# MTRN4230 - Project 1

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## Contents



# 1 Part A: Dynamic forward kinematics

You do not need to include anything in your report for this practical part of the assessment.

# 2 Part B: UR5e modelling

The DH table for the UR5e robot arm is as provided:

| | theta (rad) | a (m) | d (m) | alpha (rad) |
|---|---|---|---|---|
| Joint 1 | 0 | 0 | 0.1625 | $\pi/2$ |
| Joint 2 | 0 | -0.425 | 0 | 0 |
| Joint 3 | 0 | -0.3922 | 0 | 0 |
| Joint 4 | 0 | 0 | 0.1333 | $\pi/2$ |
| Joint 5 | 0 | 0 | 0.0997 | -$\pi/2$ |
| Joint 6 | 0 | 0 | 0.0996 | 0 |

Table 1: The DH table for the UR5e robot arm

*The home joint configuration (in degrees)*: [0.00, -75.00, 90.00, -105.00, -90.00, 0]

## 2.1 Manual Calculation of Forward Kinematic Solutions

### 2.1.1 Resultant Matrix and Output Pose

The resultant matrix derived for the home joint configuration is (in millimeters and radians):

$$^{0}T_6 = \begin{bmatrix} 0 & 1.00 & 0 & -588.53 \\ 1.00 & 0 & 0 & -133.30 \\ 0 & 0 & -1.00 & 371.91 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}$$

The resultant posed derived for the home joint configuration is (in millimeters and radians):

$$\begin{bmatrix} -588.5342 & -133.3000 & 371.9096 & 3.1416 & 0 & 1.5708 \end{bmatrix}$$

However upon cross inspection with the simulation, the 5th joint had an angle of -3.1416, but this is equivalent as it is a phase difference of 2 $\pi$

### 2.1.2 Full Written Working

From Table 1, we can derive the following DH table for the home joint configuration:

| | theta (rad) | a (m) | d (m) | alpha (rad) |
|---|---|---|---|---|
| Joint 1 | 0 | 0 | 0.1625 | 1.5708 |
| Joint 2 | 1.3439 | -0.425 | 0 | 0 |
| Joint 3 | 1.5708 | -0.3922 | 0 | 0 |
| Joint 4 | -1.8326 | 0 | 0.1333 | 1.5708 |
| Joint 5 | -1.5708 | 0 | 0.0997 | -1.5708 |
| Joint 6 | 0 | 0 | 0.0996 | 0 |

Table 2: The DH table for the UR5e robot arm at home joint configuration

From first principles, the homogenous transformation matrix ($^{n-1}T_n$) can be derived as follows:

$$
\begin{aligned}
{}^{n-1}T_n &= Rot_{z,\theta_n} Trans_{z,d_n} Trans_{x,a_n} Rot_{x,\alpha_n} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n) & 0 & 0 \\ \sin(\theta_n) & \cos(\theta_n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos(\alpha_n) & -\sin(\alpha_n) & 0 \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(\theta_n) & -\sin(\theta_n)\cos(\alpha_n) & \sin(\theta_n)\sin(\alpha_n) & a_n\cos(\theta_n) \\ \sin(\theta_n) & \cos(\theta_n)\cos(\alpha_n) & -\cos(\theta_n)\sin(\alpha_n) & a_n\sin(\theta_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

Substituting each parameter with its corresponding joint values in the DH table in Table 2 to the transformation matrix in Equation (1) will yield us the following matrices:

$$
{}^0T_1 = \begin{bmatrix} 1.00 & 0 & 0 & 0 \\ 0 & 0 & -1.00 & 0 \\ 0 & 1.00 & 0 & 162.50 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^1T_2 = \begin{bmatrix} 0.26 & -0.97 & 0 & -110.00 \\ 0.97 & 0.26 & 0 & -410.52 \\ 0 & 0 & 1.00 & 0 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^2T_3 = \begin{bmatrix} 0 & -1.00 & 0 & 0 \\ 1.00 & 0 & 0 & -392.20 \\ 0 & 0 & 1.00 & 0 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^3T_4 = \begin{bmatrix} -0.26 & 0 & -0.97 & 0 \\ -0.97 & 0 & 0.26 & 0 \\ 0 & 1.0000 & 0 & 133.30 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^4T_5 = \begin{bmatrix} 0 & 0 & 1.00 & 0 \\ -1.00 & 0 & 0 & 0 \\ 0 & -1.00 & 0 & 99.70 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^5T_6 = \begin{bmatrix} 1.00 & 0 & 0 & 0 \\ 0 & 1.00 & 0 & 0 \\ 0 & 0 & 1.00 & 99.60 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

And since coordinate frames can be compounded through the relationship ${}^AT_C = {}^AT_B\,{}^BT_C$, we can derive the resultant homogenous matrix ${}^0T_6$,

$$ {}^0T_6 = {}^0T_1\,{}^1T_2\,{}^2T_3\,{}^3T_4\,{}^4T_5\,{}^5T_6 $$

$$ {}^0T_6 = \begin{bmatrix} 0 & 1.00 & 0 & -588.53 \\ 1.00 & 0 & 0 & -133.30 \\ 0 & 0 & -1.00 & 371.91 \\ 0 & 0 & 0 & 1.00 \end{bmatrix} $$

To get the pose, we realise that the resultant homogenous matrix takes the form of:

$$ \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

So our final joint positions in millimeters are,

$$ [-588.5342, -133.3000, 371.9096] $$

And our roll, pitch and yaw values in radians respectively are can be calculated using Matlab's tr2rpy function,

```
rpy = tr2rpy(R);
% rpy = [3.1416, 0, 1.5708]
```

And thus, our final pose will be,

$$ [-588.5342, -133.3000, 371.9096, 3.1416, 0, 1.5708] $$

#### 2.1.3 Intermediate Matrices

$$
{}^0T_1 = \begin{bmatrix} 1.00 & 0 & 0 & 0 \\ 0 & 0 & -1.00 & 0 \\ 0 & 1.00 & 0 & 162.50 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^0T_2 = \begin{bmatrix} 0.26 & 0.97 & 0 & -110.00 \\ 0 & 0 & -1.00 & 0 \\ -0.97 & 0.26 & 0 & 573.02 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^0T_3 = \begin{bmatrix} 0.97 & -0.26 & 0 & -488.83 \\ 0 & 0 & -1.00 & 0 \\ 0.26 & 0.97 & 0 & 471.51 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^0T_4 = \begin{bmatrix} 0 & 0 & -1.00 & -488.83 \\ 0 & -1.00 & 0 & -133.30 \\ -1.00 & 0 & 0 & 471.51 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^0T_5 = \begin{bmatrix} 0 & 1.00 & 0 & -588.53 \\ 1.00 & 0 & 0 & -133.30 \\ 0 & 0 & -1.00 & 471.51 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
{}^0T_6 = \begin{bmatrix} 0 & 1.00 & 0 & -588.53 \\ 1.00 & 0 & 0 & -133.30 \\ 0 & 0 & -1.00 & 371.91 \\ 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

#### 2.1.4 Explanation of the Meaning of Calculated Matrices

### 2.2 Model of UR5e Robotic Arm using RVC Toolbox

#### 2.2.1 Forward Kinematic Conversion to Attain Pose with Angles in RPY

#### 2.2.2 Matrix Results and Converted Results

### 2.3 Validation of Calculations

#### 2.3.1 Screenshot Showing Pose Including the Rotation in RPY Representation

# 3 Part C: Robot Speed Limits

## 3.1 Approach to Calculation

Foo

```
Links ← DH Table;
Robot ← Links;
maxSpeed ← 0;
for i = 1 to n ;            /* Where n = number of joint positions/velocities. */
do
    jacobianMatrix ← jacobian(robot, jointPosition(i));
    toolVelocity ← jacobianMatrix * jointVelocities(i);
    linearVelocity ← First 3 rows of toolVelocity;
    speed ← Magnitude of linearVelocity;
    maxSpeed ← max(maxSpeed, speed);
end
return maxSpeed;
```

## 3.2 Jacobian Calculation for the First Location

# 4 Part D: Robot Singularities

## 4.1 Determine the DH matrix

| | theta (rad) | a (m) | d (m) | alpha (rad) |
|---|---|---|---|---|
| Joint 1 | $q_1$ | 1 | 0 | 0 |
| Joint 2 | $q_2$ | 1 | 0 | 0 |
| Joint 3 | $q_3$ | 1 | 0 | 0 |

Table 3: The DH table for the 3-Link Robot

## 4.2 Calculate the Jacobian

To calculate the Jacobian that relates the joint velocities to linear velocities of the manipulator, The forward kinematics solution is as follows,

$$T_E = \begin{pmatrix} \cos(q_1+q_2+q_3) & -\sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) \\ \sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3) & \sin(q_1+q_2+q_3)+\sin(q_1+q_2)+\sin(q_1) \\ 0 & 0 & 1 \end{pmatrix}$$

And since the forward kinematics solution for a 2D transformation matrix takes the form of,

$$\left[\begin{array}{cc|c} R & & T \\ & & \\ \hline 0 & 0 & 1 \end{array}\right]$$

We can derive that,

$$\mathbf{T} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(q_1+q_2+q_3)+\cos(q_1+q_2)+\cos(q_1) \\ \sin(q_1+q_2+q_3)+\sin(q_1+q_2)+\sin(q_1) \end{pmatrix}$$

then,

$$\frac{\delta x}{\delta q_1} = -\sin(q_1+q_2+q_3) - \sin(q_1+q_2) - \sin(q_1)$$
$$\frac{\delta x}{\delta q_2} = -\sin(q_1+q_2+q_3) - \sin(q_1+q_2)$$
$$\frac{\delta x}{\delta q_3} = -\sin(q_1+q_2+q_3)$$
$$\frac{\delta y}{\delta q_1} = \cos(q_1+q_2+q_3) + \cos(q_1+q_2) + \cos(q_1)$$
$$\frac{\delta y}{\delta q_2} = \cos(q_1+q_2+q_3) + \cos(q_1+q_2)$$
$$\frac{\delta y}{\delta q_3} = \cos(q_1+q_2+q_3)$$

So the Jacobian matrix ($\mathbf{J}_v$) that relates joint velocity $\begin{pmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{pmatrix}$ to spacial velocities $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ takes the form of,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \mathbf{J}_v \begin{pmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{pmatrix}$$

where,

$$\mathbf{J}_v = \begin{pmatrix} \frac{\delta x}{\delta q_1} & \frac{\delta x}{\delta q_2} & \frac{\delta x}{\delta q_3} \\ \frac{\delta y}{\delta q_1} & \frac{\delta y}{\delta q_2} & \frac{\delta y}{\delta q_3} \end{pmatrix}$$
$$= \begin{pmatrix} -\sin(q_1+q_2+q_3) - \sin(q_1+q_2) - \sin(q_1) & -\sin(q_1+q_2+q_3) - \sin(q_1+q_2) & -\sin(q_1+q_2+q_3) \\ \cos(q_1+q_2+q_3) + \cos(q_1+q_2) + \cos(q_1) & \cos(q_1+q_2+q_3) + \cos(q_1+q_2) & \cos(q_1+q_2+q_3) \end{pmatrix}$$

The Jacobian that relates the joint velocities with angular velocities can be derived as follows,

$$\mathbf{R} = \begin{pmatrix} \cos(q_1+q_2+q_3) & -\sin(q_1+q_2+q_3) \\ \sin(q_1+q_2+q_3) & \cos(q_1+q_2+q_3) \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

where, by inspection,

$$\theta = q_1 + q_2 + q_3$$
$$\therefore \quad \omega = \dot{\theta} = \dot{q_1} + \dot{q_2} + \dot{q_3}$$
$$\therefore \quad \omega = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{pmatrix}$$

and hence,

$$\mathbf{J}_\omega = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

Therefore the full Jacobian matrix is as follows,

$$\mathbf{J} = \begin{pmatrix} -\sin(q_1+q_2+q_3) - \sin(q_1+q_2) - \sin(q_1) & -\sin(q_1+q_2+q_3) - \sin(q_1+q_2) & -\sin(q_1+q_2+q_3) \\ \cos(q_1+q_2+q_3) + \cos(q_1+q_2) + \cos(q_1) & \cos(q_1+q_2+q_3) + \cos(q_1+q_2) & \cos(q_1+q_2+q_3) \\ 1 & 1 & 1 \end{pmatrix}$$

### 4.3 For what value(s) is the manipulator at a singularity?

To calculate the values of the manipulator at a singularity, we check the values for which the Jacobian determinant equals 0.

$$\det(J) = 0$$

### 4.4 What motion is restricted at this singularity?

### 4.5 What type of singularity is experienced?

## 5 Part E: Foo Bar