Computer Simulations Homework 1

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A. Re-inspection Probability

Student ID: r05942078, hence the re-inspection probability is

$$P_{\text{re-inspect}} = 0.9$$

B. Calculate the average number of buses successfully passing through the inspection

Let $\lambda, \lambda_1, \lambda_2$ be the external arrival rate, inspection station arrival rate, and repair station arrival rate respectively. Then we have

$$\lambda_1 = \lambda + \lambda_2 \cdot p_{\text{re-inspect}}$$

and

$$\lambda_2 = p_{\text{repair}} \cdot \lambda_1$$

Therefore we have

$$\lambda_1 = \frac{\lambda}{1 - p_{\text{repair}} \cdot p_{\text{re-inspect}}} = \frac{\lambda}{1 - 0.3 \cdot 0.9} = \frac{\lambda}{0.73}$$

C. Obtain the utilization of the Inspection and each Repair Shop via simulations

We first deduce the theoretical result, and then compare with the simulation result.

1. *Server Utilization*

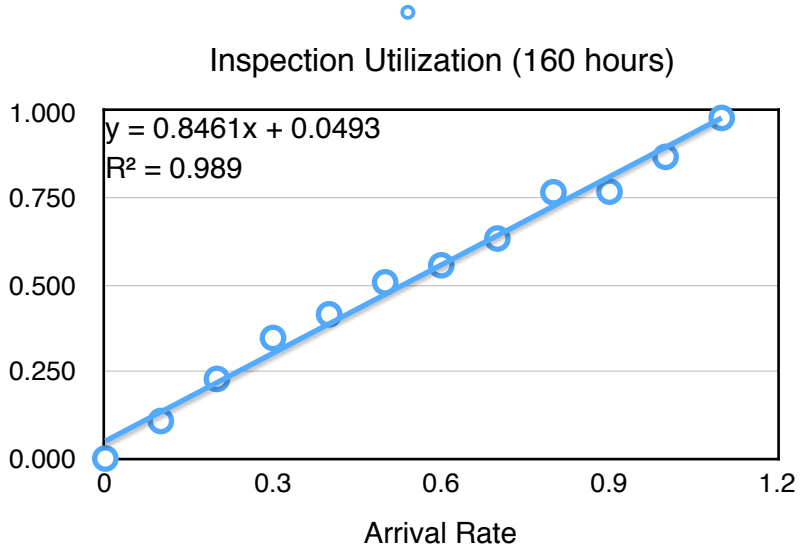
Denote the utilizations of the inspection server and the repair servers as U_1, U_2. Then follows from part B, we obtain

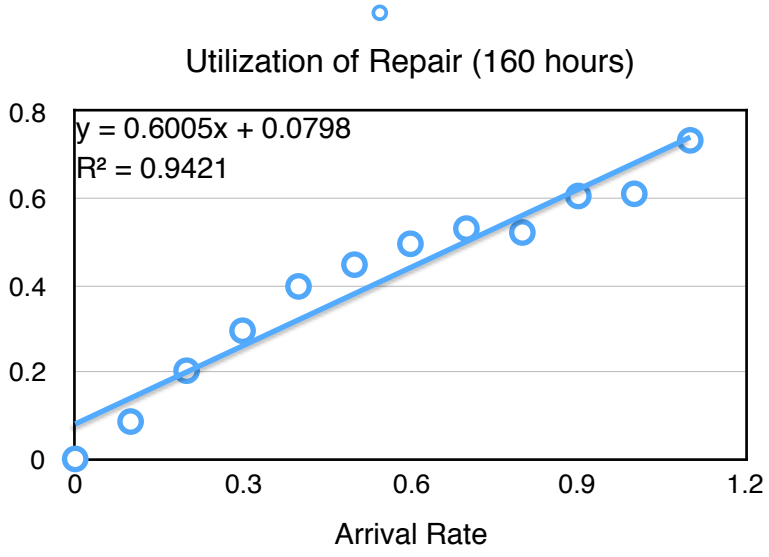
$$U_1 = \lambda_1 \cdot \frac{1}{\mu_1} = \lambda \cdot \frac{0.65}{0.73} \approx 0.89\lambda$$
$$U_2 = \lambda_2 \cdot \frac{1}{\mu_2} \cdot \frac{1}{2} = \frac{0.3 \cdot 3.3}{0.73 \cdot 2} \approx 0.678\lambda$$

2. *Simulation Result*

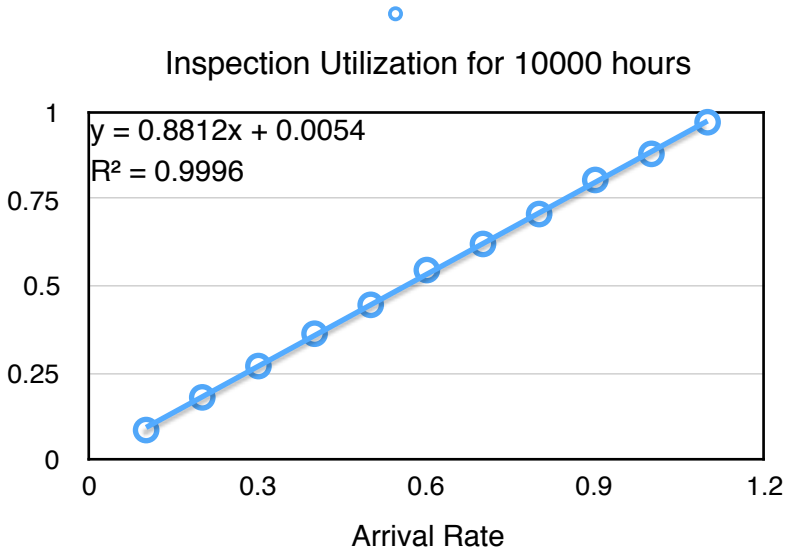
160 hours

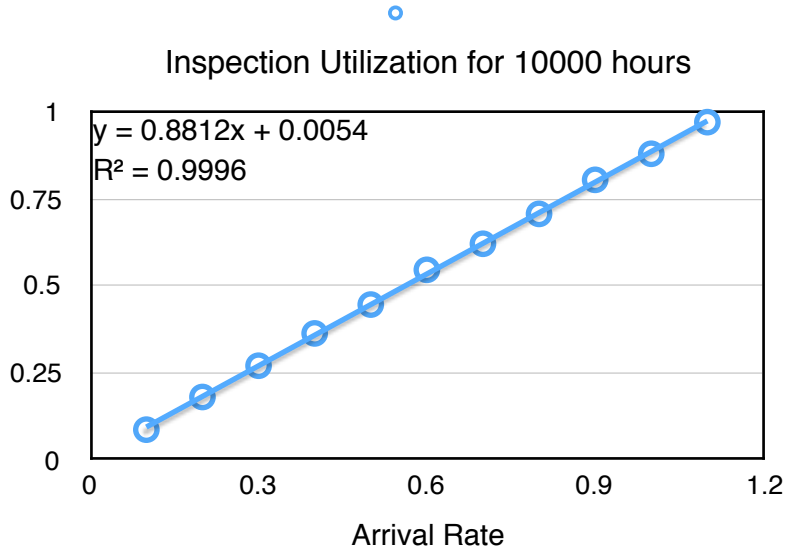
lambda	1/lambda	U1	U2
1.1	0.90909090909	0.980	0.733
1.0	1.0	0.868	0.61
0.9	1.1111111111111	0.768	0.605
0.8	1.25	0.766	0.521
0.7	1.42857142857	0.633	0.530
0.6	1.66666666666	0.556	0.495
0.5	2	0.507	0.447
0.4	2.5	0.415	0.397
0.3	3.33333333333	0.347	0.295
0.2	5	0.229	0.203
0.1	10	0.108	0.086
0.001	1000	0	0





We see that there are still some difference between simulation result and the theoretic deduction (for instance, Inspection: simulation slope is 0.846, theoretical slope is 0.89). One of the reason is that the simulation time is not long enough.
Now we consider the simulation with 10000 hours.





The simulation results are much closer to the theoretical results.

D. Estimate the maximum capacity of the whole bus inspection

From part C, we see that the (theoretical) maximum arrival rate, say, $\lambda_{\max}$ is

$$\lambda_{max} = \frac{1}{0.89} \approx 1.124$$

Therefore, we simulate 500000 hours and plot the utilization curve with λ_{max} roughly 1.12.

lambda	1/lambda	Inspect Util	Repair Util	Arrival Customers	Departure Customers
1.100	0.909	0.978	0.743	549498	549485
1.115	0.896860986	0.991	0.754	556960	556490
1.120	0.893	0.998	0.758	561291	561218
1.125	0.888888888	1	0.761	563136	562030
1.130	0.885	1	0.761	565455	561985
1.135	0.881057268	1	0.761	567557	562018

From the above result, we see that when arrival rate greater than 1.125, the inspection server utilization achieves 100%, and the the system becomes unstable (since arrival customers >> departure customers).
Thus the results indicate that the maximum capacity is roughly 1.125, which is very close to the theoretical result !