公式总结

注：此文档仅梳理了相关公式，需掌握的概念、知识点请仔细研读课件。

第一章

- 三种正交坐标系长度元，面积元和体积元表达式
- 三种正交坐标系坐标单位矢量的转换
- 标量场图和矢量场图对应的方程 $f(\vec{r}) = \text{常数值}$ $\boxed{\vec{A}(\vec{r}) \times d\vec{r} = 0}$
- 方向导数，梯度 $\boxed{\therefore \frac{df}{dl} = \vec{G} \cdot \vec{e}_l}$ $\vec{G} = \nabla f$ $\boxed{\nabla f = \vec{e}_x \frac{\partial f}{\partial x} + \vec{e}_y \frac{\partial f}{\partial y} + \vec{e}_z \frac{\partial f}{\partial z}}$
- 面元矢量： $\boxed{d\vec{S} = \vec{e}_n dS}$
- **场量穿过面元的通量：** $\boxed{\vec{A} \cdot d\vec{S} = A\cos\theta dS}$

散度 $$div\vec{A} = \nabla \cdot \vec{A} \qquad div\vec{A} = \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

高斯散度定理 $$\oint_S \vec{A} \cdot d\vec{S} = \int_V \nabla \cdot \vec{A} dV$$

环量 $$\oint_C \vec{A} \cdot d\vec{l} = \oint_C A\cos\theta dl$$

环量面密度 $$rot_n\vec{A} = rot\vec{A} \cdot \vec{e}_n$$

旋度 $$rot\vec{A} = \nabla \times \vec{A} \qquad \nabla \times \vec{A} = \vec{e}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{e}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{e}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

斯托克斯定理 $$\oint_C \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S}$$

$$\boxed{\nabla \times \nabla \phi = 0} \qquad \boxed{\nabla \cdot (\nabla \times \vec{A}) = 0}$$

第二章

$$\vec{\boldsymbol{J}} = \rho_{\boldsymbol{v}} \vec{\boldsymbol{V}}$$

$$\boldsymbol{I} = \int_{\boldsymbol{S}} \vec{\boldsymbol{J}}(\vec{\boldsymbol{r}}) \cdot \boldsymbol{d}\vec{\boldsymbol{S}}$$

$$\boldsymbol{I} = \int_{\boldsymbol{l}} \vec{\boldsymbol{J}}_{\boldsymbol{s}} \cdot \vec{\boldsymbol{e}}_{N} \boldsymbol{dl}$$

$$\overrightarrow{D} = \varepsilon_0 \overrightarrow{E} + \overrightarrow{P}$$

在线性各向同性介质中 $\overrightarrow{P} = \varepsilon_0 \chi_e \overrightarrow{E}$ $\overrightarrow{\boldsymbol{D}} == \varepsilon_0 \varepsilon_{\boldsymbol{r}} \overrightarrow{\boldsymbol{E}} = \varepsilon \overrightarrow{\boldsymbol{E}}$

$$\overrightarrow{F} = \oint_{C_2} I_2 d\vec{l}_2 \times \overrightarrow{B}$$

$$\overrightarrow{F} = q\overrightarrow{E} + q\vec{v} \times \overrightarrow{B}$$

$$\overrightarrow{H} = \frac{\overrightarrow{B}}{\mu_0} - \overrightarrow{M}$$

在线性各向同性磁介质中 $\vec{\boldsymbol{M}} = \chi_{\boldsymbol{m}} \vec{\boldsymbol{H}}$ $\vec{\boldsymbol{B}} = \mu \overrightarrow{\boldsymbol{H}}$

$$\boxed{\oint_S \vec{J}\cdot d\vec{S} = -\frac{dQ}{dt} = -\frac{d}{dt}\int_V \rho dV}$$

$$\overrightarrow{J_t} = \vec{J} + \overrightarrow{J_d} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\begin{cases} \oint_l \vec{H}\cdot d\vec{l} = \int_S (\vec{J} + \frac{\partial \vec{D}}{\partial t})\cdot d\vec{S} \\ \oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S} \\ \oint_S \vec{B}\cdot d\vec{S} = 0 \\ \oint_S \vec{D}\cdot d\vec{S} = \int_V \rho_V dV = \sum q \end{cases}$$

$$\begin{cases} \nabla\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \\ \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla\cdot\vec{B} = 0 \\ \nabla\cdot\vec{D} = \rho_v \end{cases}$$

$$\boxed{\begin{cases} H_{2t} - H_{1t} = J_{SN} \\ E_{2t} - E_{1t} = 0 \\ B_{2n} - B_{1n} = 0 \\ D_{2n} - D_{1n} = \rho \end{cases}}$$

$$\boxed{\begin{cases} \vec{e}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J} \\ \vec{e}_n \times (\vec{E}_2 - \vec{E}_1) = 0 \\ \vec{e}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0 \\ \vec{e}_n \cdot (\vec{D}_2 - \vec{D}_1) = \rho \end{cases}}$$

第三章

$$\begin{cases}\nabla\times\vec{E}=0\\ \nabla\bullet\vec{D}=\rho\\ \vec{D}=\varepsilon\vec{E}\end{cases}$$

$$d\Omega=\frac{d\vec{S}\bullet\vec{e}_R}{R^2}=\frac{dS\cos\theta}{R^2}$$

$$\vec{E}=-\nabla\phi$$

$$\phi_A-\phi_B=\int_A^B\vec{E}\bullet d\vec{l}$$

$$\phi_A=\int_A^P\vec{E}\cdot d\vec{l}$$

$$\vec{E}=\frac{1}{4\pi\varepsilon}\int_{v'}\frac{\rho_v dV'}{R^3}\vec{R}$$

$$\phi=\frac{1}{4\pi\varepsilon_0}\int_v\frac{\rho_v dv}{R}$$

$$\begin{cases}\text{束缚面电荷：}\rho'_s=\vec{P}\bullet\vec{e}_n\\ \text{束缚体电荷：}\rho'_v=-\nabla\bullet\vec{P}\end{cases}$$

$$\nabla^2\phi=-\frac{\rho_v}{\varepsilon}$$

$$\phi_1=\phi_2$$

$$\varepsilon_1\frac{\partial\phi_1}{\partial n}-\varepsilon_2\frac{\partial\phi_2}{\partial n}=\rho_s$$

电场能量 $$W_e = \frac{1}{2}\int_V \rho\phi dv \qquad W_e = \frac{1}{2}\int_V \varepsilon E^2 dv$$

电场能量密度 $$w_e = \frac{1}{2}\vec{D}\bullet\vec{E} = \frac{1}{2}\varepsilon E^2 \left(J/m^3\right)$$

恒定电场
$$\begin{cases} \nabla\times\vec{E} = 0 \\ \nabla\bullet\vec{J} = 0 \end{cases}$$
$$\vec{J} = \gamma\vec{E}$$

$$\phi_1 = \phi_2$$
$$\gamma_1 \frac{\partial\phi_1}{\partial n} = \gamma_2 \frac{\partial\phi_2}{\partial n}$$

焦耳定理 $$p = \vec{J}\bullet\vec{E}$$

第四章

$$\begin{cases} \nabla \times \vec{H} = \vec{J} \\ \nabla \cdot \vec{B} = 0 \end{cases} \qquad \vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \oint_C \frac{Id\vec{l} \times \vec{e}_R}{R^2} \qquad \vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}}{R} dV'$$

$$\begin{cases} \text{介质内部束缚体电流密度：} \vec{J}'_v = \nabla \times \vec{M} \\ \text{介质表面束缚面电流密度：} \vec{J}'_S = \vec{M} \times \vec{e}_n \end{cases}$$

$$L = \frac{\Psi}{I} (\text{单位：亨}H) \qquad W_m = \frac{1}{2} \int_V (\vec{H} \bullet \vec{B}) dV \qquad w_m = \frac{1}{2} \mu H^2$$

第六章

$$\vec{B} = \nabla \times \vec{A} \qquad \vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla \phi \qquad \nabla \cdot \vec{A} = -\mu\varepsilon \frac{\partial \phi}{\partial t}$$

$$-\nabla \cdot \vec{S} = \frac{\partial}{\partial t}\left(w_m + w_e\right) + p$$

$$-\oint_S \vec{S} \cdot d\vec{S} = \frac{d}{dt}\int_V (w_m + w_m)dv + \int_V pdv$$

$$\boxed{\vec{S}(t) = \vec{E}(t) \times \vec{H}(t)}$$

瞬时值坡印廷矢量

$$\boxed{\vec{S}_{av} = \frac{1}{T}\int_0^T \vec{S}(t)dt = \frac{1}{T}\int_0^T \left[\vec{E}(t) \times \vec{H}(t)\right]dt}$$

平均坡印廷矢量

$$\vec{E}(\vec{r},t)=\mathrm{Re}\left[\dot{\vec{E}}\,e^{j\omega t}\right]$$

复矢量 $\dot{\vec{E}}(\vec{r})=\vec{e}_x\dot{E}_{xm}(\vec{r})+\vec{e}_y\dot{E}_{ym}(\vec{r})+\vec{e}_z\dot{E}_{zm}(\vec{r})$

$$\begin{cases}\nabla\times\vec{H}=\vec{J}+j\omega\vec{D}\\ \nabla\times\vec{E}=-j\omega\vec{B}\\ \nabla\bullet\vec{B}=0\\ \nabla\bullet\vec{D}=\rho\end{cases}$$

$$\begin{cases}\oint_C\dot{\vec{H}}\bullet d\vec{l}=\int_S\left(\dot{\vec{J}}+j\omega\dot{\vec{D}}\right)\bullet d\vec{S}\\ \oint_C\dot{\vec{E}}\bullet d\vec{l}=-j\omega\int_S\dot{\vec{B}}\bullet d\vec{S}\\ \int_S\dot{\vec{B}}\bullet d\vec{S}=0\\ \oint_C\dot{\vec{D}}\bullet d\vec{S}=\int_V\dot{\rho}dv\end{cases}$$

$$\nabla\bullet\dot{\vec{J}}=-j\omega\dot{\rho}\qquad \oint_S\dot{\vec{J}}\bullet d\vec{S}=-j\omega\int_V\dot{\rho}dv$$

$$\dot{\vec{D}}=\varepsilon\dot{\vec{E}}\quad \dot{\vec{B}}=\mu\dot{\vec{H}}\quad \dot{\vec{J}}=\gamma\dot{\vec{E}}$$

$$\vec{S}_c=\frac{1}{2}(\vec{E}(\vec{r})\times\vec{H}(\vec{r})^*)\qquad \vec{S}_{av}=\mathrm{Re}\left[\frac{1}{2}\vec{E}(\vec{r})\times\vec{H}(\vec{r})^*\right]$$

$$\gamma_c=\gamma'(\omega)-j\gamma''(\omega)$$

$$\varepsilon_c=\varepsilon'(\omega)-j\varepsilon''(\omega)$$

$$\mu_c=\mu'(\omega)-j\mu''(\omega)$$

$$\begin{cases}\text{良介质：}\quad \dfrac{\gamma}{\omega\varepsilon}<10^{-2}\\ \text{有损耗介质：}\quad 10^{-2}<\dfrac{\gamma}{\omega\varepsilon}<100\\ \text{良好导体：}\quad \dfrac{\gamma}{\omega\varepsilon}>100\end{cases}$$

第七章

$$\eta = \frac{\omega\mu}{\boldsymbol{k}} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\vec{\boldsymbol{H}} = \frac{1}{\eta}\vec{\boldsymbol{e}}_z \times \vec{\boldsymbol{E}}，\quad \vec{\boldsymbol{E}} = \eta\vec{\boldsymbol{H}} \times \vec{\boldsymbol{e}}_z$$

$$\beta = \boldsymbol{k} = \omega\sqrt{\mu\varepsilon}$$

$$\lambda = \frac{2\pi}{\boldsymbol{k}} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

$$\therefore \boldsymbol{V_p} = \frac{\omega}{\boldsymbol{k}} = \frac{1}{\sqrt{\mu\varepsilon}} = \lambda\boldsymbol{f}$$

$$\boldsymbol{v_e} = \frac{|\boldsymbol{S_{av}}|}{\boldsymbol{w_{av}}}$$

导电媒质引入复介电常数$\varepsilon_c = \varepsilon - j\dfrac{\gamma}{\omega}$及复波速$K = \omega\sqrt{\mu\varepsilon_c}$

$$\vec{E} = \dot{\vec{E}}_0 e^{-\Gamma z} = \dot{\vec{E}}_0 e^{-\alpha z} e^{-j\beta z}$$

良介质

$$\begin{cases} \alpha \approx \dfrac{\gamma}{2}\sqrt{\dfrac{\mu}{\varepsilon}}, \quad \beta \approx \omega\sqrt{\mu\varepsilon}, \quad V_p \approx \dfrac{1}{\sqrt{\mu\varepsilon}} \\ \lambda \approx \dfrac{1}{f\sqrt{\mu\varepsilon}}, \quad \eta_c = \sqrt{\dfrac{\mu}{\varepsilon}}\left(1 + j\dfrac{\gamma}{2\omega\varepsilon}\right) \approx \sqrt{\dfrac{\mu}{\varepsilon}} \end{cases}$$

良导体

$$\begin{cases} \alpha \approx \beta \approx \sqrt{\dfrac{\omega\mu\gamma}{2}} = \sqrt{\pi f \mu\gamma}, \quad V_p = \dfrac{\omega}{\beta} \approx \sqrt{\dfrac{2\omega}{\mu\gamma}} = 2\sqrt{\dfrac{\pi f}{\mu\gamma}} \\ \lambda = \dfrac{2\pi}{\beta} \approx 2\pi\sqrt{\dfrac{2}{\omega\mu\gamma}} = 2\sqrt{\dfrac{\pi}{f\mu\gamma}}, \quad \eta_c \approx (1+j)\sqrt{\dfrac{\omega\mu}{2\gamma}} \end{cases}$$

沿任意方向传播的均匀平面波

$$\begin{cases} \dot{\vec{E}} = \dot{\vec{E}}_0 e^{-jk\vec{e}_n\cdot\vec{r}} = \dot{\vec{E}}_0 e^{-j\vec{k}\cdot\vec{r}} \\ \dot{\vec{H}} = \dot{\vec{H}}_0 e^{-jk\vec{e}_n\cdot\vec{r}} = \dot{\vec{H}}_0 e^{-j\vec{k}\cdot\vec{r}} \end{cases}$$

波矢量 $\vec{k} = \vec{e}_n k$

$$\vec{E} = \eta\vec{H} \times \vec{e}_n, \vec{H} = \frac{1}{\eta}\vec{e}_n \times \vec{E}$$

极化的判别方法

1、利用E_x和E_y的振幅和相位之间的关系判断

$$\vec{E} = \vec{e}_x E_{xm}\cos(\omega t - kz + \varphi_x) + \vec{e}_y E_{ym}\cos(\omega t - kz + \varphi_y)$$

$\boxed{\text{当}\varphi_y - \varphi_x = 0\text{或}\pm\pi\text{时},\rightarrow\text{线极化}}$

$\boxed{\text{当}E_{xm} = E_{xm},\text{且}\varphi_y - \varphi_x = \pm\pi/2\text{时},\rightarrow\text{圆极化}}$

若$\varphi_y - \varphi_x = +\pi/2$，沿+z(−z)传播的波为左旋（右旋 波

若$\varphi_y - \varphi_x = -\pi/2$，沿+z(−z)传播的波为右旋（左旋 波

$\boxed{\text{其他一般情形},\rightarrow\text{椭圆极化}}$

若$0 < \varphi_y - \varphi_x < \pi$，沿+z(−z)传播的波为左旋（右旋 波

若$-\pi < \varphi_y - \varphi_x < 0$，沿+z(−z)传播的波为右旋（左旋 波

2、利用复数形式判断

$$\dot{\vec{E}} = \vec{e}_x E_{xm} e^{j(-kz+\varphi_x)} + \vec{e}_y E_{ym} e^{j(-kz+\varphi_y)}$$

$$\dot{\vec{E}}(z=0) = \vec{e}_x E_{xm} e^{j\varphi_x} + \vec{e}_y E_{ym} e^{j\varphi_y}$$

$$= \vec{e}_x E_{xm}(\cos\varphi_x + j\sin\varphi_x) + \vec{e}_y E_{ym}(\cos\varphi_y + j\sin\varphi_y)$$

$$= (\vec{e}_x E_{xm}\cos\varphi_x + \vec{e}_y E_{ym}\cos\varphi_y) + j(\vec{e}_x E_{xm}\sin\varphi_x + \vec{e}_y E_{ym}\sin\varphi_y)$$

$$= \vec{E}_R + j\vec{E}_I$$

$$\vec{E}_R = \vec{e}_x E_{xm}\cos\varphi_x + \vec{e}_y E_{ym}\cos\varphi_y$$

$$\vec{E}_I = \vec{e}_x E_{xm}\sin\varphi_x + \vec{e}_y E_{ym}\sin\varphi_y$$

若：$\vec{E}_R // \vec{E}_I$　或$\vec{E}_R = 0$　或$\vec{E}_I = 0$　$\rightarrow$ 线极化

若　$\vec{E}_R \perp \vec{E}_I$ 且 $|\vec{E}_R| = |\vec{E}_I|$　$\rightarrow$ 圆极化

若$\vec{E}_I$、$\vec{E}_R$与波的传播方向符合右手螺旋关系，则为右旋波；

若$\vec{E}_I$、$\vec{E}_R$与波的传播方向符合左手螺旋关系，则为左旋波。

$$\boxed{V_p = \frac{\omega}{\beta}} \quad \boxed{V_g = \frac{d\omega}{d\beta}} \quad \boxed{V_e = \frac{S_{av}}{w_{av}}}$$

➢对于非色散媒质：

$$\frac{dV_p}{d\omega} = 0 \quad \Rightarrow \quad \boxed{V_e = V_g = V_p}$$

➢对于色散媒质： $\dfrac{dV_p}{d\omega} \neq 0$

$\boxed{\text{若}\dfrac{dV_p}{d\omega} < 0\text{或}\dfrac{dV_p}{d\lambda} > 0\text{，相速随频率增大而减小，则}V_g < V_p}$ 正常色散 $\boxed{V_g < V_p \quad V_e = V_g}$

$\boxed{\text{若}\dfrac{dV_p}{d\omega} > 0\text{或}\dfrac{dV_p}{d\lambda} < 0\text{，相速随频率增大而增大，则}V_g > V_p}$ 非正常色散 $\boxed{V_g > V_p \quad V_e \neq V_g}$

斜入射时，入射波，反射波，透射波表达式，合成波特性

Snell反射定理，折射定理

$$\Rightarrow \begin{cases} R_\perp = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ T_\perp = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{cases}$$

$$\Rightarrow \begin{cases} R_{/\!/} = \dfrac{E_{r0}}{E_{i0}} = \dfrac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \\ T_{/\!/} = \dfrac{E_{t0}}{E_{i0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \end{cases}$$

平行极化的Fresnel公式

对非磁性物质

全反射 $\boxed{|R_\perp| = |R_{/\!/}| = 1}$

$$\boxed{\begin{cases} \varepsilon_1 > \varepsilon_2，\quad \text{从光密到光疏媒质} \\ \theta_i > \theta_c = \arcsin\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = \arcsin\dfrac{n_2}{n_1} \end{cases}}$$

全折射 $\boxed{|R| = 0}$

$$\boxed{\theta_i = \theta_B = \arcsin\sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}} = \arctan\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}}$$ 仅适用平行极化波

垂直入射时，入射波，反射波，透射波表达式；合成波特性；反射系数，折射系数

$$\boxed{\rho=\frac{|E|_{\max}}{|E|_{\min}}=\frac{1+|R|}{1-|R|}\Rightarrow|R|=\frac{\rho-1}{\rho+1}}$$

对于良导体，透射波磁场的复振幅近似等于入射波磁场复振幅的**2**倍 $\boldsymbol{H}_{t0}\approx 2\boldsymbol{H}_{i0}$

良导体中：$\delta=\frac{1}{\alpha}\approx\frac{1}{\sqrt{\pi f\mu\gamma}}=\sqrt{\frac{2}{w\mu\gamma}}$

$\boldsymbol{Z}_s=\boldsymbol{\eta}_c=\boldsymbol{R}_s+\boldsymbol{jX}_s$，$\boldsymbol{R}_s$为表面电阻，$\boldsymbol{X}_s$为表面电抗

$$\boldsymbol{R}_s=\boldsymbol{X}_s=\sqrt{\frac{w\mu}{2\gamma}}=\frac{1}{\gamma\delta}$$

$$\boldsymbol{P}_l=\boldsymbol{S}_{av}|_{z=0}=\frac{1}{2}|\boldsymbol{I}|^2\boldsymbol{R}_s=\frac{1}{2}|2\boldsymbol{H}_{i0}|^2\boldsymbol{R}_s$$