Minipupper Kinematics

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CONTENTS

About this book

This book contains my notes and Jupyter notebooks that I created while gaining a better understanding of the kinematics of a quadruped robot in general and MangDang Minipupper in particular.

We cover the mathematics for 2D and 3D movements and develop different gaits for the robot. The Jupyter notebooks contain the code that will illustrate the theoretical background. But theory and reality do not always match. We provide a simulation environment using the real-time physics simulation pybullet and an execution environment that can be installed on a Minipupper so that we can upload leg trajectories to the physical robot and have them executed.

A word of caution

This work is a product of my learning effort, not the product of an expert in the field. If I would write it again I would certainly to things differently. But I still hope if will be of use to the reader.

How to install

Installation of the development environment

The goal is to focus on portable Python code to be able to run it in any Python virtual environment. We make heavy use of Jupyter Lab

- Clone the repository
- Create a Python virtual environment
- cd into the jupyternb directory within this repository
- pip install -r requirements.txt
- start Jupyter lab with "jupyter lab ."

Installation of the simulation environment

You must run in a Python virtual environment.

- cd into the controller directory within this repository
- run ./install.sh
- run "minipupper execute –help"

Installation on Minipupper

Use a SD card that has been configured with minipupper_base

- Clone this repository
- cd into the controller directory within this repository
- run ./install.sh
- minipupper walk # adjust parameters for your use case

CHAPTER

ONE

2D COORDINATE SYSTEMS

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

```
import sympy as sp
from sympy.abc import theta, gamma
from sympy.vector import CoordSys3D
```

```
import math as math
```

```
upper_leg_length = 3
lower_leg_length = 4
```

1.1 Define a coordiate system as origin of the upper leg

```
U = CoordSys3D('U')
```

1.2 Upper leg

The upper leg is connected to the origin of this coordinate system and rotated by an angle of theta. Calculate the location of the end of the upper leg

```
lu = upper_leg_length*sp.cos(theta)*U.i + upper_leg_length*sp.sin(theta)*U.j
print(lu)
```

```
(3*cos(theta))*U.i + (3*sin(theta))*U.j
```

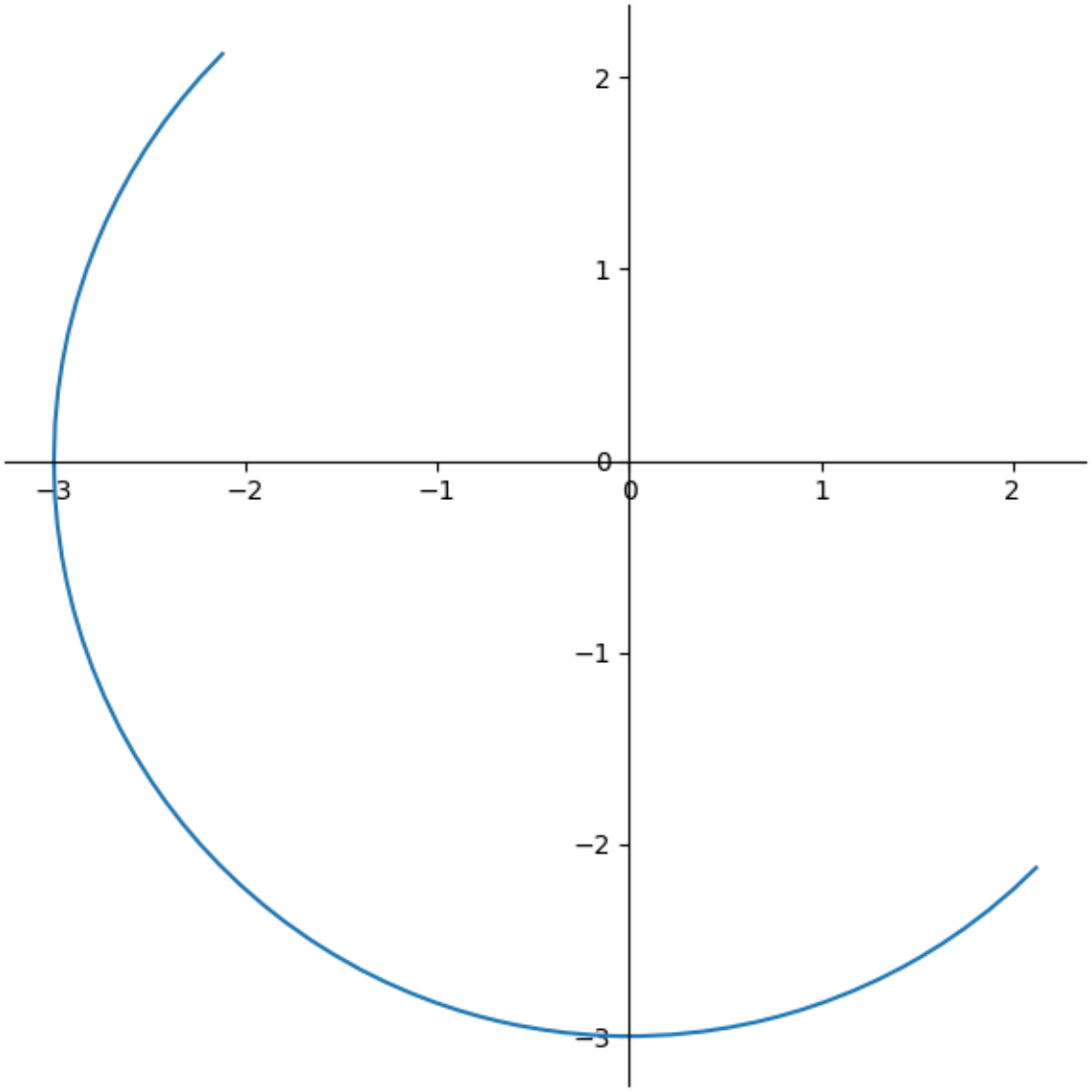
1.3 Draw reach of upper leg

We assume that neutral is -135 degree and the servo can move +/- 90 degree

```
from sympy.plotting import plot_parametric
```

```
deg135 = 3*sp.pi.evalf()/4
deg90 = sp.pi.evalf()/2
```

```
plot_parametric((upper_leg_length*sp.cos(theta), upper_leg_length*sp.sin(theta)),␣
↪(theta, -deg90-deg135, deg90-deg135), size=(6.0, 6.0))
```




```
<sympy.plotting.plot.Plot at 0x125645eb0>
```

1.4 Coordinate systen for the upper leg

Attach a new coordinate system to the end of the upper leg and the x-axis having the same direction of the upper leg

```
L = U.orient_new_axis('L', theta, U.k, location=lu)
```

1.5 Location of the foot

Calculate the location of the end of the lower leg in this coordinate system

```
ll =lower_leg_length*sp.cos(gamma)*L.i + lower_leg_length*sp.sin(gamma)*L.j
print(ll)
```

```
(4*cos(gamma))*L.i + (4*sin(gamma))*L.j
```

1.6 Make some drawings

```
t = np.radians(-135)
g = np.radians(80)
lu_example = lu.evalf(subs={theta: t})
ll_example = ll.evalf(subs={gamma: g})
```

```
# lower leg coordinate system seen in upper leg coordinate system
Lx = sp.vector.express(L.i, U)
Ly = sp.vector.express(L.j, U)
Lx_example = Lx.evalf(subs={theta: t})
Ly_example = Ly.evalf(subs={theta: t})
```

```
points = np.zeros((2, 2))
points[0,1] = float(lu_example.components.get(U.i, 0))
points[1,1] = float(lu_example.components.get(U.j, 0))

fig, ax = plt.subplots()
plt.xlim(-4,4) #<-- set the x axis limits
plt.ylim(-4,4) #<-- set the y axis limits
# plot a black point at the origin
plt.plot(0,0,'ok')
# plot upper leg
plt.plot(points[0], points[1],'-k')
# plot angle
angles = np.linspace(0, float(t))
xs = 0.5 * np.cos(angles)
ys = 0.5 * np.sin(angles)
plt.plot(xs, ys, '-b')
plt.text(xs[int(len(angles)/2)]+0.1, ys[int(len(angles)/2)]-0.2, "$\\theta$ (%s$^{\
↪circ}$ )" % math.degrees(t))
# plot a black point at the joint between upper and lower leg
plt.plot(points[0,1],points[1,1],'ok')
```

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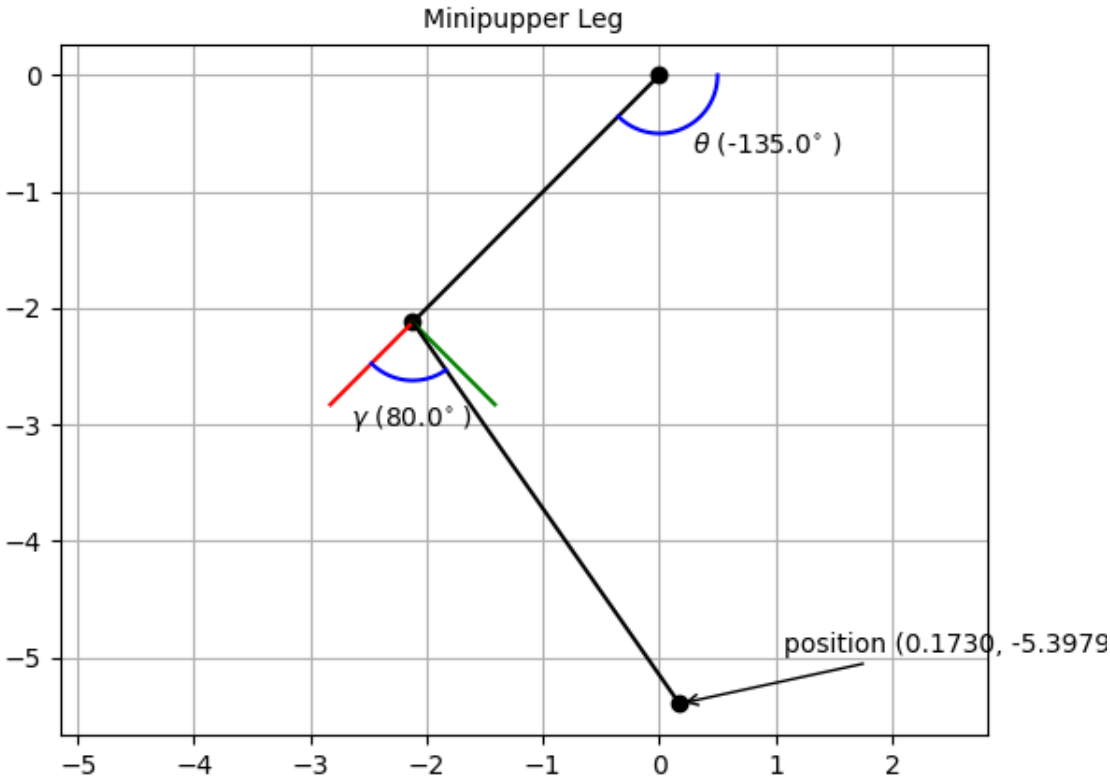
```
# show coordinate system for lower leg
points[:,0] = points[:,1]
points[0,1] = float((lu_example+Lx_example).components.get(U.i, 0))
points[1,1] = float((lu_example+Lx_example).components.get(U.j, 0))
plt.plot(points[0], points[1],'-r')
points[0,1] = float((lu_example+Ly_example).components.get(U.i, 0))
points[1,1] = float((lu_example+Ly_example).components.get(U.j, 0))
plt.plot(points[0], points[1],'-g')
# plot lower leg
points[0,1] = float((lu_example+sp.vector.express(ll_example, U).evalf(subs={theta: t}
 ↪)).components.get(U.i, 0))
points[1,1] = float((lu_example+sp.vector.express(ll_example, U).evalf(subs={theta: t}
 ↪)).components.get(U.j, 0))
plt.plot(points[0], points[1],'-k')
# plot angle
angles = np.linspace(float(t), float(t)+float(g))
xs = points[0,0] + 0.5 * np.cos(angles)
ys = points[1,0] + 0.5 * np.sin(angles)
plt.plot(xs, ys, '-b')
plt.text(xs[int(len(angles)/2)]-0.5, ys[int(len(angles)/2)]-0.4, "$\gamma$ (%s$^{\
 ↪circ}$ )" % math.degrees(g))
# plot a black point at the end of the lower leg
plt.plot(points[0,1],points[1,1],'ok')
ax.annotate(
    "position (%.4f, %.4f)" % (points[0,1],points[1,1]),
    xy=(points[0,1],points[1,1]), xycoords='data',
    xytext=(40, 20), textcoords='offset points',
    arrowprops=dict(arrowstyle="->"))



plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=True, which='major')
plt.title('Minipupper Leg',fontsize=10)
```

```
Text(0.5, 1.0, 'Minipupper Leg')
```



1.7 Inverse Kinematics

Given a point in the base coordinate system calculate theta and gamma so that the end point of the lower leg is positioned at that point

There are points with multiple solutions and there are points that can not be reached. Calculate the region that is reachable

```
P = lu + sp.vector.express(ll, U)
print(P)
```

```
(-4*sin(gamma)*sin(theta) + 4*cos(gamma)*cos(theta) + 3*cos(theta))*U.i +␣
↪(4*sin(gamma)*cos(theta) + 4*sin(theta)*cos(gamma) + 3*sin(theta))*U.j
```

1.8 Brute force solution

We can derive a system of two nonlinear equations and use a solver to find the solutions. This is quite computational intense and you have to uncomment the line to run it

```
P1 = P.components[U.i]
P2 = P.components[U.j]
print(P1)
print(P2)
```

```
-4*sin(gamma)*sin(theta) + 4*cos(gamma)*cos(theta) + 3*cos(theta)
4*sin(gamma)*cos(theta) + 4*sin(theta)*cos(gamma) + 3*sin(theta)
```

```
solution = []
#solution = sp.solvers.solve((P1-points[0,1], P2-points[1,1]), (theta, gamma))
```

```
for i in range(len(solution)):
    print("theta: %s, gamma: %s" % (math.degrees(solution[i][0]), math.
↪degrees(solution[i][1])))
```

1.9 Geometric solution

We will use trigonometric functions to solve the inverse kinematic problem

We introduce a virtual leg lv that spans from the origin of our coordinate system to the point P with coordinates (x, y).

For simplicity we denote in the following calculations lu, ll, lv as the length of the vectors lu, ll, lv.

We will use the law of cosines

The length of the virtual leg is

$$l_v = \sqrt{x^2 + y^2}$$

and the law of cosines provieds

$$l_v^2 = l_u^2 + l_l^2 - 2 l_u l_l \cos(180 - \gamma)$$

With $\cos(180 - \gamma) = -\cos(\gamma)$ and the above given length of lv we obtain

$$\cos(\gamma) = \frac{x^2 + y^2 - l_u^2 - l_l^2}{2 l_u l_l}$$

Thus

$$\gamma = \arccos(\frac{x^2 + y^2 - l_u^2 - l_l^2}{2 l_u l_l})$$

Further we know that

$$x = l_u \cos(\theta) + l_l \cos(\theta + \gamma)$$

$$y = l_u \sin(\theta) + l_l \sin(\theta + \gamma)$$

With $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ and $\sin(a+b) = \cos(a)\sin(b) + \cos(b)\sin(a)$ we obtain

$$x = l_u \cos(\theta) + l_l \cos(\theta)\cos(\gamma) - l_l \sin(\theta)\sin(\gamma)$$

$$y = l_u \sin(\theta) + l_l \cos(\theta)\sin(\gamma) + l_l \cos(\gamma)\sin(\theta)$$

We now have a system of two linear equations with two variables $\sin(\theta)$ and $\cos(\theta)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_u + l_l\cos(\gamma) & -l_l\sin(\gamma) \\ l_l\sin(\gamma) & l_u + l_l\cos(\gamma) \end{bmatrix} = \begin{bmatrix} C \\ S \end{bmatrix}$$

where $C = \cos(\theta)$ and $S = \sin(\theta)$. Knowing that

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

We calculate

$$\theta = \arctan(\frac{S}{C})$$

1.9.1 References

The following sources helped me to refine my understanding

Lecture from Howie Choset, Carnegie Mellon

Paper from Dhaivat Dholakiya et all

```
points = np.zeros((2, 2))
points[0,1] = float(lu_example.components.get(U.i, 0))
points[1,1] = float(lu_example.components.get(U.j, 0))

fig, ax = plt.subplots()
plt.xlim(-4,4) #<-- set the x axis limits
plt.ylim(-4,4) #<-- set the y axis limits
# plot a black point at the origin
plt.plot(0,0,'ok')
# plot upper leg
plt.plot(points[0], points[1],'-k')
plt.text(points[0,1]/2+0.3, points[1,1]/2, 'lu')
# plot angle
angles = np.linspace(0, float(t))
xs = 0.5 * np.cos(angles)
ys = 0.5 * np.sin(angles)
plt.plot(xs, ys, '-b')
plt.text(xs[int(len(angles)/2)]+0.1, ys[int(len(angles)/2)]-0.2, "$\\theta$ (%s$^{\
 ↪circ}$ )" % math.degrees(t))
# plot a black point at the joint between upper and lower leg
plt.plot(points[0,1],points[1,1],'ok')
# show coordinate system for lower leg
points[:,0] = points[:,1]
points[0,1] = float((lu_example+Lx_example).components.get(U.i, 0))
points[1,1] = float((lu_example+Lx_example).components.get(U.j, 0))
plt.plot(points[0], points[1],'-r')
points[0,1] = float((lu_example+Ly_example).components.get(U.i, 0))
points[1,1] = float((lu_example+Ly_example).components.get(U.j, 0))
plt.plot(points[0], points[1],'-g')
```

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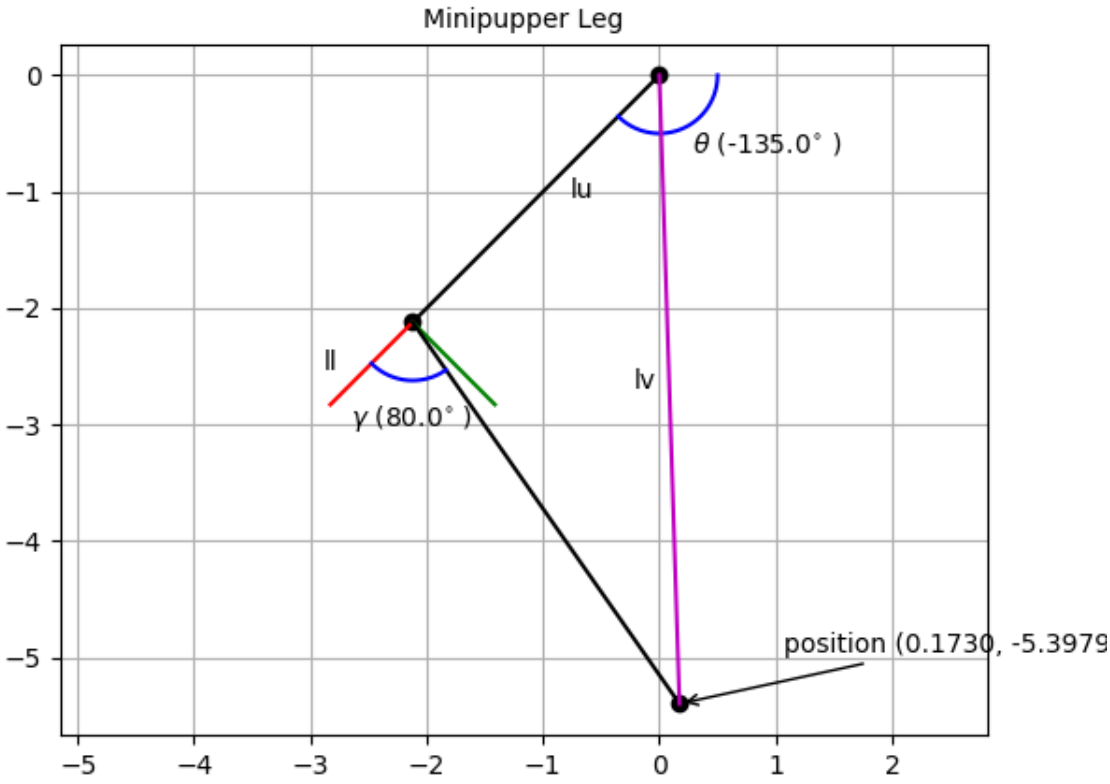
```
# plot lower leg
points[0,1] = float((lu_example+sp.vector.express(ll_example, U).evalf(subs={theta: t}
 ↪)).components.get(U.i, 0))
points[1,1] = float((lu_example+sp.vector.express(ll_example, U).evalf(subs={theta: t}
 ↪)).components.get(U.j, 0))
plt.plot(points[0], points[1],'-k')
plt.text(points[0,0]+points[1,0]/2+0.3, points[0,1]+points[1,1]/2, 'll')
# plot angle
angles = np.linspace(float(t), float(t)+float(g))
xs = points[0,0] + 0.5 * np.cos(angles)
ys = points[1,0] + 0.5 * np.sin(angles)
plt.plot(xs, ys, '-b')
plt.text(xs[int(len(angles)/2)]-0.5, ys[int(len(angles)/2)]-0.4, "$\gamma$ (%s$^{\
 ↪circ}$ )" % math.degrees(g))
# plot a black point at the end of the lower leg
plt.plot(points[0,1],points[1,1],'ok')
ax.annotate(
    "position (%.4f, %.4f)" % (points[0,1],points[1,1]),
    xy=(points[0,1],points[1,1]), xycoords='data',
    xytext=(40, 20), textcoords='offset points',
    arrowprops=dict(arrowstyle="->"))

# plot a vector from the origin to the foot
new_points = np.zeros((2, 2))
new_points[0,1] = points[0,1]
new_points[1,1] = points[1,1]
plt.plot(new_points[0], new_points[1],'-m')
plt.text(new_points[0,1]/2-0.3, new_points[1,1]/2, 'lv')


plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=True, which='major')
plt.title('Minipupper Leg',fontsize=10)
```

```
Text(0.5, 1.0, 'Minipupper Leg')
```



1.10 Calculate γ

$$\gamma = \arccos(\frac{x^2 + y^2 - l_u^2 - l_l^2}{2 l_u l_l})$$

```
gam = np.arccos((points[0,1]**2+points[1,1]**2-upper_leg_length**2-lower_leg_
↪length**2)/(2*upper_leg_length*lower_leg_length))
print(math.degrees(gam))
```

```
80.00000000000001
```

1.11 Calculate θ

Solving

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_u + l_l \cos(\gamma) & -l_l \sin(\gamma) \\ l_l \sin(\gamma) & l_u + l_l \cos(\gamma) \end{bmatrix} = \begin{bmatrix} C \\ S \end{bmatrix}$$

or

$$B = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} C \\ S \end{bmatrix}$$

$$B = AX$$

$$X = A^{-1}B$$

and calculating θ

$$\theta = \arctan(\frac{X_2}{X_1})$$

```
B = np.array([points[0,1],points[1,1]])
A11 = upper_leg_length + lower_leg_length * np.cos(gam)
A12 = -lower_leg_length * np.sin(gam)
A21 = lower_leg_length * np.sin(gam)
A22 = upper_leg_length + lower_leg_length * np.cos(gam)

A = np.array([[A11, A12], [A21, A22]])
```

```
X = np.linalg.inv(A).dot(B)
```

```
thet = np.arctan(X[1]/X[0])
print(math.degrees(thet))
print("%s in the 3rd Quadrant corresponds to %s in the first Quadrant" % (math.
 ↪degrees(np.pi -thet), math.degrees(thet)))
```

```
44.999999999999986
135.0 in the 3rd Quadrant corresponds to 44.999999999999986 in the first Quadrant
```

1.12 Draw reach of leg

We plot a wide range of servo angles without concern if a physical robot will allow these positions. We observe that some positions cause the end of the upper leg touching the ground and the foot of the leg sticks into the air. If we think about gravity we can see that some positions will put a lot of lateral stress on parts of the leg

```
def draw_leg(ax, l1, l2, t, g):
    points = np.zeros((2, 3))
    points[0,1] = l1*np.cos(t)
    points[1,1] = l1*np.sin(t)
    points[0,2] = l2*np.cos(g+t) + points[0,1]
    points[1,2] = l2*np.sin(g+t) + points[1,1]
    ax.set_aspect('equal')
    ax.set(xlim=(-5, 5), ylim=(-5, 5))
    ax.get_xaxis().set_ticks([])
    ax.get_yaxis().set_ticks([])
    ax.plot(points[0],points[1])
```

```
ncols=10
nrows=10
l1_angles = np.linspace(-5*np.pi/4, np.pi/4, num=ncols)
l2_angles = np.linspace(0.0, np.pi, num=nrows)
```

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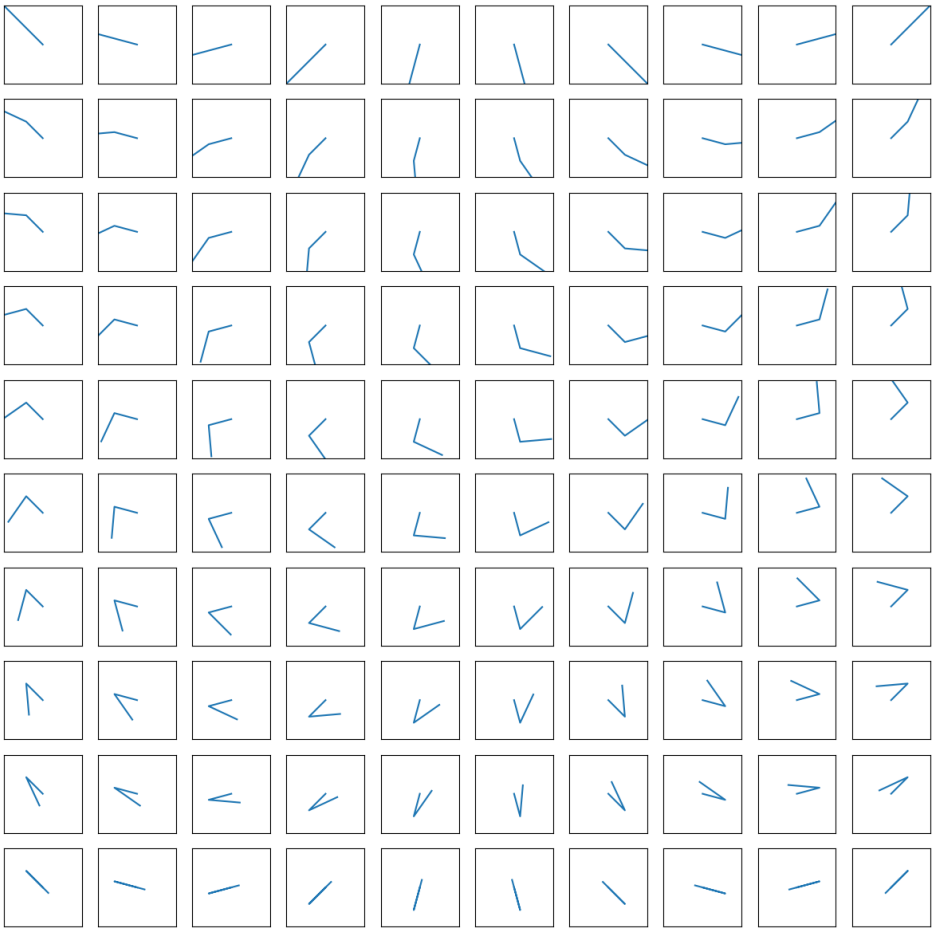
```
fig, axs = plt.subplots(ncols=ncols, nrows=nrows, figsize=(15,15))
for row in range(nrows):
    for col in range(ncols):
        draw_leg(axs[row, col], upper_leg_length, lower_leg_length, l1_angles[col],␣
↪l2_angles[row])
```



```
col=5
row=4
print("%s %s" % (np.degrees(l1_angles[col-1]), np.degrees(l2_angles[row-1])))
```

```
-105.00000000000001 59.99999999999999
```

CHAPTER

TWO

UNIFIED ROBOTICS DESCRIPTION FORMAT

URDF is an XML specification to model robots. We will have a closer look at the urdf file for minipupper

2.1 Reference

The following sources helped me to refine my understanding

Understanding URDF using MATLABR - Peter Corke

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

```
from xml.dom import minidom
```

```
with open('./simple.urdf', 'r') as f:
    xmldoc = minidom.parse(f)
```

```
print(xmldoc.toprettyxml())
```

```
<?xml version="1.0" ?>
<!-- copied from Understanding URDF using MATLABR - Peter Corke -->
<robot name="planar2">


        <link name="base_link"/>


        <link name="link1"/>


        <link name="link2"/>


        <link name="end"/>


        <joint name="q1" type="continuous">
```

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```
                <parent link="base_link"/>

                <child link="link1"/>

                <axis xyz="0 0 1"/>

        </joint>

        <joint name="q2" type="continuous">

                <parent link="link1"/>

                <child link="link2"/>

                <origin xyz="1 0 0"/>

                <axis xyz="0 0 1"/>

        </joint>

        <joint name="notajoint" type="fixed">

                <parent link="link2"/>

                <child link="end"/>

                <origin xyz="1 0 0"/>

        </joint>

</robot>
```

```
def parseDom(node):
    if node.nodeType != minidom.Node.ELEMENT_NODE:
        return
    if node.nodeName == 'transmission':
        # ignore transmission and avoid overwritten links
        return
    if len(node.getAttribute("name")) > 0:
```

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```
        urdf[node.getAttribute("name")] = node
        if node.nodeName == 'link':
            links.append(node.getAttribute("name"))
        if node.nodeName == 'joint':
            joints.append(node.getAttribute("name"))
    for child in node.childNodes:
        parseDom(child)
```

```
urdf = {}
links = []
joints = []
parseDom(xmldoc.documentElement)
```

```
for i in range(len(links)):
    print(links[i])
```

```
base_link
link1
link2
end
```

```
def geJointParent(node):
    for n in urdf[node].childNodes:
        if n.nodeType != minidom.Node.ELEMENT_NODE:
            continue
        if n.nodeName == 'parent':
            return n.getAttribute('link')
```

```
def geJointChild(node):
    for n in urdf[node].childNodes:
        if n.nodeType != minidom.Node.ELEMENT_NODE:
            continue
        if n.nodeName == 'child':
            return n.getAttribute('link')
```

```
for i in range(len(joints)):
    print("%s: parent %s child %s" % (joints[i], geJointParent(joints[i]),␣
 ↪geJointChild(joints[i])))
```

```
q1: parent base_link child link1
q2: parent link1 child link2
notajoint: parent link2 child end
```

```
with open('../minipupper_description/urdf/minipupper.urdf', 'r') as f:
    xmldoc = minidom.parse(f)
```

```
urdf = {}
links = []
joints = []
parseDom(xmldoc.documentElement)
```

```
def getJointOrigin(node):
    for n in urdf[node].childNodes:
        if n.nodeType != minidom.Node.ELEMENT_NODE:
            continue
        if n.nodeName == 'origin':
            return n.getAttribute('xyz')
```

```
joints_origin = {}
for i in range(len(joints)):
    if 'debug' in joints[i]:
        continue
    if 'inertia' in joints[i]:
        continue
    origin = getJointOrigin(joints[i])
    joints_origin[joints[i]] = [float(x) for x in origin.split(' ')]
    print("%s: parent %s child %s origin %s" % (joints[i], geJointParent(joints[i]),␣
 ↪geJointChild(joints[i]), origin))
```

```
lf_hip_joint: parent base_link child lf_hip_link origin 0.06014 0.0235 0.0171
lf_upper_leg_joint: parent lf_hip_link child lf_upper_leg_link origin 0 0.0197 0
lf_lower_leg_joint: parent lf_upper_leg_link child lf_lower_leg_link origin 0 0.
 ↪00475 -0.05
lf_foot_joint: parent lf_lower_leg_link child lf_foot_link origin 0 0 -0.056
lh_hip_joint: parent base_link child lh_hip_link origin -0.05886 0.0235 0.0171
lh_upper_leg_joint: parent lh_hip_link child lh_upper_leg_link origin 0 0.0197 0
lh_lower_leg_joint: parent lh_upper_leg_link child lh_lower_leg_link origin 0 0.
 ↪00475 -0.05
lh_foot_joint: parent lh_lower_leg_link child lh_foot_link origin 0 0 -0.056
rf_hip_joint: parent base_link child rf_hip_link origin 0.06014 -0.0235 0.0171
rf_upper_leg_joint: parent rf_hip_link child rf_upper_leg_link origin 0 -0.0197 0
rf_lower_leg_joint: parent rf_upper_leg_link child rf_lower_leg_link origin 0 -0.
 ↪00475 -0.05
rf_foot_joint: parent rf_lower_leg_link child rf_foot_link origin 0 0 -0.056
rh_hip_joint: parent base_link child rh_hip_link origin -0.05886 -0.0235 0.0171
rh_upper_leg_joint: parent rh_hip_link child rh_upper_leg_link origin 0 -0.0197 0
rh_lower_leg_joint: parent rh_upper_leg_link child rh_lower_leg_link origin 0 -0.
 ↪00475 -0.05
rh_foot_joint: parent rh_lower_leg_link child rh_foot_link origin 0 0 -0.056
```

```
locations = ['lf', 'lh', 'rf', 'rh']# robot origin
xs = [0.0]
ys = [0.0]
zs = [0.0]
# hip
for l in locations:
    loc = "%s_hip_joint" % l
    xs.append(joints_origin[loc][0])
    ys.append(joints_origin[loc][1])
    zs.append(joints_origin[loc][2])
# upper leg
for l in locations:
    loc = "%s_upper_leg_joint" % l
    loc_parent = "%s_hip_joint" % l
    xs.append(joints_origin[loc][0] + joints_origin[loc_parent][0])
    ys.append(joints_origin[loc][1] + joints_origin[loc_parent][1])
```

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```
    zs.append(joints_origin[loc][2] + joints_origin[loc_parent][2])
# lower leg
for l in locations:
    loc = "%s_lower_leg_joint" % l
    loc_parent = "%s_upper_leg_joint" % l
    loc_parent1 = "%s_hip_joint" % l
    xs.append(joints_origin[loc][0] + joints_origin[loc_parent][0] + joints_
 ↪origin[loc_parent1][0])
    ys.append(joints_origin[loc][1] + joints_origin[loc_parent][1] + joints_
 ↪origin[loc_parent1][1])
    zs.append(joints_origin[loc][2] + joints_origin[loc_parent][2] + joints_
 ↪origin[loc_parent1][2])
# foot
for l in locations:
    loc = "%s_foot_joint" % l
    loc_parent = "%s_lower_leg_joint" % l
    loc_parent1 = "%s_upper_leg_joint" % l
    loc_parent2 = "%s_hip_joint" % l
    xs.append(joints_origin[loc][0] + joints_origin[loc_parent][0] + joints_
 ↪origin[loc_parent1][0] + joints_origin[loc_parent2][0])
    ys.append(joints_origin[loc][1] + joints_origin[loc_parent][1] + joints_
 ↪origin[loc_parent1][1] + joints_origin[loc_parent2][1])
    zs.append(joints_origin[loc][2] + joints_origin[loc_parent][2] + joints_
 ↪origin[loc_parent1][2] + joints_origin[loc_parent2][2])
```

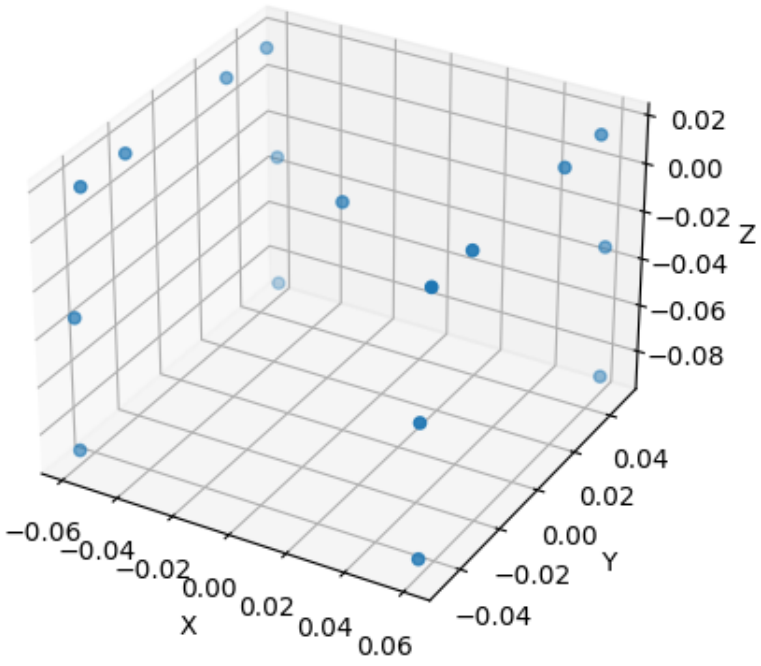
```
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure()
ax = fig.add_subplot(projection='3d')

ax.scatter(xs, ys, zs)


ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
```

```
Text(0.5, 0, 'Z')
```



2.2 Length of legs

Calculate the length of the upper and lower leg based on information from the URDF file

We will use this information for our inverse kinematics

```
print("Lenght of upper leg: %s" % np.linalg.norm(np.array(joints_origin['lh_lower_leg_
↪joint'])))
print("Lenght of lower leg: %s" % np.linalg.norm(np.array(joints_origin['lh_foot_joint
↪'])))
```

```
Lenght of upper leg: 0.05022511821787979
Lenght of lower leg: 0.056
```

2.3 Urdfpy

There is more work to do to properly parse a URDF file. We will use a library

Urdfpy Documentation

urdfpy currently requires networkx==2.2 but this version is broken on Python 3.9

pip uninstall networkx

pip install networkx

and ignore the warnings

If you want to display ur5

pip uninstall pyrender

pip install git+https://github.com/mmatl/pyrender.git

```
from urdfpy import URDF

# uncomment the urdf file you want to work on. Leave None if you want to skip this␣
 ↪step
robot=None
# we use a modified version because the original file throws an error
#robot = URDF.load('../minipupper_description/urdf/minipupper_edited.urdf')
#robot = URDF.load('./simple_visual.urdf')
#robot = URDF.load('../ur5/ur5.urdf')
```

```
if robot is not None:
    for link in robot.links:
        print(link.name)
```

```
if robot is not None:
    for joint in robot.joints:
        print('{} connects {} to {}'.format(
               joint.name, joint.parent, joint.child
              ))
```

```
if robot is not None:
    for joint in robot.actuated_joints:
        print(joint.name)
```

```
if robot is not None:
    if robot.name == 'minipupper':
        cfg={
             'lf_hip_joint': 0.0,
             'lh_hip_joint': 0.0,
             'rf_hip_joint': 0.0,
             'rh_hip_joint': 0.0,
             'lf_upper_leg_joint': math.pi/4,
             'lh_upper_leg_joint': math.pi/4,
             'rf_upper_leg_joint': math.pi/4,
             'rh_upper_leg_joint': math.pi/4,
             'lf_lower_leg_joint': -math.pi/2,
```

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```
            'lh_lower_leg_joint': -math.pi/2,
            'rf_lower_leg_joint': -math.pi/2,
            'rh_lower_leg_joint': -math.pi/2
           }
    if robot.name == 'planar2':
        cfg={'q1': 0.0, 'q2': math.pi/4}
    if robot.name == 'ur5':
        cfg={
             'shoulder_pan_joint': 0.0,
             'shoulder_lift_joint': 2.0,
             'elbow_joint': 2.0,
             'wrist_1_joint': 0.0,
             'wrist_2_joint': 0.0,
             'wrist_3_joint': 0.0,
           }
```

```
import math
# Attention, the following will open a new Window but does not terminate properly␣
↪(Mac OS)
if robot is not None:
    robot.show(cfg=cfg)
```

CHAPTER

THREE

2D LEG MOVEMENT

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

3.1 Forward Kinematics

We can express 2D Transformations as a 3-dimensional homogeneous transformation matrix

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & x \\ \sin(\theta) & \cos(\theta) & y \\ 0 & 0 & 1 \end{bmatrix}$$

Where θ is the rotation angle and x,y are the components of the translation vector

In the following we use Spatial Maths for Python from Peter Corke as our Python module

3.1.1 Reference

The following sources helped me to refine my understanding

Robotics Programming Study Guide - Tim Bower

Describing rotation and translation in 2D - Peter Corke

```
from spatialmath import *
```

```
# set parameters according 1-kinematics_2d
theta = np.radians(-135)
gamma = np.radians(80)
lu = 3
ll = 4

# set parameters according http://faculty.salina.k-state.edu/tim/robot_prog/Arm_
↪robots/forwardKin.html#simple-2-d-example
#theta = np.pi*40/180
#gamma = -np.pi*15/180
#lu = 5
#ll = 2
```

```
# create 2d rotation Matrix for theta
R = SO2(theta)
print(R)
```

```
  -0.7071    0.7071
  -0.7071   -0.7071
```

```
# create a 2d homogeneous translation matrix
# calculate the translation vector
T = SE2([lu*np.cos(theta), lu*np.sin(theta)])
print(T)
```

```
   1         0        -2.121
   0         1        -2.121
   0         0         1
```

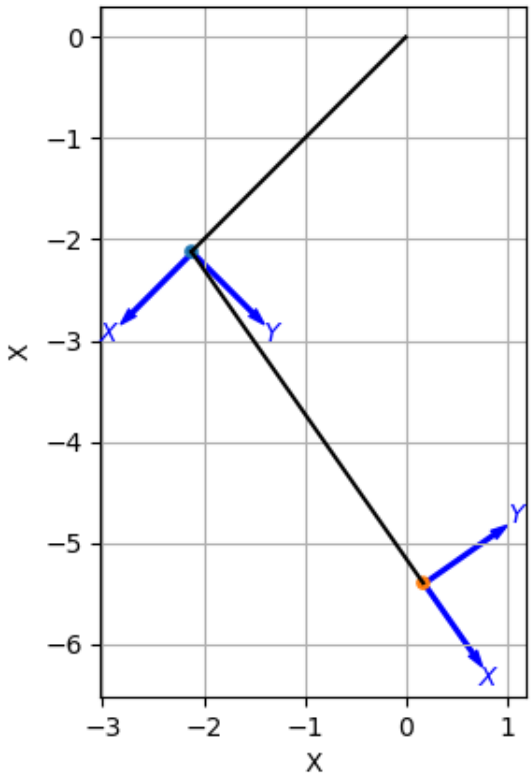
```
# combining the two
T1 = SE2([lu*np.cos(theta), lu*np.sin(theta)]) * SE2(theta)
print(T1)
```

```
  -0.7071    0.7071   -2.121
  -0.7071   -0.7071   -2.121
   0         0         1
```

```
# Transforming from origin to end of upper leg to end of lower leg
T2 = SE2([ll*np.cos(gamma), ll*np.sin(gamma)]) * SE2(gamma)
T3 = T1*T2
print(T3)

points = np.zeros((2, 2))
fig, ax = plt.subplots()
# plot upper leg
points[0,1] = lu*np.cos(theta)
points[1,1] = lu*np.sin(theta)
plt.plot(points[0], points[1],'-k')
T1.plot()
# plot lower leg
points[0,0] = points[0,1]
points[1,0] = points[1,1]
points[0,1] = lu*np.cos(theta) + ll*np.cos(theta+gamma)
points[1,1] = lu*np.sin(theta) + ll*np.sin(theta+gamma)
plt.plot(points[0], points[1],'-k')
T3.plot()
plt.grid(True)
```

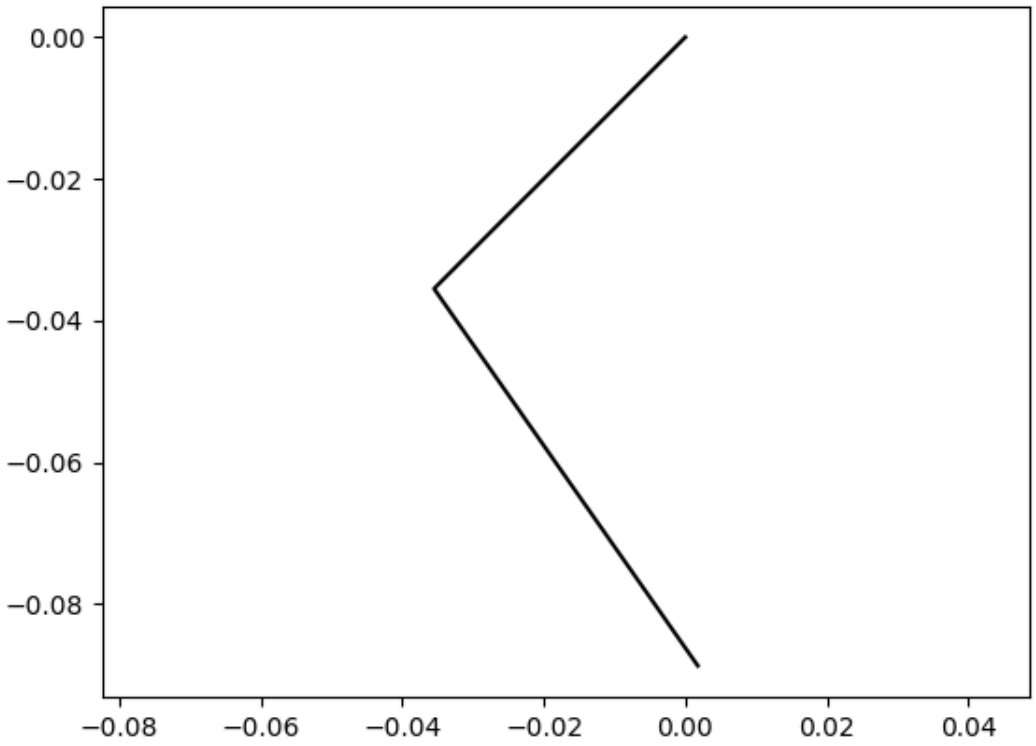
```
   0.5736    0.8192    0.173
  -0.8192    0.5736   -5.398
   0         0         1
```




```
# for Minipupper write function for forward kinematic using only np
def transformationMatrix(angle, l):
    c = np.cos(angle)
    s = np.sin(angle)
    return np.array([[c, -s, l*c], [s, c, l*s], [0, 0, 1]])
def mpForwardKin(angles):
    lu = 0.05022511821787979 # from urdf
    ll = 0.065 # measured, includes the rubber foot
    T1 = transformationMatrix(angles[0], lu)
    T2 = transformationMatrix(angles[1], ll)
    T3 = T1.dot(T2)
    P1 = T1.dot(np.array([0,0,1]))[0:2]
    P2 = T3.dot(np.array([0,0,1]))[0:2]
    return np.array([[0, P1[0], P2[0]], [0, P1[1], P2[1]]])
```

```
points = mpForwardKin([theta, gamma])
fig, ax = plt.subplots()
plt.plot(points[0], points[1],'-k')
plt.axis('equal')

plt.grid(visible=False, which='major')
```



3.2 Inverse Kinematics

We use the calculations from 1-kinematics_2d

```
def mpInverseKin(point):
    lu = 0.05022511821787979 # from urdf
    ll = 0.065 # measured, includes the rubber foot
    gam = np.arccos((point[0]**2+point[1]**2-lu**2-ll**2)/(2*lu*ll))
    B = np.array(point)
    A11 = lu + ll * np.cos(gam)
    A12 = -ll * np.sin(gam)
    A21 = ll * np.sin(gam)
    A22 = lu + ll * np.cos(gam)

    A = np.array([[A11, A12], [A21, A22]])
    X = np.linalg.inv(A).dot(B)
    thet = np.arctan(X[1]/X[0])
    # correction for 2nd and 3rd quadrant
    # we assume -np.pi/4 <= thet <= -3*np.pi/4
    if thet > -np.pi/4:
        thet = thet - np.pi
    return [thet, gam]
```

```
np.testing.assert_almost_equal([theta, gamma], mpInverseKin(mpForwardKin([theta,␣
↪gamma])[:,2]), decimal=7, err_msg='', verbose=True)
```

```
for i in range(1,20):
    theta = -np.pi/4 - i/20 * np.pi
    gamma = i/20 * np.pi
    np.testing.assert_almost_equal([theta, gamma], mpInverseKin(mpForwardKin([theta,␣
↪gamma])[:,2]), decimal=10, err_msg='', verbose=True)
```

3.3 Push Up

We start with a neutral position and then draw a line up and down the y axes. On this line we define a number of points that must be reached, each point in a configurable time interval that is constant for all points.

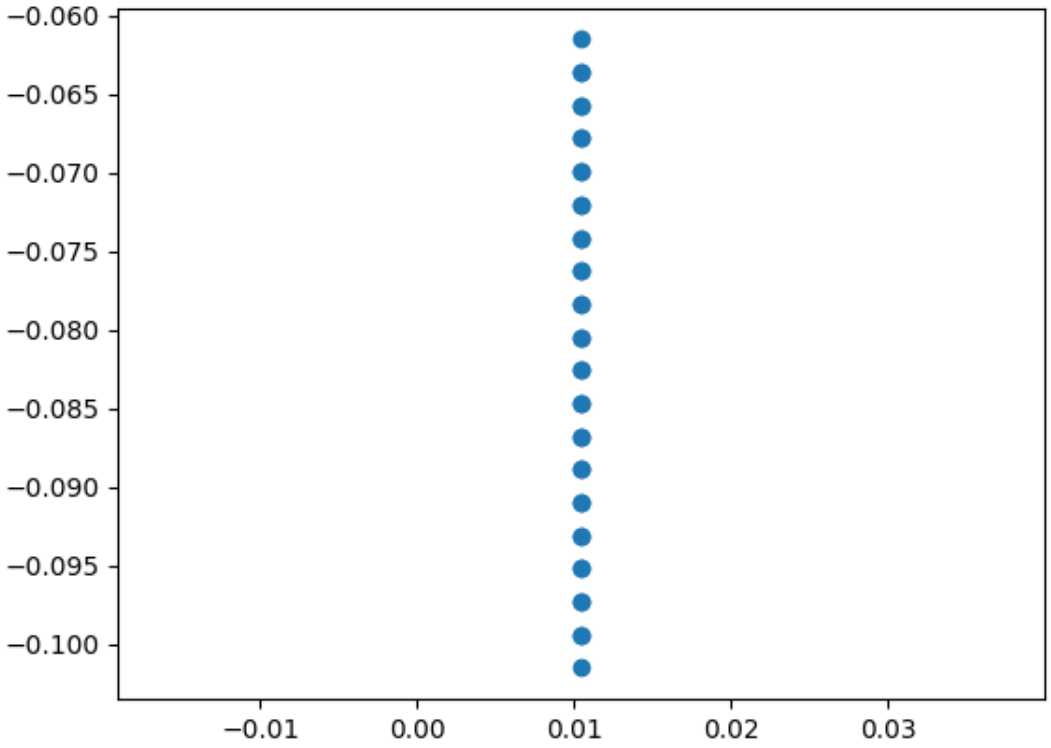
We will take care that we can repeat this leg_servo_positions to build loops

```
theta = -3*np.pi/4
gamma = np.pi/2
midpoint = mpForwardKin([theta, gamma])[:,2]
#
delta = 0.02 # how much up, down
N = 20 # number of points
line = np.linspace(midpoint[1]-delta, midpoint[1]+delta, num=N)
```

```
# create points there to position the foot. We construct a loop and make sure the end␣
↪points are only hit once
points = np.full((2, 2*N-2), midpoint[0])
points[1][0:N] = line
points[1][N:] = np.flip(line)[1:N-1]

#plot
fig, ax = plt.subplots()
plt.scatter(points[0], points[1])
plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=False, which='major')
```




```
leg_servo_positions = []
for j in range(2*N-2):
    leg_servo_positions.append(mpInverseKin(points[:,j]))
```

```
import matplotlib.animation as animation
from collections import deque

fig, ax = plt.subplots()
plt.xlim(-0.15, 0.15)
plt.ylim(-0.15, 0)
plt.axis('equal')
plt.grid(visible=False, which='major')

line, = ax.plot([], [], 'o-', lw=2)
trace, = ax.plot([], [], '.-', lw=1, ms=2)
step_template = 'step = %.1f'
step_text = ax.text(0.05, 0.9, '', transform=ax.transAxes)
history_len = 50
history_x, history_y = deque(maxlen=history_len), deque(maxlen=history_len)


def animate(i):
    p = mpForwardKin(leg_servo_positions[i])

    if i == 0:
```

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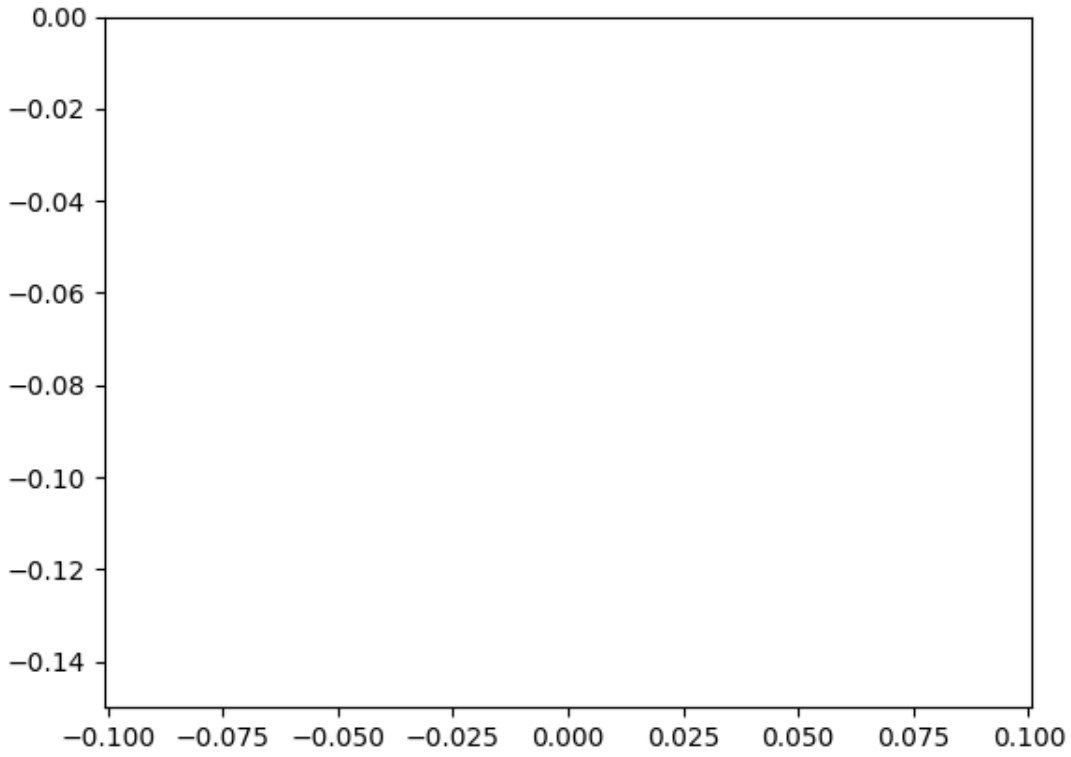
```
        history_x.clear()
        history_y.clear()

    history_x.appendleft(p[0][2])
    history_y.appendleft(p[1][2])

    line.set_data(p[0], p[1])
    trace.set_data(history_x, history_y)
    step_text.set_text(step_template % (i))
    #step_text.set_text("%s" % (p[0]))
    return line, trace, step_text


ani = animation.FuncAnimation(fig, animate, len(leg_servo_positions), interval=200,␣
↪blit=True)

plt.show()
```



3.4 Deployment

We can deploy leg trajectories to a simulated or physical minipuper by writing angles to a file for each servo.

The controller in the simulated or physical environments reads all servo command files and finds the file with the most steps in it. This is the cycle length. You can specify the duration of one cycle by a parameter.

```
# wite to disk

fh_hip = open("hip_servo", "w")
fh_upper_leg = open("upper_leg_servo", "w")
fh_lower_leg = open("lower_leg_servo", "w")
# hip will not move
fh_hip.write("%.10f\n" % 0.0)
for j in range(2*N-2):
    fh_upper_leg.write("%.10f\n" % leg_servo_positions[j][0])
    fh_lower_leg.write("%.10f\n" % leg_servo_positions[j][1])
fh_hip.close()
fh_upper_leg.close()
fh_lower_leg.close()
```

```
# Check

cycle_duration = 1 # duration in seconds

hip = []
with open("hip_servo", "r") as f:
    for line in f:
        hip.append(float(line.rstrip()))
upper_leg = []
with open("upper_leg_servo", "r") as f:
    for line in f:
        upper_leg.append(float(line.rstrip()))
lower_leg = []
with open("lower_leg_servo", "r") as f:
    for line in f:
        lower_leg.append(float(line.rstrip()))

max_steps = max(len(hip), len(upper_leg), len(lower_leg))

print("Duration per cycle: %s [msec]\nDuration per step: %s [msec]\nNumber of steps:
 ↪%s"
      % (cycle_duration*1000, cycle_duration*1000/max_steps, max_steps))
```

```
Duration per cycle: 1000 [msec]
Duration per step: 26.31578947368421 [msec]
Number of steps: 38
```

```
# copy files to controller
import shutil

joints = ['hip', 'upper_leg', 'lower_leg']
legs = ['lf', 'rf', 'lh', 'rh']

for joint in joints:
```

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```
    for leg in legs:
        shutil.copy("./%s_servo" % joint, "../controller/servos/%s_%s" % (leg, joint))
```

```
# example how to control only front legs

for joint in joints:
    for leg in legs[2:]:
        with open("../controller/servos/%s_%s" % (leg, joint), "w") as f:
            angle = 0.0
            if 'upper' in joint:
                angle = -np.pi/4
            if 'lower' in joint:
                angle = np.pi/2
            f.write("%.10f\n" % angle)
```

```
# Save foot movements

for joint in joints:
    for leg in legs:
        shutil.copy("./%s_servo" % joint, "../controller/servos/pushup/%s_%s" % (leg,␣
 ↪joint))
```

CHAPTER

FOUR

MINIPUPPER LEG MOVEMENTS

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

```
# Just a helper module, details are covered later
from gaits import GaitController
```

4.1 Minipupper leg and servos

We made the assumption that the lower leg joint moves around the endpoint of the upper leg and this is how we calculate the angles.

However the lower leg of the minipupper moves as in the figure below. The lower leg servo is connected to the hip cage and drives the lower leg through linkage.

```
upper_leg_length = 0.05022511821787979 # from urdf
lower_leg_length = 0.065 # measured, includes the rubber foot
lower_leg_extension_length = 0.015 # measured
servo_arm_length = 0.0125 # measured
servo_link_length = 0.0175 # measured
servo_delta_x = 0.0105 # measured
servo_delta_y = 0.014 # measured
gc = GaitController(1, 1, 1, 1, 1, 1)
```

```
theta = -3*np.pi/4
gamma = np.pi/2

# see below
zmax = -2.2142974355881813
#gamma = zmax - theta-1e-10 # correct rounding errors
#theta = np.round(zmax - gamma, 15)+1e-10 # correct rounding errors

leg = gc.mpForwardKin([theta, gamma])
servos = np.zeros((2,2))
servos[:, 0] = leg[:, 0]
servos[0, 1] = servos[0, 1] + servo_delta_x
servos[1, 1] = servos[1, 1] + servo_delta_y

# Black magic just for the figure
```

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```
linkage = np.zeros((2,6))
T1 = gc._transformationMatrix(theta, gc.lu)
T2 = gc._transformationMatrix(gamma+np.pi, lower_leg_extension_length)
T3 = T1.dot(T2)
P = T3.dot(np.array([0, 0, 1]))[0:2]
linkage[:, 0] = leg[:, 1]
linkage[:, 1] = P
T1 = gc._transformationMatrix(theta, 0)
T2 = gc._transformationMatrix(gamma+np.pi, lower_leg_extension_length)
T3 = T1.dot(T2)
P = T3.dot(np.array([0, 0, 1]))[0:2]
linkage[:, 2] = P
T2 = gc._transformationMatrix(gamma, servo_arm_length)
T3 = T1.dot(T2)
P = T3.dot(np.array([0, 0, 1]))[0:2]
linkage[:, 3] = P
linkage[:, 5] = servos[:, 1]
```

```
# copied from http://paulbourke.net/geometry/circlesphere/circle_intersection.py
# adapted to numpy

class Circle(object):
    """ An OOP implementation of a circle as an object """

    def __init__(self, point, radius):
        self.xpos = point[0]
        self.ypos = point[1]
        self.radius = radius

    def circle_intersect(self, circle2):
        """
        Intersection points of two circles using the construction of triangles
        as proposed by Paul Bourke, 1997.
        http://paulbourke.net/geometry/circlesphere/
        """
        X1, Y1 = self.xpos, self.ypos
        X2, Y2 = circle2.xpos, circle2.ypos
        R1, R2 = self.radius, circle2.radius

        Dx = X2-X1
        Dy = Y2-Y1
        D = np.sqrt(Dx**2 + Dy**2)
        # Distance between circle centres
        if D > R1 + R2:
            return "The circles do not intersect"
        elif D < np.fabs(R2 - R1):
            return "No Intersect - One circle is contained within the other"
        elif D == 0 and R1 == R2:
            return "No Intersect - The circles are equal and coincident"
        else:
            if D == R1 + R2 or D == R1 - R2:
                CASE = "The circles intersect at a single point"
            else:
                CASE = "The circles intersect at two points"
            chorddistance = (R1**2 - R2**2 + D**2)/(2*D)
```

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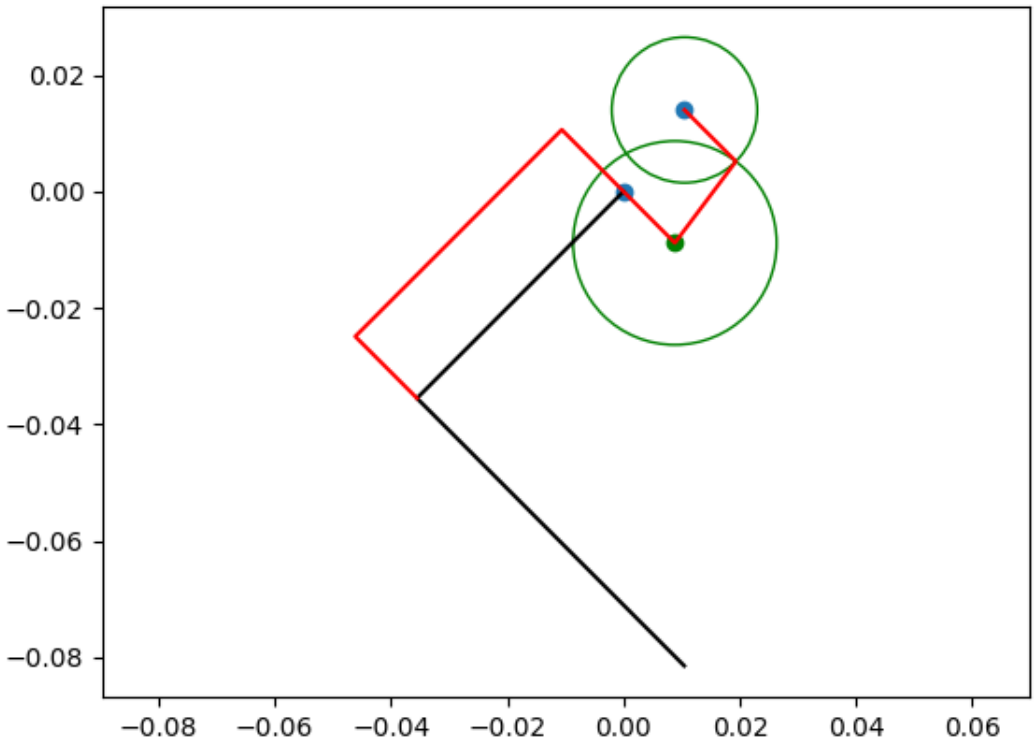
```
            # distance from 1st circle's centre to the chord between intersects
            halfchordlength = np.sqrt(np.round(R1**2 - chorddistance**2, 15))
            chordmidpointx = X1 + (chorddistance*Dx)/D
            chordmidpointy = Y1 + (chorddistance*Dy)/D
            I1 = (chordmidpointx + (halfchordlength*Dy)/D,
                  chordmidpointy - (halfchordlength*Dx)/D)
            theta1 = np.degrees(np.arctan2(I1[1]-Y1, I1[0]-X1))
            I2 = (chordmidpointx - (halfchordlength*Dy)/D,
                  chordmidpointy + (halfchordlength*Dx)/D)
            theta2 = np.degrees(np.arctan2(I2[1]-Y1, I2[0]-X1))
            if theta2 > theta1:
                I1, I2 = I2, I1
            return (I1, I2, CASE)
```

```
c1 = Circle(linkage[:, 3], servo_link_length)
c2 = Circle(servos[:, 1], servo_arm_length)
ret = c1.circle_intersect(c2)
if len(ret) == 28:
    linkage[:, 5] = [0.0, 0.0]
    raise ValueError("Invalid angles: %s" % ret)
linkage[:, 4] = ret[1]
```

```
#plot
fig, ax = plt.subplots()
plt.plot(leg[0], leg[1],'-k')
plt.scatter(servos[0], servos[1])
plt.plot(linkage[0], linkage[1],'-r')
plt.scatter(linkage[0,-3], linkage[1, -3], color='green')
circle1 = plt.Circle(linkage[:, 3], servo_link_length, color='g', fill=False)
circle2 = plt.Circle(servos[:, 1], servo_arm_length, color='g', fill=False)
ax.add_patch(circle1)
ax.add_patch(circle2)
plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=False, which='major')
```




```
# angle of servo arm (see below)
alpha = -np.arccos((linkage[0, 4] - servos[0, 1])/servo_arm_length)
print("%.5f %.5f" % (np.degrees(alpha), np.degrees(theta + gamma)))
```

```
-45.00000 -45.00000
```

4.2 Special case: Parallelogram

The above considerations are generic and by changing the length of the different part you can see the impact. At first I have taken a wrong measurement which lead me down this path.

However there is a special case where the length of servo_link is exactly the distance between the two servo motors. This is when the following condition is met:

$$servo\_link\_length^2 = servo\_delta\_x^2 + servo\_delta\_y^2$$

In this case we have a parallelogram between the lower leg servo and the linkage that revolves around the upper leg servo. There is a second parallelogram formed by the linkage spanning the part revolving around the upper leg servo and the lower leg. An element of that part is adjustable to ensure the property is met.

In this special case the lower leg servo arm is always parallel to the lower leg. We set $\zeta = \theta + \gamma$ as the angle of the lower leg servo arm. We can see that we have a dependency between the position of the upper and lower leg. Changing the position of the upper leg by maintaining the relative position of the lower leg requires also to adapt the lower leg. This is different from quadrupeds where the lower leg actuator is actually attached to the upper leg

4.3 Angle limits

The lower leg servo arm is limited in rotation. First the servo motor itself has a limited rotation of 180° This is further limited by mechanics. We will have a collision if the lower leg servo arm comes to close to the upper leg servo. We will set a limit of -90° We also do not want the end of the upper leg to touch ground before the lower get can get the the ground. We therefore limit the lower leg servo arm to 0° If the upper leg is set to -90° we can not move the lower leg less then 45°

We further limit the upper leg movement to a range between -90° and -180° and the lower leg movement to a range between 0° and 135°

In summary and $\zeta = \theta + \gamma$ as the angle of the lower leg servo arm:

$$-90^\circ \leq \zeta \leq 0^\circ$$

$$-180^\circ \leq \theta - 90^\circ$$

$$45^\circ \leq \gamma \leq 135^\circ$$

```
joints = ['hip', 'upper_leg', 'lower_leg']
legs = ['lf', 'rf', 'lh', 'rh']

angles = np.linspace(-np.pi/2, -np.pi, 20)

for joint in joints:
    for leg in legs:
        with open("../controller/servos/leg-test/%s_%s" % (leg, joint), "w") as f:
            angle = 0.0
            if 'upper' in joint:
                angle = -3*np.pi/4
            if 'lower' in joint:
                angle = np.pi/2
            f.write("%.10f\n" % angle)

with open("../controller/servos/leg-test/rf_upper_leg", "w") as f:
    for i in range(len(angles)):
        f.write("%.10f\n" % angles[i])
with open("../controller/servos/leg-test/rf_lower_leg", "w") as f:
    for i in range(len(angles)):
        if angles[i] > -3*np.pi/4:
            f.write("%.10f\n" % (np.pi/2))
        else:
            f.write("%.10f\n" % (3*np.pi/4))
```

```
for joint in joints:
    for leg in legs:
        with open("../controller/servos/lift-leg/%s_%s" % (leg, joint), "w") as f:
            angle = 0.0
            if 'upper' in joint:
                angle = -3*np.pi/4
            if 'lower' in joint:
                angle = np.pi/2
            f.write("%.10f\n" % angle)

with open("../controller/servos/lift-leg/rf_upper_leg", "w") as f:
```

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```
    angle = -np.pi
    f.write("%.10f\n" % angle)

with open("../controller/servos/lift-leg/rf_lower_leg", "w") as f:
    angle = 3*np.pi/4
    f.write("%.10f\n" % angle)
```

CHAPTER

FIVE

DISCUSSION OF A PAPER

We will study foot trajectories as descibed in Foot Trajectory Planning Method with Adjustable Parameters for Complex Environment

We will break down the foot trajectory into two phases, a swing phase and a stance phase. we will use 3rd-order Bezier curves to describe the curves of each phase and to smoothen the impact of the transition between the two phases we will use a Sigmoid function.

A Bézier curve is a mapping from $s \in [0, 1]$ to convex combinations of points $P_0, P_1, \ldots, P_n$ in some vector space:

$$B(s) = \sum_{i=0}^{n} \binom{n}{i} s^i (1-s)^{n-i} P_i$$

with $n = 3$:

$$B(s) = \sum_{i=0}^{3} \binom{3}{i} s^i (1-s)^{3-i} P_i$$

$$B(s) = (1-s)^3 P_0 + 3s(1-s)^2 P_1 + 3s^2(1-s) P_2 + s^3 P_3$$

To find the maximum we have to solve the differential equation

$$\frac{dB}{ds} = 0$$

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

```
import sympy as sp
s, P0, P1, P2, P3 = sp.symbols('s P_0 P_1 P_2 P_3')
```

```
B = (1-s)**3*P0+3*s*(1-s)**2*P1+3*s**2*(1-s)*P2+s**3*P3
B
```

$$P_0(1-s)^3 + 3P_1 s(1-s)^2 + 3P_2 s^2 \cdot (1-s) + P_3 s^3$$

```
H, L = sp.symbols('H L')
```

```
B1 = B.subs(P0, sp.Matrix([[-L/2, 0]]).transpose())
B2 = B1.subs(P1, sp.Matrix([[0, H]]).transpose())
B3 = B2.subs(P2, sp.Matrix([[L/2, H]]).transpose())
B4 = B3.subs(P3, sp.Matrix([[L/2, 0]]).transpose())
B4
```

$$\begin{bmatrix} \frac{Ls^3}{2} + \frac{3Ls^2 \cdot (1-s)}{2} - \frac{L(1-s)^3}{2} \\ 3Hs^2 \cdot (1-s) + 3Hs\,(1-s)^2 \end{bmatrix}$$

```
sp.solve(sp.diff(B4[1], s), s)
```

```
[1/2]
```

```
B4.subs(s, 1/2)
```

$$\begin{bmatrix} 0.1875L \\ 0.75H \end{bmatrix}$$

5.1 Swing Phase

The below figure shows the trajectory of the swing phase. We use from numpy the bezier.curve module to do the calculations. Red dots in the figure are the support points.

```
import bezier

step_length = 1.0
point_height = 1.0

points = np.asarray([
                     [-step_length/2, 0.0, step_length/2, step_length/2],
                     [0.0, point_height, point_height,0.0]
                    ])
curve = bezier.Curve(points, degree=3)
```

```
import matplotlib.patches as patches

fig, ax = plt.subplots()

plt.scatter(points[0], points[1], color='red')
curve.plot(50, ax=ax)
# plot step_length
l_points = np.zeros((2, 2))
l_points[0,0] = -step_length/2
l_points[1,0] = step_length/2
#plt.plot(l_points[0], l_points[1],'-k')
p1 = patches.FancyArrowPatch(l_points[0], l_points[1], arrowstyle='<->', mutation_
 ↪scale=20)
ax.add_artist(p1)
```

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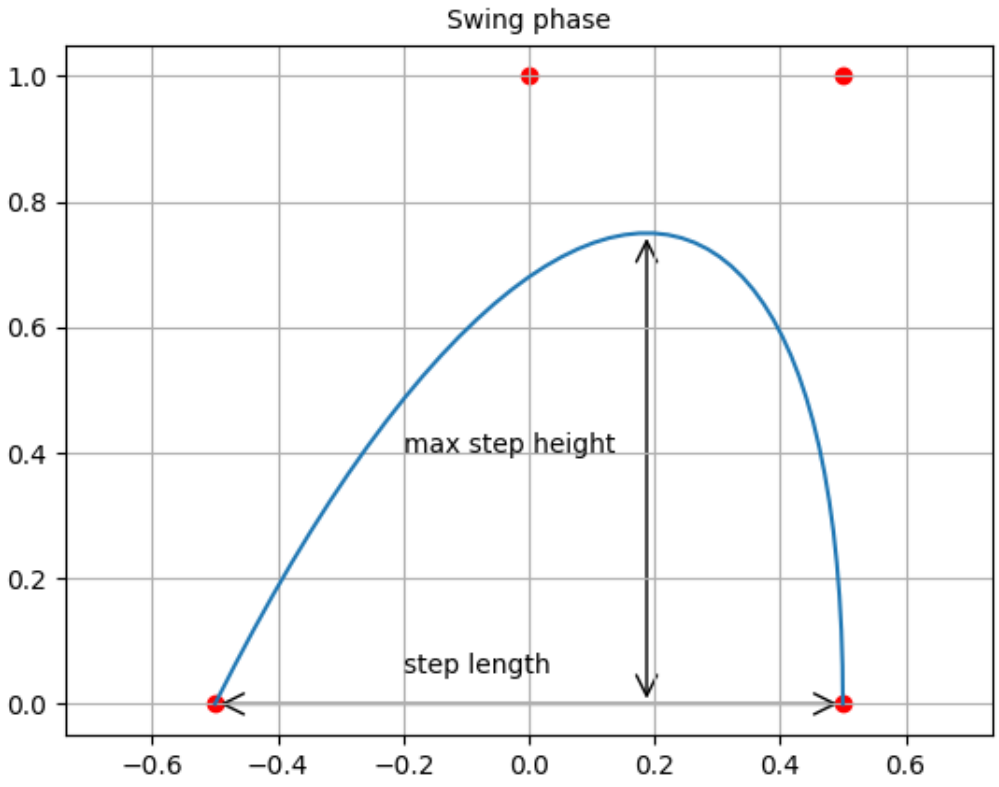
```
plt.text(-0.2, 0.05, "step length")
# plot step_height
h_points = np.zeros((2, 2))
h_points[0,0] = 0.1875*step_length
h_points[1,0] = 0.1875*step_length
h_points[1,1] = 0.75*point_height
p2 = patches.FancyArrowPatch(h_points[0], h_points[1], arrowstyle='<->', mutation_
 ↪scale=20)
ax.add_artist(p2)
plt.text(-0.2, 0.4, "max step height")


plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=True, which='major')
plt.title('Swing phase',fontsize=10)
```

```
Text(0.5, 1.0, 'Swing phase')
```

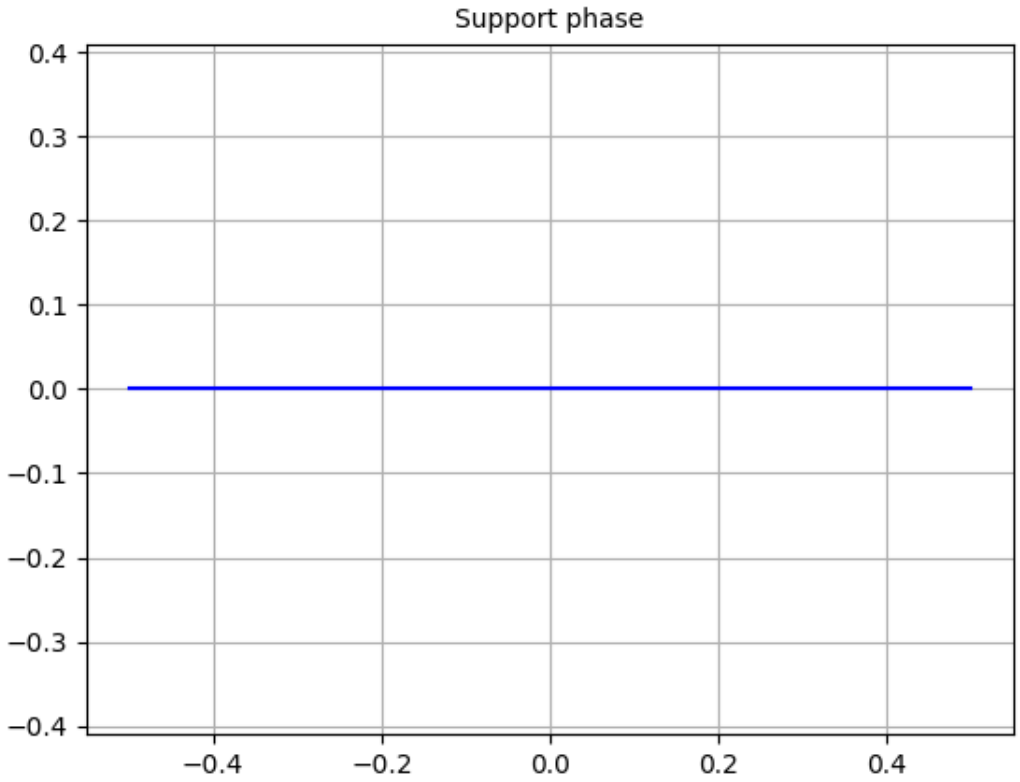


5.2 Support Phase

We will use a straight line for the stance phase

```
fig, ax = plt.subplots()

l_points = np.zeros((2, 2))
l_points[0,0] = -step_length/2
l_points[0,1] = step_length/2
plt.plot(l_points[0], l_points[1],'-b')

plt.axis('equal')  #<-- set the axes to the same scale

plt.grid(visible=True, which='major')
plt.title('Support phase',fontsize=10)
```

```
Text(0.5, 1.0, 'Support phase')
```



5.3 Sigmoid function

The sigmoid function is defined as

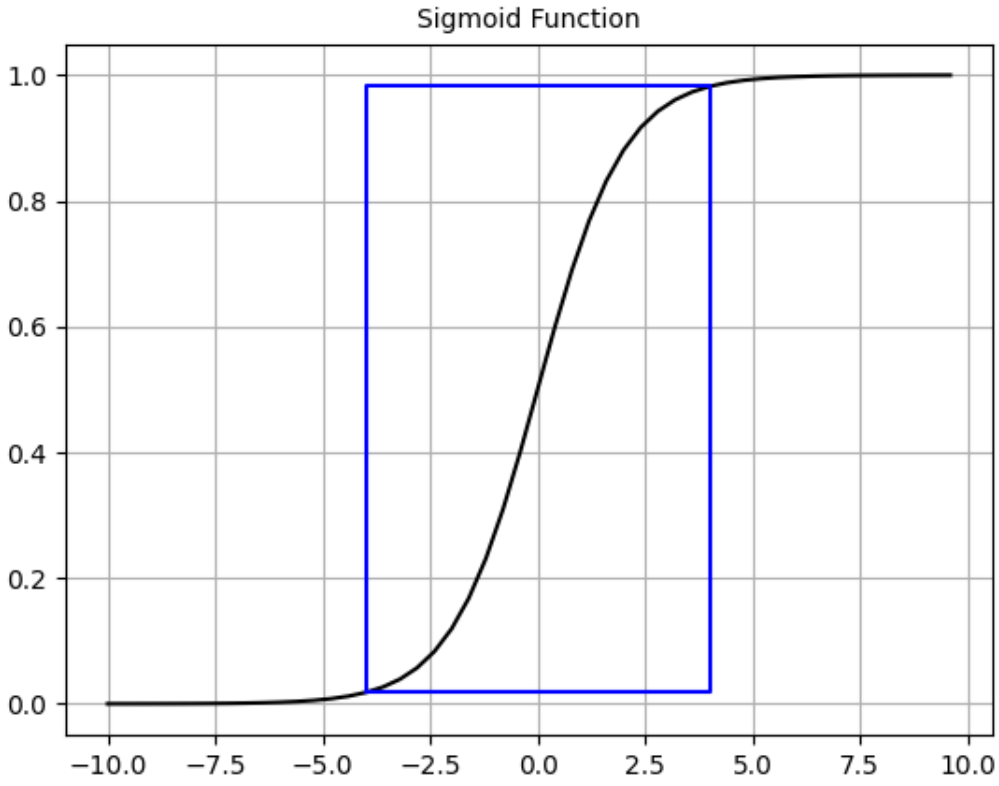
$$S(x) = \frac{1}{1 + e^{-x}}$$

We introduce a truncation parameter ϵ ($\epsilon > 0$) and map the time $t \in [0, 1]$ to the input $s \in [0, 1]$ of our Bézier curve

```
def sigmoid(x):
    return 1 / (1 + np.exp(-x))
```

```
num_points = 50
epsilon = 4
points = np.zeros((2, num_points))

points[0] = np.linspace(-10, 10, num=num_points, endpoint=False)
points[1] = np.array(list(map(sigmoid, points[0])))

e_points = np.zeros((2, 5))
e_points[0,0] = -epsilon
e_points[1,0] = sigmoid(-epsilon)
e_points[0,1] = -epsilon
e_points[1,1] = sigmoid(epsilon)
e_points[0,2] = epsilon
e_points[1,2] = sigmoid(epsilon)
e_points[0,3] = epsilon
e_points[1,3] = sigmoid(-epsilon)
e_points[0,4] = e_points[0,0]
e_points[1,4] = e_points[1,0]

fig, ax = plt.subplots()
plt.plot(points[0], points[1],'-k')
plt.plot(e_points[0], e_points[1],'-b')

plt.grid(visible=True, which='major')
plt.title('Sigmoid Function',fontsize=10)
```

```
Text(0.5, 1.0, 'Sigmoid Function')
```

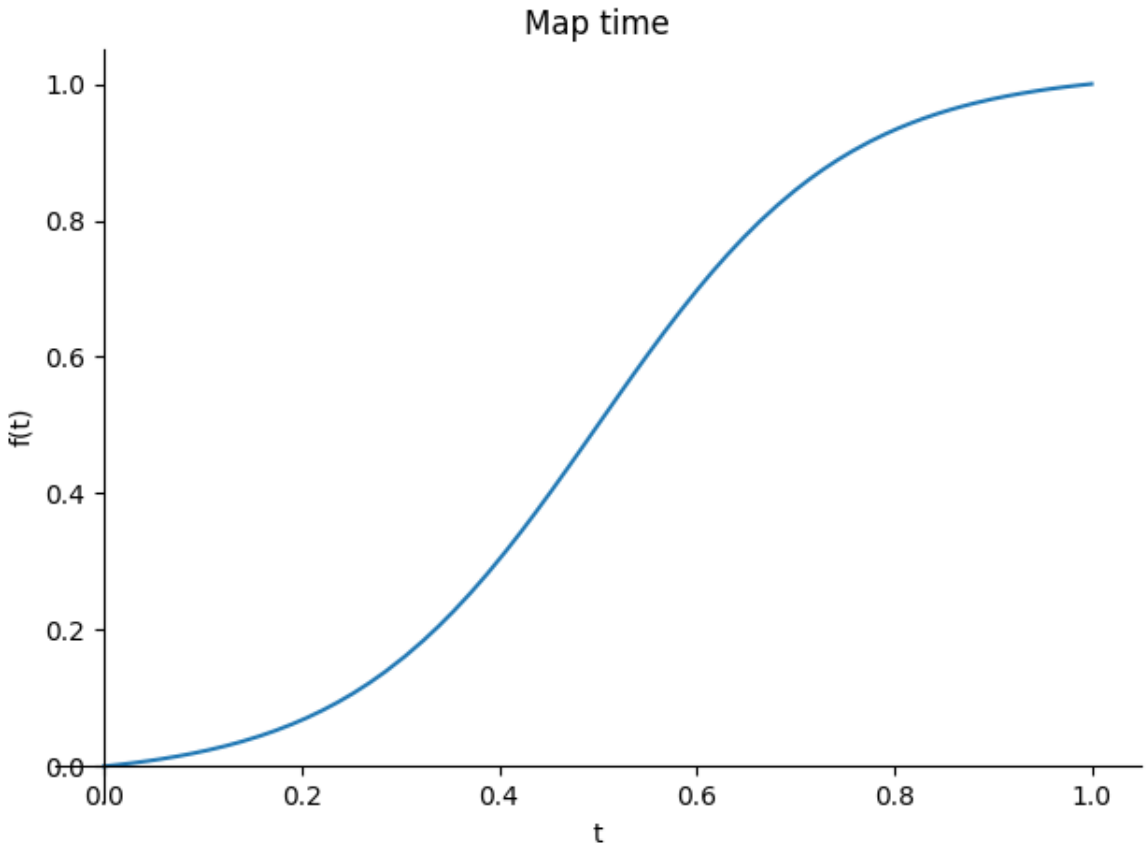



```
x, t, epsilon = sp.symbols('x t epsilon')
```

```
m = ((sp.exp(epsilon) + 1)/(sp.exp(epsilon) - 1))*(1/(sp.exp(-2*epsilon*(t-1/2))+1) -␣
↪1/(sp.exp(epsilon) + 1))
m
```

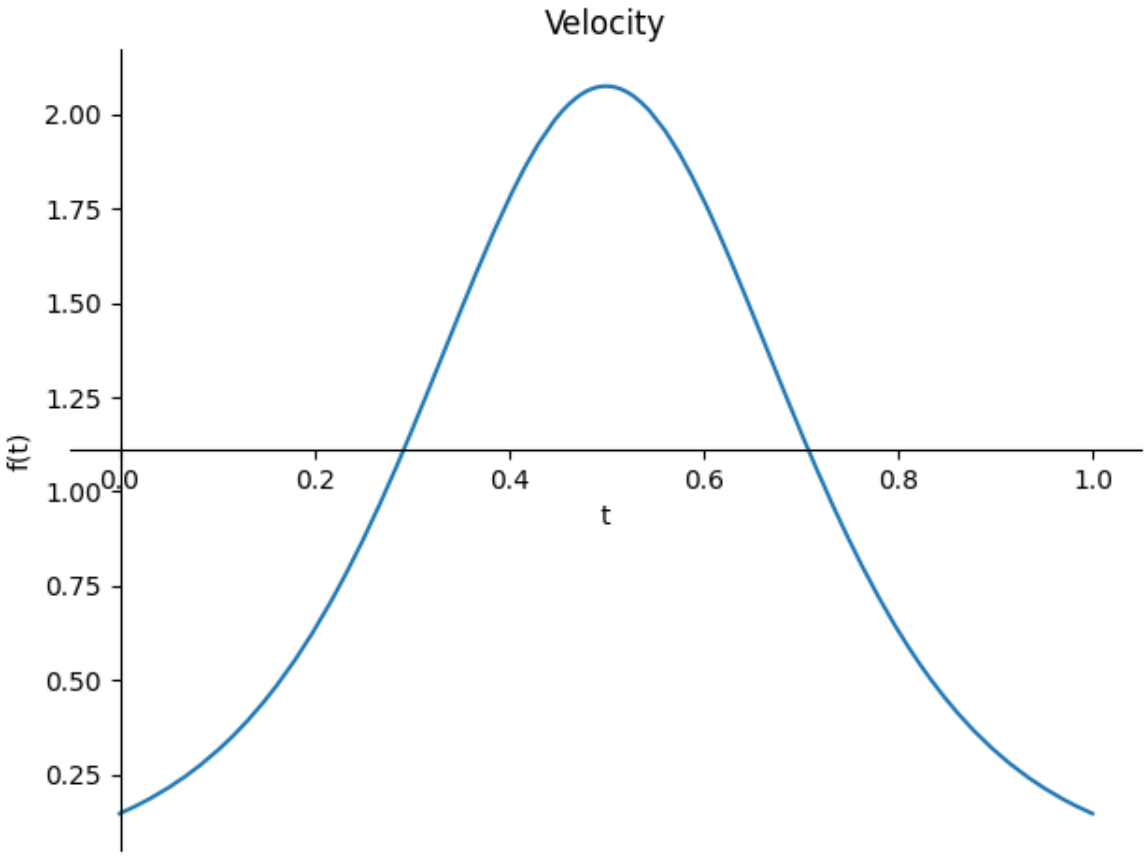
$$\frac{\left(-\frac{1}{e^{\epsilon}+1}+\frac{1}{1+e^{-2\epsilon(t-0.5)}}\right)\left(e^{\epsilon}+1\right)}{e^{\epsilon}-1}$$

```
from sympy.plotting import plot

plot(m.subs(epsilon, 4), (t, 0, 1), title='Map time')
```




```
<sympy.plotting.plot.Plot at 0x124c23e20>
```

```
plot(sp.diff(m.subs(epsilon, 4), t), (t, 0, 1), title='Velocity')
```




```
<sympy.plotting.plot.Plot at 0x124c6a880>
```

5.4 Putting everything together

We start with the stance phase and select a number of points from the interval $[0, 1]$ using our sigmoid function. Then we select the same number of points for the swing phase but we do not include the endpoints since they are already included in the stance phase

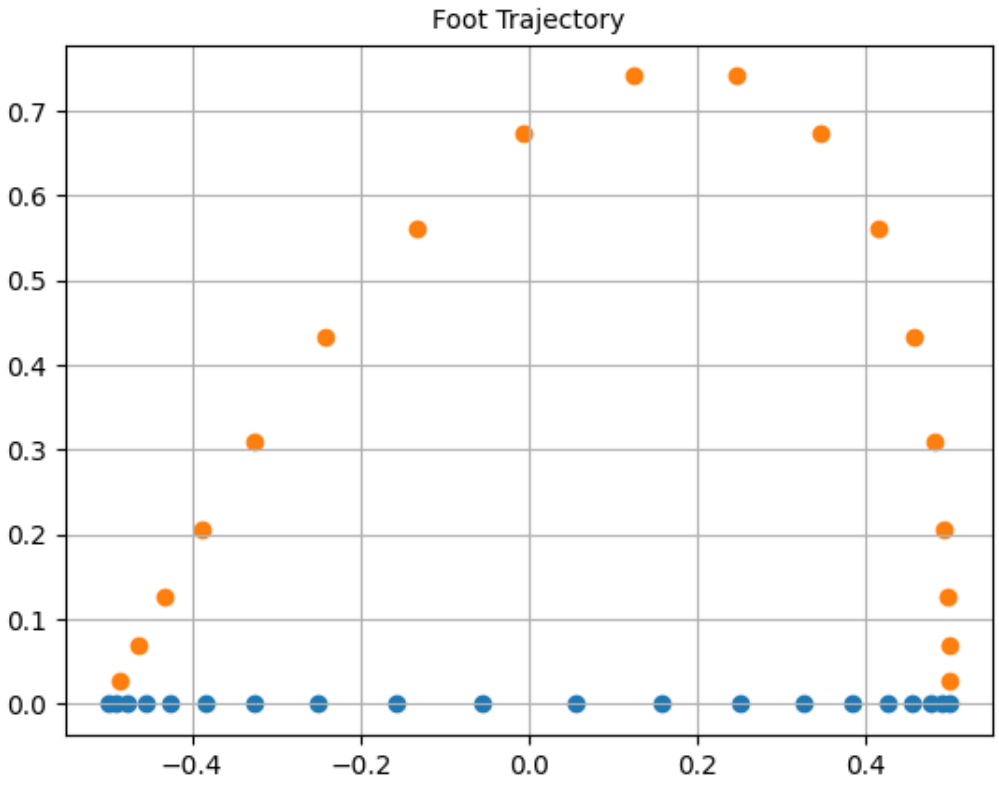
```
def adapt_velocity(eps, times):
    return [((np.exp(eps) + 1)/(np.exp(eps) - 1))*(1/(np.exp(-2*eps*(t-1/2))+1) - 1/
 ↪(np.exp(eps) + 1)) for t in times]
```

```
def stance_phase(l, eps, times):
    return l * np.array(adapt_velocity(eps, np.linspace(0, 1, num=N))) - l/2
```

```
def swing_phase(l, h, eps, times):
    points = np.asarray([
                         [-l/2, 0.0, l/2, l/2],
                         [0.0, h/0.75, h/0.75,0.0]
                         ])
    curve = bezier.Curve(points, degree=3)
    return curve.evaluate_multi(times)
```

```
N = 20 # number of points
step_length = 1.0
step_height = 0.75
eps = 4

v_adj = np.array(adapt_velocity(4, np.linspace(0, 1, num=N)))

l_points = np.zeros((2, N))
l_points[0] = step_length * v_adj - step_length/2

# remove first and lest element from v_adj because it is already covered by the␣
↪stance phase
s_points = np.array(swing_phase(step_length, step_height, eps, v_adj[1:-1]))

fig, ax = plt.subplots()
plt.scatter(l_points[0], l_points[1])
plt.scatter(s_points[0], s_points[1])

plt.grid(visible=True, which='major')
plt.title('Foot Trajectory',fontsize=10)
```

```
Text(0.5, 1.0, 'Foot Trajectory')
```



CHAPTER

SIX

GAITS FOR MINIPUPPER

We moved the calculation of the foot trajectory to a Python class

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib widget
```

```
from gaits import GaitController
```

```
# set values for minipupper
step_length = 0.05
step_height = 0.03
velocity_adapt = 4
number_of_points = 20
theta = -3*np.pi/4  # upper leg angle for origin of bezier curve
gamma = np.pi/2
```

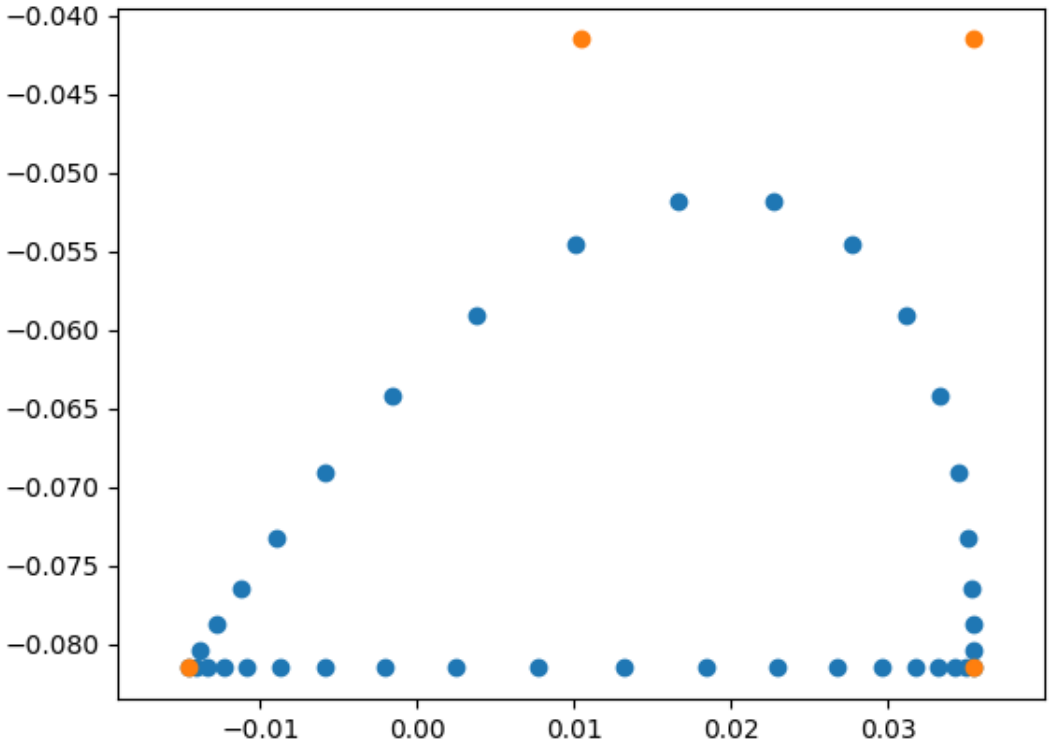
```
gc = GaitController(step_length, step_height, velocity_adapt, number_of_points, theta,
 ↪ gamma)
trajectory = gc.get_trajectory()
```

```
gc.v_adj
```

```
array([0.        , 0.00950315, 0.02364571, 0.04444672, 0.07451808,
       0.11692519, 0.17468076, 0.24971679, 0.34144914, 0.44560524,
       0.55439476, 0.65855086, 0.75028321, 0.82531924, 0.88307481,
       0.92548192, 0.95555328, 0.97635429, 0.99049685, 1.        ])
```

```
fig, ax = plt.subplots()
plt.xlim(-0.15, 0.15)
plt.ylim(-0.15, 0)
plt.axis('equal')
plt.grid(visible=False)
plt.scatter(trajectory[0], trajectory[1])
bezier_points = gc.get_bezier_points()
plt.scatter(bezier_points[0], bezier_points[1])
```

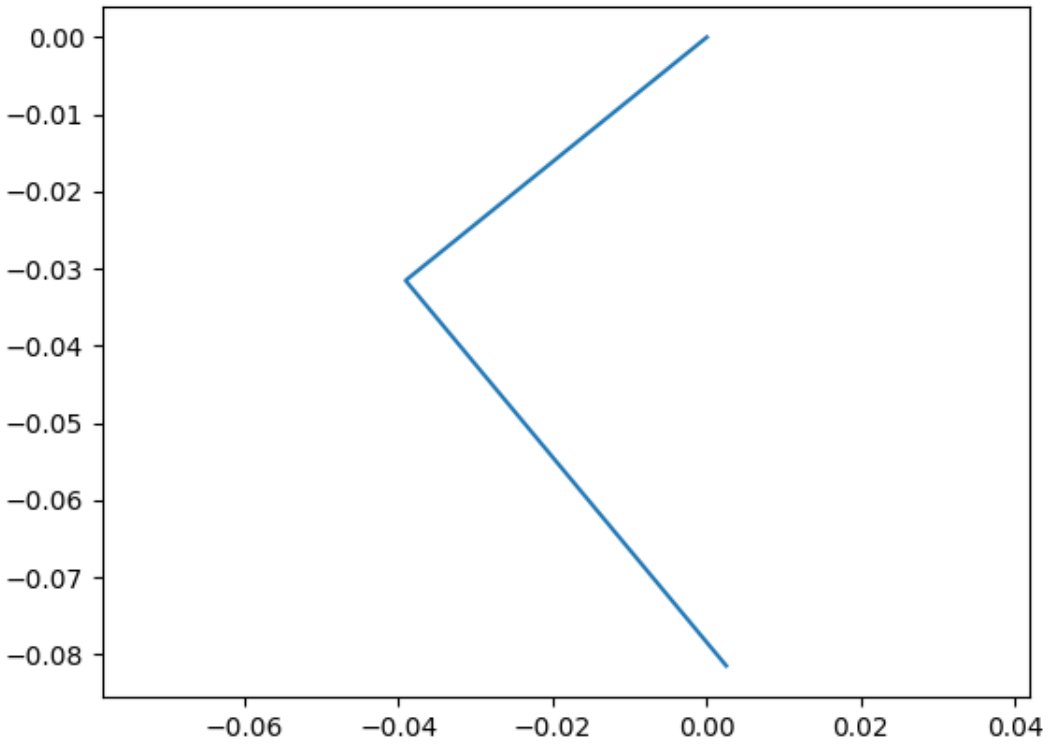
```
<matplotlib.collections.PathCollection at 0x111011fa0>
```




```
leg_servo_positions = gc.get_leg_servo_positions()
```

```
p = gc.mpForwardKin(leg_servo_positions[11])
fig, ax = plt.subplots()
plt.xlim(-0.15, 0.15)
plt.ylim(-0.15, 0)
plt.axis('equal')
plt.grid(visible=False)
plt.plot(p[0], p[1])
```

```
[<matplotlib.lines.Line2D at 0x1111266d0>]
```




```
import matplotlib.animation as animation
from collections import deque

fig, ax = plt.subplots()
plt.xlim(-0.15, 0.15)
plt.ylim(-0.15, 0)
plt.axis('equal')
plt.grid(visible=False)

line, = ax.plot([], [], 'o-', lw=2)
trace, = ax.plot([], [], '.-', lw=1, ms=2)
step_template = 'step = %.1f'
step_text = ax.text(0.05, 0.9, '', transform=ax.transAxes)
history_len = 50
history_x, history_y = deque(maxlen=history_len), deque(maxlen=history_len)


def animate(i):
    p = gc.mpForwardKin(leg_servo_positions[i])

    if i == 0:
        history_x.clear()
        history_y.clear()

    history_x.appendleft(p[0][2])
    history_y.appendleft(p[1][2])
```

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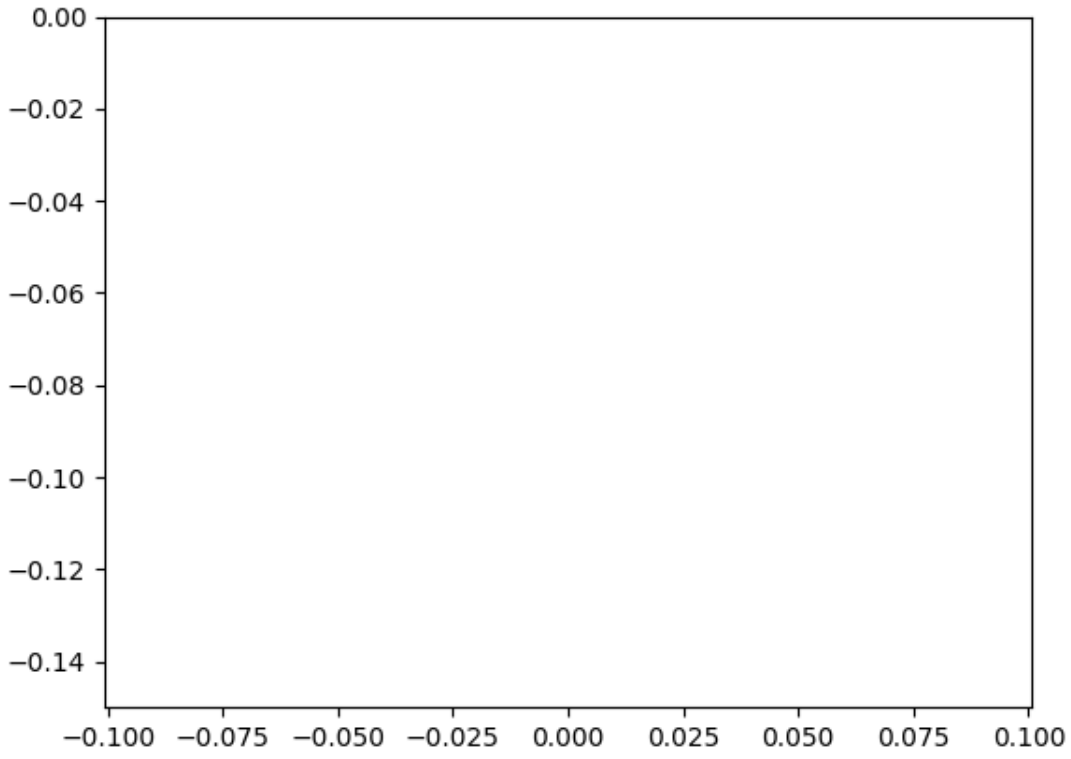
```
    line.set_data(p[0], p[1])
    trace.set_data(history_x, history_y)
    step_text.set_text(step_template % (i))
    #step_text.set_text("%s" % (p[0]))
    return line, trace, step_text


ani = animation.FuncAnimation(fig, animate, len(leg_servo_positions), interval=200,␣
 ↪blit=True)

plt.show()
```




```
joints = ['hip', 'upper_leg', 'lower_leg']
legs = ['lf', 'rf', 'lh', 'rh']

for joint in joints:
    for leg in legs:
        fn = "%s_%s" % (leg, joint)
        if 'lf' in fn or 'rh' in fn:
            lp = leg_servo_positions
        else:
            lp = np.roll(leg_servo_positions, number_of_points, axis=0)
        with open("../controller/servos/walk/%s" % fn, "w") as f:
```

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```
            if 'hip' in fn:
                f.write("%.10f\n" % 0.0)
            elif 'upper' in fn:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][0])
            else:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][1])
```

6.1 Leg position for testing

```
# neutral position
lp = [[-3*np.pi/4, np.pi/2]]

for joint in joints:
    for leg in legs:
        fn = "%s_%s" % (leg, joint)
        with open("../controller/servos/neutral/%s" % fn, "w") as f:
            if 'hip' in fn:
                f.write("%.10f\n" % 0.0)
            elif 'upper' in fn:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][0])
            else:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][1])
```

```
# streched position (max allowed on minipupper)
lp = [[-np.pi/2, np.pi/4]]

for joint in joints:
    for leg in legs:
        fn = "%s_%s" % (leg, joint)
        with open("../controller/servos/streched_legs/%s" % fn, "w") as f:
            if 'hip' in fn:
                f.write("%.10f\n" % 0.0)
            elif 'upper' in fn:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][0])
            else:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][1])
```

```
# sitting position (max allowed on minipupper)
lp = [[-np.pi/2, np.pi/2]]

for joint in joints:
    for leg in legs:
        fn = "%s_%s" % (leg, joint)
        with open("../controller/servos/sitting/%s" % fn, "w") as f:
            if 'hip' in fn:
```

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```
                f.write("%.10f\n" % 0.0)
            elif 'upper' in fn:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][0])
            else:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][1])
```

```
# retracted position (max allowed on minipupper)
lp = [[-np.pi, 3*np.pi/4]]

for joint in joints:
    for leg in legs:
        fn = "%s_%s" % (leg, joint)
        with open("../controller/servos/retracted_legs/%s" % fn, "w") as f:
            if 'hip' in fn:
                f.write("%.10f\n" % 0.0)
            elif 'upper' in fn:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][0])
            else:
                for j in range(len(lp)):
                    f.write("%.10f\n" % lp[j][1])
```