



SUMMARY

- Decomposition of signals in orthogonal dimensions.
- Exponentials e^{st} and z^n are eigenfunctions of LTI systems.
- Representation and analysis of LTI systems.



Decomposition of signals in orthogonal dimensions(1)

- An arbitrary sequence being represented as a linear combination of shifted unit samples $\delta[n-k]$:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \delta[n-k] = x[n] * \delta[n]$$

- An arbitrary continuous-time signal being represented as a continuous combination of shifted unit impulses $\delta(t-\tau)$:

$$x(t) = \int_{-\infty}^{+\infty} x(\tau) \delta(t-\tau) d\tau = x(t) * \delta(t)$$



Decomposition of signals in orthogonal dimensions(2)

- Decomposing a continuous-time periodic signal in

orthogonal dimensions $\{e^{jk\omega_0 t}\}$:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

where

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$$



Decomposition of signals in orthogonal dimensions(3)

- Decomposing a discrete-time periodic signal into a sum of N harmonically related complex exponentials $e^{jk\omega_0 n}$:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

where

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$



Decomposition of signals in orthogonal dimensions(4)

- Decomposing a continuous-time non-periodic signal in orthogonal dimensions $\{e^{j\omega t}\}$:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

where

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$



Decomposition of signals in orthogonal dimensions(5)

- Representing a discrete-time non-periodic signal as a integration of $X(e^{j\omega})e^{j\omega n}$ over a frequency interval of

length 2π :

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

where

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$



Decomposition of signals in orthogonal dimensions(6)

- Representing a continuous-time signal as a weighted integral of complex exponentials e^{st} :

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s) e^{st} ds$$

where

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt \quad a < \operatorname{Re}(s) < b$$



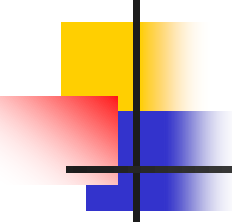
Decomposition of signals in orthogonal dimensions(7)

- Representing a discrete-time signal as a weighted contour integral of complex exponentials z^n :

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

where

$$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} \quad |a| < |z| < |b|$$



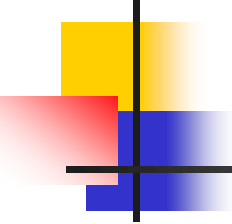
Eigenfunctions of LTI systems (1)

- Responses of a causal continuous-time LTI system to $\delta(t)$ and e^{st} :

$$\delta(t) \rightarrow h(t)$$

$$\left. \begin{array}{l} e^{jk\omega_0 t} \rightarrow H(jk\omega_0)e^{jk\omega_0 t} \\ e^{j\omega t} \rightarrow H(j\omega)e^{j\omega t} \end{array} \right\} \Rightarrow \text{premise : } \operatorname{Re}(s_i)_{\max} < 0$$

$$e^{st} \rightarrow H(s)e^{st} \Rightarrow \operatorname{Re}(s) > \operatorname{Re}(s_i)_{\max}$$



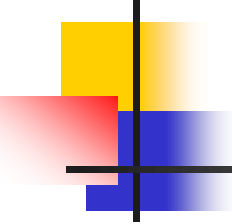
Eigenfunctions of LTI systems (2)

- Responses of a causal discrete-time LTI system to $\delta[n]$ and z^n :

$$\delta[n] \rightarrow h[n]$$

$$\left. \begin{array}{l} e^{jk\omega_0 n} \rightarrow H(e^{jk\omega_0})e^{jk\omega_0 n} \\ e^{j\omega n} \rightarrow H(e^{j\omega})e^{j\omega n} \end{array} \right\} \Rightarrow \text{premise : } |z_j|_{\max} < 1$$

$$z^n \rightarrow H(z)z^n \Rightarrow |z| > |z_j|_{\max}$$



Representation and analysis of LTI Systems (1)

- Input-output representation

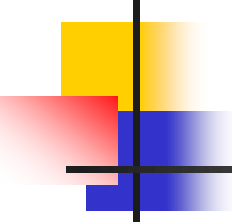
$$y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{j=0}^M b_j x^{(j)}(t) \quad \text{for continuous-time systems}$$

$$y[n+N] + \sum_{i=0}^{N-1} a_i y[n+i] = \sum_{j=0}^M b_j x[n+i] \quad \text{for discrete-time systems}$$

- State-model representation

$$\dot{\vec{x}}(t) = \vec{A}\vec{x}(t) + \vec{B}\vec{v}(t) \quad \vec{x}[n+1] = \vec{A}\vec{x}[n] + \vec{B}\vec{v}[n]$$

$$\vec{y}(t) = \vec{C}\vec{x}(t) + \vec{D}\vec{v}(t) \quad \vec{y}[n] = \vec{C}\vec{x}[n] + \vec{D}\vec{v}[n]$$



Representation and analysis of LTI Systems (2)

- Analysis in the time domain (convolution analysis method)

$$y_{zs}(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

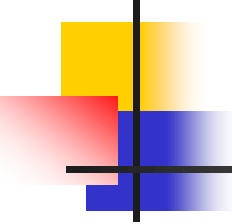
$$y_{zs}[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n] * h[n]$$

$$\vec{x}(t) = e^{\vec{A}t} \vec{x}(0) + e^{\vec{A}t} * \vec{B} \vec{v}(t)$$

$$\vec{y}(t) = \vec{C} e^{\vec{A}t} \vec{x}(0) + \left[\vec{C} e^{\vec{A}t} \vec{B} + \vec{D} \delta(t) \right] * \vec{v}(t)$$

$$\vec{x}[n] = \vec{A}^n \vec{x}[0] + \vec{A}^{n-1} \vec{B} * \vec{v}[n]$$

$$\vec{y}[n] = \vec{C} \vec{A}^n \vec{x}[0] + \left[\vec{C} \vec{A}^{n-1} \vec{B} + \vec{D} \delta[n] \right] * \vec{v}[n]$$

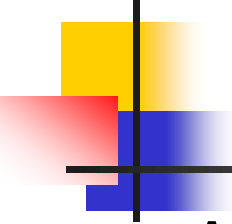


Representation and analysis of LTI Systems (3)

- Analysis in the frequency domain (Fourier analysis method)
(only for stable systems)

$$y_{zs}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)H(j\omega)e^{j\omega t} d\omega = \mathcal{F}^{-1} \{ X(j\omega)H(j\omega) \}$$

$$y_{zs}[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})H(e^{j\omega})e^{j\omega n} d\omega = \mathcal{F}^{-1} \{ X(e^{j\omega})H(e^{j\omega}) \}$$



Representation and analysis of LTI Systems (4)

- Analysis in the complex-frequency domain (transformation analysis method)

$$y_{zs}(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)H(s)e^{st} ds = \mathcal{L}^{-1} \{X(s)H(s)\}$$

$$y_{zs}[n] = \frac{1}{2\pi j} \oint X(z)H(z)z^{n-1} dz = \mathcal{Z}^{-1} \{X(z)H(z)\}$$

$$\vec{X}(s) = (s\vec{I} - \vec{A})^{-1} \vec{x}(0) + (s\vec{I} - \vec{A})^{-1} \vec{B}\vec{V}(s)$$

$$\vec{Y}(s) = \vec{C}(s\vec{I} - \vec{A})^{-1} \vec{x}(0) + [\vec{C}(s\vec{I} - \vec{A})^{-1} \vec{B} + \vec{D}]\vec{V}(s)$$

$$\vec{X}(z) = (z\vec{I} - \vec{A})^{-1} z\vec{x}[0] + (z\vec{I} - \vec{A})^{-1} \vec{B}\vec{V}(z)$$

$$\vec{Y}(z) = \vec{C}(z\vec{I} - \vec{A})^{-1} z\vec{x}[0] + [\vec{C}(z\vec{I} - \vec{A})^{-1} \vec{B} + \vec{D}]\vec{V}(z)$$

Network about contents

