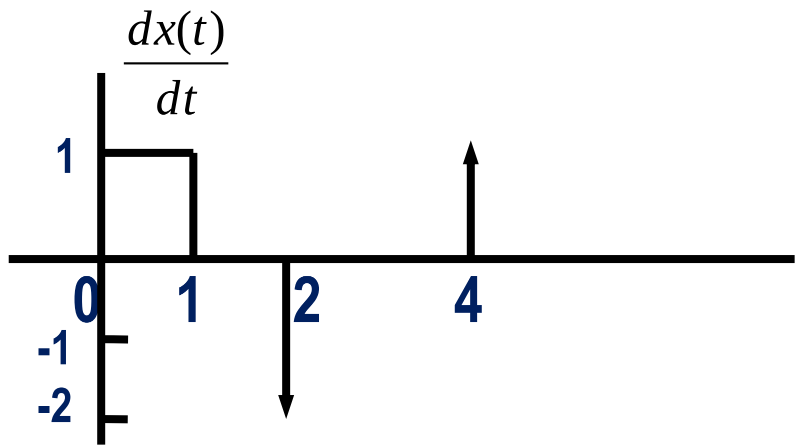
$$\delta(-t) = \delta(t), \quad \delta(-t-t_0) = \delta(t+t_0)$$

✓ Scaling property $\delta(at) = \dfrac{1}{|a|}\delta(t)$

1.3 Representation of Signals Using Basic Functions

Example 1.3 Determine and sketch the derivative of $x(t)$ depicted in the following figure.





$$x(t) = t[u(t) - u(t-1)] + [u(t-1) - u(t-2)] - [u(t-2) - u(t-4)]$$

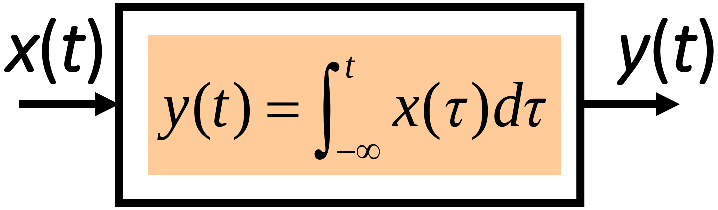
$$x'(t) = [u(t) - u(t-1)] + t[\delta(t) - \delta(t-1)] + [\delta(t-1) - \delta(t-2)] - [\delta(t-2) - \delta(t-4)]$$

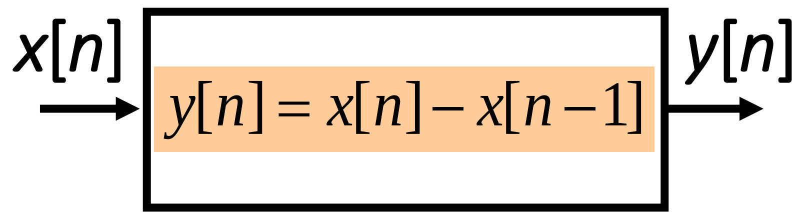
$$= [u(t) - u(t-1)] - \delta(t-1) + [\delta(t-1) - \delta(t-2)] - [\delta(t-2) - \delta(t-4)]$$

$$= [u(t) - u(t-1)] - 2\delta(t-2) + \delta(t-4)$$

1.4 Concept and Interconnections of Systems

- In broad sense, physical systems are an interconnection of components, devices or subsystems.
- In contexts from signal processing to communications, a *system* can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs.
- Continuous-time system: both input signals and output signals are continuous-time signals: $x(t) \rightarrow y(t)$
- Discrete-time system: transform discrete-time inputs into discrete-time outputs: $x[n] \rightarrow y[n]$

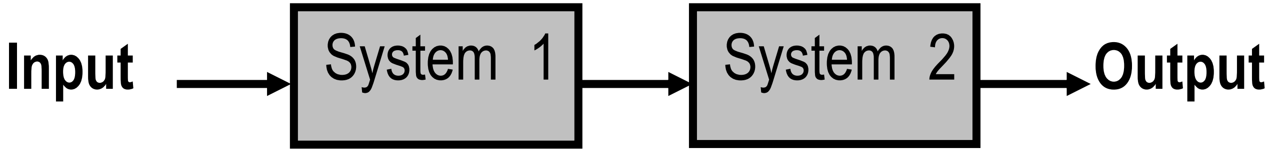




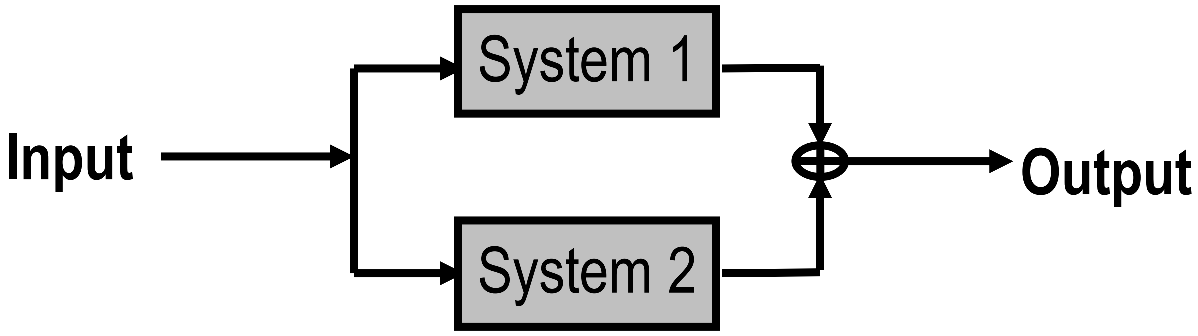
1.4 Concept and Interconnections of Systems

➢ Basic interconnections of subsystems

✓ *series* or *cascade interconnection*（串联或级联互连）

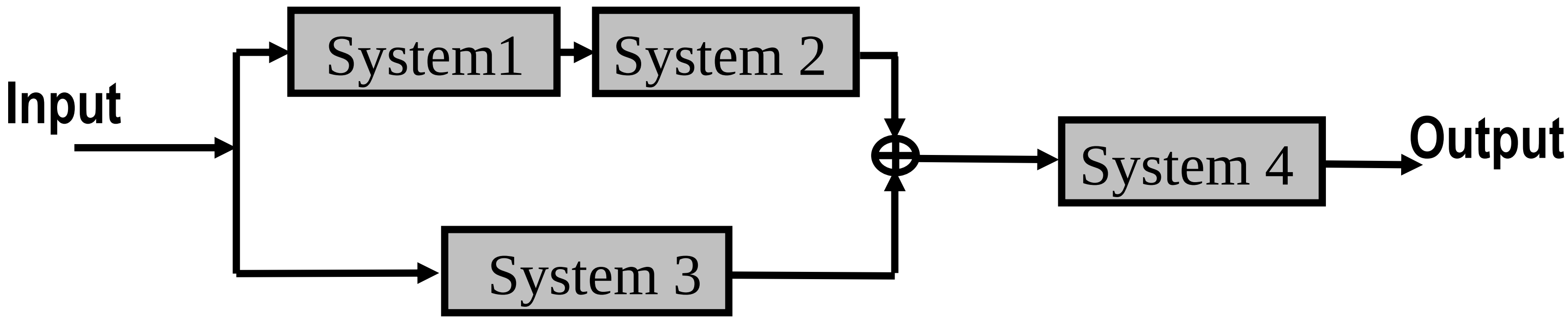


✓ *parallel interconnection*（并联互连）

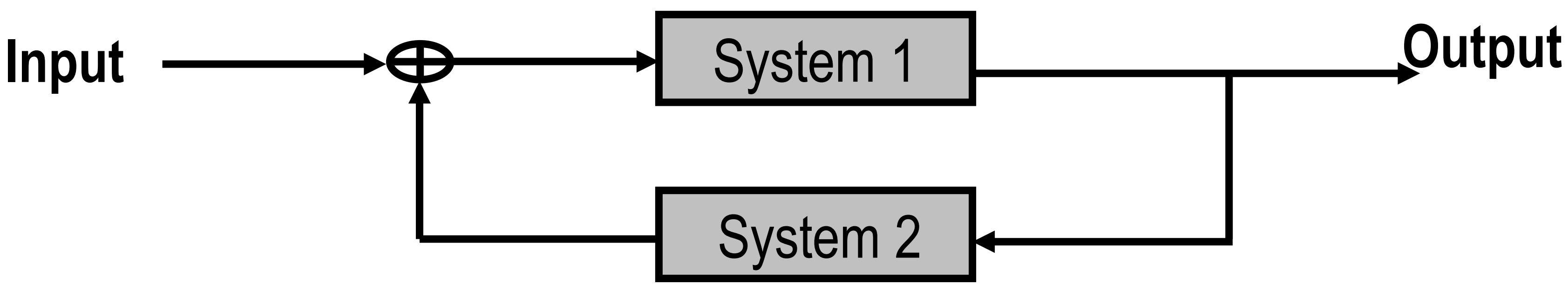


1.4 Concept and Interconnections of Systems

- complicated interconnections which combine the former two interconnections （混合互连）



- *feedback interconnection*（反馈互连）



1.5 Basic System Properties and Classification of Systems

- Systems with and without Memory

(有记忆系统和无记忆系统，有记忆系统也称为动态系统)

A system with *memory* means it can retain or store information about input values at times other than the current time.

example: $y(t) = tx(t)$ $\quad y[n] = \sum_{k=-\infty}^{n} x[k]$ $\quad y[n] = x[-n]$

Memoryless

With memory

- Invertibility and Inverse Systems (可逆性和逆系统)

A system is said to be *invertible* if distinct inputs lead to distinct outputs.

example: $y(t) = tx(t)$ $\quad y[n] = x^2[-n]$ $\quad y[n] = \sum_{k=0}^{n} x[k]$

Non-Invertible

Invertible

$x(t) \rightarrow$ S1 $\rightarrow$ Inverse of S1 $\rightarrow x(t)$

- Causality (因果性)

A system is *causal* if the output at any time depends only on values of the input at the present time and in the past.

example:

$$y(t)=\int_{-\infty}^{t}x(\tau)d\tau \qquad y(t)=x(t)\cos(t+1) \qquad y[n]=x[-n]$$

Causal

Non-causal

- Stability (稳定性)

If the input to a *stable* system is bounded (i.e., if its magnitude does not grow without bound), then the output must also be bounded (BIBO Stability).

example:

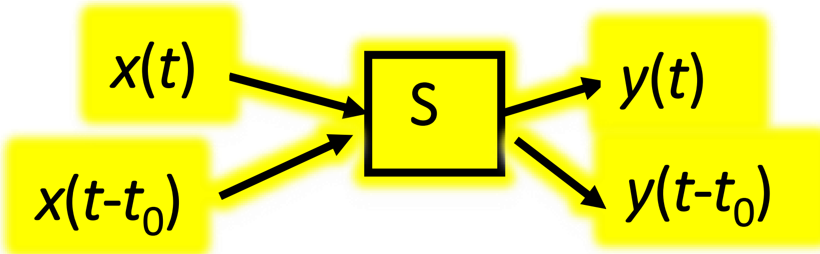
$$y(t)=x(t)\cos(t+1) \qquad y[n]=\sum_{k=0}^{n}x[k] \qquad y(t)=tx(t)$$

Stable

Unstable

- Time Invariance (时不变性)

A system is *time invariant* if the behavior and characteristics of the system are fixed over time.



example: $y[n]=nx[n]$

Sol: $x_1[n]\rightarrow y_1[n]=nx_1[n]$

$$x_2[n]=x_1[n-n_0]\rightarrow y_2[n]=nx_2[n]=nx_1[n-n_0]$$

However, $y_1[n-n_0]=(n-n_0)x_1[n-n_0]\neq y_2[n]$

Time-varying

- Linearity (线性) = *additivity* + *homogeneity*

（可加性） （齐次性）

Continuous-time: $ax_1(t)+bx_2(t)\rightarrow ay_1(t)+by_2(t)$

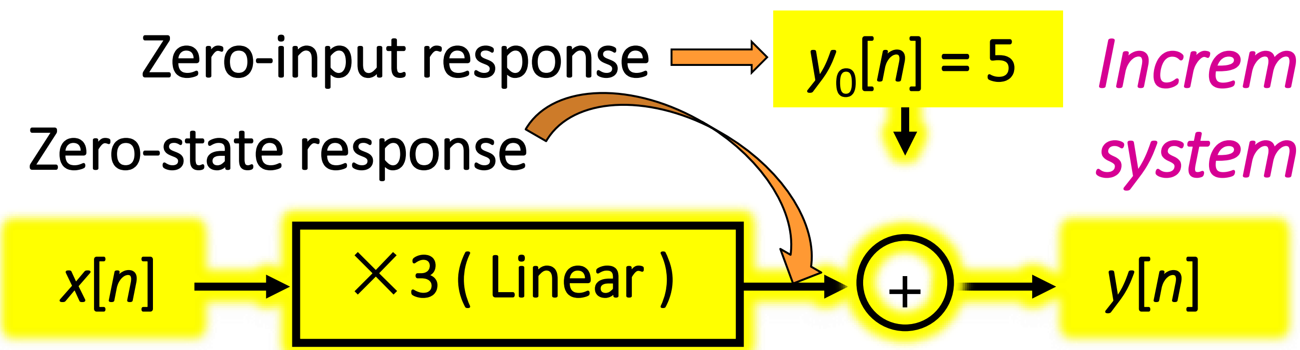
Discrete-time: $ax_1[n]+bx_2[n]\rightarrow ay_1[n]+by_2[n]$

example: $y[n]=3x[n]+5$

Sol: $x_1[n]\rightarrow y_1[n]=3x_1[n]+5 \quad x_2[n]\rightarrow y_2[n]=3x_2[n]+5$

Let $x_3[n]=ax_1[n]+bx_2[n]$ Nonlinear !

$$y_3[n]=3x_3[n]+5=3ax_1[n]+3bx_2[n]+5\neq ay_1[n]+by_2[n]$$



Incrementally linear system(增量线性系统)

Example:

Consider an LTI system whose response to the signal $x_1(t)$ in Figure (a) is the signal $y_1(t)$ illustrated in Figure (b). Determine and sketch the responses of the system to the inputs $x_2(t)$ and $x_3(t)$ depicted in Figure (c) and (d) respectively.

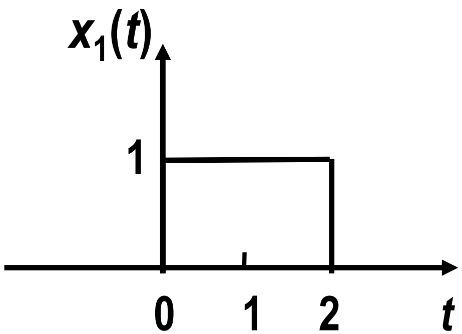


Figure (a)

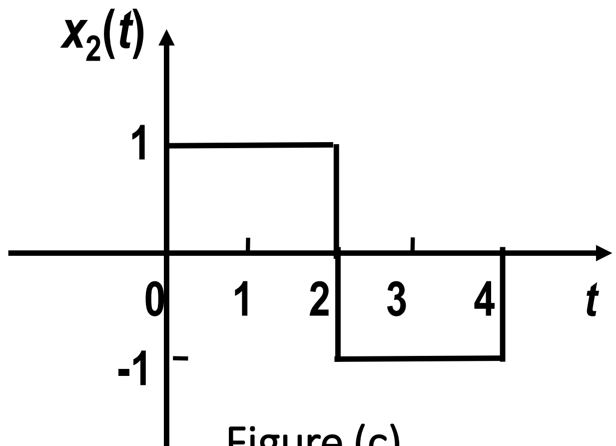


Figure (c)

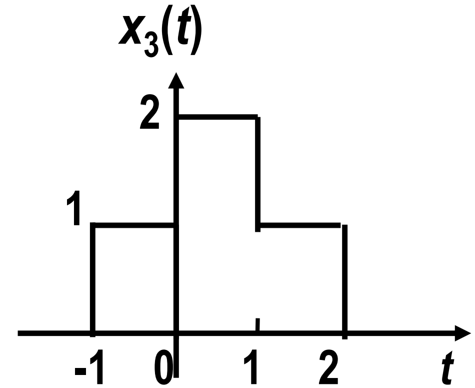


Figure (d)

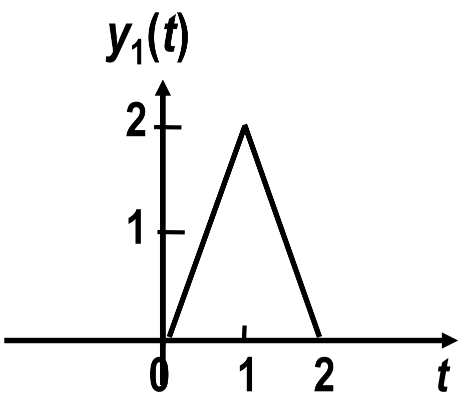
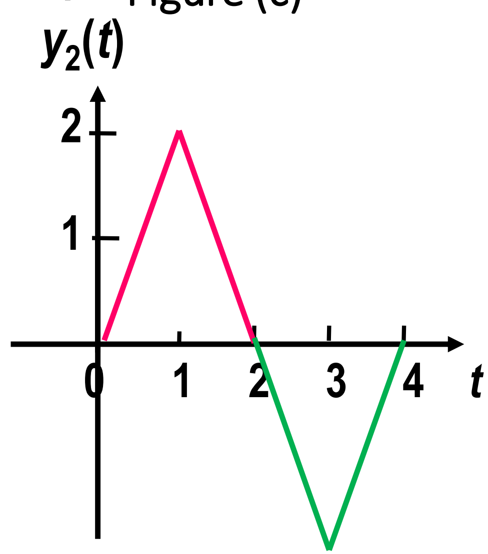
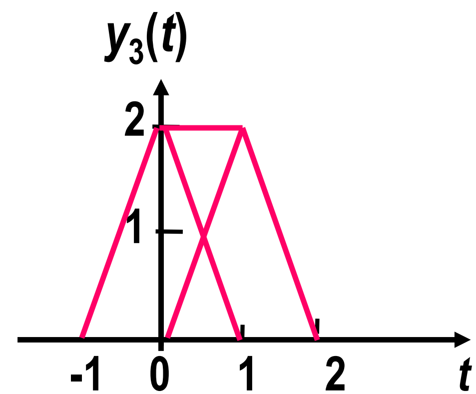


Figure (b)





1.6 SUMMARY

- The concepts of signals and systems ;
- The graphical and mathematical representation of signals ;
- The classifications of signals ;
- Transformations of the independent variable of signals ;
- Several basic signals ;
- Properties of systems .

Homework

1.22 (d)(h) 1.23 (b) 1.24 (a)

1.25 (b) (d) (f) 1.26 (b) (d) (e)

1.27 (b) (e) (f) 1.28 (b) (c) (e)