SUMMARY

- Decomposition of signals in orthogonal dimensions.
- Exponentials e^{st} and z^n are eigenfunctions of LTI systems.
- Representation and analysis of LTI systems.

Decomposition of signals in orthogonal dimensions(1)

- An arbitrary sequence being represented as a linear combination of shifted unit samples $\delta[n-k]$:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] = x[n] * \delta[n]$$

- An arbitrary continuous-time signal being represented as a continuous combination of shifted unit impulses $\delta(t-\tau)$:

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau = x(t) * \delta(t)$$

Decomposition of signals in orthogonal dimensions(2)

- Decomposing a continuous-time periodic signal in

  orthogonal dimensions $\left\{e^{jk\omega_0 t}\right\}$:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

where

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Decomposition of signals in orthogonal dimensions(3)

- Decomposing a discrete-time periodic signal into a sum of N harmonically related complex exponentials $e^{jk\omega_0 n}$:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

where

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

Decomposition of signals in orthogonal dimensions(4)

- Decomposing a continuous-time non-periodic signal in orthogonal dimensions $\left\{e^{j\omega t}\right\}$:

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega$$

where

$$X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$$

Decomposition of signals in orthogonal dimensions(5)

- Representing a discrete-time non-periodic signal as a integration of $X(e^{j\omega})e^{j\omega n}$ over a frequency interval of length 2π:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

where

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Decomposition of signals in orthogonal dimensions(6)

- Representing a continuous-time signal as a weighted integral of complex exponentials e^{st} :

$$x(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} X(s)e^{st}ds$$

where

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt \qquad a < \mathrm{Re}(s) < b$$

Decomposition of signals in orthogonal dimensions(7)

- Representing a discrete-time signal as a weighted contour integral of complex exponentials z^n :

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

$where$

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} \qquad |a| < |z| < |b|$$

Eigenfunctions of LTI systems (1)

- Responses of a causal continuous-time LTI system to $\delta(t)$ and e^{st}:

$$\delta(t) \rightarrow h(t)$$

$$\left.\begin{array}{l} e^{jk\omega_0 t} \rightarrow H(jk\omega_0)e^{jk\omega_0 t} \\ e^{j\omega t} \rightarrow H(j\omega)e^{j\omega t} \end{array}\right\} \Rightarrow premise: \mathrm{Re}(s_i)_{\max} < 0$$

$$e^{st} \rightarrow H(s)e^{st} \Rightarrow \mathrm{Re}(s) > \mathrm{Re}(s_i)_{\max}$$

Eigenfunctions of LTI systems (2)

- Responses of a causal discrete-time LTI system to $\delta[n]$ and z^n :

$$\delta[n] \rightarrow h[n]$$

$$\left.\begin{aligned} e^{jk\omega_0 n} &\rightarrow H(e^{jk\omega_0})e^{jk\omega_0 n} \\ e^{j\omega n} &\rightarrow H(e^{j\omega})e^{j\omega n} \end{aligned}\right\} \Rightarrow premise: \left|z_j\right|_{\max} < 1$$

$$z^n \rightarrow H(z)z^n \Rightarrow \left|z\right| > \left|z_j\right|_{\max}$$

Representation and analysis of LTI Systems (1)

- Input-output representation

$$y^{(N)}(t)+\sum_{i=0}^{N-1}a_i y^{(i)}(t)=\sum_{j=0}^{M}b_j x^{(j)}(t) \quad \textit{for continuous}-\textit{time systems}$$

$$y[n+N]+\sum_{i=0}^{N-1}a_i y[n+i]=\sum_{j=0}^{M}b_j x[n+i] \quad \textit{for discrete}-\textit{time systems}$$

- State-model representation

$$\dot{\vec{x}}(t)=\vec{A}\vec{x}(t)+\vec{B}\vec{v}(t) \qquad \vec{x}[n+1]=\vec{A}\vec{x}[n]+\vec{B}\vec{v}[n]$$

$$\vec{y}(t)=\vec{C}\vec{x}(t)+\vec{D}\vec{v}(t) \qquad \vec{y}[n]=\vec{C}\vec{x}[n]+\vec{D}\vec{v}[n]$$

Representation and analysis of LTI Systems (2)

- Analysis in the time domain (convolution analysis method)

$$y_{zs}(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=x(t)*h(t)$$

$$y_{zs}[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]=x[n]*h[n]$$

$$\vec{x}(t)=e^{\vec{A}t}\vec{x}(0)+e^{\vec{A}t}*\vec{B}\vec{v}(t)$$

$$\vec{y}(t)=\vec{C}e^{\vec{A}t}\vec{x}(0)+\left[\vec{C}e^{\vec{A}t}\vec{B}+\vec{D}\vec{\delta}(t)\right]*\vec{v}(t)$$

$$\vec{x}[n]=\vec{A}^{n}\vec{x}[0]+\vec{A}^{n-1}\vec{B}*\vec{v}[n]$$

$$\vec{y}[n]=\vec{C}\vec{A}^{n}\vec{x}[0]+\left[\vec{C}\vec{A}^{n-1}u[n-1]\vec{B}+\vec{D}\vec{\delta}[n]\right]*\vec{v}[n]$$

Representation and analysis of LTI Systems (3)

- Analysis in the frequency domain (Fourier analysis method)

  (only for stable systems)

$$y_{zs}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)H(j\omega)e^{j\omega t}d\omega=\mathcal{F}^{-1}\left\{X(j\omega)H(j\omega)\right\}$$

$$y_{zs}[n]=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})H(e^{j\omega})e^{j\omega n}d\omega=\mathcal{F}^{-1}\left\{X(e^{j\omega})H(e^{j\omega})\right\}$$

Representation and analysis of LTI Systems (4)

- Analysis in the complex-frequency domain (transformation analysis method)

$$y_{zs}(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)H(s)e^{st}ds = \mathcal{L}^{-1}\left\{X(s)H(s)\right\}$$

$$y_{zs}[n] = \frac{1}{2\pi j}\oint X(z)H(z)z^{n-1}dz = \mathcal{Z}^{-1}\left\{X(z)H(z)\right\}$$

$$\vec{X}(s) = (s\vec{I}-\vec{A})^{-1}\vec{x}(0) + (s\vec{I}-\vec{A})^{-1}\vec{B}\vec{V}(s)$$

$$\vec{Y}(s) = \vec{C}(s\vec{I}-\vec{A})^{-1}\vec{x}(0) + [\vec{C}(s\vec{I}-\vec{A})^{-1}\vec{B}+\vec{D}]\vec{V}(s)$$

$$\vec{X}(z) = (z\vec{I}-\vec{A})^{-1}z\vec{x}[0] + (z\vec{I}-\vec{A})^{-1}\vec{B}\vec{V}(z)$$

$$\vec{Y}(z) = \vec{C}(z\vec{I}-\vec{A})^{-1}z\vec{x}[0] + [\vec{C}(z\vec{I}-\vec{A})^{-1}\vec{B}+\vec{D}]\vec{V}(z)$$

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