

Wavefunction Simulation and Experimental Validation in VCSEL Systems

Wei-Hsiang Pan

Advisor: Prof. Hsing-Chih Liang

July 15, 2025

1 Introduction

Vertical-Cavity Surface-Emitting Lasers (VCSELs) represent a class of semiconductor lasers distinguished by their vertical emission of light from the surface of the wafer, as opposed to traditional edge-emitting configurations. Their compactness, scalability, low power consumption, and compatibility with planar manufacturing processes render them highly suitable for a broad spectrum of applications, ranging from high-speed optical communication and biometric authentication (such as Apple Face ID) to LiDAR and proximity sensing systems. The operational principles of VCSELs intertwine with quantum mechanical behavior, particularly the formation of confined optical modes within a microcavity structure.

Structural Overview of VCSELs

A canonical VCSEL device comprises multiple epitaxially grown layers, which together constitute an optically resonant microcavity. The principal layers include:

- **Top Metal Contact:** Facilitates the injection of current into the upper p-type region.
- **p-type DBR (Distributed Bragg Reflector):** A periodic multilayer stack exhibiting high reflectivity for the operational wavelength.
- **Oxide Aperture Layers:** These layers provide both optical confinement and current localization via selective oxidation.
- **Active Region (MQW, Multiple Quantum Wells):** Quantum-confined structures where radiative recombination of carriers results in photon emission.
- **n-type DBR:** The bottom mirror of the optical cavity, completing the resonant structure.
- **Bottom Metal Contact:** Completes the electrical circuit for forward-bias operation.

Light Emission Mechanism

Under forward bias conditions, electrons and holes are injected into the active MQW region from the n-type and p-type regions, respectively. Carrier recombination within the quantum wells emits photons with energies approximately equal to the bandgap energy:

$$E_\gamma = h\nu = E_c - E_v = E_g \quad (1)$$

These emitted photons are amplified through multiple reflections between the DBR mirrors, enabling coherent stimulated emission predominantly in the vertical direction.

2 Theoretical Framework and Wavefunction Analysis

This study examines the quantum mechanical behavior of confined optical modes in VCSEL microcavities by modeling them using solutions to the time-independent Schrödinger equation in bounded domains. We explore the formation of wavefunctions, eigenstates, and their superpositions to simulate classical-like trajectories and interference paths.

Quantum Mechanical Foundations

The wave-particle duality, central to quantum mechanics, implies that particles can exhibit wave-like properties characterized by their de Broglie wavelength $\lambda = h/p$. The system's dynamics are governed by the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi, \quad \text{with } \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(x) \quad (2)$$

The wavefunction $\psi(x, t)$ encapsulates the probabilistic nature of quantum mechanics, with $|\psi(x)|^2$ representing the spatial probability density. Physical observables, such as momentum and energy, are represented as Hermitian operators acting on ψ .

Wavefunctions in Bounded Domains

Two prototypical geometries relevant to microcavity confinement are the square and circular wells:

- **Square Potential Well:** The eigenfunctions satisfying Dirichlet boundary conditions on a square domain $[0, L]^2$ are given by
$$\psi_{n_x, n_y}(x, y) = \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right), \quad n_x, n_y \in \mathbb{Z}^+ \quad (3)$$

- **Circular Well:** With radial symmetry, the solutions are Bessel functions:

$$\psi_{m,n}(r, \phi) = J_m\left(\frac{\alpha_{mn} r}{a}\right) e^{im\phi}, \quad (4)$$

where α_{mn} denotes the n -th zero of the Bessel function J_m .

These eigenfunctions form a complete orthonormal basis for constructing wave-packets and analyzing modal behavior.

Wave-Packet Construction and Trajectory Emulation

Time-evolving quantum states can be expressed as linear combinations of energy eigenstates:

$$\Psi(x, t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar} \quad (5)$$

By appropriately selecting the coefficients c_n , one can simulate localized wave-packet dynamics that closely follow classical trajectories within the confining geometry.

3 Modal Interference and Eigenlength Analysis

Mathematical Formulation of Modal Superposition

Constructing synthetic wavefunctions using modal combinations allows us to identify dominant interference paths within the cavity:

$$\psi(x) = \sum_{n_x, n_y} A_{n_x, n_y} \exp \left(i \sqrt{n_x^2 + n_y^2} \cdot x \right) \quad (6)$$

These superpositions encode spatial frequencies which, upon Fourier transformation, yield distinct peaks corresponding to eigenlengths.

Fourier Analysis

To extract these eigenlengths:

1. Simulate the spectral response over the target wavelength range (e.g., 800–815 nm).
2. Normalize and convert the wavelength spectrum to wavenumber domain $k = 2\pi/\lambda$.
3. Resample data to a uniform grid and apply a discrete Fourier transform.
4. Analyze the resulting power spectrum to determine dominant spatial frequencies.

The location of spectral peaks provides insight into constructive interference path lengths inside the VCSEL cavity.

4 Experimental Apparatus and Procedure

Setup Description

The experimental validation is performed using the following optical configuration:

- A precision power supply provides current to the VCSEL source.
- The emitted beam passes through an intermediate object to introduce spatial modulation.
- A lens (focal length $f = 15$ mm) focuses the beam onto the detection apparatus.
- A spectrometer captures and resolves the wavelength components of the output.

Measurement Protocol

Spectral measurements are processed as follows:

1. Acquire raw spectral data from the VCSEL output.
2. Normalize the intensity profile.
3. Convert wavelength data to wavenumber domain.
4. Apply interpolation and Fourier analysis.
5. Compare extracted eigenlengths with theoretical predictions.

5 Results and Discussion

The experimental spectrum reveals a well-defined emission band in the range of 800–815 nm. Fourier analysis of the normalized wavenumber spectrum uncovers discrete spatial frequency components, which correspond closely to theoretically predicted eigenlengths derived from modal superposition analysis. This agreement substantiates the validity of our theoretical framework and simulation methodology.

6 Conclusion

By comparing the experimentally measured VCSEL spectrum with the simulated modal interference pattern, we observe a strong correspondence between dominant spectral peaks and the predicted eigenlengths of ideal square and circular resonators. Minor deviations may arise from limited mode orders (n , m), interpolation artifacts, insufficient spectral resolution, or intrinsic energy losses such as cavity absorption and boundary leakage.

These results further support the quantum–classical correspondence, showing that the experimentally observed mode distribution reflects not only theoretical boundary conditions but also the influence of real-world losses and mode coupling within the cavity.

Reference

Yu, Y.-T., Tuan, P.-H., Su, K.-W., & Huang, K.-F. (2013). *52297528592797627dda9663521776846a2164ec6a79d162808a0a606f*, 35(2), 54–70.

Wavefunction Simulation and Experimental Verification in VCSEL Systems



學生:潘偉翔
指導教授:梁興弛 老師

Introduction

VCSEL stands for Vertical-Cavity Surface-Emitting Laser, with its emission direction perpendicular to the wafer surface. The VCSEL structure mainly consists of a top DBR (Distributed Bragg Reflector), an active region, and a bottom DBR. The multiple quantum wells (MQWs) in the middle are responsible for photon generation, while the top and bottom DBRs form the optical cavity, providing resonance and amplification. An oxide layer is typically added to confine both light and current.

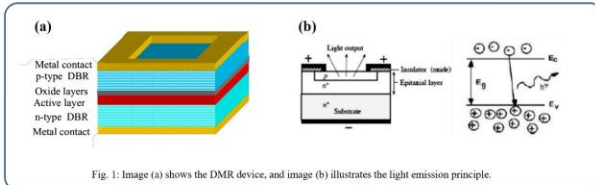


Fig. 1: Image (a) shows the DMR device, and image (b) illustrates the light emission principle.

Principle Overview

Matter-Wave Duality

According to de Broglie's hypothesis, a particle of momentum p exhibits wave like behavior with a wavelength:

$$\frac{h}{p}$$

This concept lays the foundation for describing quantum particles as wavefunctions:

Time-independent Schrödinger Equation
The evolution of the quantum state $\psi(x,t)$ is governed by:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

This equation dictate how wavefunctions evolve in time under the influence of a Hamiltonian operator.

Hamiltonian Operator

The Hamiltonian \hat{H} describes total energy as defined as:

$$\hat{H} = \frac{1}{2m} \hat{p}^2 + V(x)$$

Time-dependent Schrödinger Equation

The evolution of quantum state $\psi(x,t)$ governed by:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

This equation dictate how wavefunctions evolve in time under the influence of a Hamiltonian operator.

Time-independent Schrödinger Equation
For systems with static potentials, stationary states (eigenstates) satisfy:

$$\hat{H} \psi_n = E_n \psi_n$$

Each solution ψ_n corresponds to a standing wave with quantized energy E_n .

Wave Packet Superposition

General quantum states can be constructed by superposing eigenstates:

From Modes to Trajectories

This section presents the simulated eigenstates and their associated classical trajectories under ideal boundary conditions. These theoretical results serve as a reference for comparing with experimental data such as observed emission patterns or Fourier-transformed spectra.

1. define how wavefunctions behave at the edges and ensure physically valid solutions. Different geometries (square, circular) lead to different sets of allowed wave modes.

- Boundary conditions for a square geometry

$$\text{At } 0 \leq x \leq L, 0 \leq y \leq L \quad \psi(0,y)=\psi(L,y)=\psi(x,0)=\psi(x,L)=0$$

- Bessel boundary conditions on a circular domain

$$\text{At } 0 \leq r \leq a, 0 \leq \theta \leq 2\pi \quad \psi(r=a,\theta)=0$$

2. An eigenstate is a standing wave that satisfies both the Schrödinger equation and boundary conditions. It represents a stable quantum mode with a specific energy.

- Eigenstates in a square domain

$$\psi_{nm}(x,y) = \sin(n\pi x/L) \sin(m\pi y/L)$$

- Eigenstates in a circular domain

$$\psi_{nm}(r,\phi) = J_n(n_{nm}r/a) e^{im\phi}$$

3. In the semiclassical regime, where the quantum numbers are large, eigenstates begin to exhibit spatial patterns that strongly resemble classical particle paths. This is known as quantum-classical correspondence.

4. As quantum numbers grow, the system behaves more classically. Superposition of many modes forms localized wave packets that follow classical trajectories.

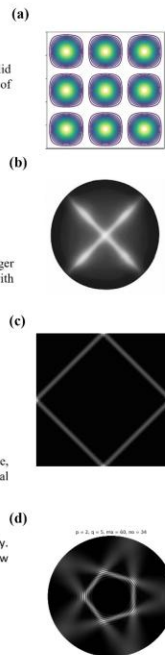
Fig. 1: Simulations demonstrating quantum-classical correspondence.

(a) Simulated eigenmode intensity in a square cavity.

(b) Far-field interference pattern from circular cavity modes.

(c) Classical trajectory reflection in a square domain.

(d) Wave packet formed by multi-mode superposition following a classical path.



Experimental simulation

Constructing Eigenlengths

1. Simulation Objective

To identify dominant interference path lengths (eigenlengths) from modal combinations.

2. Simulation Method

Construct total field using modal phase sum:

$$\psi(x) = \sum \exp(i \sqrt{n_x^2 + n_y^2} \cdot x)$$

3. Fourier Transform

Experimental spectra are Fourier-transformed to extract effective path lengths for comparison.

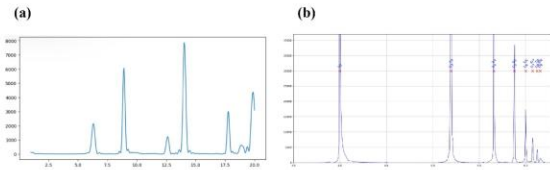


Fig. 3 : Simulated eigenlength spectra under ideal boundary conditions: (a) Square cavity modes. (b) Circular (Bessel) modes.

Experimental setup

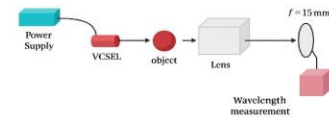


Fig.4 : Schematic of the experimental setup showing VCSEL emission driven by a power supply, passing through an object and lens ($f = 15 \text{ mm}$) before wavelength measurement.

Experimental result

This script processes the experimental VCSEL spectrum (800-815 nm), normalizes the intensity, and prepares the data for interpolation and Fourier analysis in the wavenumber domain.

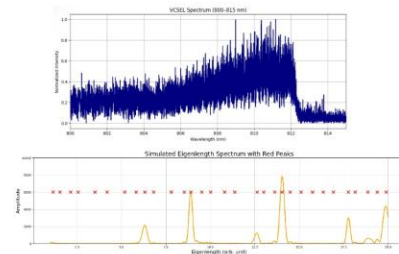


Fig.5 : Eigenlength Extraction Procedure :

1. Normalize the VCSEL spectrum.
2. Convert wavelength to wavenumber.
3. Interpolate to uniform spacing.
4. Perform Fourier transform.
5. Extract dominant eigenlengths from the result.

Conclusion

By comparing the experimentally measured VCSEL spectrum with the simulated modal interference pattern, we observe a strong correspondence between dominant spectral peaks and the predicted eigenlengths of ideal square and circular resonators. Minor deviations may arise from limited mode orders (n, m), interpolation artifacts, insufficient spectral resolution, or intrinsic energy losses such as cavity absorption and boundary leakage.

These results further support the quantum-classical correspondence, showing that the experimentally observed mode distribution reflects not only theoretical boundary conditions but also the influence of real-world losses and mode coupling within the cavity.

Reference

Yu, Y.-T., Tuan, P.-H., Su, K.-W., & Huang, K.-F. (2013). 利用大面積面射型雷射類比研究二維量子彈子球壘的特徵. 科儀新知, 35(2), 54-70.