

BASIC NEURAL NETWORK

Hand-writing digit recognition

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Principal Component Analysis (PCA) is a dimensionality reduction technique used to reduce high-dimensional data from p dimensions to k dimensions (where $k \leq p$) while minimizing the reconstruction error.

1. Define μ as the mean of the data
2. Centralize the data into y_i
3. Determine the $\arg\min E$ is equal to $\arg\max \text{trace}(U^T \Sigma U)$
4. Determine the U and β_i

$$\mu = \frac{1}{n} \sum_{i=1}^n x_i \quad y_i = x_i - \mu$$

$$U^T U = I_k \quad \beta_i = U^T y_i = \sum_{j=1}^k \langle u_j, y_i \rangle$$

$\arg \min_U \sum_{i=1}^n \|x_i - (\mu + U\beta_i)\|^2$ is equivalent to $\arg \max_U \text{trace}(U^T \Sigma U)$

Spectral Clustering is a technique used in data analysis to group similar items into the same subset or cluster. The key idea behind spectral clustering is to use the eigenvalues and eigenvectors of a similarity matrix to reduce the dimensionality of the data.

1. Construct a similarity matrix S using methods like Gaussian similarity or ϵ -neighborhood
2. Compute the degree matrix D from S , and then compute the Laplacian matrix $L = D - S$
3. Construct the eigenspace and represent the data in low dimensions

- Gaussian Similarity (RBF kernel):

$$S_{ij} = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right)$$

- ϵ -Neighborhood:

$$S_{ij} = \begin{cases} 1 & \text{if } \|x_i - x_j\| < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

$$D_{ii} = \sum_j S_{ij} \quad LU = \Lambda U$$

$$T_{ij} = \frac{U_{ij}}{\sqrt{\sum_j U_{ij}^2}}$$

Convolutional Neural Network (CNN) is a type of deep learning model particularly well-suited for image recognition tasks such as 0-9 handwritten digit recognition. CNNs use convolutional layers to automatically and adaptively learn spatial hierarchies of features from input images.

1. Input layer by receiving features after PCA dimensionality reduction (MNIST dataset)
2. Convolutional layers
3. Pooling Layers
4. Fully connected layers
5. Output layer

$$S(i, j) = (X * K)(i, j) = \sum_m \sum_n X(i + m, j + n) K(m, n)$$

$$P(i, j) = \max_{m, n} X(i + m, j + n)$$

$$y = W \cdot x + b$$

We combine PCA, spectral clustering, and CNN for 0-9 handwritten digit recognition. PCA reduces data dimensionality while preserving its main features. Spectral clustering groups similar data points to capture intrinsic structures. Finally, a CNN extracts features and classifies the data. This multi-stage approach improves model accuracy and efficiency. Additionally, I will implement the process using Python.