Machine Learning

Deep Learning

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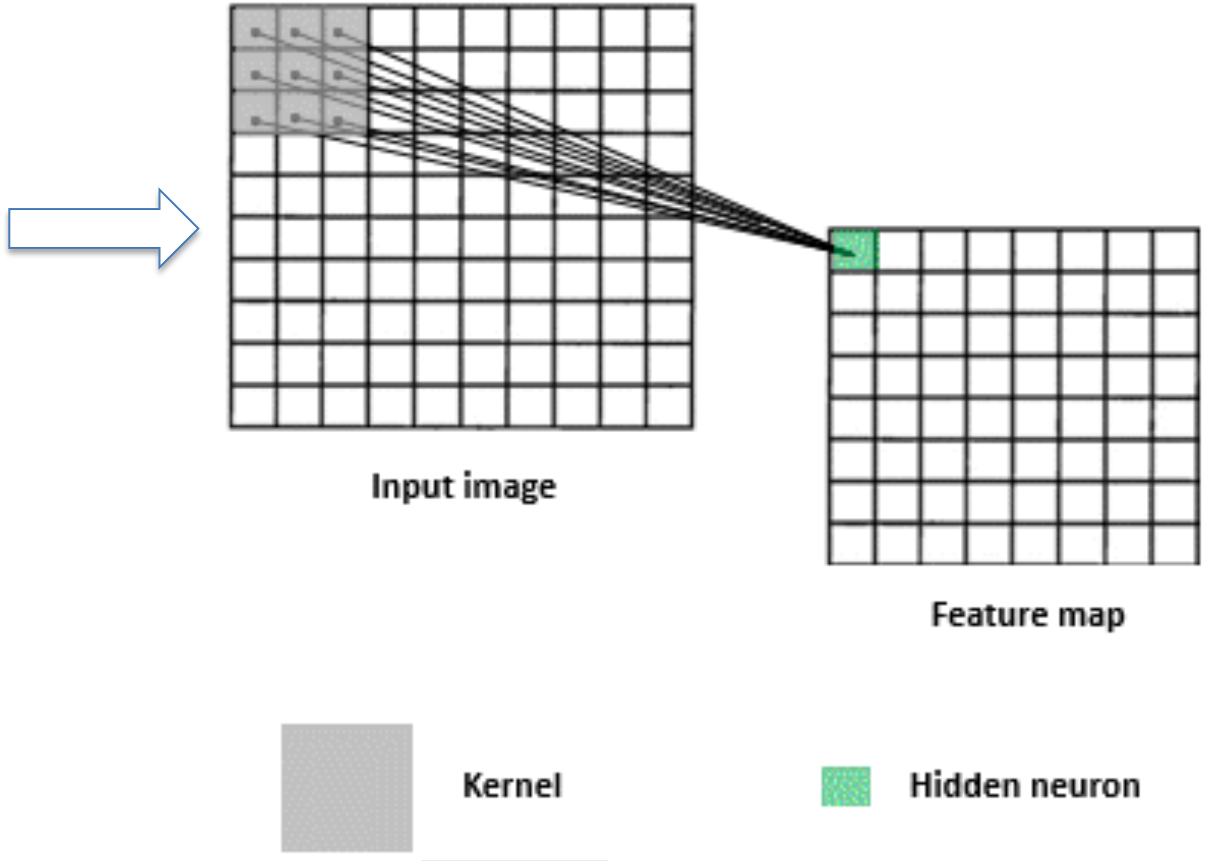
Acknowledge: The slides are partially referred to the online http:indoml.com (https://indoml.com/2018/03/07/student-notes-convolutional-neural-networks-cnn-introduction/) and Prof.

CNN Architectures

- Basics of CNN
- Classic Networks

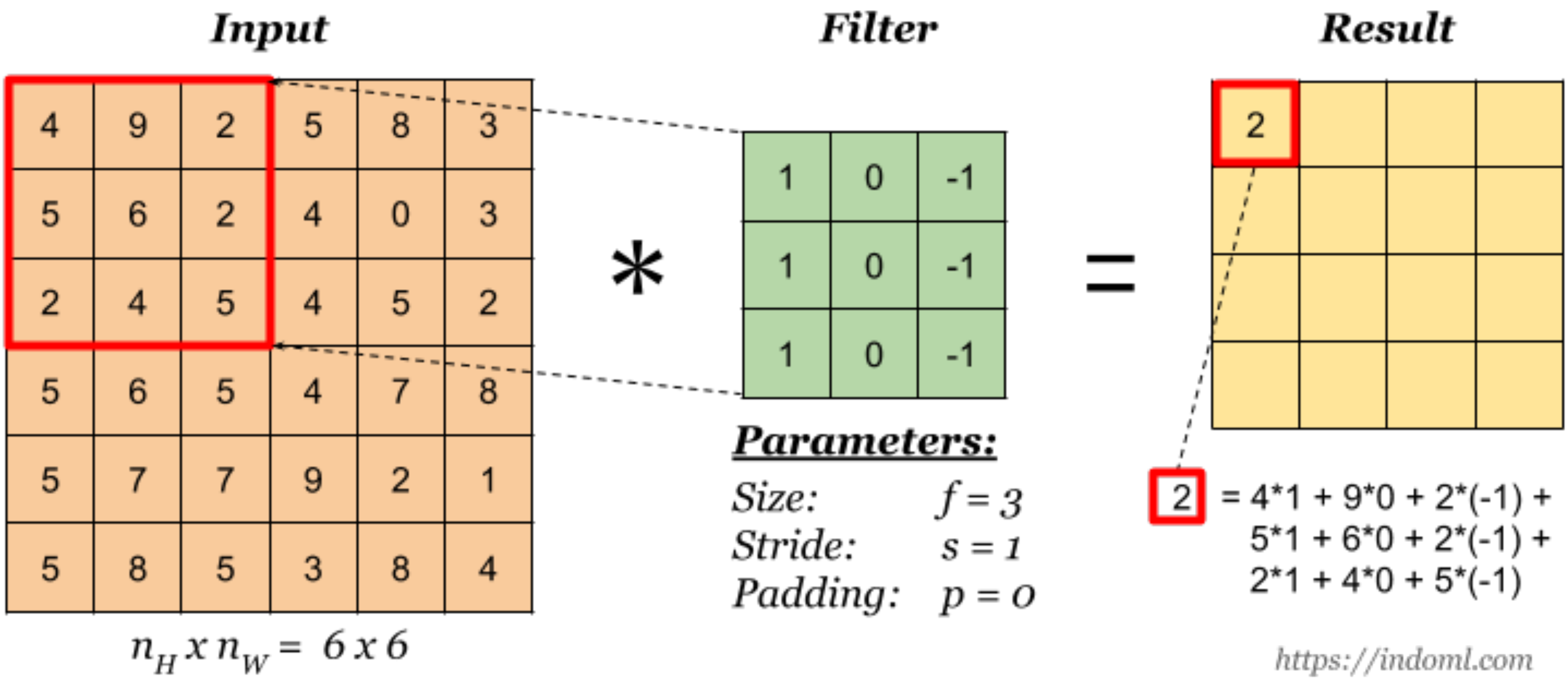
Standard Convolution Operation



Source: https://www.ahmedbesbes.com/blog/introduction-to-cnns

CNN Basics

Basic Convolution Operation



Input **Filter** **Result**

$$\begin{bmatrix} 4 & 9 & 2 & 5 & 8 & 3 \\ & 6 & 2 & 4 & 0 & 3 \\ 2 & 4 & 5 & 4 & 5 & 2 \\ 5 & 6 & 5 & 4 & 7 & 8 \\ 5 & 7 & 7 & 9 & 2 & 1 \\ 5 & 8 & 5 & 3 & 8 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & \boxed{6} & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

$n_H \, x \, n_W = \; 6 \, x \, 6$

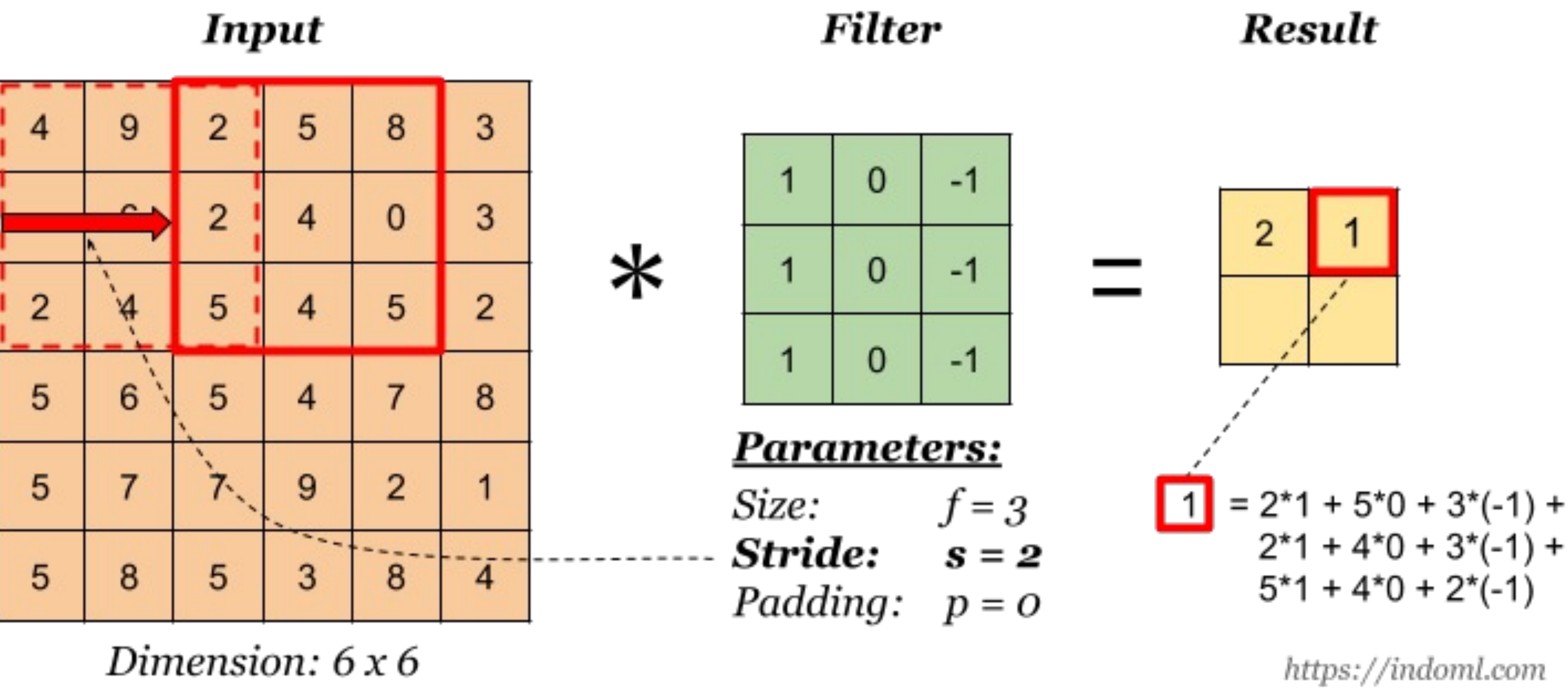
Parameters:

Size: $f = 3$
Stride: $s = 1$
Padding: $p = 0$

$\boxed{6}$ = 9*1 + 2*0 + 5*(-1) +
6*1 + 2*0 + 4*(-1) +
4*1 + 5*0 + 4*(-1)

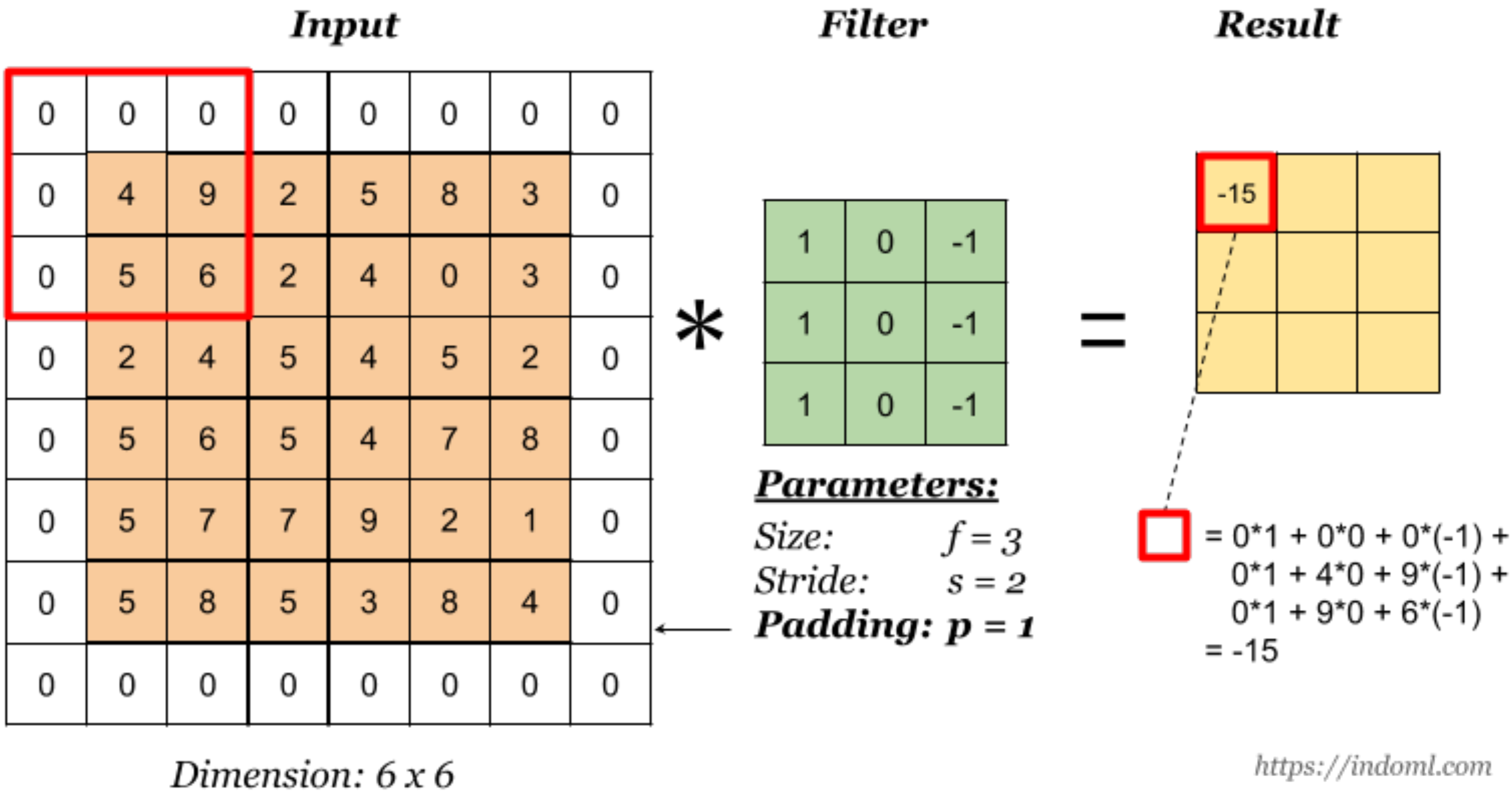
https://indoml.com

Stride:

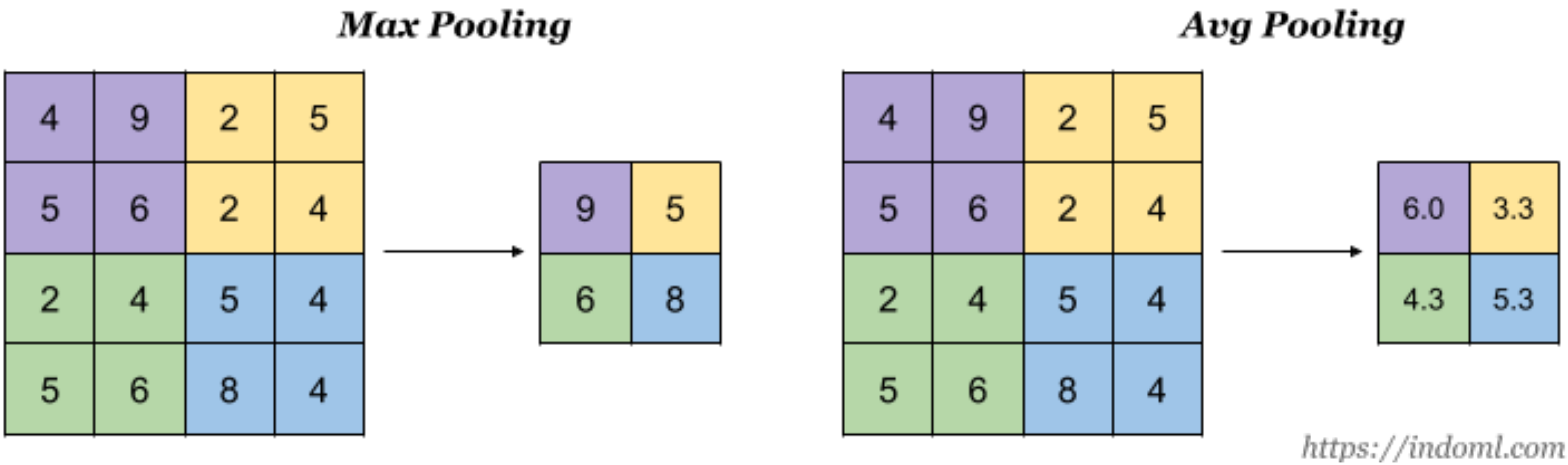


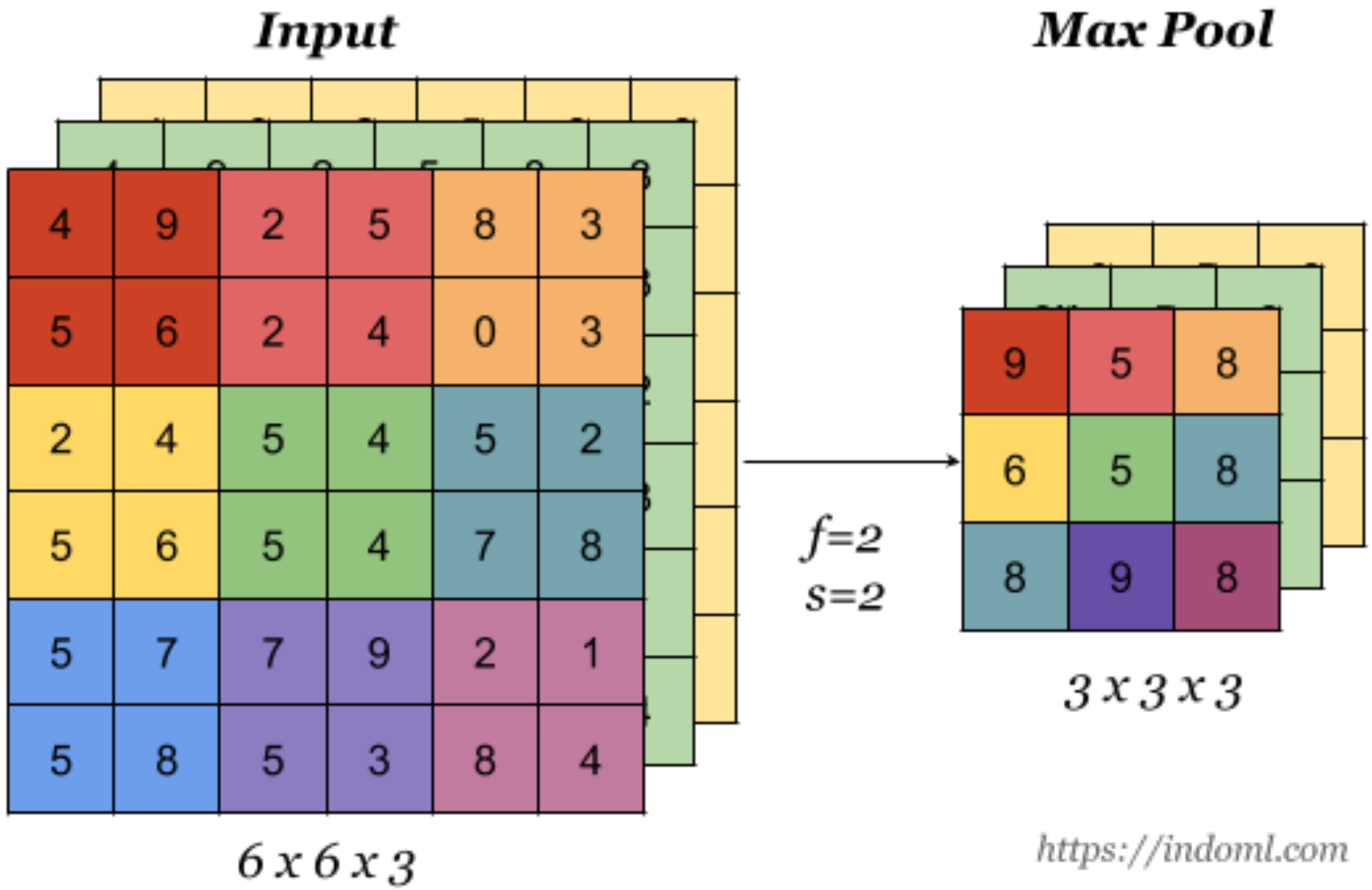
Input
4 9 2 5 8 3
2 4 0 3
2 4 5 4 5 2
5 6 5 4 7 8
5 7 7 9 2 1
5 8 5 3 8 4
Dimension: 6 x 6
*
Filter
1 0 -1
1 0 -1
1 0 -1
Parameters:
Size: f = 3
Stride: s = 2
Padding: p = 0
=
Result
2 1
1 = 2*1 + 5*0 + 3*(-1) +
2*1 + 4*0 + 3*(-1) +
5*1 + 4*0 + 2*(-1)
https://indoml.com

Padding:


Input
Filter
Result
0 0 0 0 0 0 0 0
0 4 9 2 5 8 3 0
0 5 6 2 4 0 3 0
0 2 4 5 4 5 2 0
0 5 6 5 4 7 8 0
0 5 7 7 9 2 1 0
0 5 8 5 3 8 4 0
0 0 0 0 0 0 0 0
*
1 0 -1
1 0 -1
1 0 -1
=
-15
Parameters:
Size: f = 3
Stride: s = 2
Padding: p = 1
= 0*1 + 0*0 + 0*(-1) +
0*1 + 4*0 + 9*(-1) +
0*1 + 9*0 + 6*(-1)
= -15
Dimension: 6 x 6
https://indoml.com

Pooling layer:




Input
4 9 2 5 8 3
5 6 2 4 0 3
2 4 5 4 5 2
5 6 5 4 7 8
5 7 7 9 2 1
5 8 5 3 8 4
6 x 6 x 3
f=2
s=2
Max Pool
9 5 8
6 5 8
8 9 8
3 x 3 x 3
https://indoml.com

Softmax layer:

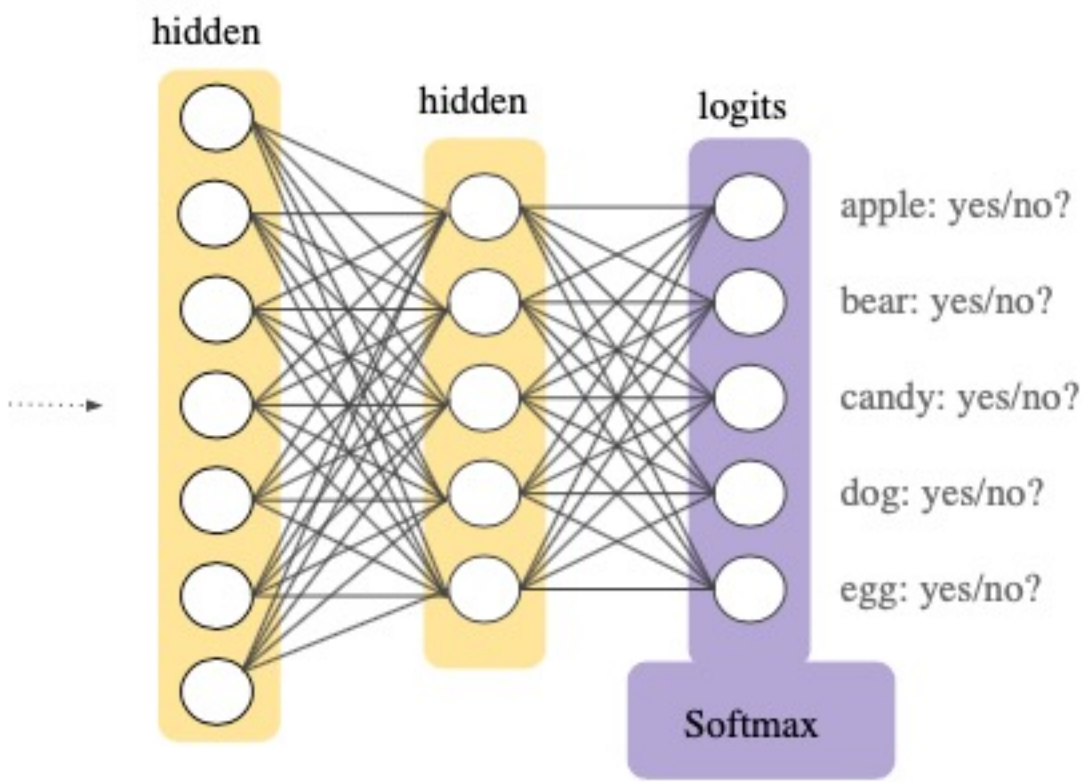
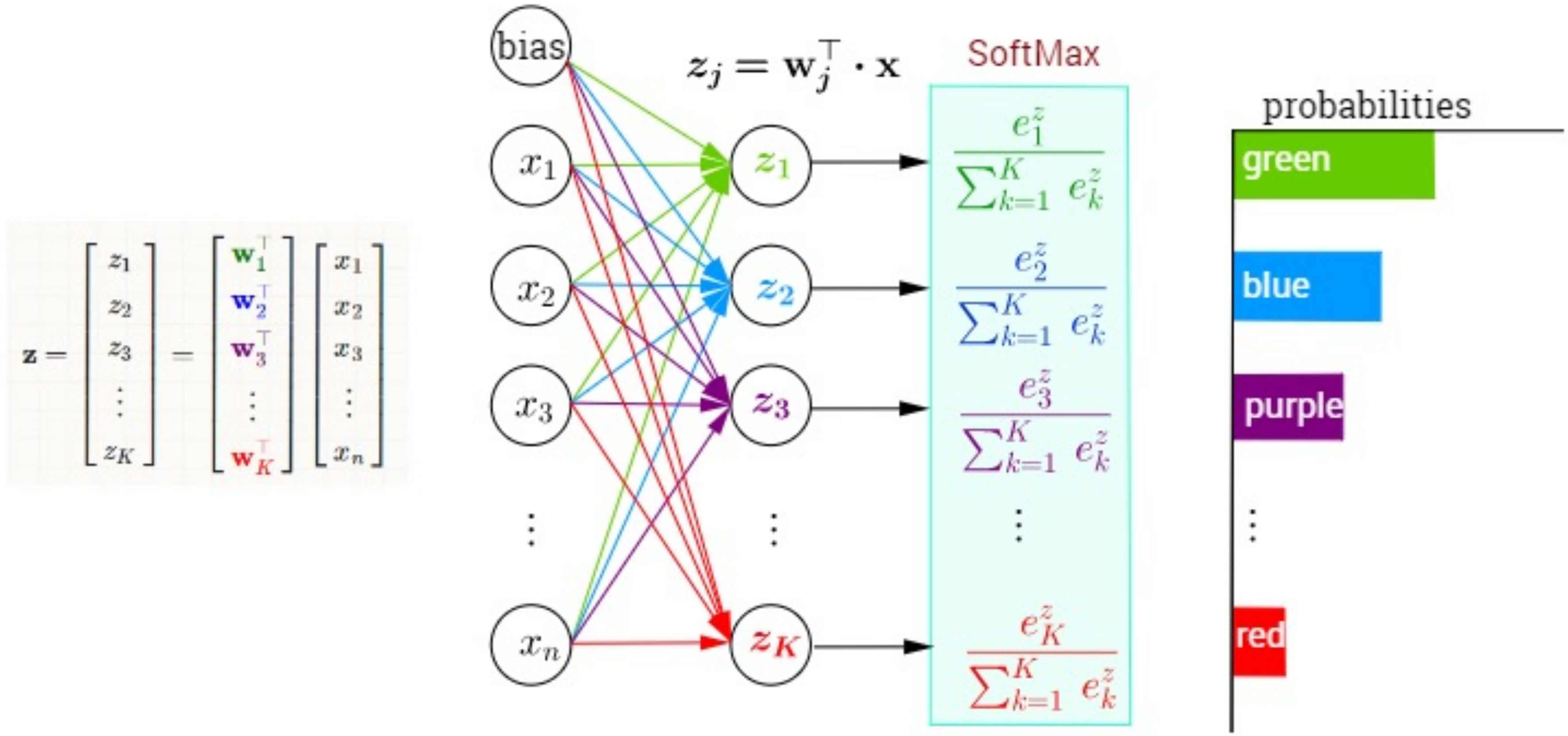
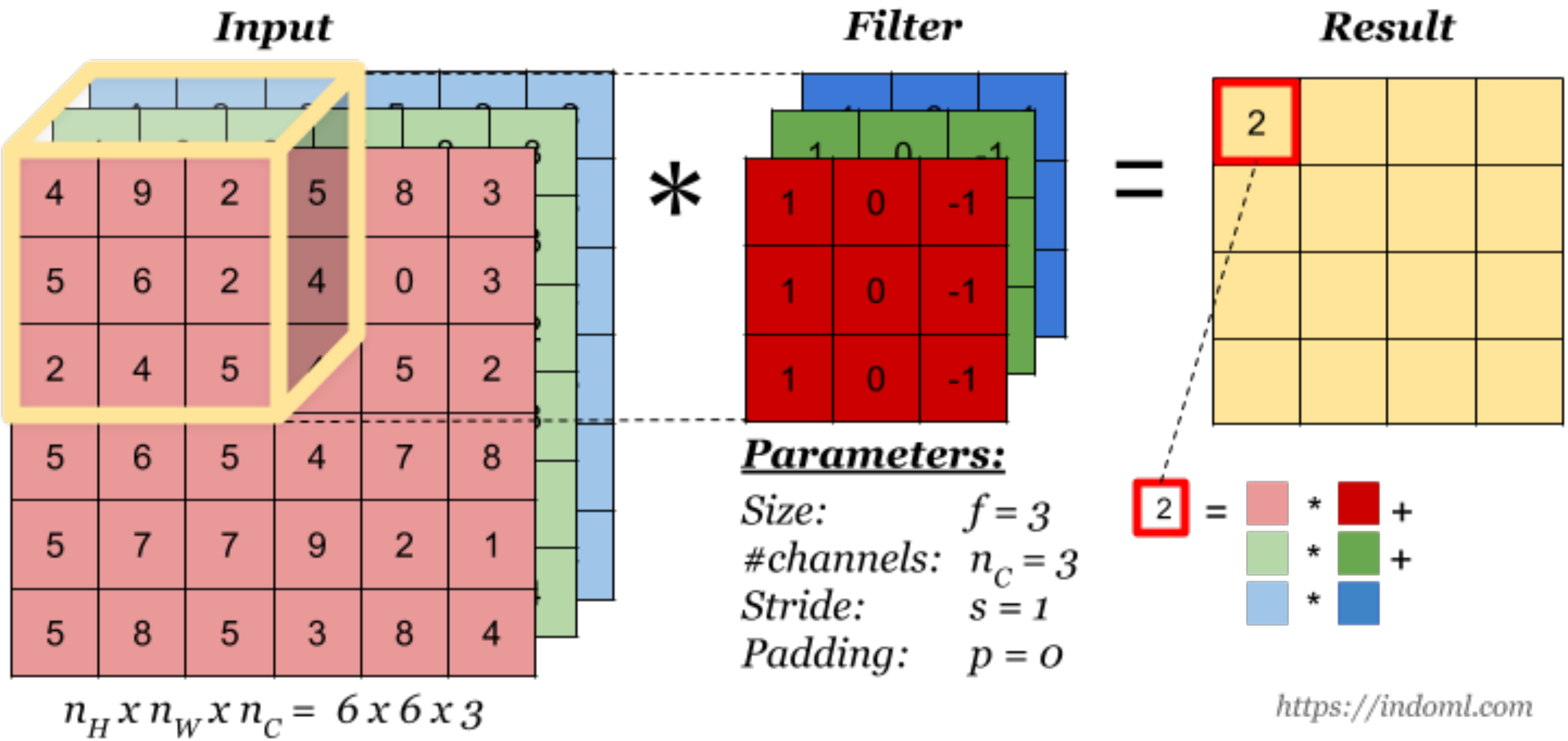


Image source: https://developers.google.com/machine-learning/crash-course/multi-class-neural-networks/softmax

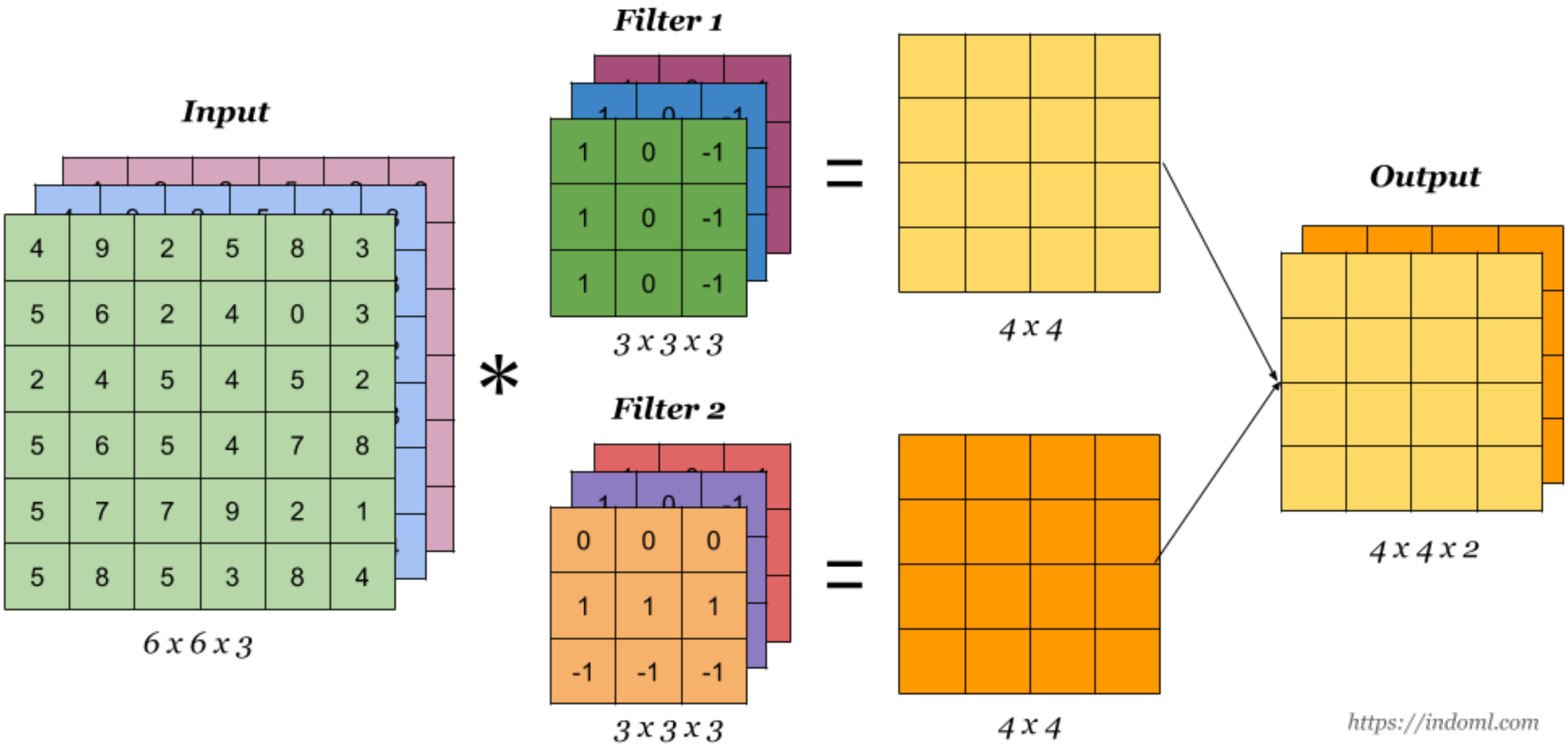


Source: https://stats.stackexchange.com/questions/273465/neural-network-softmax-activation

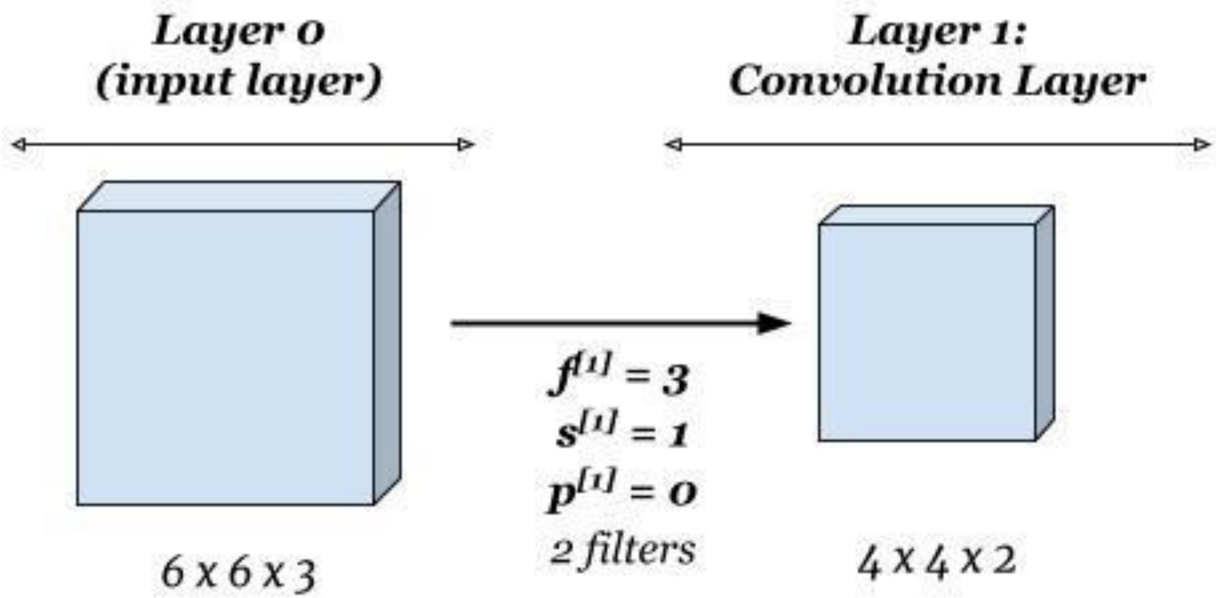
CNN on Volume (Tensors)



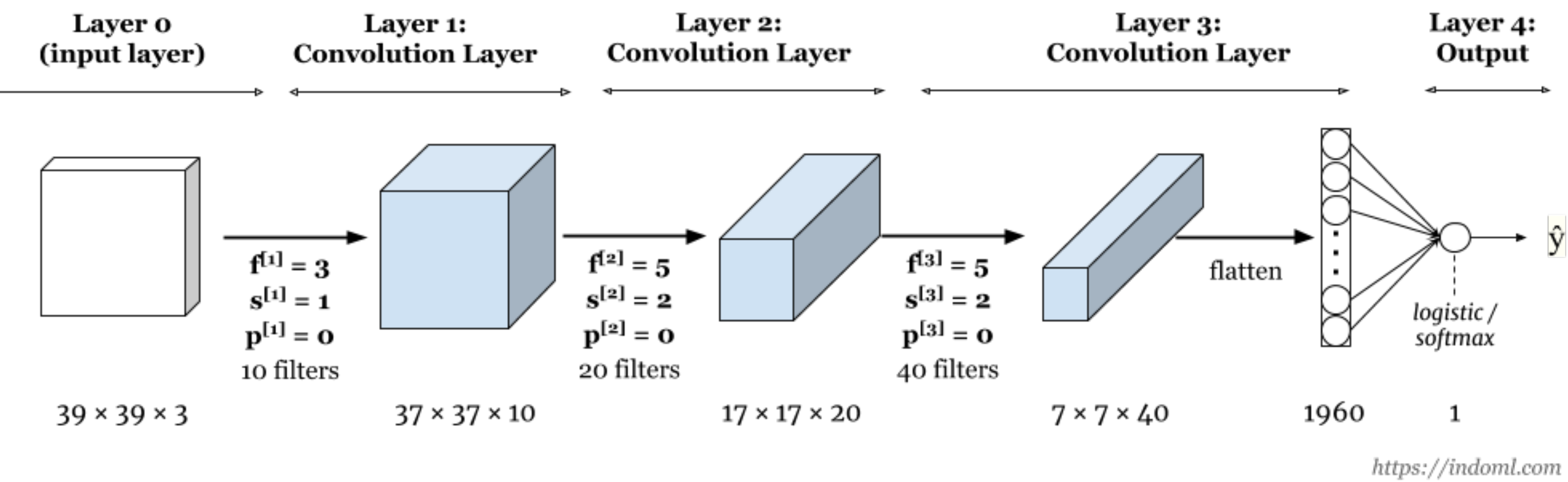
Multiple Filters



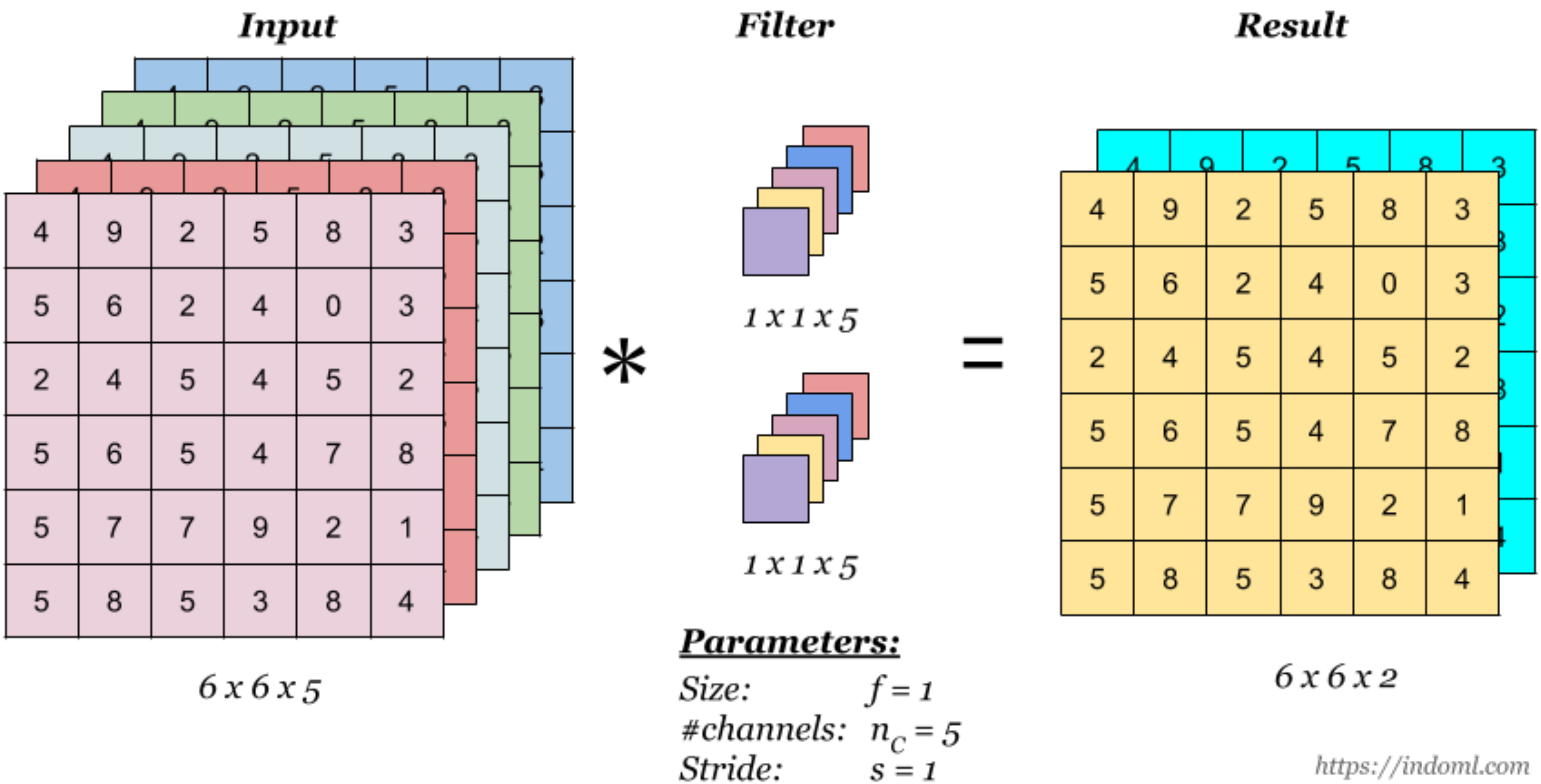
Shorthand Representation



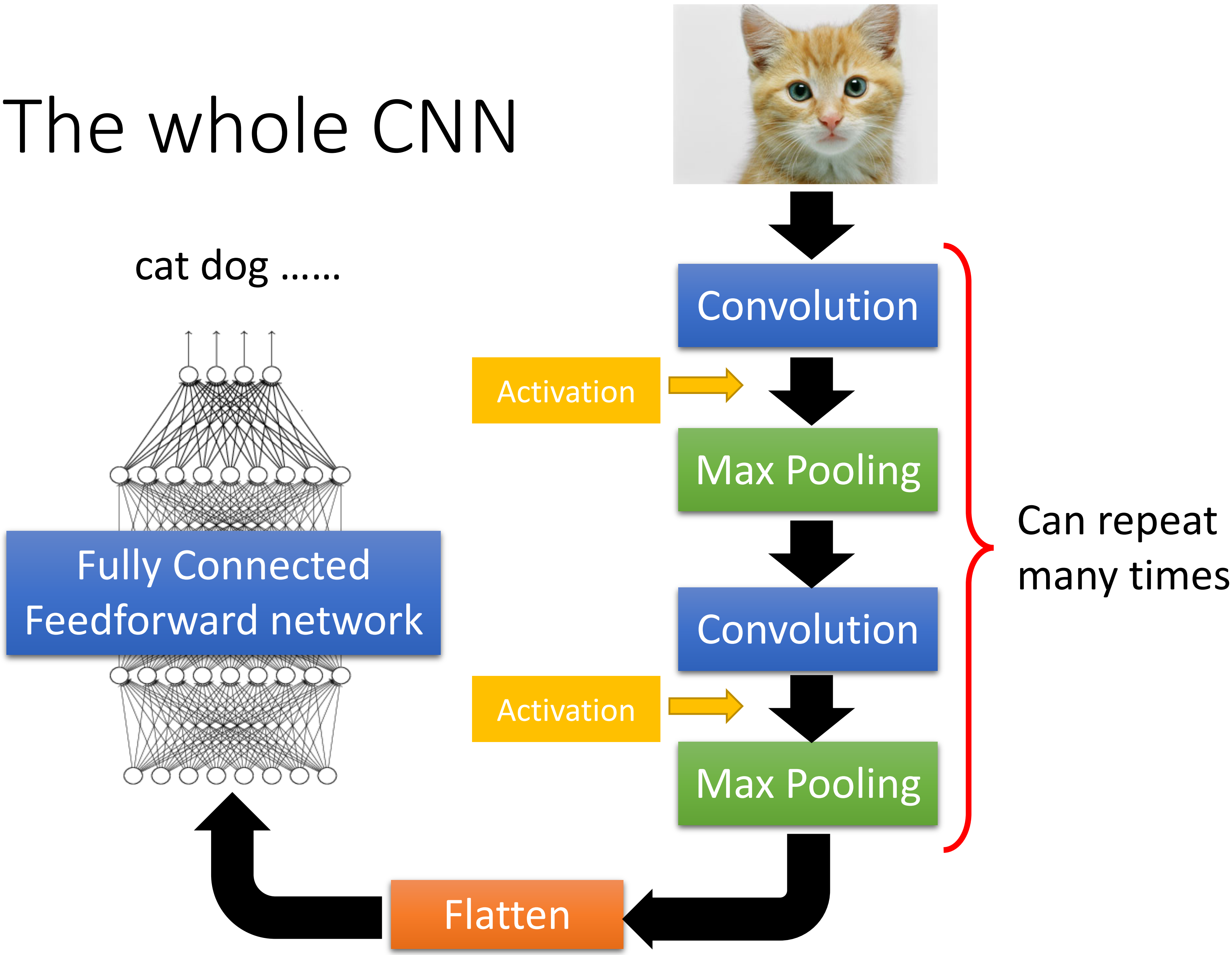
Sample Network Structure



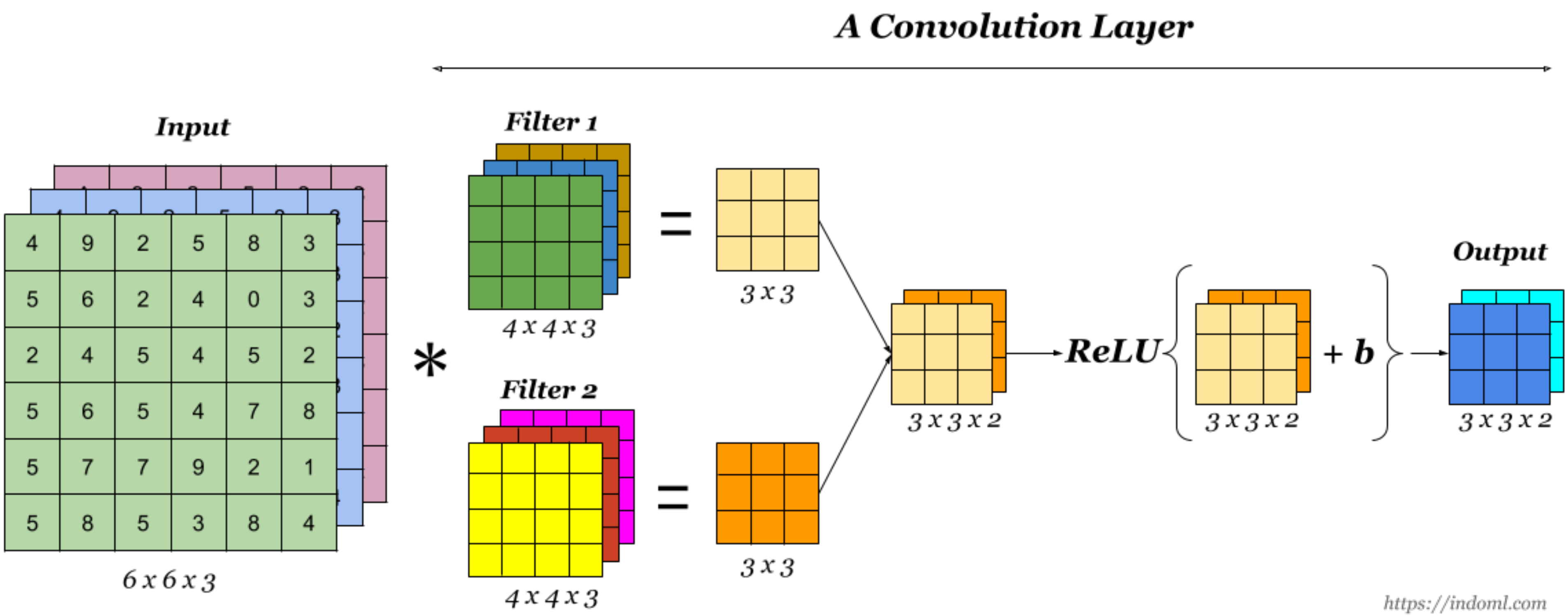
1*1 Convolutions



The whole CNN

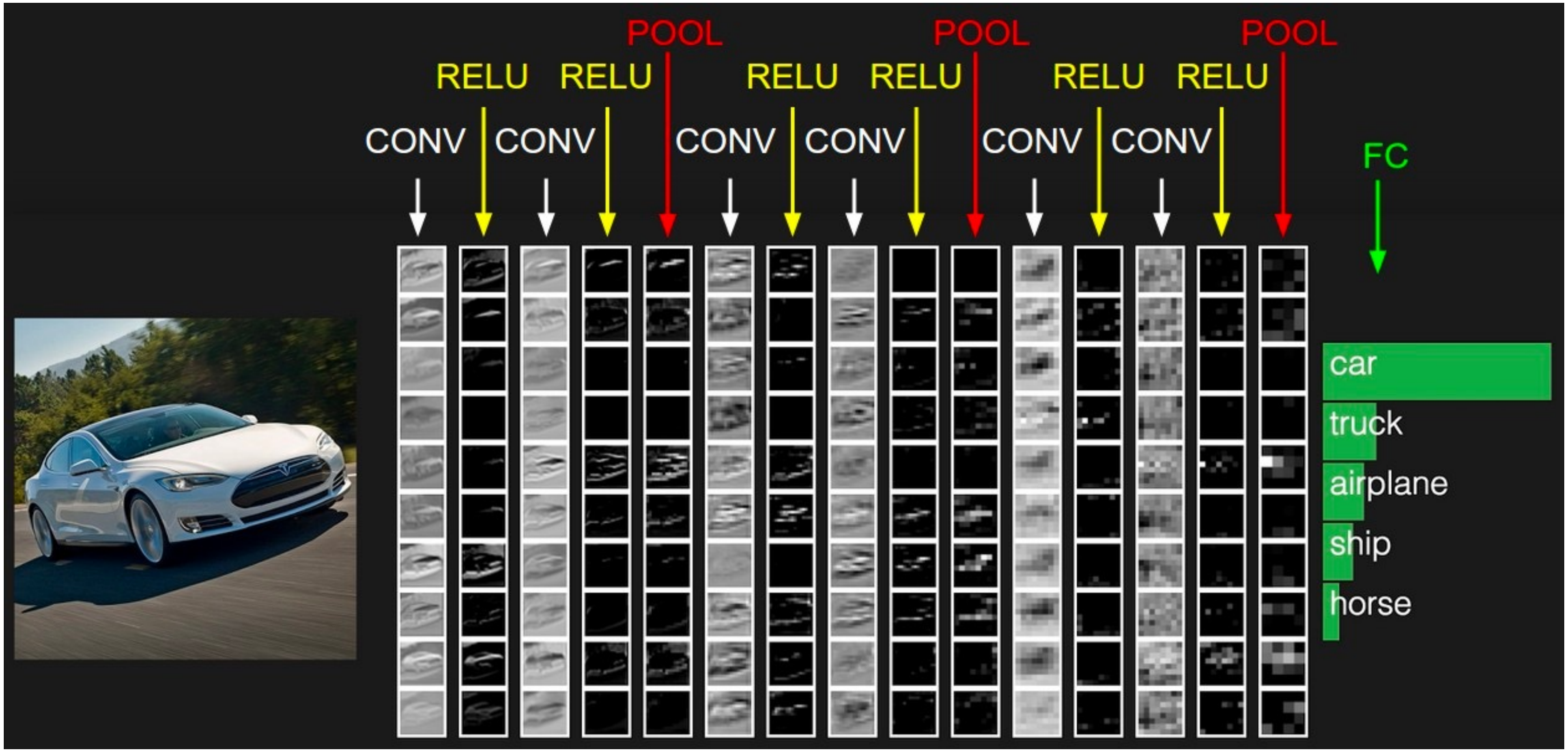


One Layer Representation



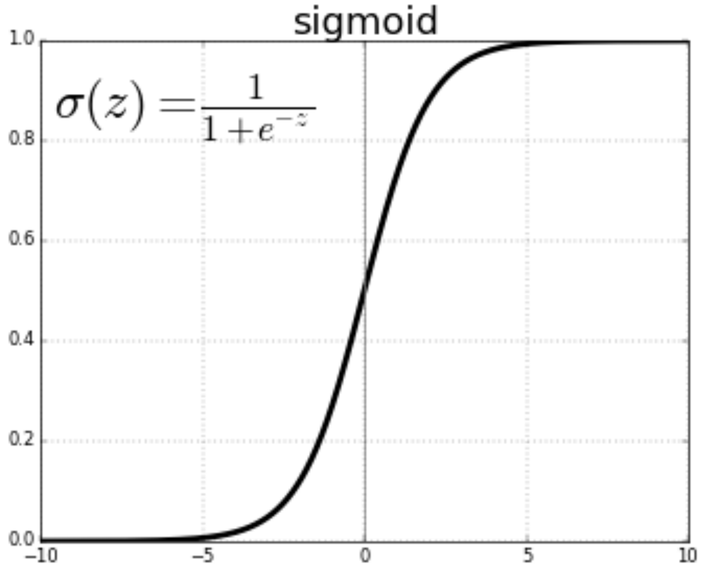
Convolution Neural Network

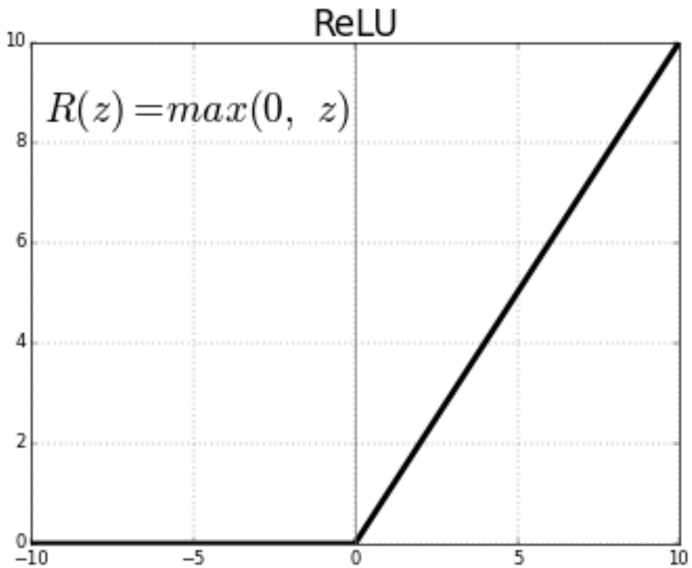
- An example of ConvNet architecture.

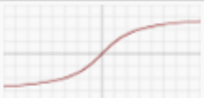
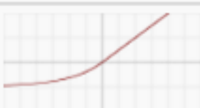


Activation Functions

- Sigmoid Function
- ReLU Function
- More

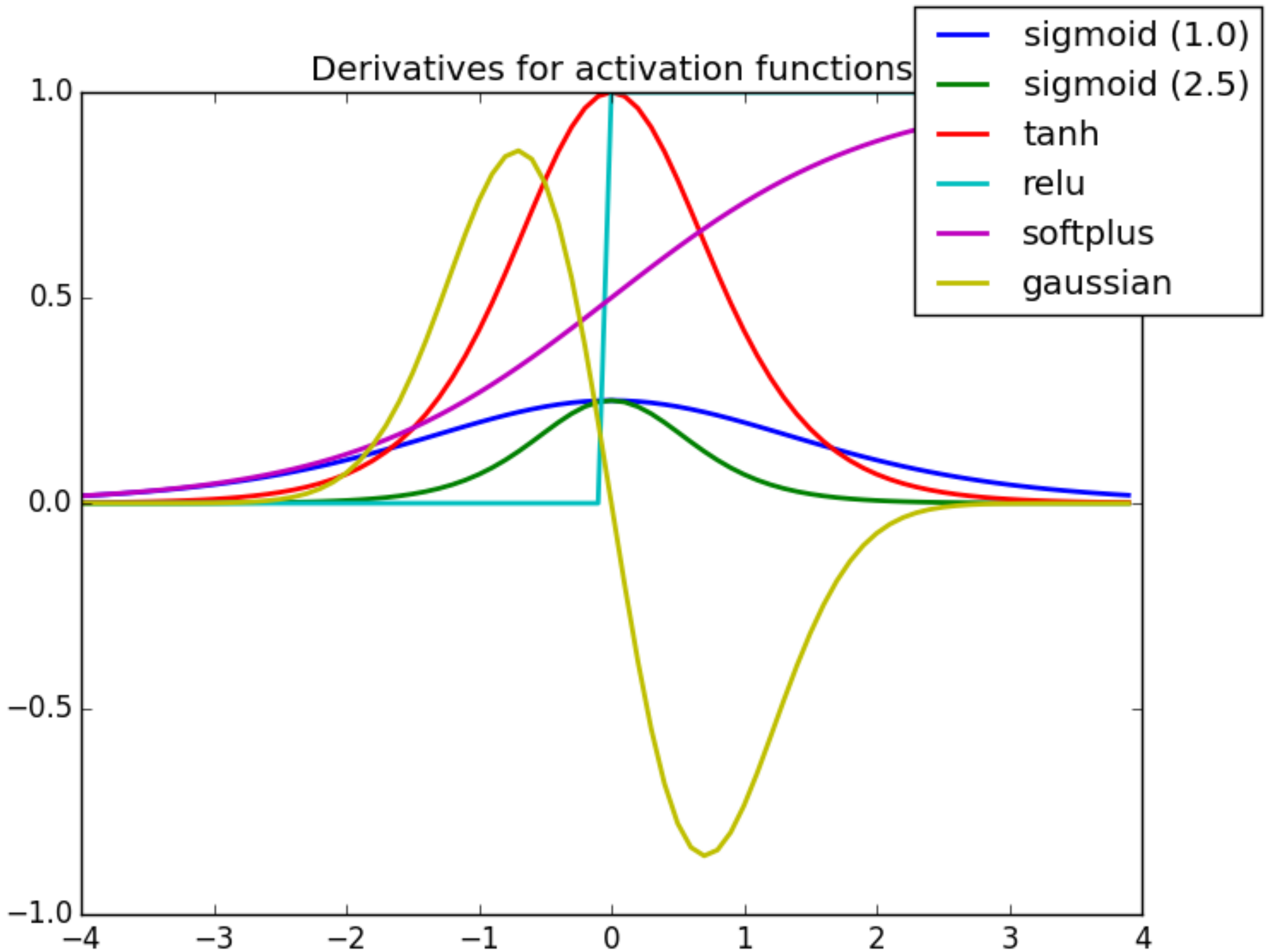




Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU)[2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU)[3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$

Source: https://towardsdatascience.com/activation-functions-neural-networks-1cbd9f8d91d6

Derivatives of Activation Functions

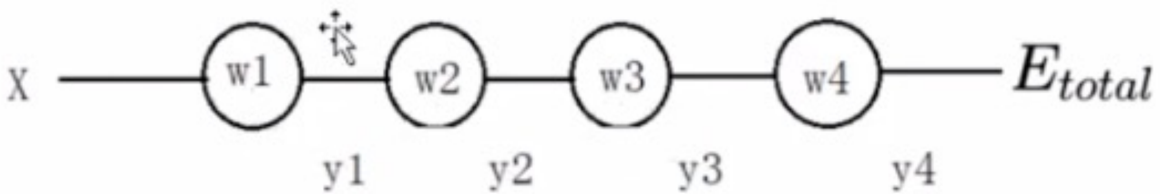


Source: https://www.picswe.com/pics/relu-activation-7d.html

Gradient Vanishing and Exploding

$$w_1^+ = w_1 - \eta \frac{\partial E_{total}}{\partial w_1}$$

$$y_i = \sigma(z_i) = \sigma(w_i x_i + b_i)$$



$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial y_4} \frac{\partial y_4}{\partial z_4} \frac{\partial z_4}{\partial x_4} \frac{\partial x_4}{\partial z_3} \frac{\partial z_3}{\partial x_3} \frac{\partial x_3}{\partial z_2} \frac{\partial z_2}{\partial x_2} \frac{\partial x_2}{\partial z_1} \frac{\partial z_1}{\partial w_1}$$

$$= \frac{\partial E_{total}}{\partial y_4} \sigma'(z_4) w_4 \sigma'(z_3) w_3 \sigma'(z_2) w_2 \sigma'(z_1) x_1$$

Data Normalization for Deep Learning training

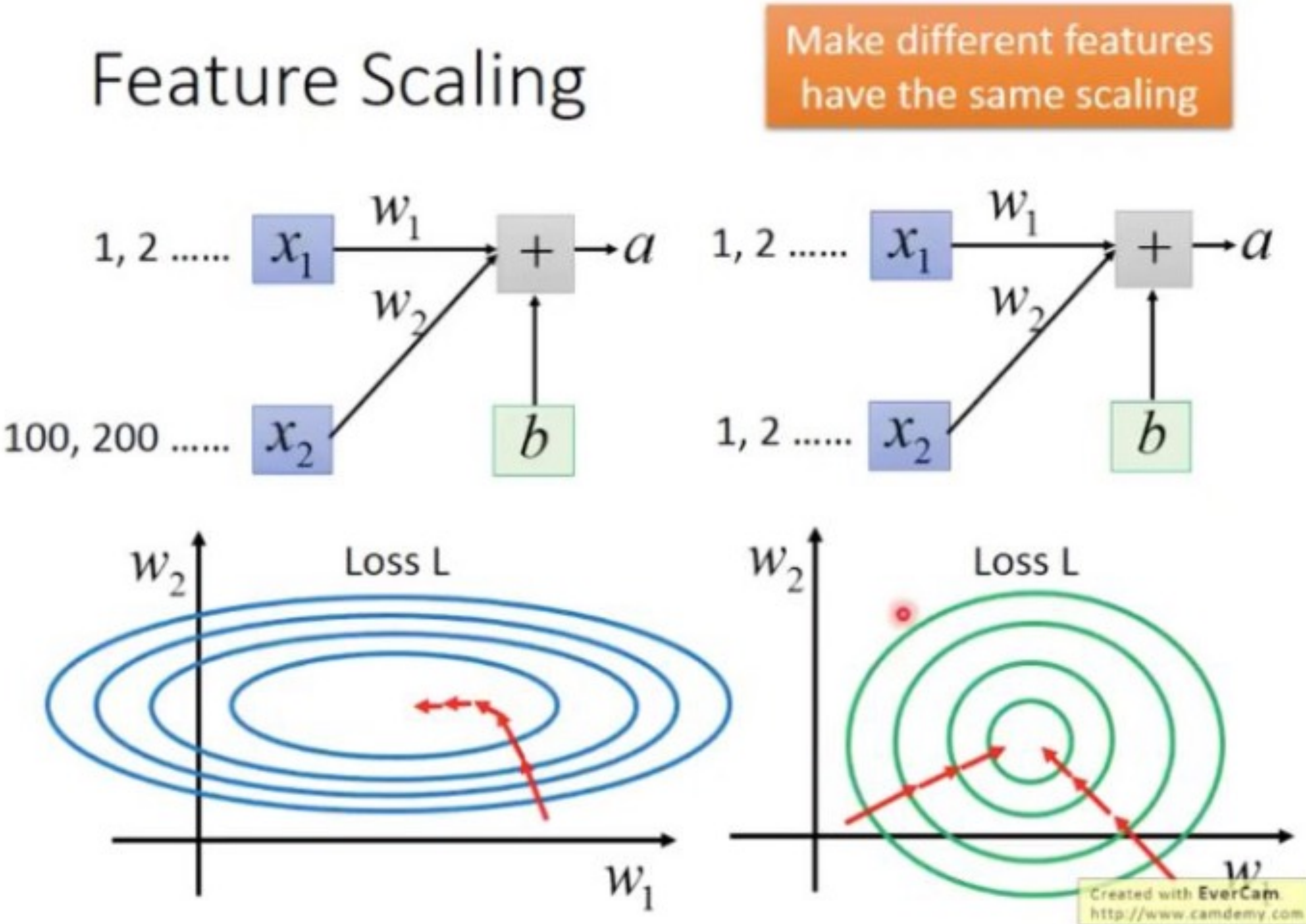
Batch Normalization

- Batch normalization speeds up the training by setting high learning rate

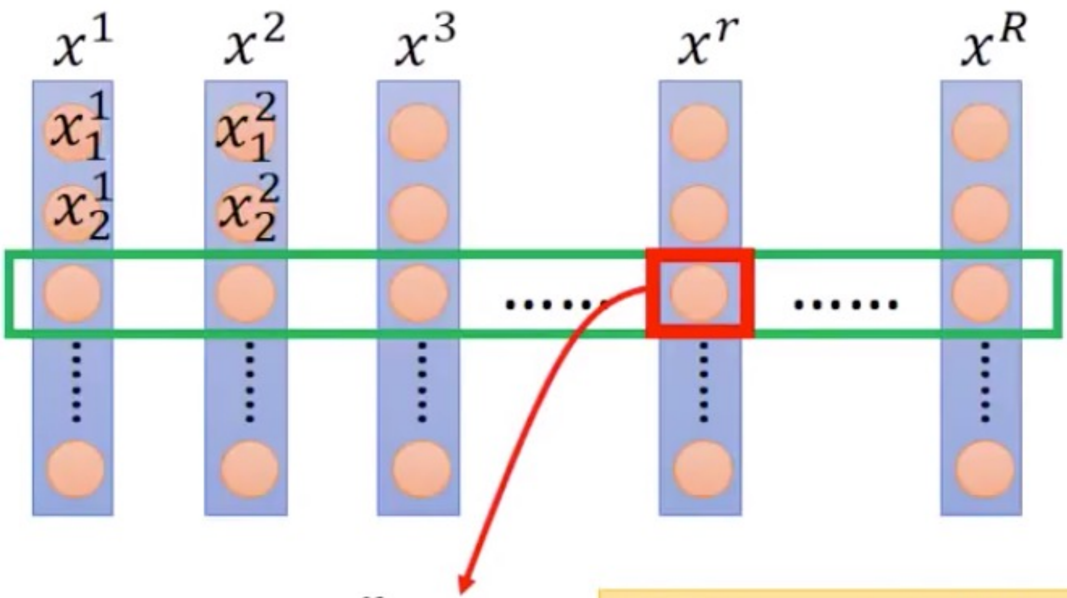
- Batch normalization prevents the gradients from getting too small or too large by normalizing data across each batch, as the name suggests. It also acts as a regularization method, similar to dropout.

Cited: https://zhuanlan.zhihu.com/p/34480619 and Prof. Lee from TW Univ.

Feature Scaling



Feature Scaling



For each dimension i:

mean: m_i

standard deviation: σ_i

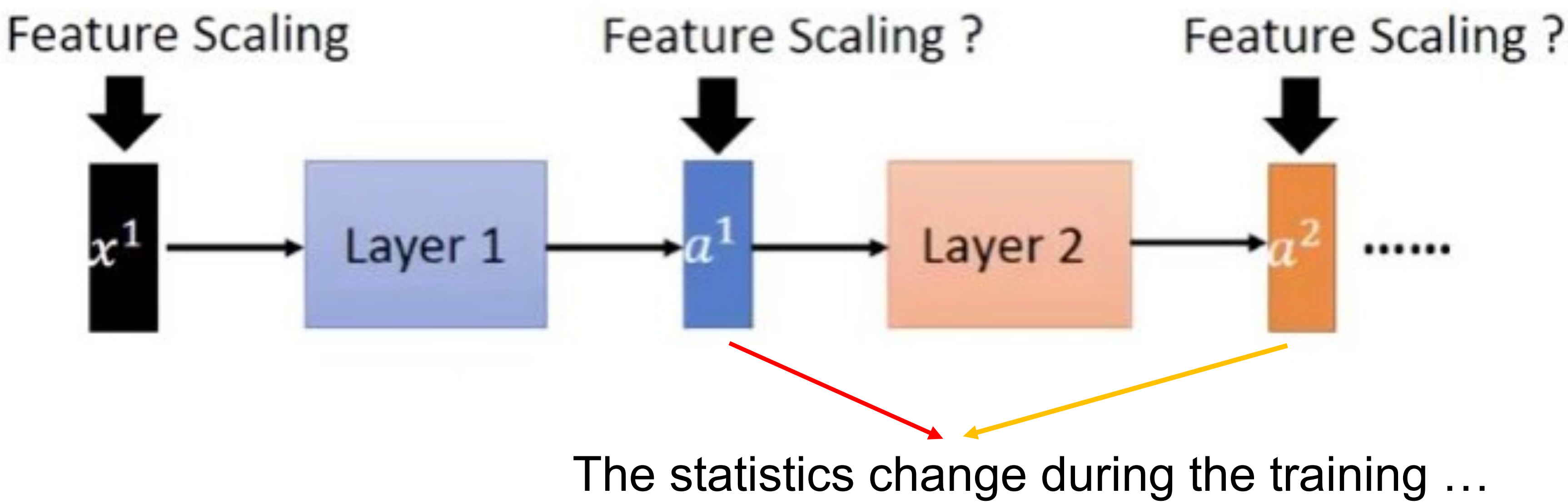
$$x_i^r \leftarrow \frac{x_i^r - m_i}{\sigma_i}$$

The means of all dimensions are 0, and the variances are all 1

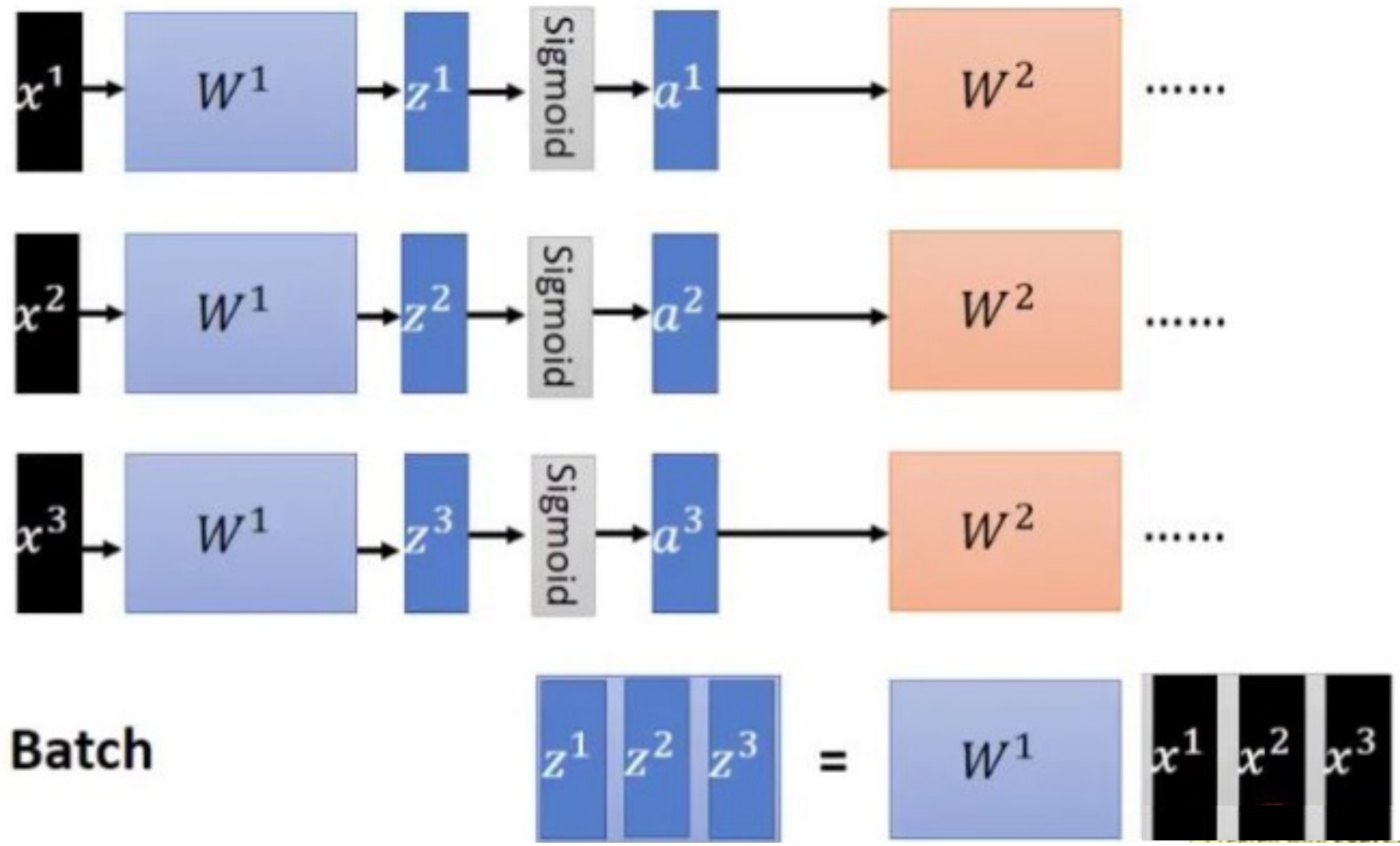
In general, gradient descent converges much faster with feature scaling than without it.

Scaling at Hidden Layer

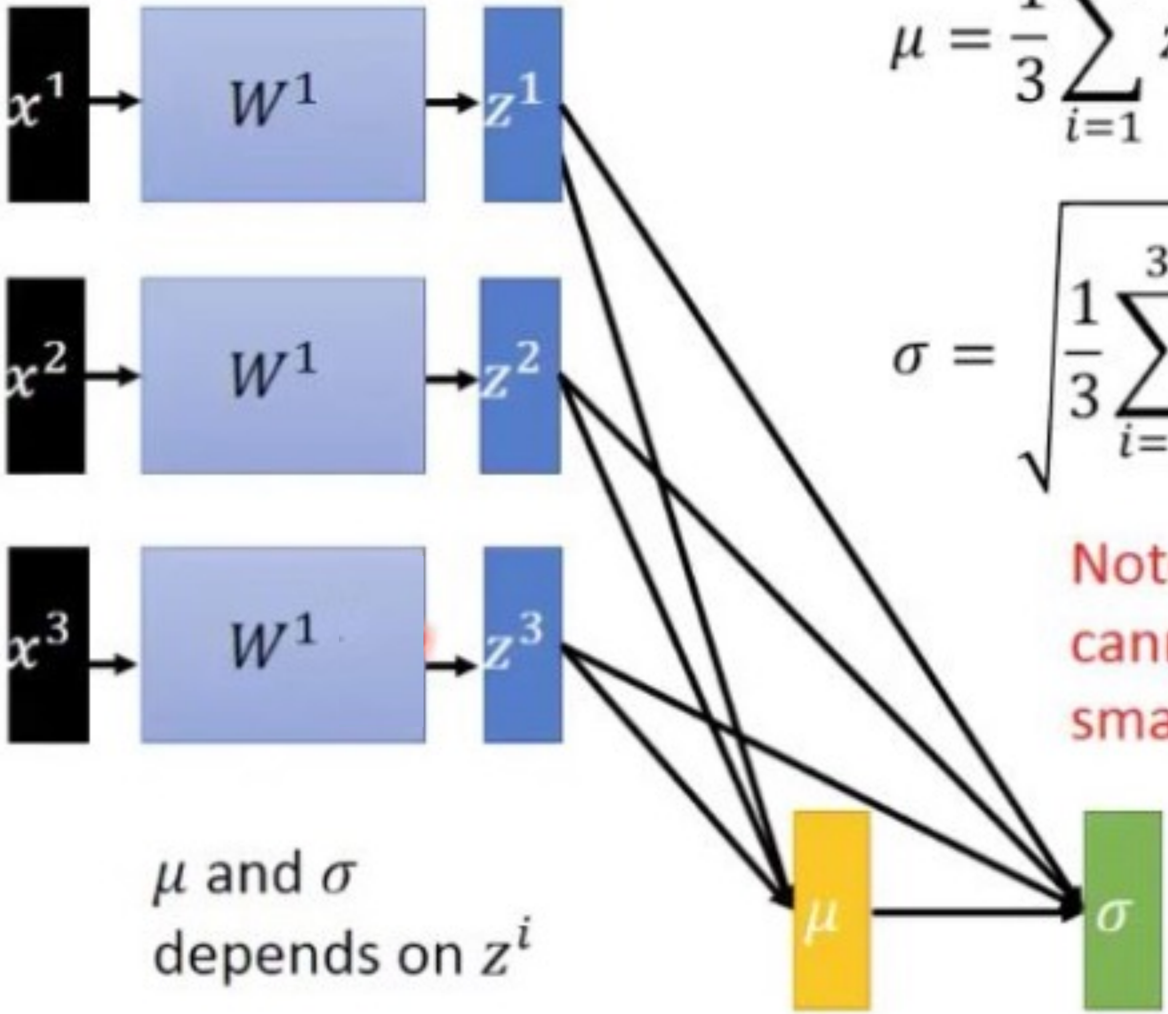
- Internal Covariate Shift



Batch of Data



Batch normalization



$$\mu = \frac{1}{3}\sum_{i=1}^{3} z^i$$

$$\sigma = \sqrt{\frac{1}{3}\sum_{i=1}^{3}(z^i - \mu)^2}$$

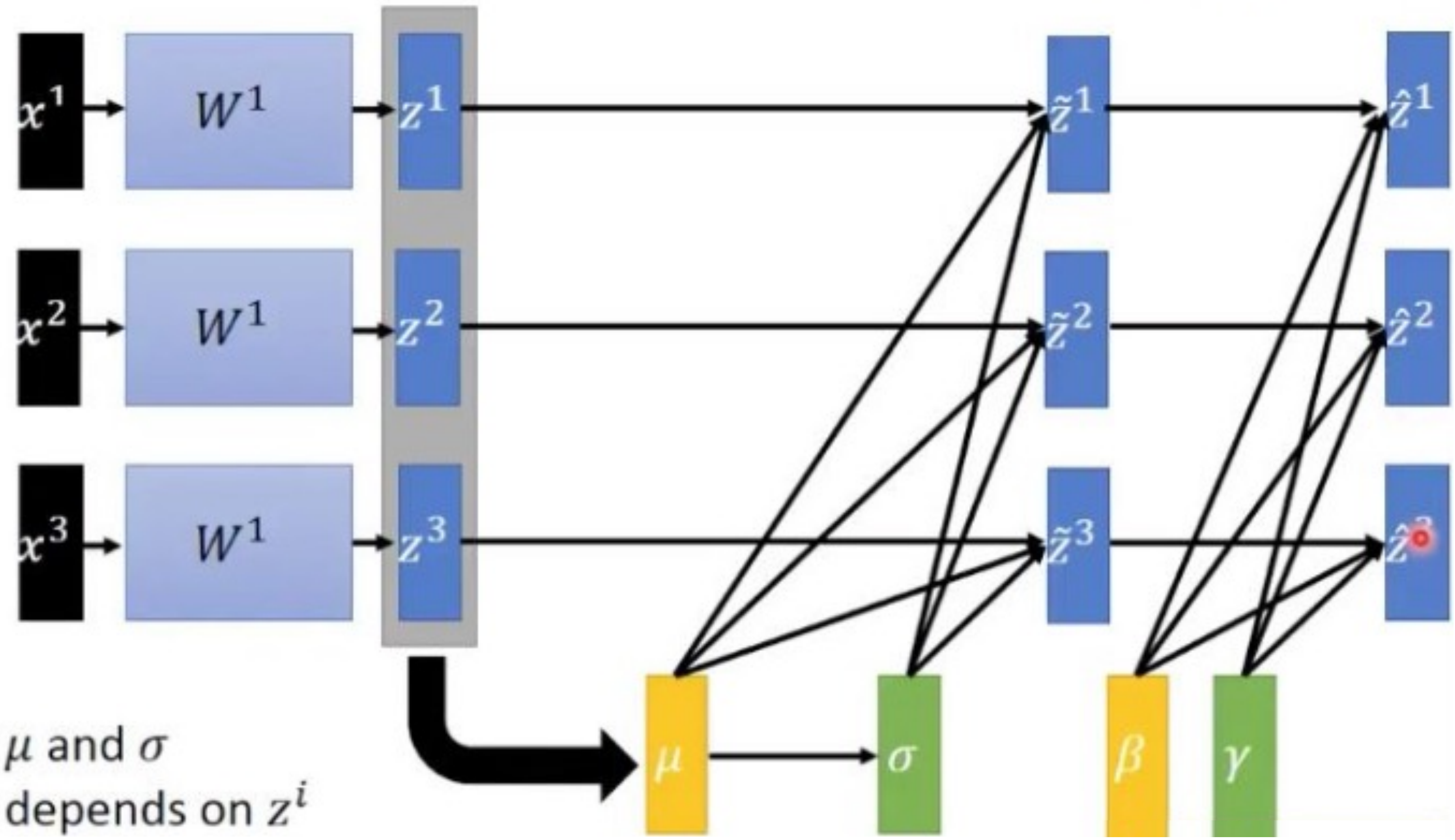
Note: Batch normalization cannot be applied on small batch.

μ and σ depends on z^i

Batch normalization

$$\tilde{z}^i = \frac{z^i - \mu}{\sigma}$$

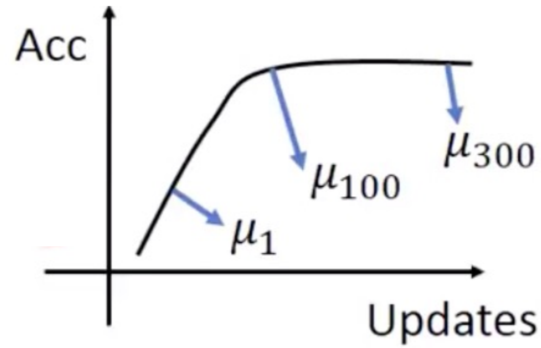
$$\hat{z}^i = \gamma \odot \tilde{z}^i + \beta$$



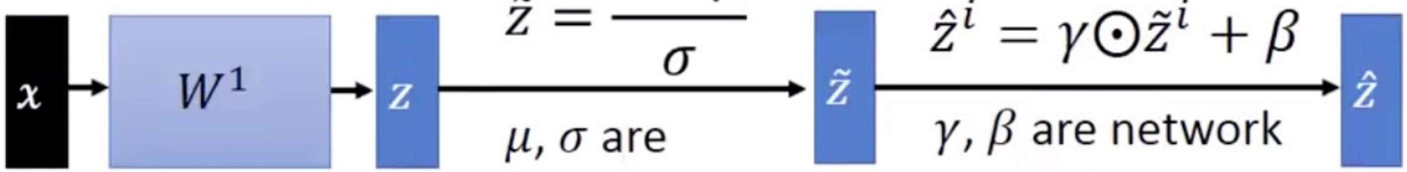
μ and σ depends on z^i

BN at Testing

Batch normalization



- At testing stage:



$$\tilde{z} = \frac{z - \mu}{\sigma}$$

μ, σ are from ***batch***

$$\hat{z}^i = \gamma \odot \tilde{z}^i + \beta$$

γ, β are network parameters

We do not have ***batch*** at testing stage.

Ideal solution:

Computing μ and σ using the whole training dataset.

Practical solution:

Computing the moving average of μ and σ of the batches during training.

BN: Benefits

- Reduce the covariate shift, speed up the training

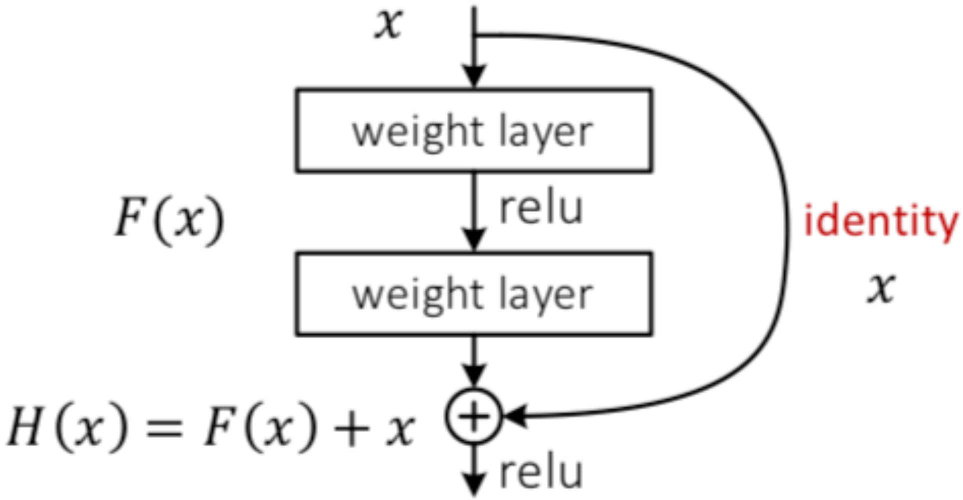
- Reduce the vanishing/exploding gradients

- Less affected by weights initialization

Residual Training

Deep Residual Learning

- Residual net



$H(x)$ is any desired mapping,

~~hope the 2 weight layers fit $H(x)$~~

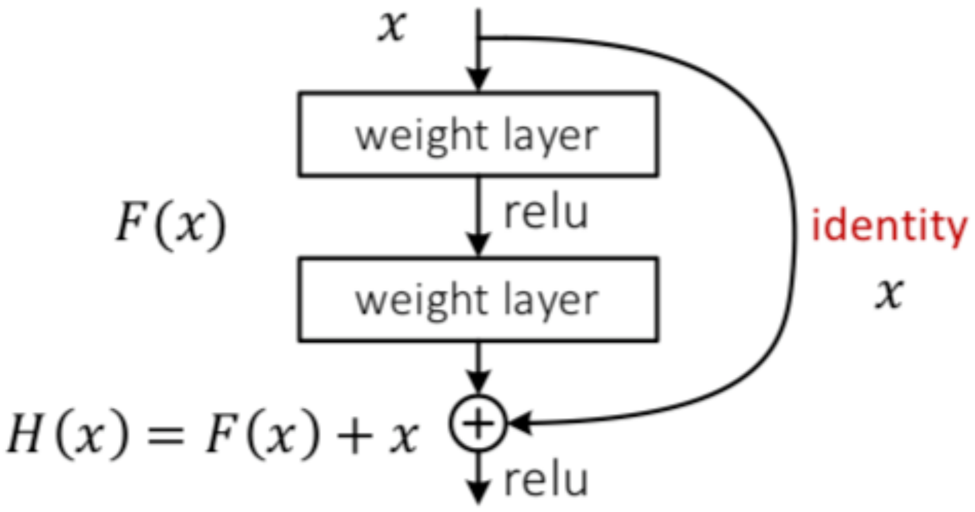
hope the 2 weight layers fit $F(x)$

let $H(x) = F(x) + x$

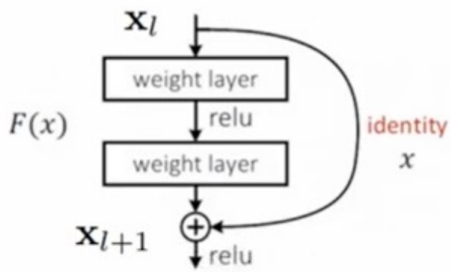
Cite: Kaiming He, etc, Deep Residual Learning for Image Recognition, CVPR 21016

Deep Residual Learning

- $F(x)$ is a residual mapping w.r.t. identity



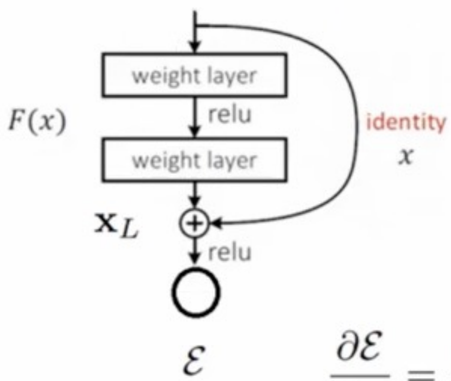
- If identity were optimal, easy to set weights as 0
- If optimal mapping is closer to identity, easier to find small fluctuations



$$\mathbf{x}_{l+1} = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l)$$

a term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}$ that propagates information directly without concerning any weight layers, and another term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\left(\frac{\partial}{\partial \mathbf{x}_l}\sum_{i=l}^{L-1}\mathcal{F}\right)$ that propagates through the weight layers. The additive term of $\frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}$ ensures that information is directly propagated back to *any shallower unit* l.

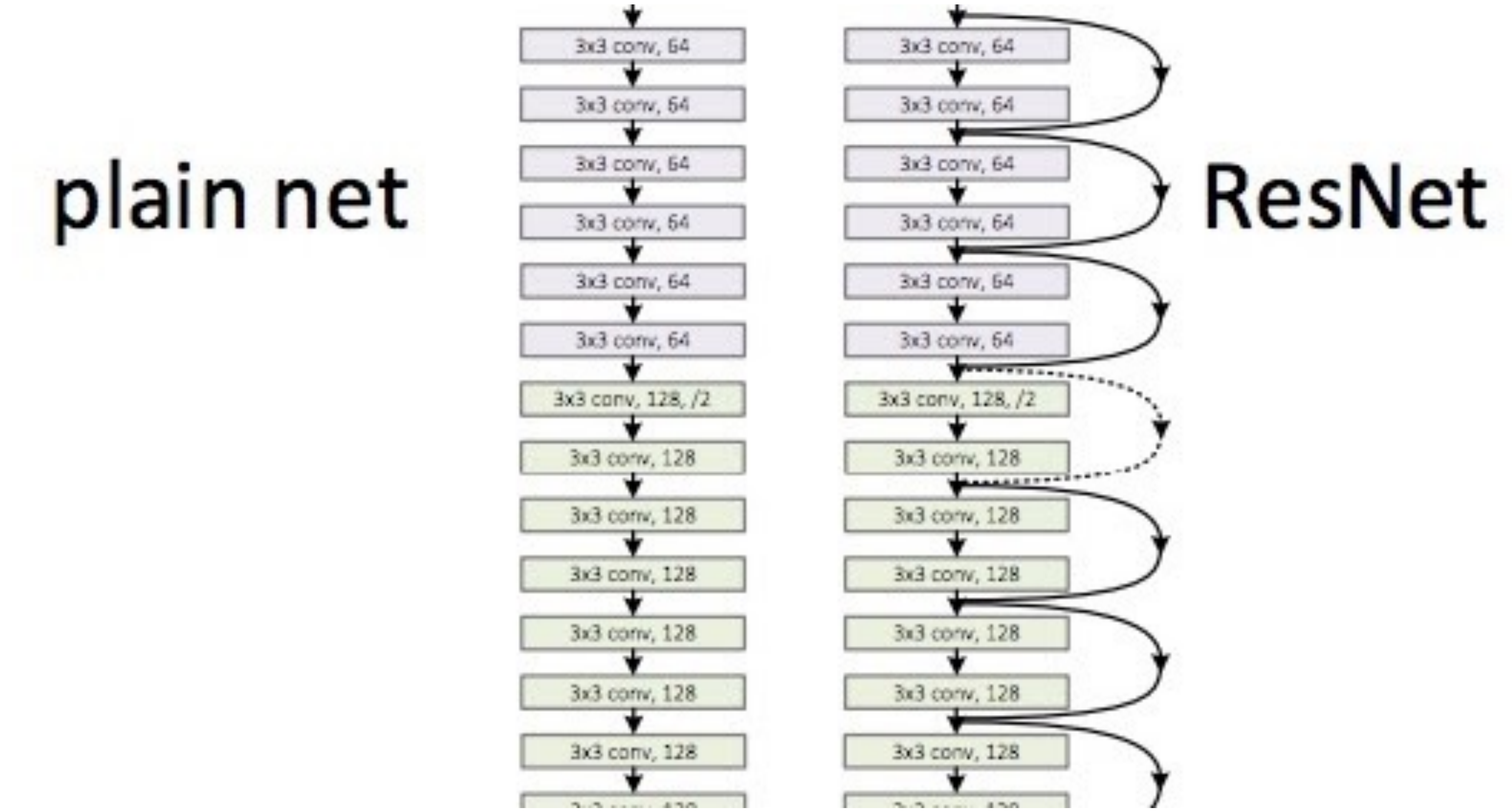
$$\mathbf{x}_{l+2} = \mathbf{x}_{l+1} + \mathcal{F}(\mathbf{x}_{l+1}, \mathcal{W}_{l+1}) = \mathbf{x}_l + \mathcal{F}(\mathbf{x}_l, \mathcal{W}_l) + \mathcal{F}(\mathbf{x}_{l+1}, \mathcal{W}_{l+1})$$



$$\mathbf{x}_L = \mathbf{x}_l + \sum_{i=l}^{L-1}\mathcal{F}(\mathbf{x}_i, \mathcal{W}_i)$$

$$\frac{\partial \mathcal{E}}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\frac{\partial \mathbf{x}_L}{\partial \mathbf{x}_l} = \frac{\partial \mathcal{E}}{\partial \mathbf{x}_L}\left(1 + \frac{\partial}{\partial \mathbf{x}_l}\sum_{i=l}^{L-1}\mathcal{F}(\mathbf{x}_i, \mathcal{W}_i)\right)$$

Cite: Kaiming He, etc, Deep Residual Learning for Image Recognition, CVPR 21016 & Zhubo Jiang Youtube Resnet Explained



Network Design

Benefits

- Speed up the training
- Deeper network with reduced gradient vanishing

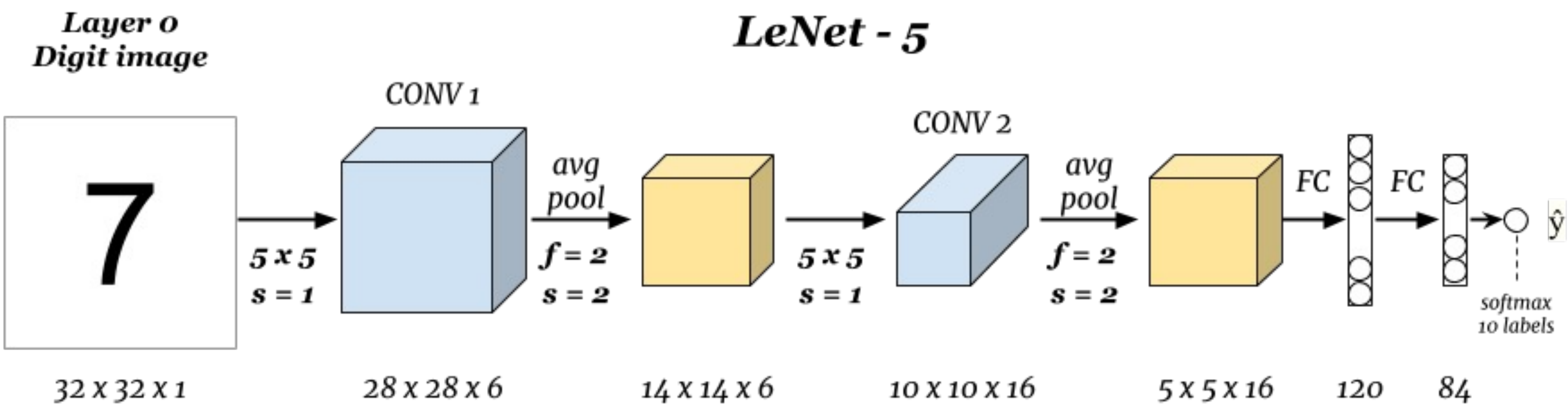
References

- CNN: ML Lecture 10: Convolutional Neural Network by Hongyi Li (youtube)
- Chapter 20 from Berkeley Artificial Intelligence: A Modern Approach (link: http://aima.eecs.berkeley.edu/)
- Module 2: Convolutional Neural Networks from course notes of Stanford CS231n (link: http://cs231n.github.io/convolutional-networks/)
- Full connection blog (link: https://www.superdatascience.com/blogs/convolutional-neural-networks-cnn-step-4-full-connection)
- http://aima.eecs.berkeley.edu/
- http://cs231n.github.io/convolutional-networks/

Classic Networks

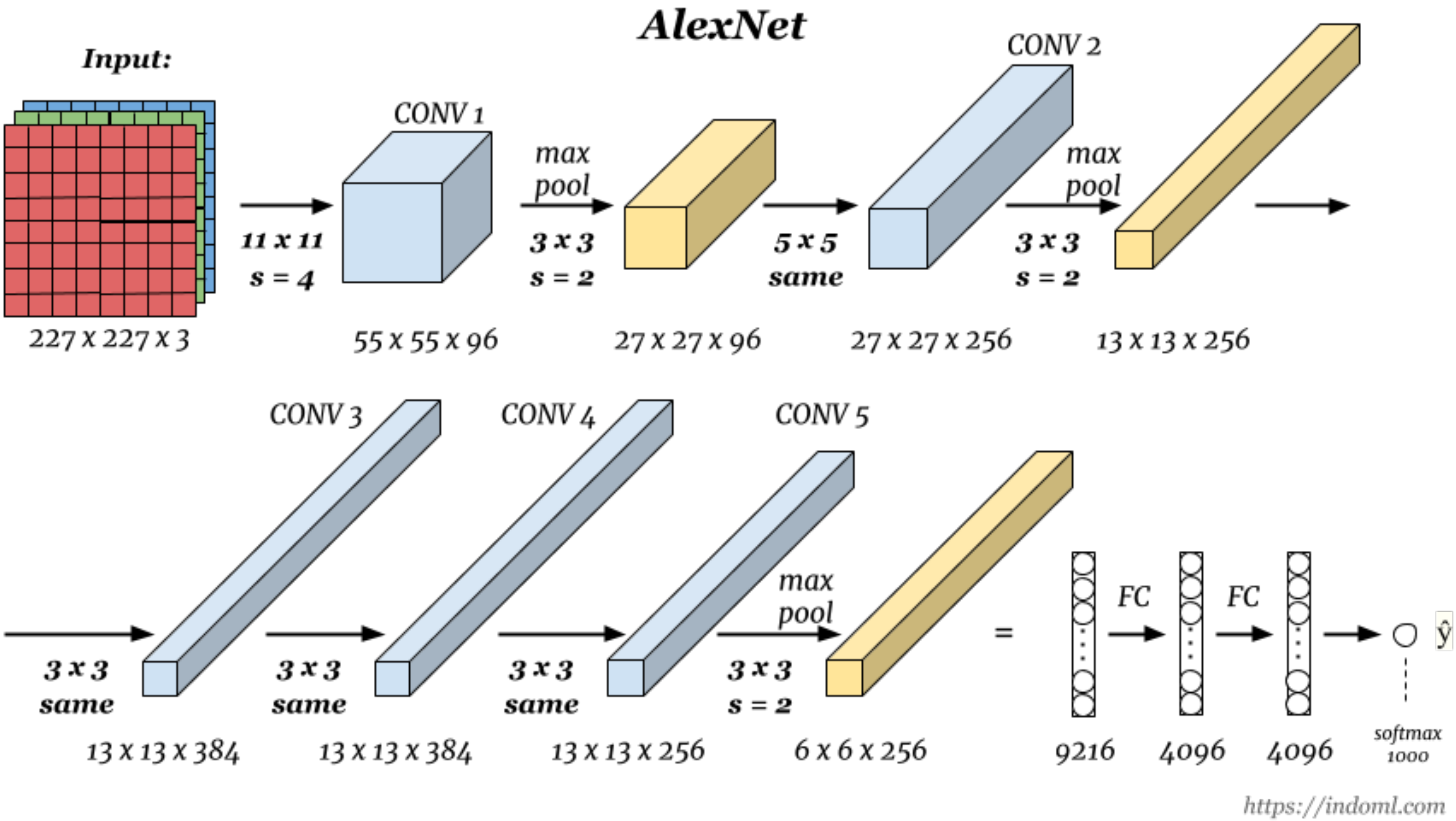
- LeNet
- AlexNet
- VGG Net
- ResNet
- GooLeNet

LeNet

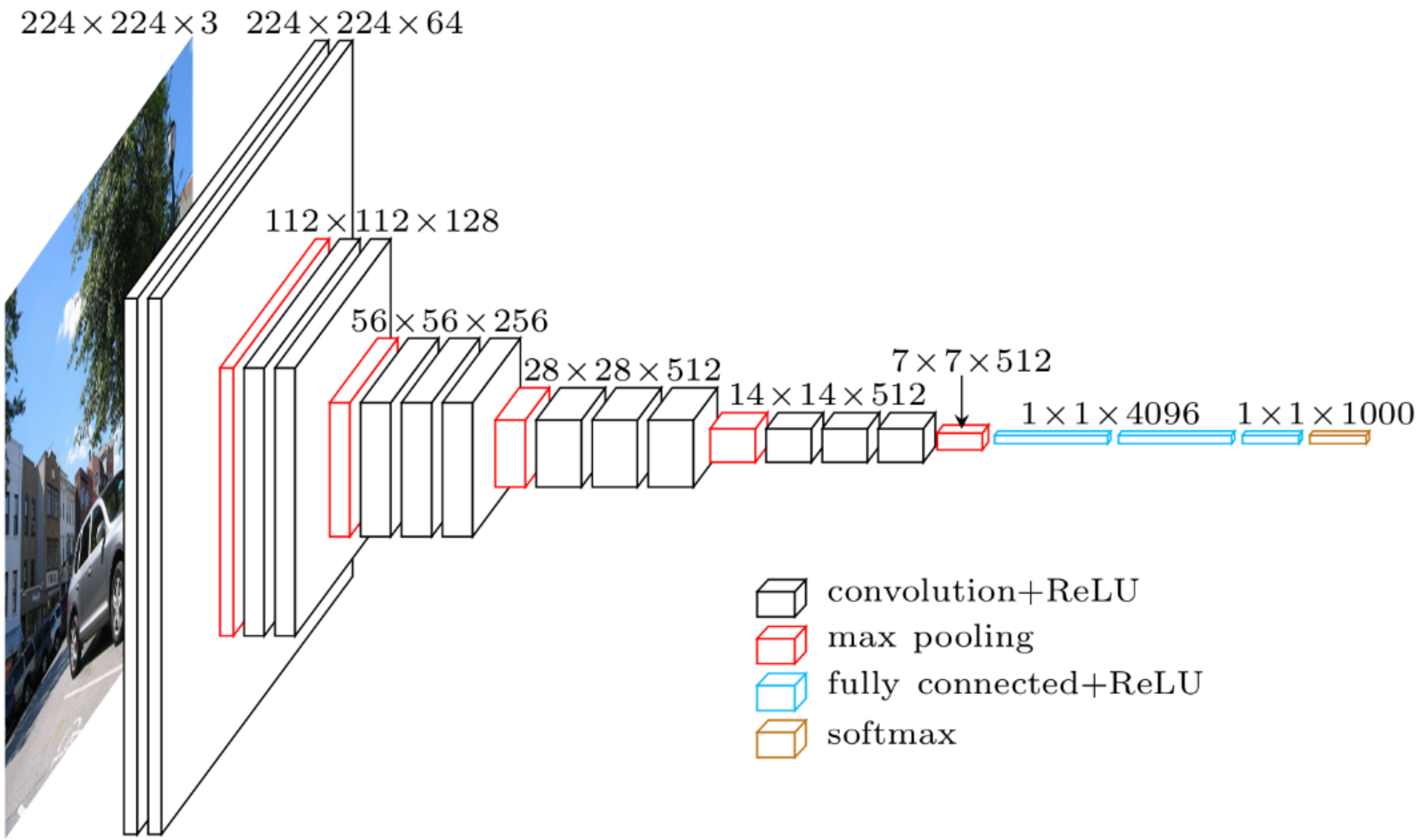


LeNet - 5
Layer 0
Digit image
7
CONV 1
CONV 2
5 x 5
s = 1
avg
pool
f = 2
s = 2
5 x 5
s = 1
avg
pool
f = 2
s = 2
FC
FC
ŷ
softmax
10 labels
32 x 32 x 1
28 x 28 x 6
14 x 14 x 6
10 x 10 x 16
5 x 5 x 16
120
84

AlexNet

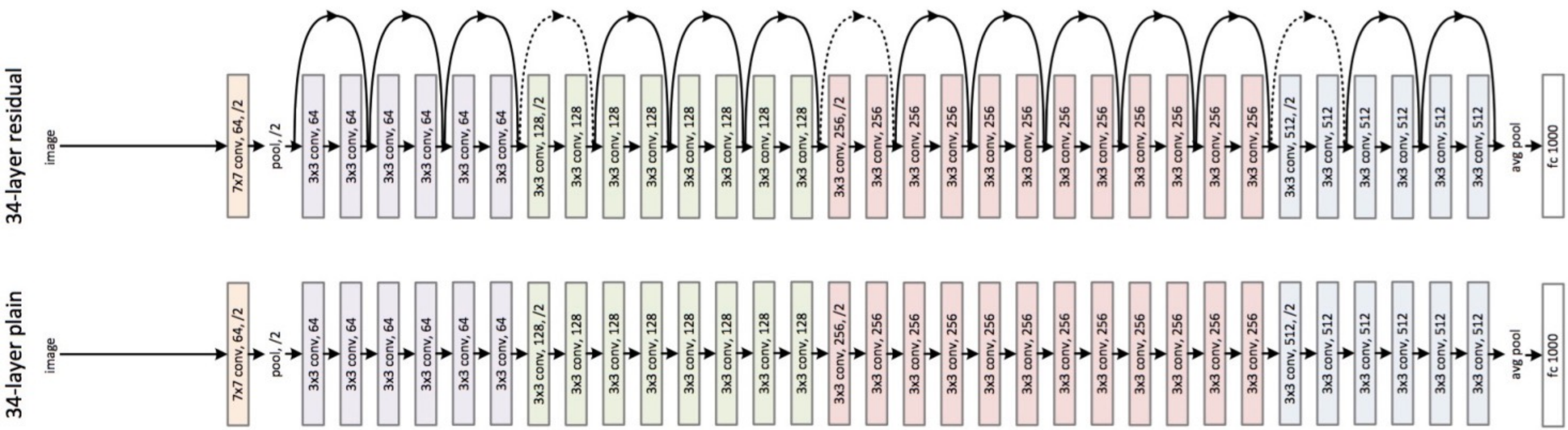

AlexNet
Input:
227 x 227 x 3
11 x 11
s = 4
CONV 1
55 x 55 x 96
max pool
3 x 3
s = 2
27 x 27 x 96
5 x 5
same
CONV 2
27 x 27 x 256
max pool
3 x 3
s = 2
13 x 13 x 256
3 x 3
same
CONV 3
13 x 13 x 384
3 x 3
same
CONV 4
13 x 13 x 384
3 x 3
same
CONV 5
13 x 13 x 256
max pool
3 x 3
s = 2
6 x 6 x 256
=
9216
FC
4096
FC
4096
ŷ
softmax
1000
https://indoml.com

VGG-16

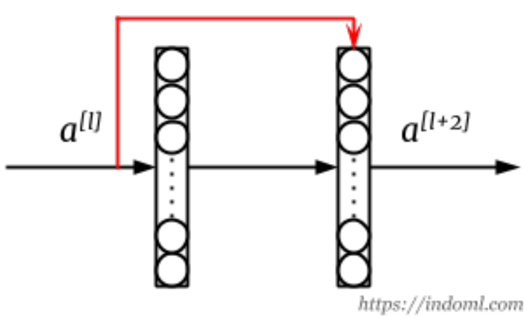


224 × 224 × 3
224 × 224 × 64
112 × 112 × 128
56 × 56 × 256
28 × 28 × 512
14 × 14 × 512
7 × 7 × 512
1 × 1 × 4096
1 × 1 × 1000
convolution+ReLU
max pooling
fully connected+ReLU
softmax

ResNet

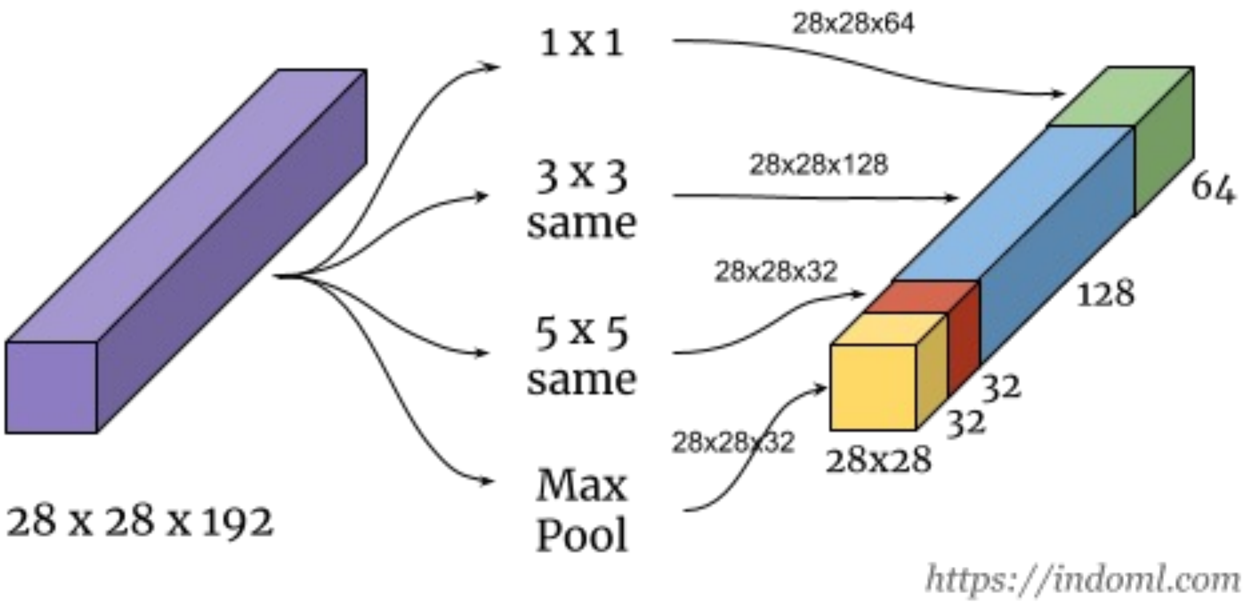


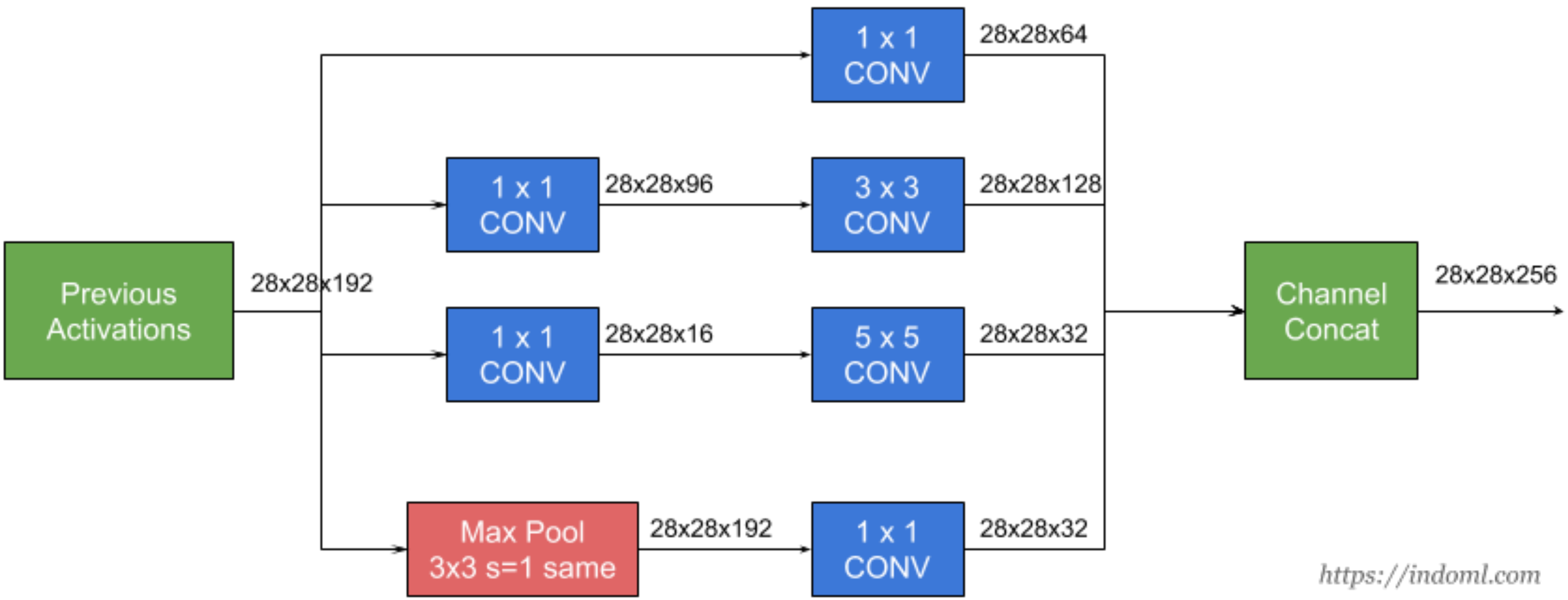
Residual Module



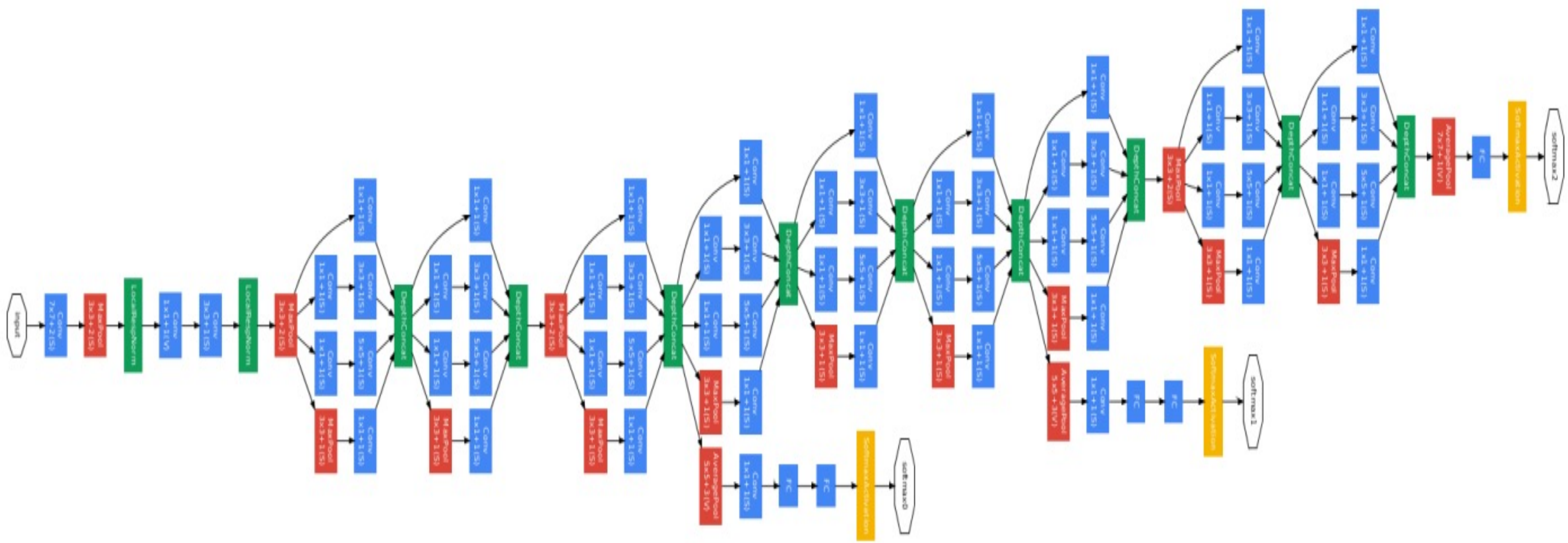
GoogLeNet

Inception Module




Previous Activations
28x28x192
1 x 1 CONV
28x28x64
1 x 1 CONV
28x28x96
3 x 3 CONV
28x28x128
1 x 1 CONV
28x28x16
5 x 5 CONV
28x28x32
Max Pool 3x3 s=1 same
28x28x192
1 x 1 CONV
28x28x32
Channel Concat
28x28x256
https://indoml.com

GoogLeNet:



Summary of Networks

Year	CNN	Developed by	Place	Top-5 error rate	No. of parameters
1998	LeNet(8)	Yann LeCun et al			60 thousand
2012	AlexNet(7)	Alex Krizhevsky, Geoffrey Hinton, Ilya Sutskever	1st	15.3%	60 million
2014	GoogLeNet(19)	Google	1st	6.67%	4 million
2014	VGG Net(16)	Simonyan, Zisserman	2nd	7.3%	138 million
2015	ResNet(152)	Kaiming He	1st	3.6%	