

MTRN4230 - Project 1

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1 Part A: Dynamic forward kinematics

You do not need to include anything in your report for this practical part of the assessment.

2 Part B: UR5e modelling

The DH table for the UR5e robot arm is as provided:

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	0	0	0.1625	$\pi/2$
Joint 2	0	-0.425	0	0
Joint 3	0	-0.3922	0	0
Joint 4	0	0	0.1333	$\pi/2$
Joint 5	0	0	0.0997	$-\pi/2$
Joint 6	0	0	0.0996	0

Table 1: The DH table for the UR5e robot arm

The home joint configuration (in millimeters and degrees): [0.00, 77.00, 90.00, -105.00, -90.00, 0.00]

2.1 Manual Calculation of Forward Kinematic Solutions

2.1.1 Resultant Matrix and Output Pose

The resultant matrix derived for the home joint configuration is (in millimeters and radians):

$${}^0T_6 = \begin{bmatrix} 0 & -0.8829 & 0.4695 & 421.3330 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0.4695 & 0.8829 & -298.6978 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

The resultant posed derived for the home joint configuration is (in millimeters and radians):

$$[421.3330 \quad -133.3000 \quad -298.6978 \quad 0.4887 \quad 0 \quad 1.5708]$$

2.1.2 Full Written Working

From Table 1, we can derive the following DH table for the home joint configuration:

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	0	0	0.1625	1.5708
Joint 2	1.3439	-0.425	0	0
Joint 3	1.5708	-0.3922	0	0
Joint 4	-1.8326	0	0.1333	1.5708
Joint 5	-1.5708	0	0.0997	-1.5708
Joint 6	0	0	0.0996	0

Table 2: The DH table for the UR5e robot arm at home joint configuration

From first principles, the homogenous transformation matrix (${}^{n-1}T_n$) can be derived as follows:

$$\begin{aligned}
{}^{n-1}T_n &= Rot_{z,\theta_n} Trans_{z,d_n} Trans_{x,a_n} Rot_{x,\alpha_n} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n) & 0 & 0 \\ \sin(\theta_n) & \cos(\theta_n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos(\alpha_n) & -\sin(\alpha_n) & 0 \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & \sin(\theta_n)\sin(\alpha_n) & a_n\cos(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n)\cos(\alpha_n) & -\cos(\theta_n)\sin(\alpha_n) & a_n\sin(\theta_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}$$

Substituting each parameter with its corresponding joint values in the DH table in Table 2 to the transformation matrix in Equation (1) will yield us the following matrices:

$$\begin{aligned}
{}^0T_1 &= \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & -1.0000 & 0 \\ 0 & 1.0000 & 0 & 162.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^1T_2 &= \begin{bmatrix} 0.2250 & -0.9744 & 0 & -95.6042 \\ 0.9744 & 0.2250 & 0 & -414.1073 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^2T_3 &= \begin{bmatrix} 0 & -1.0000 & 0 & 0 \\ 1.0000 & 0 & 0 & -392.2000 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^3T_4 &= \begin{bmatrix} -0.2588 & 0 & -0.9659 & 0 \\ -0.9659 & 0 & 0.2588 & 0 \\ 0 & 1.0000 & 0 & 133.3000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^4T_5 &= \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ -1.0000 & 0 & 0 & 0 \\ 0 & -1.0000 & 0 & 99.7000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^5T_6 &= \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 99.6000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\end{aligned}$$

And since coordinate frames can be compounded through the relationship ${}^AT_C = {}^AT_B {}^BT_C$,

we can derive the resultant homogenous matrix 0T_6 ,

$${}^0T_6 = {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6$$

$${}^0T_6 = \begin{bmatrix} 0 & -0.8829 & 0.4695 & 421.3330 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0.4695 & 0.8829 & -298.6978 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

To get the pose, we realise that the resultant homogenous matrix takes the form of:

$$\left[\begin{array}{ccc|c} & & & \\ & R & & T \\ & & & \\ \hline 0 & 0 & 0 & 1 \end{array} \right]$$

So our final joint positions in millimeters are,

$$[421.3330, -133.3000, -298.6978]$$

And our roll, pitch and yaw values in radians respectively are can be calulated using Matlab's tr2rpy function,

$$\begin{aligned} \text{rpy} &= \text{tr2rpy}(R); \\ \% \text{ rpy} &= [0.4887, 0, 1.5708] \end{aligned}$$

And thus, our final pose will be,

$$[421.3330, -133.3000, -298.6978, 0.4887, 0, 1.5708]$$

2.1.3 Intermediate Matrices

$$\begin{aligned}
{}^0T_1 &= \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 0 & -1.0000 & 0 \\ 0 & 1.0000 & 0 & 162.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^0T_2 &= \begin{bmatrix} 0.2250 & -0.9744 & 0 & -95.6042 \\ 0 & 0 & -1.0000 & 0 \\ 0.9744 & 0.2250 & 0 & -251.6073 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^0T_3 &= \begin{bmatrix} -0.9744 & -0.2250 & 0 & 286.5437 \\ 0 & 0 & -1.0000 & 0 \\ 0.2250 & -0.9744 & 0 & -339.8331 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^0T_4 &= \begin{bmatrix} 0.4695 & 0 & 0.8829 & 286.5437 \\ 0 & -1.0000 & 0 & -133.3000 \\ 0.8829 & 0 & -0.4695 & -339.8331 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^0T_5 &= \begin{bmatrix} 0 & -0.8829 & 0.4695 & 374.5736 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0.4695 & 0.8829 & -386.6394 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \\
{}^0T_6 &= \begin{bmatrix} 0 & -0.8829 & 0.4695 & 421.3330 \\ 1.0000 & 0 & 0 & -133.3000 \\ 0 & 0.4695 & 0.8829 & -298.6978 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\end{aligned}$$

2.1.4 Explanation of the Meaning of Calculated Matrices

2.2 Model of UR5e Robotic Arm using RVC Toolbox

2.2.1 Forward Kinematic Conversion to Attain Pose with Angles in RPY

2.2.2 Matrix Results and Converted Results

2.3 Validation of Calculations

2.3.1 Screenshot Showing Pose Including the Rotation in RPY Representation

3 Part C: Robot Speed Limits

3.1 Approach to Calculation

3.2 Jacobian Calculation

4 Part D: Robot Singularities

4.1 Determine the DH matrix

	theta (rad)	a (m)	d (m)	alpha (rad)
Joint 1	θ_1	1	0	0
Joint 2	θ_2	1	0	0

Table 3: The DH table for the 3-Link Robot

4.2 Calculate the Jacobian

4.3 For what value(s) is the manipulator at a singularity?

4.4 What motion is restricted at this singularity?

4.5 What type of singularity is experienced?