Personal Statement

Looking back on my journey in the world of mathematics evokes many memorable experiences. Back in primary school, I was extremely curious about the proof of volume formula of a cone, $V = \frac{1}{3}\pi r^2 h$. Seeking guidance, I turned to my mother, who handed me a weathered book titled *Calculus*. I then spent months self-studying this book. Eventually, I mastered the method of calculating volumes using integrals. This small exploration sparked my interest in mathematics and led me onto a journey of exploring its depths.

In high school, I participated in the training for the Mathematical Olympiad. During that time, I was particularly fascinated by number theory problems. I kept records of my musings on number theory problems, including many solutions I discovered. A mathematics professor at Zhejiang University found my notes immensely valuable and was willing to help me publish them. Thus, in the spring of 2022, my book *Congruence Problems in Number Sequences* was published by Zhejiang University Press. Throughout my three years in high school, my youthful years, I really got into math. I loved the challenge of solving problems—it was like uncovering little puzzles that brought me so much joy. Gradually, a natural and ambitious idea took root in my mind — to become a mathematician.

After entering University, I felt like a fish finally swimming into the vast ocean. It was a new world to me and I thoroughly enjoyed learning modern mathematics. During my first year, I spent most of my time studying algebra and topology. To establish a solid foundation in algebra, I read Hungerford and Serge Lang's *Algebra*. What greatly impressed me was the proof that there is no radical solution to general polynomial equations of degree five or higher. I was impressed by the way that mathematicians transformed this problem into a field extension problem, coupled with Galois's profound insights, linking solvability by radicals to the solvability of Galois groups. After a year of study, I discovered a strong inclination toward algebra. Nevertheless, I studied various courses in geometry during my second year, covering topics like manifolds, Riemannian geometry, and algebraic topology. Through these courses, I acquired knowledge about fiber bundles, connections, curvatures, cohomology groups, and characteristic classes. This knowledge proved invaluable as I started to study the geometric Langlands program later on.

The rigorous training in fundamentals during the initial two years provided me with a solid foundation in algebra and geometry, empowering me to delve into more advanced topics. My undergraduate journey through modern mathematics resembled a symphony, orchestrating the melodies of number theory, representation theory, and automorphic forms, reaching its crescendo in the Langlands program and the geometric Langlands conjecture. In the subsequent paragraphs, I will elaborate on my understanding of these domains.

Continuing my passion from high school, my earliest in-depth study was focused on number theory. I took a two-semester graduate course on algebra and algebraic number theory, instructed by Professor Wong Kayue. Moreover, I self-studied Marcus's *Number Field* and Neukirch's *Algebraic Number Theory*. At that time, I found myself dissatisfied with the unmotivated proofs regarding the analytic continuation and functional equations of Dedekind zeta functions. Under the suggestion of my advisor, I study the elegant theory presented in Tate's thesis. Due to my diligent studies, I achieved a bronze medal in algebra and number theory on the 13th Shing-Tung Yau College Student Mathematics Contest. Following this, I furthered my studies in class field theory, primarily referring to Guillot's *A Gentle Course in Local Class Field Theory* and Milne's *Class Field Theory*. Admittedly, class field theory, precisely describing all abelian extensions of a given number field, holds paramount importance in number theory. However, even after going through the entire proof, which took me half a year, I still remained deeply perplexed as to why class field theory naturally holds. This confusion lingered until I encountered the insights of the geometric Langlands.

Representation theory is a fascinating world with subtle connections to both analysis and geometry. At first, I studied representation theory of finite and compact groups by reading Serre's *Linear Representations of Finite Groups*. I was amazed by the hidden link between representation theory and Fourier theory, as the following theorem indicates: the L^2 function space on a finite/compact group G is generated by an orthogonal basis consisting of matrix coefficients of irreducible representations. Then, I took a two-semester course topics in representations of Lie group and Lie algebra, instructed by Professor Huang Jingsong. I also actively participated in his seminar discussing topics on Minuscule representations. Furthermore, I learned about Galois representations when I studied theory of modular forms. Initially, I found myself quite perplexed as to why mathematics delve into Galois representations. One night, as I was on the brink of falling asleep but with my mind racing, I stumbled upon a great explanation. Although Galois groups don't naturally act on vector spaces, they do act on varieties and consequently act on their cohomology groups, or more generally, spaces of global sections of any sheaf on them! This idea extends readily to any group action on geometric spaces, thereby generating a vast array of group representations. As an example, the Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, acting on the étale cohomology group of elliptic curves over $\mathbb{Q}$, gives rise to two-dimensional Galois representations. Later, I learned that representations of Lie groups/Lie algebras could also be realized through this method, as the Borel-Weil-Bott theorem indicates.

Automorphic forms also play an important role in my study. During my time at Berkeley, I engaged in the direct reading program, where my mentor guided me through the study of Diamond's "A First Course in Modular Forms." We mainly discuss the Modularity theorem, elliptic curve over $\mathbb{Q}$ associated to modular forms, and Galois representations associated to modular forms. Besides, I wrote a paper to prove that various definitions of Hecke operators on modular forms are equivalent. In this semester, with the guidance of Professor Luo Caihua, I led a seminar to discuss automorphic forms and representations over $\mathrm{GL}_2(\mathbb{A}_{\mathbb{Q}})$, referencing Goldfeld's *Automorphic Representations and L-functions for the General Linear Group*. Throughout these studies, a grander picture began to emerge in my mind—one that intricately connected automorphic forms/representations, Galois representations, L-functions, and geometry. Naturally, Langlands program came into my sight.

Professor Edward Frenkel's Abel Prize Lectures *Langlands program and Unification* introduced me to the realm of the geometric Langlands program. The mystery connection between Langlands program and physics deeply captivated me, inspiring me a profound desire to know the world of geometric Langlands. I began with reading Frenkel's notes *Lectures on the Langlands Program and Conformal Field Theory*. There, I learned several intriguing geometric interpretations of algebraic concepts. For instance, Galois groups were interpreted as fundamental groups, and the double quotient space $\mathrm{GL}_n(F)\backslash\mathrm{GL}_n(\mathbb{A}_F)/\mathrm{GL}_n(\mathcal{O})$

equated to the moduli space of rank n vector bundles on a curve. In particular, the case when $n = 1$ is exactly an analogue of the class field theory, which now has a natural geometric interpretation. These findings not only cleared up my previous confusion about the class field theory but also fueled my excitement to further understand the geometric Langlands.

When I studied in Berkeley, I chose Frenkel's course without hesitation. His lectures began with the commutativity of Gaudin Hamiltonians, leading to two questions: Why do they commute, and how to find their common eigenvectors? The first question led to the center theorem, involving various algebraic concepts like deformation quantization, Poisson algebra, and vertex algebras. The second question led to a construction by free field realization and conformal blocks. At last, I composed a term paper titled *Opers and the Center of Affine Kac-Moody Vertex Algebras*. My paper studied the G-oper space on a formal disk and discussed the isomorphism between the center of the affine Kac-Moody vertex algebra and the function space on $\mathrm{Op}_G(D)$. At that time, I was able to view the space of opers as a moduli space of enhanced connections on G-bundles, yet I struggled to fully comprehend the rationale behind our interest in this space.

After coming back from Berkeley, I studied Frenkel's video lecture *Langlands program and Quantum Field Theory*. This lecture proved pivotal as I finally grasped the statement of the geometric Langlands conjecture in parallel to the classical Langlands program. The lecture focused on how to construct Hecke eigensheaves on Bun_G for a given local system. For $G = \mathrm{GL}_1$, I learned the construction due to Deligne. For general group, I learned the construction due to Beilinson and Drinfeld based on the idea of 2D CFT. Specifically, the local correspondence associates each point χ in $\mathrm{Op}_G(D^\times)$ with a categorical representation $\mathcal{C}_\chi$ of loop group $G((t))$, together with a spherical object V_χ. And Beilinson and Drinfeld's theorem claims that if this local oper χ_x can be extended to a global oper on the curve, then the localization $\Delta(V_{\chi_x})$ is a Hecke eigensheaf with eigenvalue corresponds to χ_x. In addition, fibers of this sheaf are coinvariant space. All fancy concepts such as opers and comformal blocks I learned in last semester now become natural and powerful. Hence, I reviewed these concepts and write a lecture note for last semester course, titled *Gaudin model, Center Theorem, and Vertex algebras*. Besides, I also study David Ben-Zvi's video lecture on geometric Langlands which based on the viewpoint that it is Fourier transform for sheaves on Bun_G. I found his geometric function theory very enlightening, which unifies ideas behind many theories such as Fourier Mukai transform and action of Hecke algebras.

The PhD program in Mathematics at University of California, Berkeley, as one of the most prestigious programs in this field, is definitely the dream place for me to pursue my graduate study. I believe with my great passion and solid foundation in mathematics, I have been well-prepared for my graduate study. Therefore, I genuinely hope that UC Berkeley will offer me the opportunity to study further in mathematics as well as contribute positively to the mathematics community.