MTRN4230 - Project 1

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Contents

1 Part A: Dynamic forward kinematics

You do not need to include anything in your report for this practical part of the assessment.

2 Part B: UR5e modelling

The DH table for the UR5e robot arm is as provided:

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	0	0	0.1625	$\pi/2$
Joint 2	0	-0.425	0	0
Joint 3	0	-0.3922	0	0
Joint 4	0	0	0.1333	$\pi/2$
Joint 5	0	0	0.0997	$-\pi/2$
Joint 6	0	0	0.0996	0

Table 1: The DH table for the UR5e robot arm

The home joint configuration (in millimeters and radians): [-588.5342, -133.3000, 371.9096, 3.1416, 0, 1.5708] However upon cross inspection with the simulation, the 5th joint had an angle of -3.1416 but this is equivalent as it is a phase difference of 2 π

2.1 Manual Calculation of Forward Kinematic Solutions

2.1.1 Resultant Matrix and Output Pose

The resultant matrix derived for the home joint configuration is (in millimeters and radians):

$$^{0}T_6 = \begin{bmatrix} 0 & 1.0000 & 0 & -588.5342 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0 & -1.0000 & 371.9096 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

The resultant posed derived for the home joint configuration is (in millimeters and radians):

$$\begin{bmatrix} -588.5342 & -133.3000 & 371.9096 & 3.1416 & 0 & 1.5708 \end{bmatrix}$$

2.1.2 Full Written Working

From Table 1, we can derive the following DH table for the home joint configuration:

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	0	0	0.1625	1.5708
Joint 2	1.3439	-0.425	0	0
Joint 3	1.5708	-0.3922	0	0
Joint 4	-1.8326	0	0.1333	1.5708
Joint 5	-1.5708	0	0.0997	-1.5708
Joint 6	0	0	0.0996	0

Table 2: The DH table for the UR5e robot arm at home joint configuration

From first principles, the homogenous transformation matrix ($^{n-1}T_n$) can be derived as follows:

$$
\begin{aligned}
{}^{n-1}T_n &= Rot_{z,\theta_n} Trans_{z,d_n} Trans_{x,a_n} Rot_{x,\alpha_n} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n) & 0 & 0 \\ \sin(\theta_n) & \cos(\theta_n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos(\alpha_n) & -\sin(\alpha_n) & 0 \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & \sin(\theta_n)\sin(\alpha_n) & a_n\cos(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n)\cos(\alpha_n) & -\cos(\theta_n)\sin(\alpha_n) & a_n\sin(\theta_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

Substituting each parameter with its corresponding joint values in the DH table in Table 2 to the transformation matrix in Equation (1) will yield us the following matrices:

$$
{}^0T_1 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & -1.0000 & 0 \\ 0 & 1.0000 & 0 & 162.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^1T_2 = \begin{bmatrix} 0.2588 & -0.9659 & 0 & -109.9981 \\ 0.9659 & 0.2588 & 0 & -410.5185 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^2T_3 = \begin{bmatrix} 0 & -1.0000 & 0 & 0 \\ 1.0000 & 0 & 0 & -392.2000 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^3T_4 = \begin{bmatrix} -0.2588 & 0 & -0.9659 & 0 \\ -0.9659 & 0 & 0.2588 & 0 \\ 0 & 1.0000 & 0 & 133.3000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^4T_5 = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ -1.0000 & 0 & 0 & 0 \\ 0 & -1.0000 & 0 & 99.7000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^5T_6 = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 99.6000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

And since coordinate frames can be compounded through the relationship ${}^AT_C = {}^AT_B{}^BT_C$, we can derive the resultant homogenous matrix ${}^0T_6$,

$$
\begin{aligned}
{}^0T_6 &= {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 \\
{}^0T_6 &= \begin{bmatrix} 0 & -0.8829 & 0.4695 & 421.3330 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0.4695 & 0.8829 & -298.6978 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\end{aligned}
$$

To get the pose, we realise that the resultant homogenous matrix takes the form of:

$$
\left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

So our final joint positions in millimeters are,

$$[421.3330, -133.3000, -298.6978]$$

And our roll, pitch and yaw values in radians respectively are can be calulated using Matlab's tr2rpy function,

```
rpy = tr2rpy(R);
% rpy = [0.4887, 0, 1.5708]
```

And thus, our final pose will be,

$$[421.3330, -133.3000, -298.6978, 0.4887, 0, 1.5708]$$

2.1.3 Intermediate Matrices

$$
{}^{0}T_{1} = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & -1.0000 & 0 \\ 0 & 1.0000 & 0 & 162.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^{0}T_{2} = \begin{bmatrix} 0.2588 & 0.9659 & & 0 - 109.9981 \\ 0 & 0 & -1.0000 & 0 \\ -0.9659 & 0.2588 & 0 & 573.0185 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^{0}T_{3} = \begin{bmatrix} 0.9659 & -0.2588 & 0 & -488.8342 \\ 0 & 0 & -1.0000 & 0 \\ 0.2588 & 0.9659 & 0 & 471.5096 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^{0}T_{4} = \begin{bmatrix} 0 & 0 & -1.0000 & -488.8342 \\ 0 & -1.0000 & 0 & -133.3000 \\ -1.0000 & 0 & 0 & 471.5096 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^{0}T_{5} = \begin{bmatrix} 0 & 1.0000 & 0 & -588.5342 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0 & -1.0000 & 471.5096 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^{0}T_{6} = \begin{bmatrix} 0 & 1.0000 & 0 & -588.5342 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0 & -1.0000 & 371.9096 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

2.1.4 Explanation of the Meaning of Calculated Matrices

2.2 Model of UR5e Robotic Arm using RVC Toolbox

2.2.1 Forward Kinematic Conversion to Attain Pose with Angles in RPY

2.2.2 Matrix Results and Converted Results

2.3 Validation of Calculations

2.3.1 Screenshot Showing Pose Including the Rotation in RPY Representation

3 Part C: Robot Speed Limits

3.1 Approach to Calculation

3.2 Jacobian Calculation

4 Part D: Robot Singularities

4.1 Determine the DH matrix

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	q_1	1	0	0
Joint 2	q_2	1	0	0
Joint 3	q_3	1	0	0

Table 3: The DH table for the 3-Link Robot

4.2 Calculate the Jacobian

To calculate the Jacobian that relates the joint velocities to linear velocities of the manipulator,
The forward kinematics solution is as follows,

$$T_E = \begin{pmatrix} \cos(q_1+q_2+q_3) & -\sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) \\ \sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3) & \sin(q_1+q_2+q_3)+\sin(q_1+q_2)+\sin(q_1) \\ 0 & 0 & 1 \end{pmatrix}$$

And since the forward kinematics solution for a 2D transformation matrix takes the form of,

$$\left[\begin{array}{cc|c} R & & T \\ \hline 0 & 0 & 1 \end{array}\right]$$

We can derive that,

$$\mathbf{T} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) \\ \sin(q_1+q_2+q_3)+\sin(q_1+q_2)+\sin(q_1) \end{pmatrix}$$

then,

$$\frac{\delta x}{\delta q_1} = -\sin(q_1+q_2+q_3)-\sin(q_1+q_2)-\sin(q_1)$$
$$\frac{\delta x}{\delta q_2} = -\sin(q_1+q_2+q_3)-\sin(q_1+q_2)$$
$$\frac{\delta x}{\delta q_3} = -\sin(q_1+q_2+q_3)$$
$$\frac{\delta y}{\delta q_1} = \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1)$$
$$\frac{\delta y}{\delta q_2} = \cos(q_1+q_2+q_3)+\cos(q_1+q_2)$$
$$\frac{\delta y}{\delta q_3} = \cos(q_1+q_2+q_3)$$

So the Jacobian matrix ($\mathbf{J}_v$) that relates joint velocity $\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix}$ to spacial velocities $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ takes the form of,

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \mathbf{J}_v \begin{pmatrix}\dot{q_1}\\ \dot{q_2}\\ \dot{q_3}\end{pmatrix}$$

where,

$$\begin{aligned}\mathbf{J}_v &= \begin{pmatrix}\frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \frac{\delta x}{\delta q_3}\\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \frac{\delta y}{\delta q_3}\end{pmatrix}\\ &= \begin{pmatrix}-\sin(q_1+q_2+q_3)-\sin(q_1+q_2)-\sin(q_1) & -\sin(q_1+q_2+q_3)-\sin(q_1+q_2) & -\sin(q_1+q_2+q_3)\\ \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) & \cos(q_1+q_2+q_3)+\cos(q_1+q_2) & \cos(q_1+q_2+q_3)\end{pmatrix}\end{aligned}$$

The Jacobian that relates the joint velocities with angular velocities can be derived as follows,

$$\begin{aligned}\mathbf{R} &= \begin{pmatrix}\cos(q_1+q_2+q_3) & -\sin(q_1+q_2+q_3)\\ \sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3)\end{pmatrix}\\ &= \begin{pmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{pmatrix}\end{aligned}$$

where, by inspection,

$$\begin{aligned}&\theta = q_1+q_2+q_3\\ \therefore\quad &\omega = \dot{\theta} = \dot{q_1}+\dot{q_2}+\dot{q_3}\\ \therefore\quad &\omega = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}\dot{q_1}\\ \dot{q_2}\\ \dot{q_3}\end{pmatrix}\end{aligned}$$

and hence,

$$\mathbf{J}_\omega = \begin{pmatrix}1 & 1 & 1\end{pmatrix}$$

Therefore the full Jacobian matrix is as follows,

$$\mathbf{J} = \begin{pmatrix}-\sin(q_1+q_2+q_3)-\sin(q_1+q_2)-\sin(q_1) & -\sin(q_1+q_2+q_3)-\sin(q_1+q_2) & -\sin(q_1+q_2+q_3)\\ \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) & \cos(q_1+q_2+q_3)+\cos(q_1+q_2) & \cos(q_1+q_2+q_3)\\ 1 & 1 & 1\end{pmatrix}$$

4.3 For what value(s) is the manipulator at a singularity?

To calculate the values of the manipulator at a singularity, we check the values for which the Jacobian determinant equals 0.

$$\det(J) = 0$$

4.4 What motion is restricted at this singularity?

4.5 What type of singularity is experienced?

5 Part E: Foo Bar