Data Mining
Association Analysis: Basic Concepts and Algorithms

Lecture Notes for Chapter 6

Introduction to Data Mining

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What is Pattern Discovery?

- What are patterns?
  - Patterns: A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
  - Patterns represent intrinsic and important properties of datasets
- Pattern discovery: Uncovering patterns from massive data sets
- Motivation examples:
  - What products were often purchased together?
  - What are the subsequent purchases after buying an iPad?
  - What word sequences likely form phrases in this corpus?

Why is it important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Mining sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: Discriminative pattern-based analysis
  - Cluster analysis: Pattern-based subspace clustering
- Broad applications
  - Market basket analysis, cross-marketing, sale campaign analysis, Web log analysis, biological sequence analysis

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers

- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items

- **A classic rule:**
  - If someone buys diaper and milk, then he/she is likely to buy beer
  - Don’t be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A large set of **items**
  - e.g., things sold in a supermarket
- A **large set** of **baskets**
- Each basket is a **small subset of items**
  - e.g., the things one customer buys on one day
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!

Input:

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:
{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

Applications – (1)

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store

- **Real market baskets:** Chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer

  Amazon's people who bought *X* also bought *Y*

Applications – (2)

- **Baskets** = patients; **Items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects
  - **But requires extension:** Absence of an item needs to be observed as well as presence

Association Rule Mining

- Given a set of transactions, find rules that will predict <u>the occurrence of an item</u> based on <u>the occurrences of other items</u> in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

$\{\text{Diaper}\} \rightarrow \{\text{Beer}\}$,
$\{\text{Milk, Bread}\} \rightarrow \{\text{Eggs,Coke}\}$,
$\{\text{Beer, Bread}\} \rightarrow \{\text{Milk}\}$,

Implication means co-occurrence, not causality!

Two key issues

- Discovering patterns from a large transaction data set can be computationally expensive.

- Some of the discovered patterns may be spurious
  - Evaluating the discovered pattern

Outline

- **First: Define**
  - **Frequent itemsets**
  - **Association rules:**
    - Confidence, Support
- **Then: Algorithms for finding frequent itemsets**
  - **A-Priori algorithm**

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. $\sigma(\{Milk, Bread, Diaper\}) = 2$

$$\sigma(X) = \left|\{t_i \mid X \subseteq t_i,\ t_i \in T\}\right|,$$

Definition: Frequent Itemset

- **Simplest question:** Find sets of items that appear together “frequently” in baskets

- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5

- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* **threshold**

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Support threshold** = 3 baskets

$\mathbf{B_1}$ = {m, c, b}	$\mathbf{B_2}$ = {m, p, j}
$\mathbf{B_3}$ = {m, b}	$\mathbf{B_4}$ = {c, j}
$\mathbf{B_5}$ = {m, p, b}	$\mathbf{B_6}$ = {m, c, b, j}
$\mathbf{B_7}$ = {c, b, j}	$\mathbf{B_8}$ = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j},

  {m,b}, {b,c}, {c,j}.

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are disjoint itemsets
  - Example:
    $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- **Rule Evaluation Metrics**
  - **Support (s)**
    - Fraction of transactions that contain both X and Y

$$s(X \longrightarrow Y) = \frac{\sigma(X \cup Y)}{N};$$

  - **Confidence (c)**
    - Measures how often items in Y appear in transactions that contain X

$$c(X \longrightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}.$$

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Why use support and confidence?

- Support is often used to eliminate uninteresting rules.

- Confidence measures the reliability of the inference made by a rule.
  - The higher the confidence, the more likely it is for Y to be present in transactions that contains X.

- The inference made by an association rule does not necessary imply causality. It suggests a strong co-occurrence relationship between items in X and Y.

Association Rule Mining Task

- **Association Rules:**
  If-then rules about the contents of baskets

- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"

- **Problem:** Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

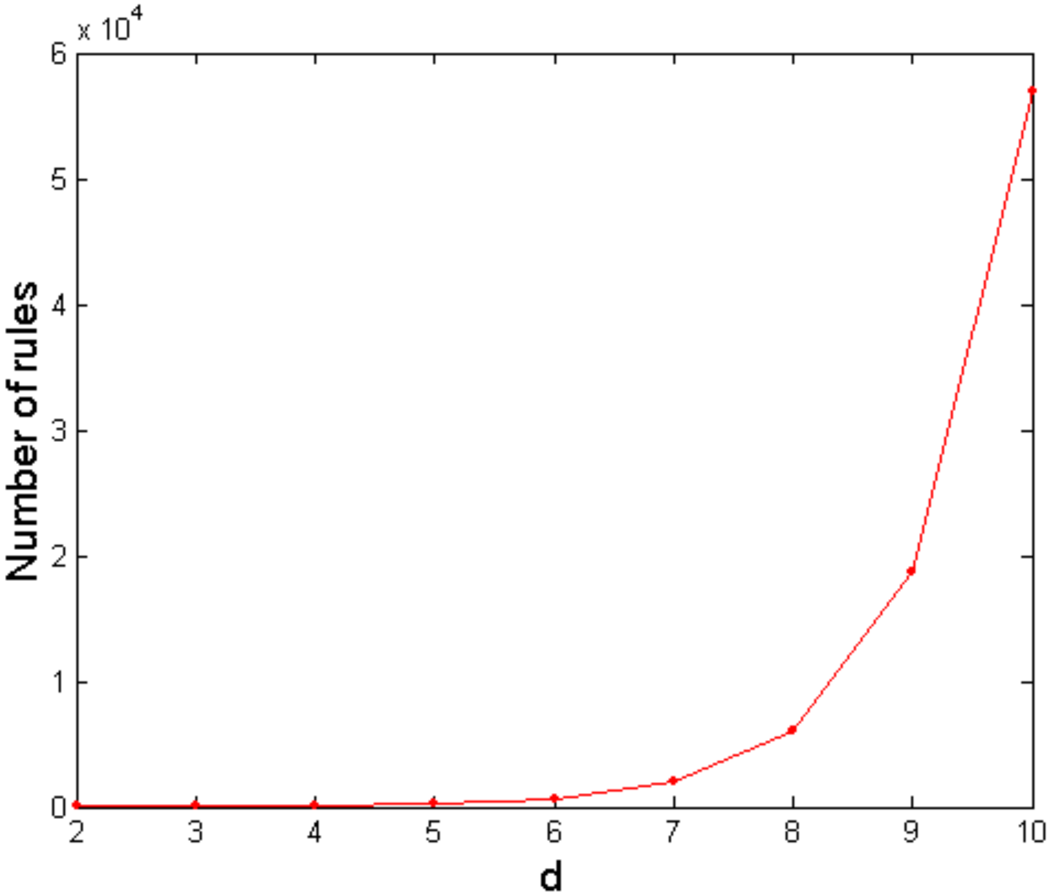
Association Rule Mining Task

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  $\Rightarrow$ Computationally prohibitive!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = 2^d
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1} \left[\binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} \right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Mining Association Rules

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- **Problem: Find all association rules with support $\geq s$ and confidence $\geq c$**

- **Hard part: Finding the frequent itemsets!**
  - If $\{i_1, i_2,\ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2,\ldots, i_k\}$ and $\{i_1, i_2,\ldots,i_k, j\}$ will be "frequent"

Mining Association Rules

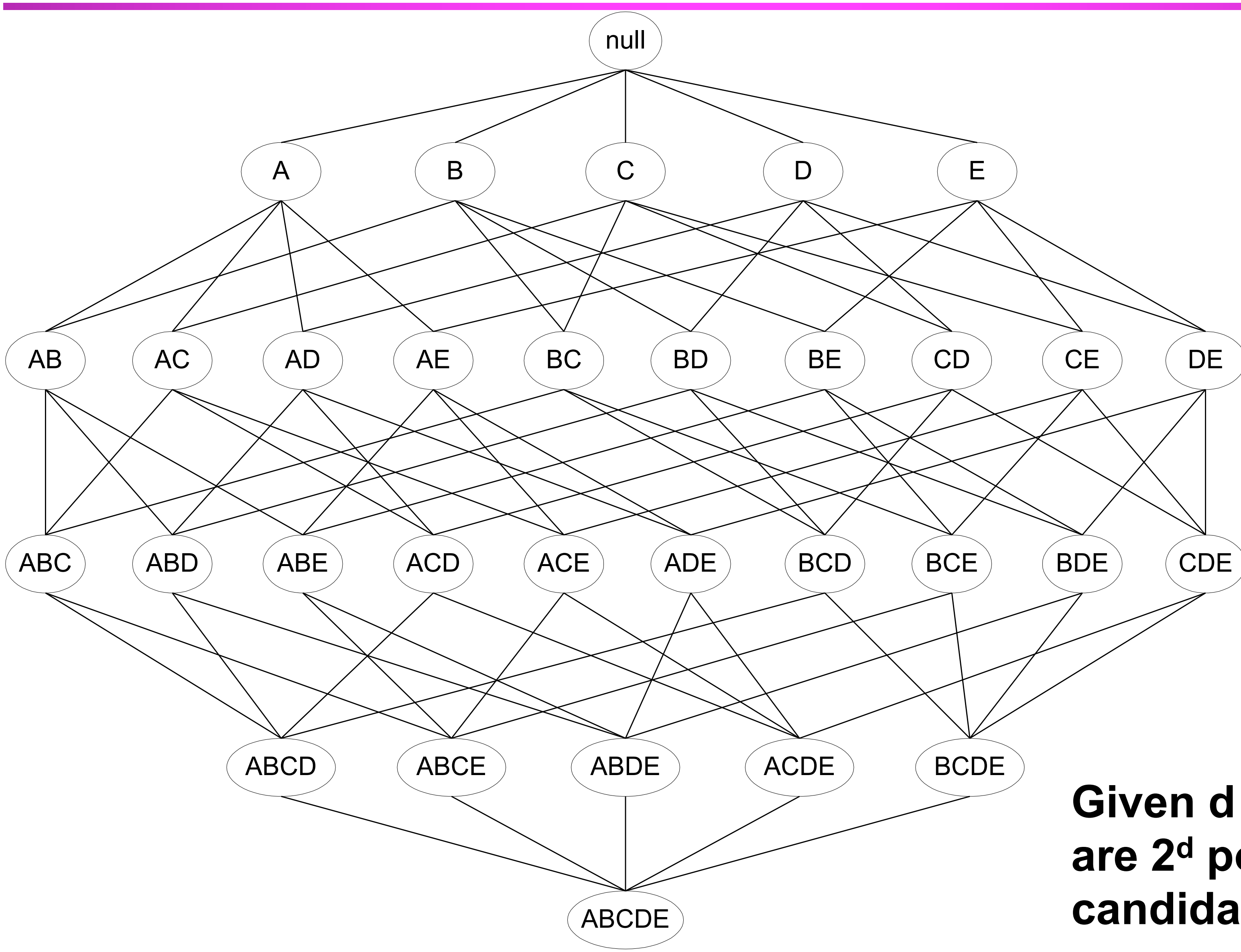
- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets I whose support $\geq$ minsup

  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

     - For every subset A of I, generate a rule $A \to I \setminus A$
       - Since I is frequent, A is also frequent

- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



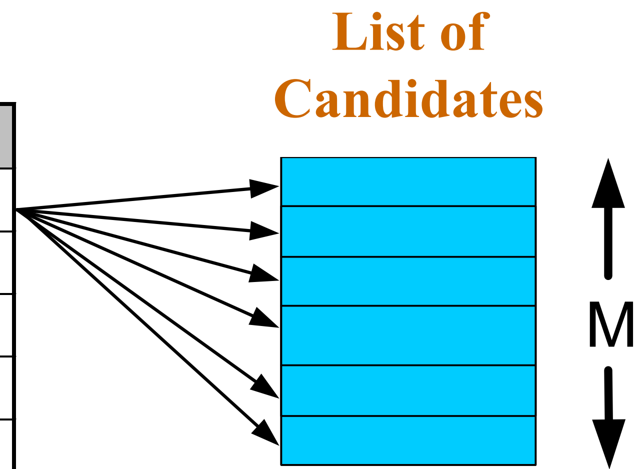
Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke



  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Quiz

- Let c1, c2, and c3 be the confidence values of the rules $\{p\} \rightarrow \{q\}$, $\{p\} \rightarrow \{q,r\}$, and $\{p,r\} \rightarrow \{q\}$, respectively.

If we assume that c1, c2, and c3 have different values, what are the possible relationships that may exist among c1, c2, and c3? Which rule has the lowest confidence?

Reducing Number of Candidates

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

Illustrating Apriori Principle

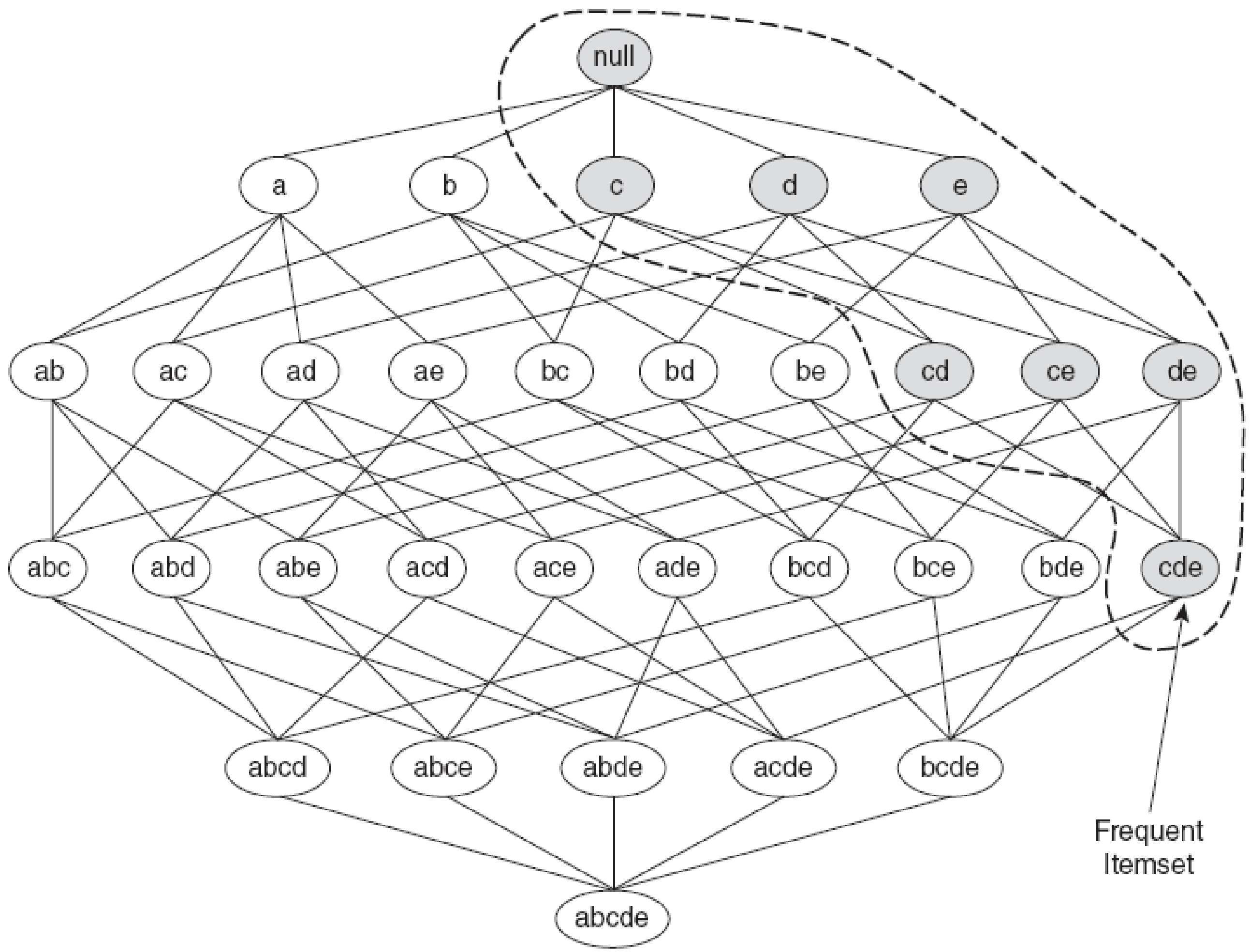


Figure 6.3. An illustration of the *Apriori* principle. If $\{c, d, e\}$ is frequent, then all subsets of this itemset are frequent.