CS-446 - Digital 3D Geometry Processing
Assignment 4

Maxime Pisa, Lucas Ramirez, Philippe Weier

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1 Theory Exercise

1.1 Curvature of Curves

- $k_1(s) = \frac{s^2-1}{s^2+1}$

  This function corresponds to curve **d**. We see that the function is symmetric around 0, meaning that $\forall s \in \mathbb{R}^+,\ k_1(s) = k_1(-s)$. Hence the representation must also be symmetric around the line $x = 0$. Secondly, we observe that $\lim_{s\to\infty} k_1(s) = \lim_{s\to-\infty} k_1(s) = 1$, so on both ends the curve must shape like a non-shrinking unit circle, which is only the case for curve **d**.

- $k_2(s) = s$

  This function corresponds to curve **a**. Again the function is symmetric around 0, so the representation must also be symmetric around the line $x = 0$. The curvature linearly tends to $\pm\infty$ when $s \to \pm\infty$, so on both ends the curve must shape like a shrinking unit circle, which is only the case for curve **a**.

- $k_3(s) = s^3 - 4s$

  This function corresponds to curve **c**. The funcion is not symmetric around 0, so the representation must also not be symmetric around the line $x = 0$. This is only the case for curve **c**.

- $k_4(s) = \sin(s) \cdot s$

  This function corresponds to curve **b** (by default). The curvature is divergent at both infinities because of the sin function. It also makes the curvature equal 0 periodically (every 2π), changing its sign every period. This is only the case for curve **b**.

1.2 Surfaces Area

We directly consider the general problem with N slices. We denote by S_i the slice of index $i \in \{1, 2, \ldots, N\}$, starting from the half-dome's top (this means that slice S_1 is the upper slice).

We need to compute the area $f(i)$ associated to each slice S_i. We denote by R the half-dome's radius.

$$f(i) = \int_{\frac{i-1}{R}}^{\frac{i}{R}} (2\pi \cdot R)\, dr = \pi \left[\left(\frac{i}{R}\right)^2 - \left(\frac{i-1}{R}\right)^2 \right] = \pi \cdot \frac{2i+1}{R^2}$$

The most profitable slice is the one with the lowest area:

$$\min_{i\in\{1,\ldots,N\}} f(i) = f(1) = \frac{3\pi}{R^2}$$

Hence the upper slice S_1 is always the most profitable one. In the $N = 2$ case one must pick *Part A* to make the most profit.

2 Coding Exercise