Safety Audit Scheduling

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Outline

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3. Small Instance
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Introduction(1/2)

What is it?

The problem is mainly about planning the **best schedule for safety audit**, which takes the **total inspection time** and **the growth of workers' experienced level** into consideration.

Why it may matter?

A safety audit is **a process that evaluates the health and safety of a work setting**. During an audit, a group gathers data about a location's operations. They identify hazards and illustrate how to make the area safer for employees.

Why is It Important?

- Adhering to federal regulations
- Showing dedication to safety in the organization
- Determining the cause of workplace injuries
- Updating safety protocols

Introduction(2/2)

Why solving it may help?

It helps because it turns a messy, high-stakes planning task ("who audits what, when, with which team?") into a **repeatable decision process** that is **faster, cheaper, and safer**—while explicitly balancing **total inspection time** and **worker experience growth**.

Here are the main ways it helps:

1. Cuts total inspection time (and overtime risk)
2. Accelerates worker learning in a controlled way
3. Makes trade-offs explicit and defensible

Which parts do you expect it may be difficult?

The most challenging part of the Safety Audit Scheduling problem is not the scheduling itself, but rather **how to model workers' experience growth** in a way that is both realistic and computationally tractable.

Model(1/3)

Sets	
E	Experienced workers i=1..n
B	Inexperienced workers j=1..n
K	Projects k=1..n
T	Days, 1 project 1 day, no ordering t=1..n

Parameters	
f_i	Efficiency of $i \in E$
g_i	Initial efficiency of j $\in B$
δ_i^k	$\begin{cases}0\\1\end{cases}$, whether i can do project k
δ_j^k	$\begin{cases}0\\1\end{cases}$, whether j can do project k
β_j	Efficiency gain when working with some i
b_j	Upper bounds on number times of working with i
H_i	Upper bounds on the number of projects i can do
H_j	Upper bounds on the number of projects j can do
d_k	Number of workers required for project k

Decision variables	
x_i^{kt}	Whether k is assigned to i in day t
y_i^{kt}	Whether k is assigned to j in day t
z_{ij}^{kt}	Whether i work with j on project k at day t
q_j^t	Efficiency of j at the end of t
v^{kt}	Whether the project k is done on day t
w_j^{kt}	$= q_j^t \, y_j^{kt}$

Model(2/3)

Objective:

$$max \sum_t \sum_k \sum_i f_i x_i^{kt} + \sum_t \sum_k \sum_j q_j^t y_j^{kt}$$

Maximize efficiency for experienced & inexperienced

Non-linear

Constraints:

$$\sum_t \sum_i x_i^{kt} + \sum_t \sum_j y_j^{kt} = d_k v^{kt}, \quad k \in K$$

How many people are required

$$\sum_i x_i^{kt} + \sum_j y_j^{kt} \le (|E| + |B|) v^{kt}, \quad \forall k \in K, t \in T$$

$$\sum_k v^{kt} = 1, \quad \forall t \in T$$

1 task a day

$$\sum_t v^{kt} = 1, \quad \forall k \in K$$

1 day a task

$$\sum_t \sum_k x_i^{kt} \le H_i, \quad \forall i \in E$$

$$\sum_t \sum_k y_j^{kt} \le H_j, \quad \forall j \in B$$

How many tasks can the worker do

$$\sum_t \sum_k \sum_i z_{ij}^{kt} \le b_j, \quad \forall j \in B$$

Limits on the number of times of working with i

$$q_j^t = q_j^{t-1} + \beta_j \sum_k \sum_i \delta_j^k z_{ij}^{kt}, \quad \forall j \in B, \forall t \in T$$

Update efficiency

$$q_j^0 = g_j, \quad \forall j \in B$$

$$x_i^{kt} \le \delta_i^k, \quad \forall i, j, k, t$$

$$y_j^{kt} \le \delta_j^k, \quad \forall i, j, k, t$$

Whether the worker can do the job

$$\forall t, k, i, j, if\ x_i^{kt} = 1\ and\ y_j^{kt}, then\ z_{ij}^{kt} = 1: \ z_{ij}^{kt} = x_i^{kt} \cdot y_j^{kt}$$

Non-linear

Model(3/3)

Objective:

$$max \sum_t \sum_k \sum_i f_i x_i^{kt} + \sum_t \sum_k \sum_j w_j^{kt}$$

Add $w_j^{kt} = q_j^t y_j^{kt}$

Constraints:

$$\sum_t \sum_i x_i^{kt} + \sum_t \sum_j y_j^{kt} = d_k, \qquad k \in K$$

$$\sum_i x_i^{kt} + \sum_j y_j^{kt} \le (|E| + |B|) v^{kt}, \qquad \forall k \in K, t \in T$$

$$\sum_k v^{kt} = 1, \qquad \forall t \in T$$

$$\sum_t v^{kt} = 1, \qquad \forall k \in K$$

$$\sum_t \sum_k x_i^{kt} \le H_i, \qquad \forall i \in E$$

$$\sum_t \sum_k y_j^{kt} \le H_j, \qquad \forall j \in B$$

$$\sum_t \sum_k \sum_i z_{ij}^{kt} \le b_j, \qquad \forall j \in B$$

$$q_j^t = q_j^{t-1} + \beta_j \sum_k \sum_i \delta_j^k z_{ij}^{kt}, \qquad \forall j \in B, \forall t \in T$$

$$q_j^0 = g_j, \qquad \forall j \in B$$

$$x_i^{kt} \le \delta_i^k, \qquad \forall i, j, k, t$$

$$y_j^{kt} \le \delta_j^k, \qquad \forall i, j, k, t$$

~~$\forall t, k, i, j, if\ x_i^{kt} = 1\ and\ y_j^{kt}, then\ z_{ij}^{kt} = 1$: $z_{ij}^{kt} = x_i^{kt} \cdot y_j^{kt}$~~

→

$$z_{ij}^{kt} \le x_i^{kt}, \qquad \forall i, j, k, t$$

$$z_{ij}^{kt} \le y_j^{kt}, \qquad \forall i, j, k, t$$

$$z_{ij}^{kt} \ge x_i^{kt} + y_j^{kt} - 1, \qquad \forall i, j, k, t$$

→

$$0 \le q_j^t \le g_j + \beta_j b_j$$

$$w_j^{kt} \le (g_j + \beta_j b_j) y_j^{kt}$$

$$w_j^{kt} \ge 0$$

$$w_j^{kt} \le q_j^t$$

$$w_j^{kt} \ge q_j^t - (g_j + \beta_j b_j)(1 - y_j^{kt})$$

At most b_j learning events, with each event increasing by β_j ;if a worker can be matched with multiple i in a day, a larger value is required.

Small instance-Result(1/1)

Settings

```
# ----------------------
# 0) Small instance data
# ----------------------
E = [1, 2]          # experienced workers
B = [1, 2]          # inexperienced workers
K = [1, 2, 3]       # projects
T_days = [1, 2, 3]  # days

# efficiency of experienced
f = {1: 10.0, 2: 8.0}

# initial efficiency of inexperienced
g = {1: 2.0, 2: 3.0}

# learning gain
beta = {1: 1.0, 2: 0.5}

# workers required per project
d = {1: 2, 2: 2, 3: 2}

# capacity: max number of projects a worker can do over horizon
H_i = {1: 2, 2: 2}
H_j = {1: 2, 2: 2}

# pairing upper bound per inexperienced (total number of z events)
b = {1: 2, 2: 2}

# qualification delta
delta_i = {(1, 1): 1, (1, 2): 1, (1, 3): 1,
           (2, 1): 1, (2, 2): 0, (2, 3): 1}
delta_j = {(1, 1): 1, (1, 2): 1, (1, 3): 0,
           (2, 1): 1, (2, 2): 1, (2, 3): 1}
```

```
Optimal solution found (tolerance 1.00e-04)
Best objective 4.350000000000e+01, best bound 4.350000000000e+01, gap 0.0000%

=== Optimal objective ===
43.5

=== Schedule (day -> project) ===
Day 1: Project 3
  Experienced assigned: [1, 2]
  Inexperienced assigned: []
Day 2: Project 1
  Experienced assigned: [2]
  Inexperienced assigned: [2]
Day 3: Project 2
  Experienced assigned: [1]
  Inexperienced assigned: [2]

=== q (efficiency) over time ===
q[1,0] = 2.000
q[1,1] = 2.000
q[1,2] = 2.000
q[1,3] = 2.000
q[2,0] = 3.000
q[2,1] = 3.000
q[2,2] = 3.500
q[2,3] = 4.000
```

growth

Large instance: Parameters (1/6)

Settings	#
Experienced	8
inexperienced	14
Day	20

```
--- f (experienced efficiency) ---
f[1] = 9.6192
f[2] = 8.7542
f[3] = 11.2547
f[4] = 8.3622
f[5] = 10.6794
f[6] = 9.8284
f[7] = 8.2900
f[8] = 10.5372

--- g (inexperienced initial efficiency) ---
g[1] = 1.1125
g[2] = 2.3009
g[3] = 1.2096
g[4] = 1.2721
g[5] = 2.2736
g[6] = 3.4806
g[7] = 1.3714
g[8] = 1.6697
g[9] = 2.8823
g[10] = 3.8431
g[11] = 2.7313
g[12] = 2.1900
g[13] = 3.9288
g[14] = 1.1397

--- beta (learning gain) ---
beta[1] = 1.0726
beta[2] = 0.5606
beta[3] = 0.4298
beta[4] = 0.4060
beta[5] = 0.5776
beta[6] = 1.0345
beta[7] = 0.4627
beta[8] = 0.8234
beta[9] = 0.8750
beta[10] = 0.6352
beta[11] = 0.7930
beta[12] = 0.3565
beta[13] = 0.3536
beta[14] = 0.4854
```

```
--- d (workers required per project) ---
d[1] = 6
d[2] = 6
d[3] = 5
d[4] = 5
d[5] = 5
d[6] = 6
d[7] = 5
d[8] = 5
d[9] = 5
d[10] = 4
d[11] = 4
d[12] = 6
d[13] = 4
d[14] = 4
d[15] = 6
d[16] = 5
d[17] = 6
d[18] = 5
d[19] = 5
d[20] = 6

--- H_i (max projects per experienced) ---
H_i[1] = 11
H_i[2] = 10
H_i[3] = 12
H_i[4] = 8
H_i[5] = 8
H_i[6] = 12
H_i[7] = 11
H_i[8] = 9

--- H_j (max projects per inexperienced) ---
H_j[1] = 10
H_j[2] = 9
H_j[3] = 11
H_j[4] = 11
H_j[5] = 8
H_j[6] = 8
H_j[7] = 12
H_j[8] = 12
H_j[9] = 10
H_j[10] = 10
H_j[11] = 10
H_j[12] = 12
H_j[13] = 11
H_j[14] = 12
```

```
--- b (pairing cap per inexperienced; sum z_{i,j,k,t} <= b_j) ---
b[1] = 55
b[2] = 44
b[3] = 32
b[4] = 56
b[5] = 32
b[6] = 60
b[7] = 38
b[8] = 45
b[9] = 52
b[10] = 51
b[11] = 32
b[12] = 31
b[13] = 53
b[14] = 52
```

Large instance: Result (2/6)

```
Time limit reached
Best objective 1.303171948932e+03, best bound 3.701674509940e+03, gap 184.0511%

=== Status === 9 (OPTIMAL=2, TIME_LIMIT=9)
Obj = 1303.1719489315763
```

```
--- Schedule (Day -> Project) ---
Day 1: Project 13
Day 2: Project 10
Day 3: Project 12
Day 4: Project 9
Day 5: Project 15
Day 6: Project 3
Day 7: Project 16
Day 8: Project 2
Day 9: Project 1
Day 10: Project 8
Day 11: Project 17
Day 12: Project 19
Day 13: Project 4
Day 14: Project 20
Day 15: Project 18
Day 16: Project 5
Day 17: Project 11
Day 18: Project 14
Day 19: Project 6
Day 20: Project 7
```

```
--------------------------------------------------------------------------------
Assignments per day
--------------------------------------------------------------------------------
Day 1 / Project 13:  |E|=3, |B|=1, total=4 (d=4)
  Experienced: [1, 3, 6]
  Inexperienced: [8]
Day 2 / Project 10:  |E|=2, |B|=2, total=4 (d=4)
  Experienced: [1, 3]
  Inexperienced: [8, 11]
Day 3 / Project 12:  |E|=4, |B|=2, total=6 (d=6)
  Experienced: [1, 3, 6, 8]
  Inexperienced: [8, 9]
Day 4 / Project 9:  |E|=4, |B|=1, total=5 (d=5)
  Experienced: [2, 3, 5, 8]
  Inexperienced: [9]
Day 5 / Project 15:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [1, 4, 5]
  Inexperienced: [8, 9, 11]
Day 6 / Project 3:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [1, 3, 6]
  Inexperienced: [8, 10]
Day 7 / Project 16:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [3, 4, 6]
  Inexperienced: [8, 9]
Day 8 / Project 2:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [2, 5, 6]
  Inexperienced: [8, 9, 10]
Day 9 / Project 1:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [1, 5, 8]
  Inexperienced: [8, 9, 11]
Day 10 / Project 8:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [2, 5, 6]
  Inexperienced: [10, 11]
Day 11 / Project 17:  |E|=4, |B|=2, total=6 (d=6)
  Experienced: [1, 3, 6, 8]
  Inexperienced: [10, 11]
Day 12 / Project 19:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [1, 5, 8]
  Inexperienced: [9, 11]
```

```
Day 13 / Project 4:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [3, 6, 8]
  Inexperienced: [10, 11]
Day 14 / Project 20:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [3, 5, 6]
  Inexperienced: [8, 10, 11]
Day 15 / Project 18:  |E|=4, |B|=1, total=5 (d=5)
  Experienced: [1, 2, 3, 8]
  Inexperienced: [9]
Day 16 / Project 5:  |E|=2, |B|=3, total=5 (d=5)
  Experienced: [3, 5]
  Inexperienced: [9, 10, 11]
Day 17 / Project 11:  |E|=1, |B|=3, total=4 (d=4)
  Experienced: [3]
  Inexperienced: [8, 10, 11]
Day 18 / Project 14:  |E|=2, |B|=2, total=4 (d=4)
  Experienced: [1, 6]
  Inexperienced: [9, 10]
Day 19 / Project 6:  |E|=4, |B|=2, total=6 (d=6)
  Experienced: [4, 6, 7, 8]
  Inexperienced: [1, 8]
Day 20 / Project 7:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [1, 6, 8]
  Inexperienced: [8, 10]
```

Large instance: Result (3/6)

```
--- Worker loads (assigned project count) ---
Experienced:
  i=1: load=11  (cap H_i=11)
  i=2: load=4  (cap H_i=10)
  i=3: load=12  (cap H_i=12)
  i=4: load=3  (cap H_i=8)
  i=5: load=8  (cap H_i=8)
  i=6: load=12  (cap H_i=12)
  i=7: load=1  (cap H_i=11)
  i=8: load=9  (cap H_i=9)
Inexperienced:
  j=1: load=1  (cap H_j=10)
  j=2: load=0  (cap H_j=9)
  j=3: load=0  (cap H_j=11)
  j=4: load=0  (cap H_j=11)
  j=5: load=0  (cap H_j=8)
  j=6: load=0  (cap H_j=8)
  j=7: load=0  (cap H_j=12)
  j=8: load=12  (cap H_j=12)
  j=9: load=10  (cap H_j=10)
  j=10: load=10  (cap H_j=10)
  j=11: load=10  (cap H_j=10)
  j=12: load=0  (cap H_j=12)
  j=13: load=0  (cap H_j=11)
  j=14: load=0  (cap H_j=12)
```

Limits too large

```
--------------------------------------------------------------------------------
q (efficiency) over time
--------------------------------------------------------------------------------
j=1  g=1.112  beta=1.073  b=55  U=60.107
  t0:1.112, t1:1.112, t2:1.112, t3:1.112, t4:1.112, t5:1.112, t6:1.112, t7:1.112, t8:1.112, t9:1.112, t10:1.112, t11:1.112, t12:1.112, t13:1.112, t14:1.
112, t15:1.112, t16:1.112, t17:1.112, t18:1.112, t19:5.403, t20:5.403
j=2  g=2.301  beta=0.561  b=44  U=26.969
  t0:2.301, t1:2.301, t2:2.301, t3:2.301, t4:2.301, t5:2.301, t6:2.301, t7:2.301, t8:2.301, t9:2.301, t10:2.301, t11:2.301, t12:2.301, t13:2.301, t14:2.
301, t15:2.301, t16:2.301, t17:2.301, t18:2.301, t19:2.301, t20:2.301
j=3  g=1.210  beta=0.430  b=32  U=14.964
  t0:1.210, t1:1.210, t2:1.210, t3:1.210, t4:1.210, t5:1.210, t6:1.210, t7:1.210, t8:1.210, t9:1.210, t10:1.210, t11:1.210, t12:1.210, t13:1.210, t14:1.
210, t15:1.210, t16:1.210, t17:1.210, t18:1.210, t19:1.210, t20:1.210
j=4  g=1.272  beta=0.406  b=56  U=24.009
  t0:1.272, t1:1.272, t2:1.272, t3:1.272, t4:1.272, t5:1.272, t6:1.272, t7:1.272, t8:1.272, t9:1.272, t10:1.272, t11:1.272, t12:1.272, t13:1.272, t14:1.
272, t15:1.272, t16:1.272, t17:1.272, t18:1.272, t19:1.272, t20:1.272
j=5  g=2.274  beta=0.578  b=32  U=20.758
  t0:2.274, t1:2.274, t2:2.274, t3:2.274, t4:2.274, t5:2.274, t6:2.274, t7:2.274, t8:2.274, t9:2.274, t10:2.274, t11:2.274, t12:2.274, t13:2.274, t14:2.
274, t15:2.274, t16:2.274, t17:2.274, t18:2.274, t19:2.274, t20:2.274
j=6  g=3.481  beta=1.035  b=60  U=65.551
  t0:3.481, t1:3.481, t2:3.481, t3:3.481, t4:3.481, t5:3.481, t6:3.481, t7:3.481, t8:3.481, t9:3.481, t10:3.481, t11:3.481, t12:3.481, t13:3.481, t14:3.
481, t15:3.481, t16:3.481, t17:3.481, t18:3.481, t19:3.481, t20:3.481
j=7  g=1.371  beta=0.463  b=38  U=18.952
  t0:1.371, t1:1.371, t2:1.371, t3:1.371, t4:1.371, t5:1.371, t6:1.371, t7:1.371, t8:1.371, t9:1.371, t10:1.371, t11:1.371, t12:1.371, t13:1.371, t14:1.
371, t15:1.371, t16:1.371, t17:1.371, t18:1.371, t19:1.371, t20:1.371
j=8  g=1.670  beta=0.823  b=45  U=38.725
  t0:1.670, t1:4.140, t2:5.787, t3:9.081, t4:9.081, t5:11.551, t6:14.021, t7:16.492, t8:18.962, t9:21.432, t10:21.432, t11:21.432, t12:21.432, t13:21.43
2, t14:23.903, t15:23.903, t16:23.903, t17:24.726, t18:24.726, t19:28.020, t20:30.490
j=9  g=2.882  beta=0.875  b=52  U=48.383
  t0:2.882, t1:2.882, t2:2.882, t3:6.382, t4:9.882, t5:12.508, t6:12.508, t7:15.133, t8:17.758, t9:20.383, t10:20.383, t11:20.383, t12:23.008, t13:23.00
8, t14:23.008, t15:26.508, t16:28.258, t17:28.258, t18:30.008, t19:30.008, t20:30.008
j=10  g=3.843  beta=0.635  b=51  U=36.236
  t0:3.843, t1:3.843, t2:3.843, t3:3.843, t4:3.843, t5:3.843, t6:5.749, t7:5.749, t8:7.654, t9:7.654, t10:9.560, t11:12.100, t12:12.100, t13:14.006, t1
4:15.911, t15:15.911, t16:17.181, t17:17.817, t18:19.087, t19:19.087, t20:20.992
j=11  g=2.731  beta=0.793  b=32  U=28.106
  t0:2.731, t1:2.731, t2:4.317, t3:4.317, t4:4.317, t5:6.696, t6:6.696, t7:6.696, t8:6.696, t9:9.075, t10:11.454, t11:14.626, t12:17.005, t13:19.384, t1
4:21.763, t15:21.763, t16:23.349, t17:24.141, t18:24.141, t19:24.141, t20:24.141
j=12  g=2.190  beta=0.357  b=31  U=13.242
  t0:2.190, t1:2.190, t2:2.190, t3:2.190, t4:2.190, t5:2.190, t6:2.190, t7:2.190, t8:2.190, t9:2.190, t10:2.190, t11:2.190, t12:2.190, t13:2.190, t14:2.
190, t15:2.190, t16:2.190, t17:2.190, t18:2.190, t19:2.190, t20:2.190
j=13  g=3.929  beta=0.354  b=53  U=22.672
  t0:3.929, t1:3.929, t2:3.929, t3:3.929, t4:3.929, t5:3.929, t6:3.929, t7:3.929, t8:3.929, t9:3.929, t10:3.929, t11:3.929, t12:3.929, t13:3.929, t14:3.
929, t15:3.929, t16:3.929, t17:3.929, t18:3.929, t19:3.929, t20:3.929
j=14  g=1.140  beta=0.485  b=52  U=26.379
  t0:1.140, t1:1.140, t2:1.140, t3:1.140, t4:1.140, t5:1.140, t6:1.140, t7:1.140, t8:1.140, t9:1.140, t10:1.140, t11:1.140, t12:1.140, t13:1.140, t14:1.
140, t15:1.140, t16:1.140, t17:1.140, t18:1.140, t19:1.140, t20:1.140
```

Large instance: Parameters (4/6)

Settings	#
Experienced	8
inexperienced	14
Day	20

-- f (experienced efficiency) ---
f[1] = 9.6192
f[2] = 8.7542
f[3] = 11.2547
f[4] = 8.3622
f[5] = 10.6794
f[6] = 9.8284
f[7] = 8.2900
f[8] = 10.5372

--- g (inexperienced initial efficiency) ---
g[1] = 1.1125
g[2] = 2.3009
g[3] = 1.2096
g[4] = 1.2721
g[5] = 2.2736
g[6] = 3.4806
g[7] = 1.3714
g[8] = 1.6697
g[9] = 2.8823
g[10] = 3.8431
g[11] = 2.7313
g[12] = 2.1900
g[13] = 3.9288
g[14] = 1.1397

--- beta (learning gain) ---
beta[1] = 1.0726
beta[2] = 0.5606
beta[3] = 0.4298
beta[4] = 0.4060
beta[5] = 0.5776
beta[6] = 1.0345
beta[7] = 0.4627
beta[8] = 0.8234
beta[9] = 0.8750
beta[10] = 0.6352
beta[11] = 0.7930
beta[12] = 0.3565
beta[13] = 0.3536
beta[14] = 0.4854

--- d (workers required per project) ---
d[1] = 6
d[2] = 6
d[3] = 5
d[4] = 5
d[5] = 5
d[6] = 6
d[7] = 5
d[8] = 5
d[9] = 5
d[10] = 4
d[11] = 4
d[12] = 6
d[13] = 4
d[14] = 4
d[15] = 6
d[16] = 5
d[17] = 6
d[18] = 5
d[19] = 5
d[20] = 6

--- H_i (max projects per experienced) ---
H_i[1] = 7
H_i[2] = 6
H_i[3] = 4
H_i[4] = 4
H_i[5] = 7
H_i[6] = 5
H_i[7] = 6
H_i[8] = 5

--- H_j (max projects per inexperienced) ---
H_j[1] = 11
H_j[2] = 11
H_j[3] = 8
H_j[4] = 8
H_j[5] = 12
H_j[6] = 12
H_j[7] = 10
H_j[8] = 10
H_j[9] = 10
H_j[10] = 12
H_j[11] = 11
H_j[12] = 12
H_j[13] = 11
H_j[14] = 8

--- b (pairing cap per inexperienced; sum z_{i,j,k,t} <= b_j) ---
b[1] = 56
b[2] = 32
b[3] = 60
b[4] = 38
b[5] = 45
b[6] = 52
b[7] = 51
b[8] = 32
b[9] = 31
b[10] = 53
b[11] = 52
b[12] = 39
b[13] = 50
b[14] = 48

Large instance: Result (5/6)

```
Time limit reached
Best objective 1.282168614904e+03, best bound 3.700621709472e+03, gap 188.6221%

=== Status === 9 (OPTIMAL=2, TIME_LIMIT=9)
Obj = 1282.1686149039674
```

```
--- Worker loads (assigned project coun
t) ---
Experienced:
  i=1: load=7  (cap H_i=7)
  i=2: load=6  (cap H_i=6)
  i=3: load=4  (cap H_i=4)
  i=4: load=4  (cap H_i=4)
  i=5: load=7  (cap H_i=7)
  i=6: load=5  (cap H_i=5)
  i=7: load=6  (cap H_i=6)
  i=8: load=5  (cap H_i=5)
Inexperienced:
  j=1: load=0  (cap H_j=11)
  j=2: load=0  (cap H_j=11)
  j=3: load=0  (cap H_j=8)
  j=4: load=0  (cap H_j=8)
  j=5: load=12  (cap H_j=12)
  j=6: load=12  (cap H_j=12)
  j=7: load=0  (cap H_j=10)
  j=8: load=0  (cap H_j=10)
  j=9: load=10  (cap H_j=10)
  j=10: load=12  (cap H_j=12)
  j=11: load=11  (cap H_j=11)
  j=12: load=0  (cap H_j=12)
  j=13: load=2  (cap H_j=11)
  j=14: load=0  (cap H_j=8)
```

```
------------------------------------------------------------------------------
Assignments per day
------------------------------------------------------------------------------
Day 1 / Project 17:  |E|=4, |B|=2, total=6 (d=6)
  Experienced: [1, 2, 5, 8]
  Inexperienced: [9, 10]
Day 2 / Project 16:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [4, 5, 7]
  Inexperienced: [6, 10]
Day 3 / Project 11:  |E|=2, |B|=2, total=4 (d=4)
  Experienced: [3, 7]
  Inexperienced: [5, 10]
Day 4 / Project 19:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [2, 3, 6]
  Inexperienced: [5, 11]
Day 5 / Project 9:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [2, 5, 6]
  Inexperienced: [5, 9]
Day 6 / Project 5:  |E|=2, |B|=3, total=5 (d=5)
  Experienced: [1, 5]
  Inexperienced: [5, 10, 11]
Day 7 / Project 6:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [4, 5, 6]
  Inexperienced: [5, 9, 11]
Day 8 / Project 1:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [3, 4, 7]
  Inexperienced: [6, 10, 11]
Day 9 / Project 2:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [2, 5, 7]
  Inexperienced: [6, 10, 11]
Day 10 / Project 18:  |E|=3, |B|=2, total=5 (d=5)
  Experienced: [1, 3, 6]
  Inexperienced: [6, 10]
```

```
Day 11 / Project 13:  |E|=2, |B|=2, total=4 (d=4)
  Experienced: [1, 6]
  Inexperienced: [6, 11]
Day 12 / Project 10:  |E|=1, |B|=3, total=4 (d=4)
  Experienced: [1]
  Inexperienced: [5, 10, 11]
Day 13 / Project 8:  |E|=2, |B|=3, total=5 (d=5)
  Experienced: [4, 7]
  Inexperienced: [6, 9, 11]
Day 14 / Project 15:  |E|=3, |B|=3, total=6 (d=6)
  Experienced: [2, 7, 8]
  Inexperienced: [6, 9, 10]
Day 15 / Project 20:  |E|=2, |B|=4, total=6 (d=6)
  Experienced: [2, 8]
  Inexperienced: [5, 6, 9, 10]
Day 16 / Project 7:  |E|=1, |B|=4, total=5 (d=5)
  Experienced: [1]
  Inexperienced: [5, 6, 9, 11]
Day 17 / Project 14:  |E|=0, |B|=4, total=4 (d=4)
  Experienced: []
  Inexperienced: [5, 6, 10, 11]
Day 18 / Project 4:  |E|=2, |B|=3, total=5 (d=5)
  Experienced: [1, 8]
  Inexperienced: [5, 6, 9]
Day 19 / Project 12:  |E|=2, |B|=4, total=6 (d=6)
  Experienced: [5, 8]
  Inexperienced: [5, 9, 10, 13]
Day 20 / Project 3:  |E|=0, |B|=5, total=5 (d=5)
  Experienced: []
  Inexperienced: [5, 6, 9, 11, 13]
```

Large instance: Result (6/6)

Growth

j=1 g=1.112 beta=1.073 b=56 U=61.179
t0:1.112, t1:1.112, t2:1.112, t3:1.112, t4:1.112, t5:1.112, t6:1.112, t7:1.112, t8:1.112, t9:1.112, t10:1.112, t11:1.112, t12:1.112, t13:1.112, t14:1.112, t15:1.112, t16:1.112, t17:1.112, t18:1.112, t19:1.112, t20:1.112
j=2 g=2.301 beta=0.561 b=32 U=20.242
t0:2.301, t1:2.301, t2:2.301, t3:2.301, t4:2.301, t5:2.301, t6:2.301, t7:2.301, t8:2.301, t9:2.301, t10:2.301, t11:2.301, t12:2.301, t13:2.301, t14:2.301, t15:2.301, t16:2.301, t17:2.301, t18:2.301, t19:2.301, t20:2.301
j=3 g=1.210 beta=0.430 b=60 U=26.999
t0:1.210, t1:1.210, t2:1.210, t3:1.210, t4:1.210, t5:1.210, t6:1.210, t7:1.210, t8:1.210, t9:1.210, t10:1.210, t11:1.210, t12:1.210, t13:1.210, t14:1.210, t15:1.210, t16:1.210, t17:1.210, t18:1.210, t19:1.210, t20:1.210
j=4 g=1.272 beta=0.406 b=38 U=16.701
t0:1.272, t1:1.272, t2:1.272, t3:1.272, t4:1.272, t5:1.272, t6:1.272, t7:1.272, t8:1.272, t9:1.272, t10:1.272, t11:1.272, t12:1.272, t13:1.272, t14:1.272, t15:1.272, t16:1.272, t17:1.272, t18:1.272, t19:1.272, t20:1.272
j=5 g=2.274 beta=0.578 b=45 U=28.267
t0:2.274, t1:2.274, t2:2.274, t3:3.429, t4:5.162, t5:6.895, t6:8.050, t7:9.783, t8:9.783, t9:9.783, t10:9.783, t11:9.783, t12:10.360, t13:10.360, t14:10.360, t15:11.516, t16:12.093, t17:12.093, t18:13.249, t19:14.404, t20:14.404
j=6 g=3.481 beta=1.035 b=52 U=57.275
t0:3.481, t1:3.481, t2:6.584, t3:6.584, t4:6.584, t5:6.584, t6:6.584, t7:6.584, t8:9.688, t9:12.791, t10:15.895, t11:17.964, t12:17.964, t13:20.033, t14:23.136, t15:25.205, t16:26.240, t17:26.240, t18:28.309, t19:28.309, t20:28.309
j=7 g=1.371 beta=0.463 b=51 U=24.967
t0:1.371, t1:1.371, t2:1.371, t3:1.371, t4:1.371, t5:1.371, t6:1.371, t7:1.371, t8:1.371, t9:1.371, t10:1.371, t11:1.371, t12:1.371, t13:1.371, t14:1.371, t15:1.371, t16:1.371, t17:1.371, t18:1.371, t19:1.371, t20:1.371
j=8 g=1.670 beta=0.823 b=32 U=28.020
t0:1.670, t1:1.670, t2:1.670, t3:1.670, t4:1.670, t5:1.670, t6:1.670, t7:1.670, t8:1.670, t9:1.670, t10:1.670, t11:1.670, t12:1.670, t13:1.670, t14:1.670, t15:1.670, t16:1.670, t17:1.670, t18:1.670, t19:1.670, t20:1.670
j=9 g=2.882 beta=0.875 b=31 U=30.008
t0:2.882, t1:6.382, t2:6.382, t3:6.382, t4:6.382, t5:9.007, t6:9.007, t7:11.633, t8:11.633, t9:11.633, t10:11.633, t11:11.633, t12:11.633, t13:13.383, t14:16.008, t15:17.758, t16:18.633, t17:18.633, t18:20.383, t19:22.133, t20:22.133
j=10 g=3.843 beta=0.635 b=53 U=37.506
t0:3.843, t1:6.384, t2:8.289, t3:9.560, t4:9.560, t5:9.560, t6:10.830, t7:10.830, t8:12.735, t9:14.641, t10:16.546, t11:16.546, t12:17.181, t13:17.181, t14:19.087, t15:20.357, t16:20.357, t17:20.357, t18:20.357, t19:21.628, t20:21.628
j=11 g=2.731 beta=0.793 b=52 U=43.966
t0:2.731, t1:2.731, t2:2.731, t3:2.731, t4:5.110, t5:5.110, t6:6.696, t7:9.075, t8:11.454, t9:13.833, t10:13.833, t11:15.419, t12:16.212, t13:17.798, t14:17.798, t15:17.798, t16:18.591, t17:18.591, t18:18.591, t19:18.591, t20:18.591
j=12 g=2.190 beta=0.357 b=39 U=16.094
t0:2.190, t1:2.190, t2:2.190, t3:2.190, t4:2.190, t5:2.190, t6:2.190, t7:2.190, t8:2.190, t9:2.190, t10:2.190, t11:2.190, t12:2.190, t13:2.190, t14:2.190, t15:2.190, t16:2.190, t17:2.190, t18:2.190, t19:2.190, t20:2.190
j=13 g=3.929 beta=0.354 b=50 U=21.611
t0:3.929, t1:3.929, t2:3.929, t3:3.929, t4:3.929, t5:3.929, t6:3.929, t7:3.929, t8:3.929, t9:3.929, t10:3.929, t11:3.929, t12:3.929, t13:3.929, t14:3.929, t15:3.929, t16:3.929, t17:3.929, t18:3.929, t19:4.636, t20:4.636
j=14 g=1.140 beta=0.485 b=48 U=24.437
t0:1.140, t1:1.140, t2:1.140, t3:1.140, t4:1.140, t5:1.140, t6:1.140, t7:1.140, t8:1.140, t9:1.140, t10:1.140, t11:1.140, t12:1.140, t13:1.140, t14:1.140, t15:1.140, t16:1.140, t17:1.140, t18:1.140, t19:1.140, t20:1.140

Further Discussion

In the current formulation, each worker (or team) is assumed to perform at most one task per day. This assumption is commonly adopted in the literature because it significantly simplifies the model structure by avoiding intra-day sequencing and time-overlap considerations. Such a setting is reasonable when tasks are highly demanding, geographically dispersed, or require a full working day to complete.

However, in real-world applications, especially in audit, inspection, or service operations, it is often feasible and even necessary for a worker to complete multiple tasks within the same day. In these cases, the binding constraint is not the number of tasks assigned per day, but rather the total available working time.

To better reflect operational reality, the model can be extended by replacing the “one-task-per-day” restriction with a daily working-time capacity constraint. Specifically, instead of limiting each worker to a single task per day, the total duration of all tasks assigned to that worker on a given day is constrained to be less than or equal to the daily working-hour limit. This modification allows multiple short tasks to be performed within one day while preserving model tractability.