Chap 5:
convection-diffusion equation

$$\underbrace{\frac{d}{dx}\left(\rho u \phi\right)}_{\text{convection}} = \underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion, Chapter 4}}$$

$$\text{e.g., } \phi = T, \ \Gamma = \frac{k}{c_p}$$

Energy equation

Finite-Volume (FV) Integration

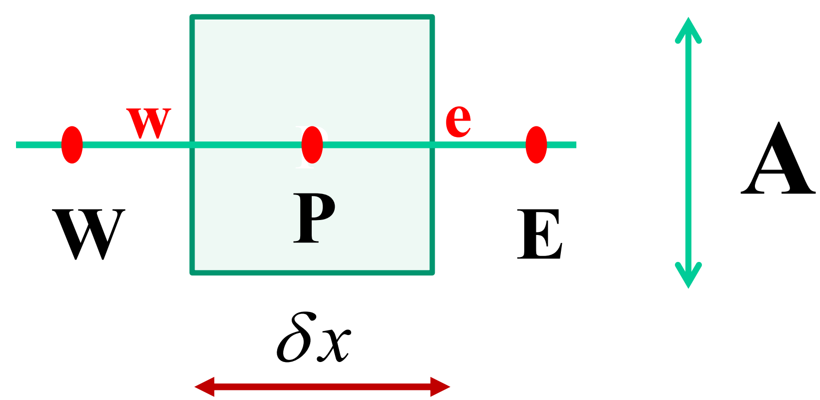
ODE

$$\begin{cases} \underbrace{\int_w^e \left[\frac{d}{dx}(\rho u \phi) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx} \right) \right] \underbrace{dxA}_{\text{differential volume}} = \underbrace{0}_{\text{source}}}_{} \\ \text{source} = \underbrace{S_u}_{\text{constant}} + \underbrace{S_p}_{\text{slope}} \phi_P \Rightarrow \boxed{S_u = S_p = 0} \end{cases}$$

for FV away from boundaries

Other topics:

1. Stability/Boundedness
2. Accuracy

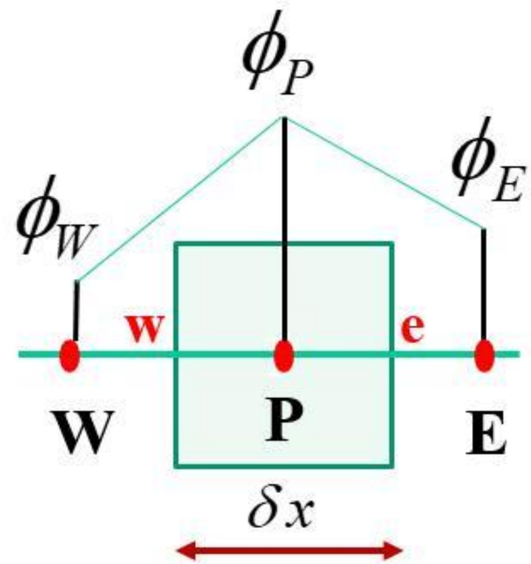


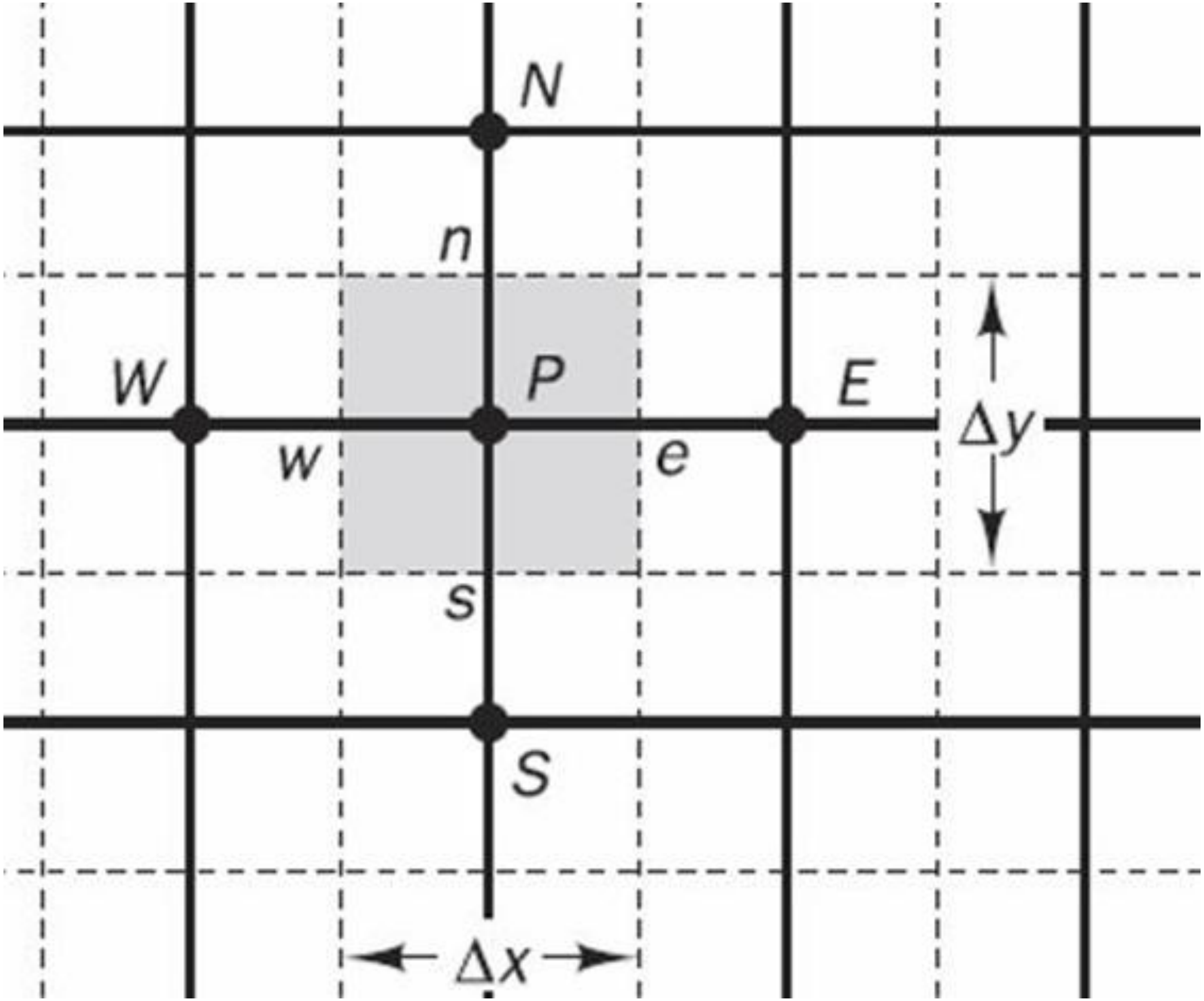
1st-order derivative 2nd-order derivative

$$\int_w^e \left[\underbrace{\frac{d}{dx}(\rho u \phi)}_{\text{convection}} - \underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion}} \right] dxA = 0 \quad (1)$$

$$\Rightarrow \underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}(\rho u \phi)}_{\text{convection}} \right] dxA}_{(1.1)} - \boxed{\underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion}} \right] dxA}_{(1.2)}} = 0$$

Chap 4



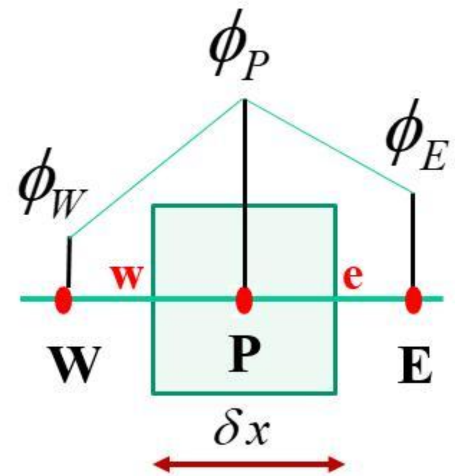

N
n
W
P
E
Δy
w
e
s
S
Δx

Diffusion (Chap. 4)

$$\Gamma\left(\frac{\phi_E-\phi_P}{\delta x}\right) \qquad \Gamma\left(\frac{\phi_P-\phi_W}{\delta x}\right)$$

$$\underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right)}_{\text{diffusion}}\right] dx A}_{(1.2)} = \left[\left(\Gamma\frac{d\phi}{dx}\right)_e - \left(\Gamma\frac{d\phi}{dx}\right)_w\right] A$$

$$= \left[\underbrace{\left(\frac{\Gamma}{\delta x}\right)}_{D_e}\left(\phi_E-\phi_P\right) - \underbrace{\left(\frac{\Gamma}{\delta x}\right)}_{D_w}\left(\phi_P-\phi_W\right)\right] A$$

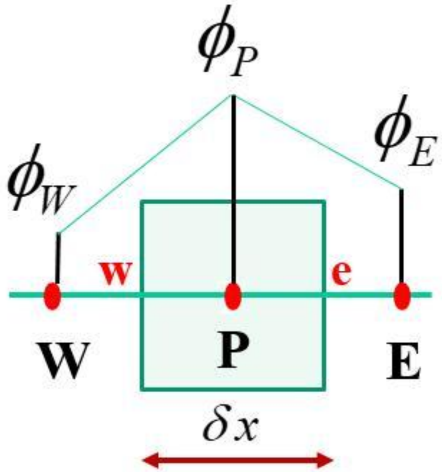


Diffusion (Chap. 4)

$$(1.2) = \text{Diffusion} = \left[\underbrace{\left(\frac{\Gamma}{\delta x}\right)}_{D_e} \left(\phi_E - \phi_P\right) - \underbrace{\left(\frac{\Gamma}{\delta x}\right)}_{D_w} \left(\phi_P - \phi_W\right) \right] A$$

$$= \boxed{\left[D_e \phi_E + D_w \phi_W - \left(D_e + D_w\right) \phi_P \right] A}$$

where $D_e = D_w = \dfrac{\Gamma}{\delta x}$



$$\int_w^e \left[\underbrace{\frac{d}{dx}\left(\rho u \phi\right)}_{\text{convection}} - \underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion}} \right] dxA = 0 \quad (1)$$

$$\Rightarrow \boxed{\underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}\left(\rho u \phi\right)}_{\text{convection}} \right] dxA}_{(1.1)}} - \underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion}} \right] dxA}_{(1.2)} = 0$$

$$\left(1.2\right) = \text{Diffusion}$$

$$= \left[D_e \phi_E + D_w \phi_W - \left(D_e + D_w\right) \phi_P \right] A$$

Convection (Chap. 5)

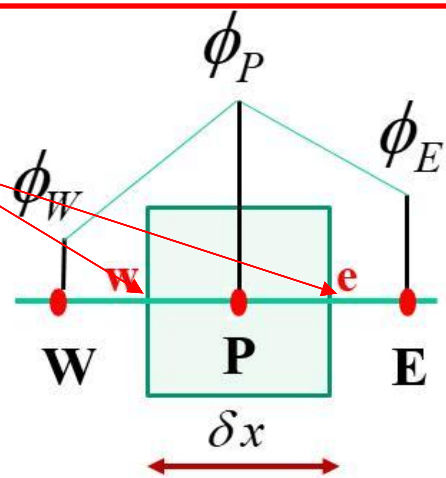
$$\underbrace{\int_w^e \left[\underbrace{\frac{d}{dx}(\rho u\phi)}_{\text{convection}}\right] dx A}_{(1.1)} = \left[(\rho u\phi)_e - (\rho u\phi)_w\right] A$$

How to determine ϕ_e, ϕ_w?
(Sec. 5.3, 5.6, 5.7, 5.9)

$$= \left[\underbrace{(\rho u)}_{F_e}\phi_e - \underbrace{(\rho u)}_{F_w}\phi_w\right] A$$

Mass flux per unit area

where $F_e = F_w = \rho u$



(1.1)

(1.2)

$$\left[F_e\phi_e - F_w\phi_w\right]A = \underbrace{\left[\Gamma\left(\frac{d\phi}{dx}\right)_e - \Gamma\left(\frac{d\phi}{dx}\right)_w\right]A}_{=\left[D_e\phi_E + D_w\phi_W - \left(D_e + D_w\right)\phi_P\right]A}$$

Convection

$$F_e = F_w = \rho u$$

Diffusion

$$D_e = D_w = \frac{\Gamma}{\delta x}$$

Central Differencing Scheme (CDS), Sec. 5.3

$$\left[F_e\phi_e - F_w\phi_w\right]A = \underbrace{\left[\Gamma\left(\frac{d\phi}{dx}\right)_e - \Gamma\left(\frac{d\phi}{dx}\right)_w\right]A}_{=\left[D_e\phi_E + D_w\phi_W - \left(D_e + D_w\right)\phi_P\right]A}$$

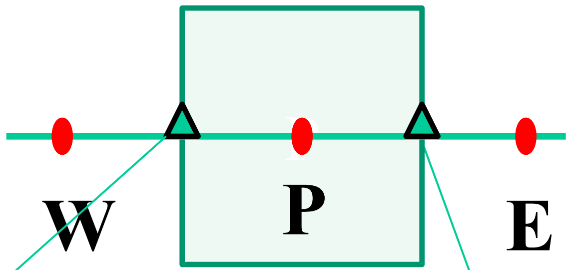
Convection

$$F_e = F_w = \rho u$$

Diffusion

$$D_e = D_w = \frac{\Gamma}{\delta x}$$

Central Differencing Scheme (CDS), Sec. 5.3

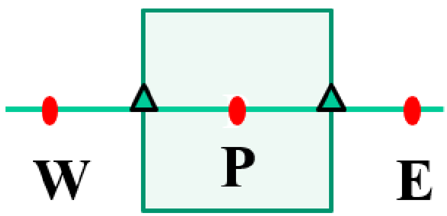


$$\phi_w \approx \frac{1}{2}\left(\phi_W + \phi_P\right) \qquad \phi_e \approx \frac{1}{2}\left(\phi_E + \phi_P\right)$$

$$F_e = F_w = \rho u$$

CDS

$$\left.\begin{aligned} \phi_e &\approx \frac{1}{2}\left(\phi_E + \phi_P\right) \\ \phi_w &\approx \frac{1}{2}\left(\phi_W + \phi_P\right) \end{aligned}\right\} \Rightarrow \left[F_e\phi_e - F_w\phi_w\right]A$$



$$\Rightarrow \left\{ F_e \underbrace{\left[\frac{1}{2}\left(\phi_E + \phi_P\right)\right]}_{\phi_e} - F_w \underbrace{\left[\frac{1}{2}\left(\phi_W + \phi_P\right)\right]}_{\phi_w} \right\} A$$

$$\Rightarrow \underbrace{\left\{ \left(\frac{F_e}{2}\right)\phi_E + \left(\frac{-F_w}{2}\right)\phi_W + \left[\left(\frac{F_e}{2}\right) + \left(\frac{-F_w}{2}\right)\right]\phi_P \right\} A}_{\text{convection}}$$

convection=diffusion (CDS)

$$\underbrace{\left\{\left(\frac{F_e}{2}\right)\phi_E+\left(\frac{-F_w}{2}\right)\phi_W+\left[\left(\frac{F_e}{2}\right)+\left(\frac{-F_w}{2}\right)\right]\phi_P\right\}\cancel{A}}_{\text{convection}}$$

$$=\underbrace{\left[D_e\phi_E+D_w\phi_W-\left(D_e+D_w\right)\phi_P\right]\cancel{A}}_{\text{diffusion}}$$

$$\Rightarrow\underbrace{\left\{\left(D_e+D_w\right)+\left[\left(\frac{F_e}{2}\right)+\left(\frac{-F_w}{2}\right)\right]\right\}}_{a_P}\phi_P=\boxed{\underbrace{\left[D_e+\left(\frac{-F_e}{2}\right)\right]}_{a_E}\phi_E+\underbrace{\left[D_w+\left(\frac{F_w}{2}\right)\right]}_{a_W}\phi_W}$$

FVM standard template

$$\frac{d}{dx}\underbrace{(\rho u \phi)}_{\text{convection}} = \underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion, Chapter 4}} \boxed{+ \underbrace{S}_{=0}}$$

$$\begin{cases} a_P \phi_P = \sum_{nb=E,W} a_{nb}\phi_{nb} + \underbrace{S_u}_{=0} \\ a_P = a_E + a_W - \underbrace{S_P}_{=0} \end{cases}$$

$$\Rightarrow a_P = \left\{ (D_e + D_w) + \left[\left(\frac{F_e}{2}\right) + \left(\frac{-F_w}{2}\right) \right] \right\}$$

$$= \left[D_e + \left(\frac{F_e}{2}\right) \right] + \left[D_w + \left(\frac{-F_w}{2}\right) \right]$$

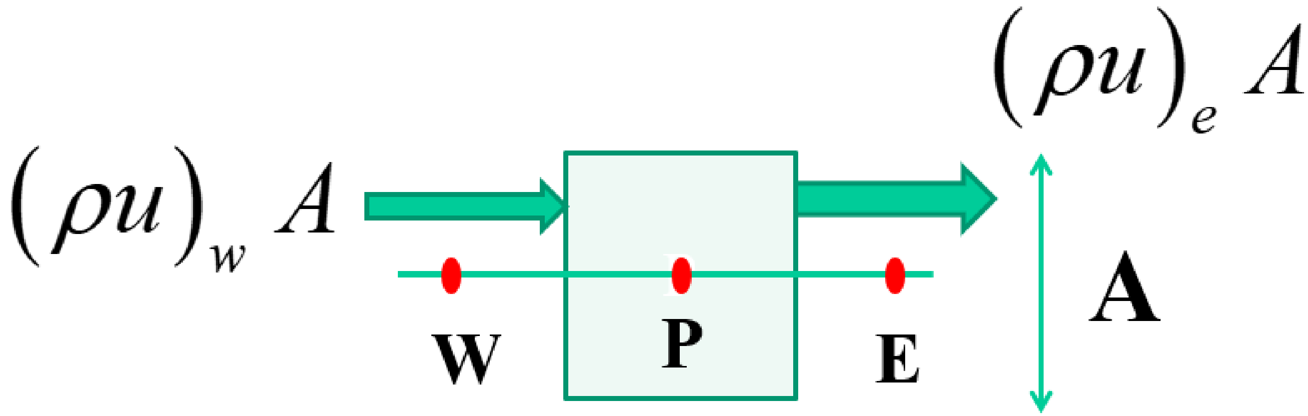
$$\Rightarrow \begin{cases} a_E = \left[D_e + \left(\frac{-F_e}{2}\right) \right] \\ a_W = \left[D_w + \left(\frac{F_w}{2}\right) \right] \end{cases}$$

$$a_P = \left[D_e + \left(\frac{F_e}{2}\right)\right] + \left[D_w + \left(\frac{-F_w}{2}\right)\right]$$

$$= \underbrace{\left[D_e + \left(\frac{-F_e}{2}\right)\right]}_{a_E} + F_e + \underbrace{\left[D_w + \left(\frac{F_w}{2}\right)\right]}_{a_W} - F_w$$

$$= a_E + a_W + \underbrace{\cancel{\left(F_e - F_w\right)}}_{=0,\ \text{continuity}} = \boxed{a_E + a_W = D_e + D_w}$$

Mass conservation
[**continuity equation**]



$$(\rho u)_e = (\rho u)_w \quad \text{, or} \quad \boxed{F_e = F_w}$$

convective flux

$$a_P = a_E + a_W + \underbrace{\cancel{(F_e - F_w)}}_{=0,\ \text{continuity}}$$

Alternative approach

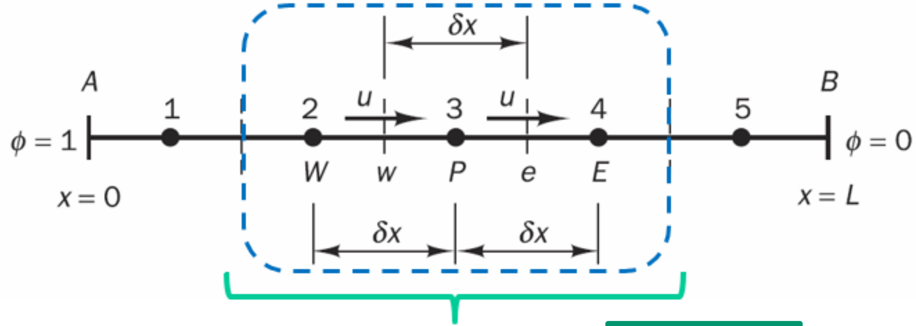
$$\Rightarrow a_P = \left\{ \left(D_e + D_w\right) + \left[\left(\frac{F_e}{2}\right) + \left(\frac{-F_w}{2}\right) \right] \right\} = \boxed{\left(D_e + D_w\right)} + \frac{1}{2} \cancel{\underbrace{\left(F_e - F_w\right)}_{=0,\text{continuity}}}$$

$$\begin{cases} a_E = \left[D_e + \left(\dfrac{-F_e}{2}\right) \right] \\ a_W = \left[D_w + \left(\dfrac{F_w}{2}\right) \right] \end{cases}$$

$$\Rightarrow a_E + a_W = \boxed{\left(D_e + D_w\right)} - \frac{1}{2} \cancel{\underbrace{\left(F_e - F_w\right)}_{=0,\text{continuity}}}$$

Therefore,

$$a_P = a_E + a_W = \sum_{nb} a_{nb} = \boxed{\left(D_e + D_w\right)}$$



CV #2,3,4

FVM

$$a_P\phi_P = \sum_{nb=E,W} a_{nb}\phi_{nb} + S_u$$

$$\frac{d}{dx}(\rho u\phi) - \frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right) = 0$$

No source

$$a_E = D_e - \frac{F_e}{2},\ a_W = D_w + \frac{F_w}{2}$$

$$a_P = a_E + a_W - S_P,\ S_P = 0$$

$$S_u = 0$$

$$D_e = D_w = D = \frac{\Gamma}{\delta x}$$

$$F_e = F_w = F = \rho u$$

Sec. 5.3: The central differencing scheme (**CDS**)

Identifying the coefficients of ϕ_W and ϕ_E as a_W and a_E, the **central differencing** expressions for the discretised convection–diffusion equation are

$$\boxed{a_P\phi_P = a_W\phi_W + a_E\phi_E} \tag{5.14}$$

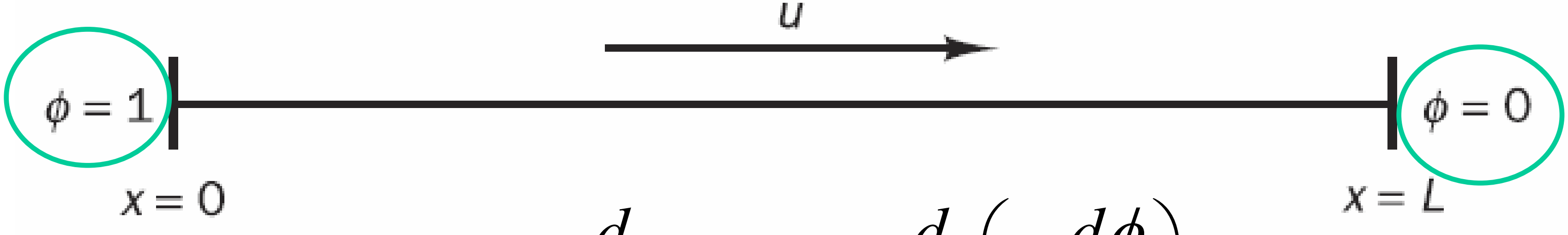
where

a_W	a_E	a_P
$D_w + \dfrac{F_w}{2}$	$D_e - \dfrac{F_e}{2}$	$a_W + a_E + \cancel{(F_e - F_w)}$

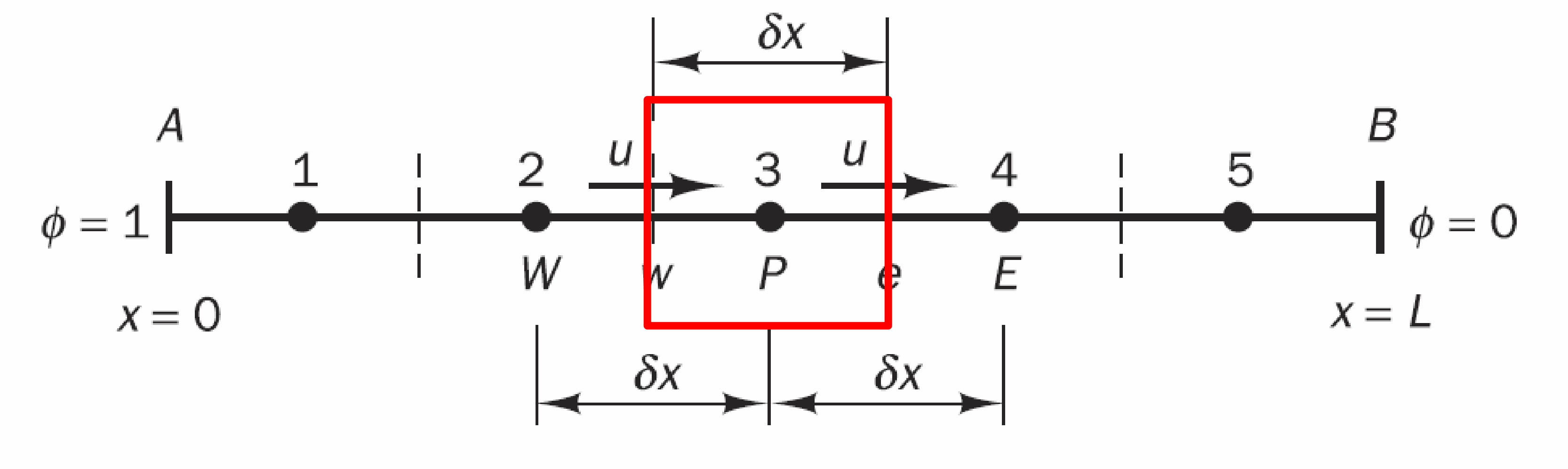
$$F_e - F_w = 0 \tag{5.10}$$

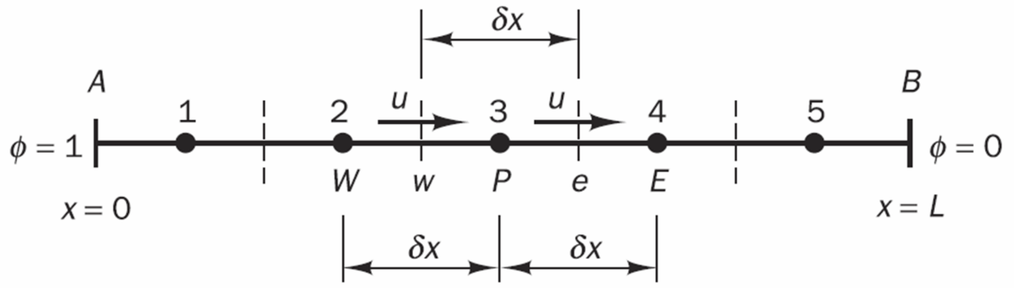
Problem 5.1
Central Differencing Scheme (CDS)

Dirichlet



$$\frac{d}{dx}(\rho u \phi) = \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)$$





$$\text{CASE I}\begin{cases} u = \boxed{0.1\ \text{m/s}} \\ \rho = 1\ \text{kg/m}^3 \\ \Gamma = 0.1\ \text{kg/(m}\cdot\text{s)} \\ L = 1\ \text{m},\ \delta x = 0.2\ \text{m}\ \ (= L/5) \end{cases}$$

$$\text{CASE II}\begin{cases} u = \boxed{2.5\ \text{m/s}} \\ \rho = 1\ \text{kg/m}^3 \\ \Gamma = 0.1\ \text{kg/(m}\cdot\text{s)} \\ L = 1\ \text{m},\ \delta x = 0.2\ \text{m} \end{cases}$$

Case I

$$D_e = D_w = D = \frac{\Gamma}{\delta x} = \frac{0.1}{0.2} = 0.5$$

$$F_e = F_w = F = \rho u = (1)(0.1) = 0.1$$

$$\begin{cases} a_E = D_e - \frac{F_e}{2} = 0.5 - 0.05 = 0.45 \\ a_W = D_w + \frac{F_w}{2} = 0.5 + 0.05 = 0.55 \\ \boxed{S_u = S_P = 0} \\ a_P = a_E + a_W - S_P = 0.45 + 0.55 - 0 = 1.0 \end{cases}$$

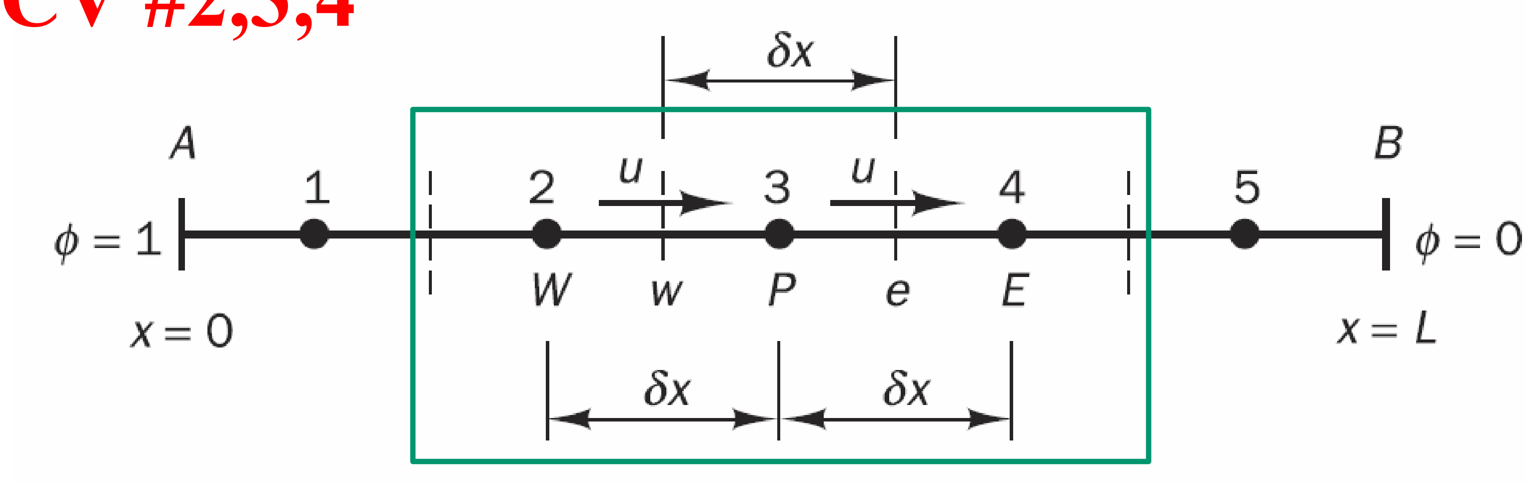
$a_E \neq a_W$

due to convection

$$\begin{bmatrix} -a_W & a_P & -a_E \end{bmatrix} = \begin{bmatrix} -0.55 & 1.0 & -0.45 \end{bmatrix}$$

$$\begin{bmatrix} -a_W & a_P & -a_E \end{bmatrix} = \begin{bmatrix} -0.55 & 1.0 & -0.45 \end{bmatrix}$$

CV #2,3,4



$$\frac{d}{dx}\left(\rho u \phi\right) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) = 0$$

ODE or PDE

FV integration

$$a_P \phi_P = \sum_{nb=E,W} a_{nb}\phi_{nb} + S_u$$

Discretized Equation

Matrix Assembly

Boundary Condition

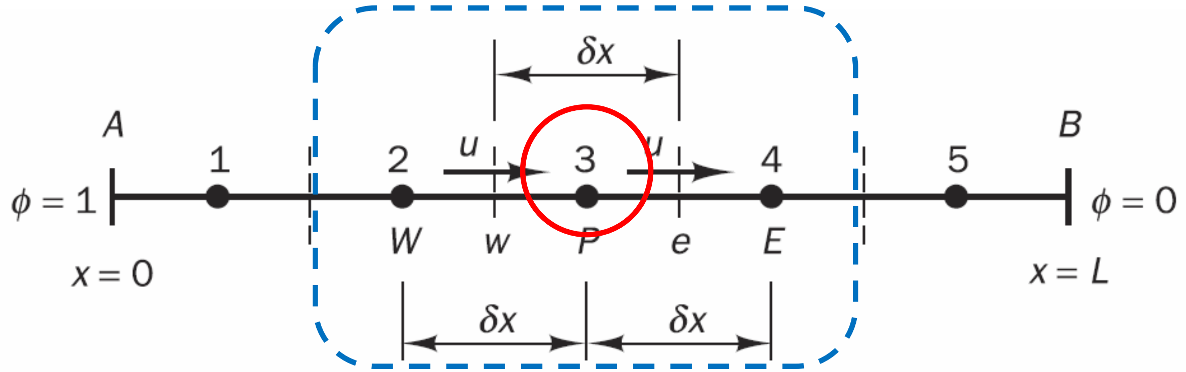
Direct or Iterative Solver

$$a_P\phi_P = \underbrace{a_E\phi_E + a_W\phi_W} + \underset{=0}{S_u}$$

$$\Rightarrow -\underset{0.55}{a_W}\,\phi_W + \underset{1.0}{a_P}\,\phi_P - \underset{0.45}{a_E}\,\phi_E = \underset{=0}{S_u}$$

vector dot product

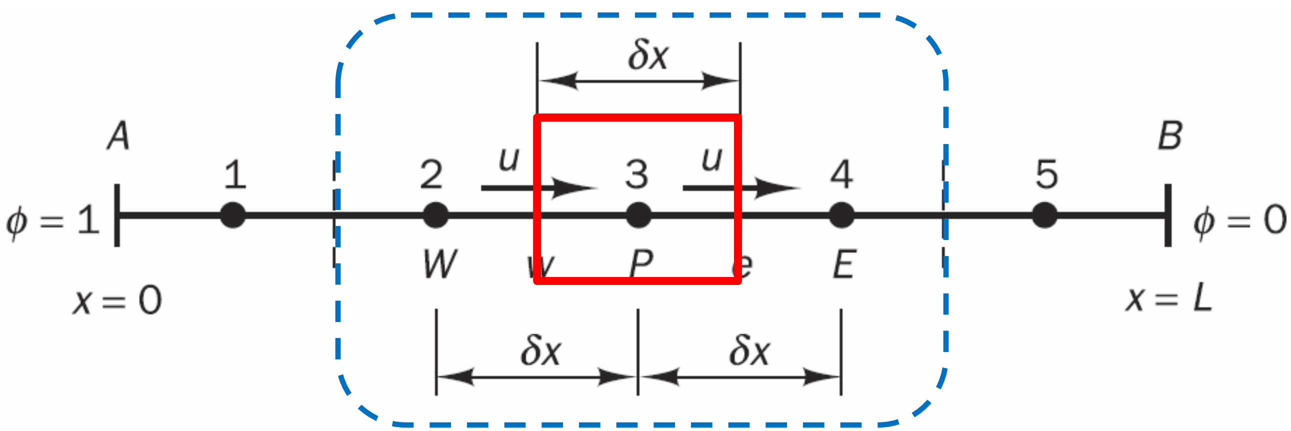
$$\Rightarrow \underbrace{\begin{bmatrix} \underset{a_W}{-0.55} & \underset{a_P}{1.0} & \underset{a_E}{-0.45} \end{bmatrix}}_{\text{row vector}} \bullet \underset{\text{column vector}}{\begin{bmatrix} \phi_W \\ \phi_P \\ \phi_E \end{bmatrix}} = 0$$



Example 5.1

$$\underbrace{\begin{bmatrix} -0.55 & 1.0 & -0.45 \end{bmatrix}}_{\text{row vector}} \bullet \underset{\text{column vector}}{\begin{bmatrix} \phi_W \\ \phi_P \\ \phi_E \end{bmatrix}} = 0$$

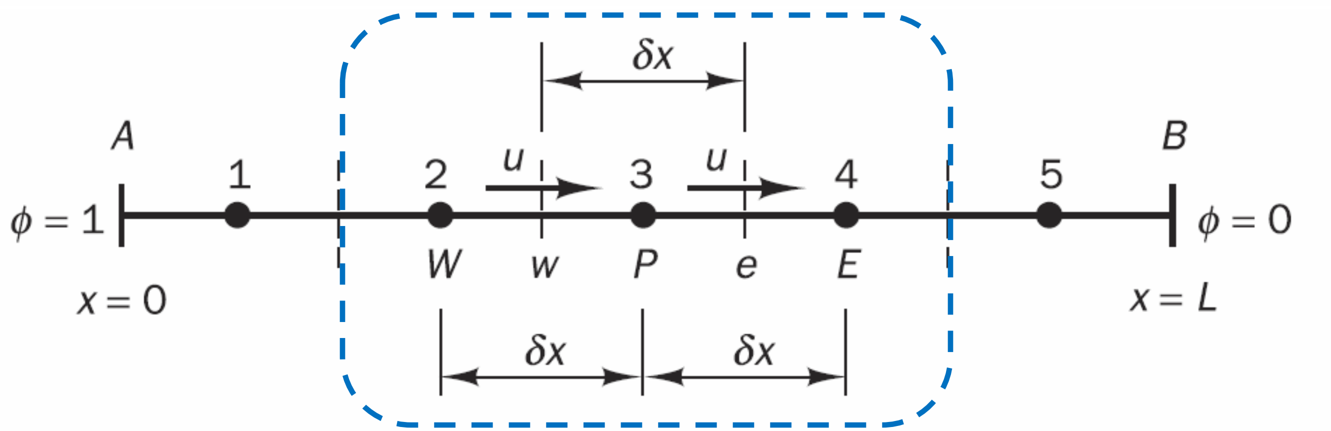
$$\begin{bmatrix} & & & 0 & 0 \\ 0 & \boxed{-0.55} & \boxed{1.0} & \boxed{-0.45} & 0 \\ 0 & 0 & & & \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \boxed{0} \\ 0 \end{bmatrix}$$

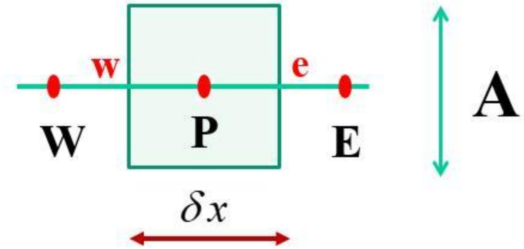


Example 5.1
CVs #2,3,4

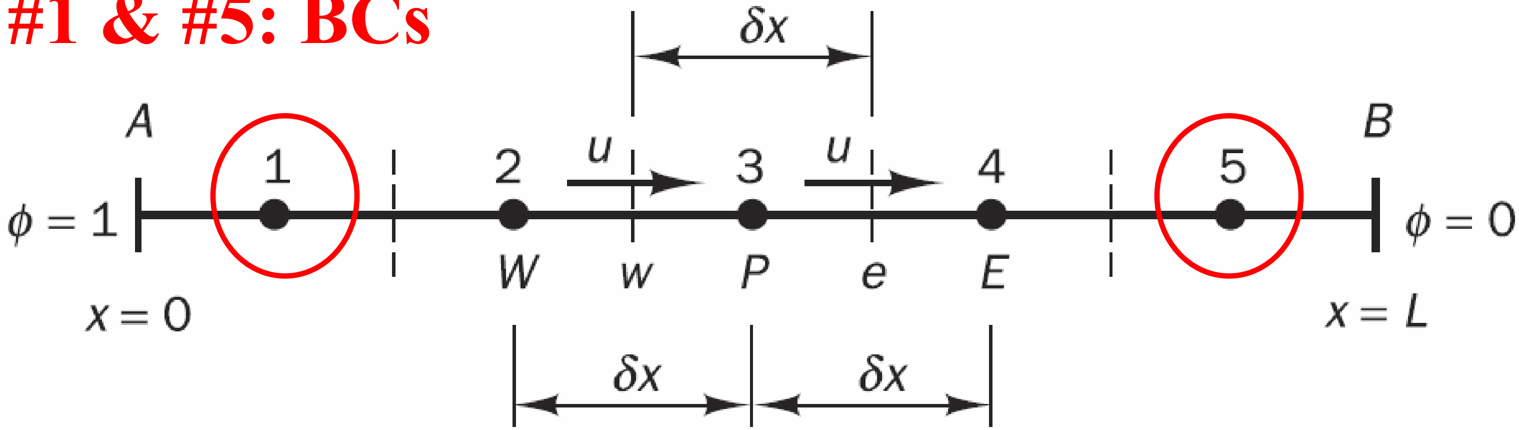
$$\underbrace{\begin{bmatrix} -0.55 & 1.0 & -0.45 \end{bmatrix}}_{\text{row vector}} \bullet \underbrace{\begin{bmatrix} \phi_W \\ \phi_P \\ \phi_E \end{bmatrix}}_{\text{column vector}} = 0$$

$$\begin{bmatrix} & & & & \\ -0.55 & 1.0 & -0.45 & 0 & 0 \\ 0 & -0.55 & 1.0 & -0.45 & 0 \\ 0 & 0 & -0.55 & 1.0 & -0.45 \\ & & & & \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \\ 0 \\ \\ \end{bmatrix}$$





CV #1 & #5: BCs

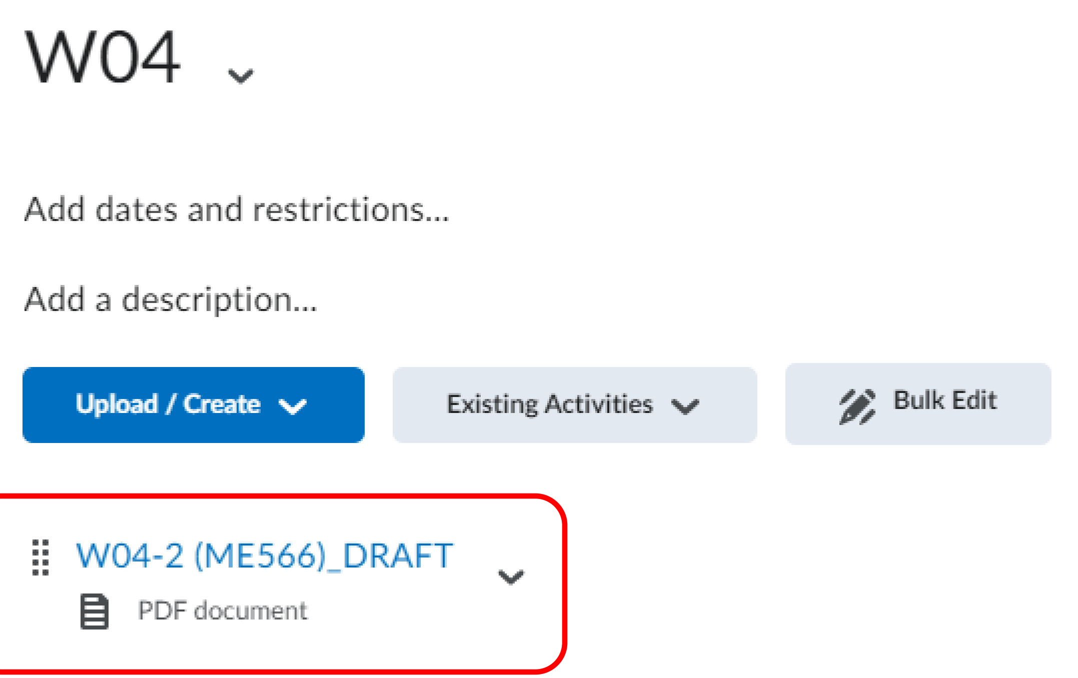


$$\int_w^e \left[\frac{d}{dx}(\rho u \phi) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx} \right) \right] dx A = 0$$

$$\Rightarrow \left[F_e \phi_e - F_w \phi_w \right] \cancel{A} = \left[\Gamma \left(\frac{d\phi}{dx} \right)_e - \Gamma \left(\frac{d\phi}{dx} \right)_w \right] \cancel{A}$$

Convection, CDS, EX 5.1

ME 566
W4-2, May 30



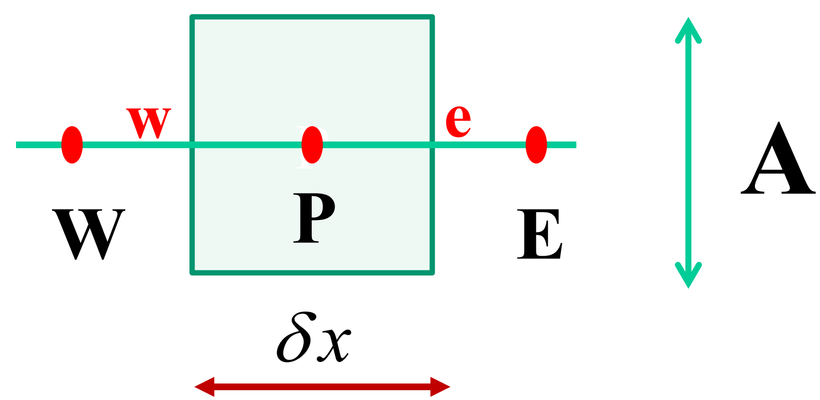
Finite-Volume (FV) Integration

ODE

$$\int_w^e \left[\underbrace{\frac{d}{dx}(\rho u \phi)}_{\text{convection}} - \underbrace{\frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right)}_{\text{diffusion}} \right] \underbrace{dxA}_{\text{differential volume}} = \underbrace{0}_{\text{source}}$$

$$\text{source} = \underbrace{S_u}_{\text{constant}} + \underbrace{S_p}_{\text{slope}} \phi_P \Rightarrow \boxed{S_u = S_p = 0}$$

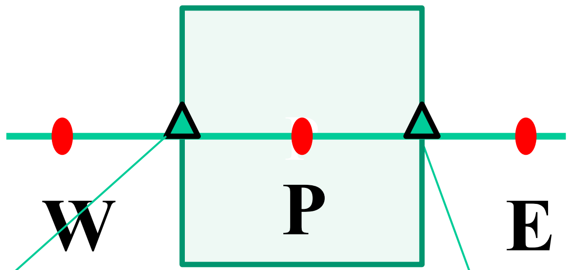
for FV away from boundaries



Other topics:

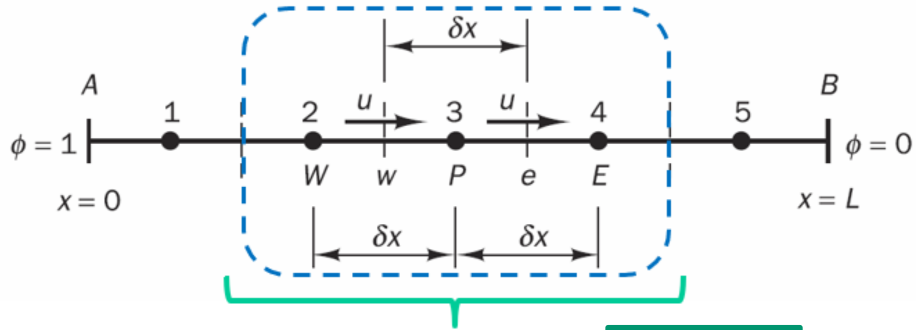
1. Stability/Boundedness
2. Accuracy

Central Differencing Scheme (CDS), Sec. 5.3



$$\phi_w \approx \frac{1}{2}\left(\phi_W + \phi_P\right)$$

$$\phi_e \approx \frac{1}{2}\left(\phi_E + \phi_P\right)$$



CV #2,3,4

FVM

$$a_P \phi_P = \sum_{nb=E,W} a_{nb}\phi_{nb} + S_u$$

$$\frac{d}{dx}(\rho u \phi) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) = 0$$

No source

$$a_E = D_e - \frac{F_e}{2},\ a_W = D_w + \frac{F_w}{2}$$

$$a_P = a_E + a_W - S_P,\ S_P = 0$$

$$S_u = 0$$

$$D_e = D_w = D = \frac{\Gamma}{\delta x}$$

$$F_e = F_w = F = \rho u$$

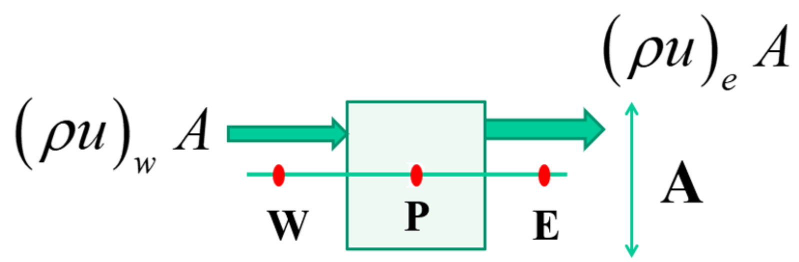
Alternative approach

$$\Rightarrow a_P = \left\{ (D_e + D_w) + \left[\left(\frac{F_e}{2} \right) + \left(\frac{-F_w}{2} \right) \right] \right\} = \boxed{(D_e + D_w)} + \frac{1}{2} \underbrace{(F_e - F_w)}_{=0,\text{continuity}}$$

$$= a_E + a_W$$

$$\begin{cases} a_E = \left[D_e + \left(\frac{-F_e}{2} \right) \right] \\ a_W = \left[D_w + \left(\frac{F_w}{2} \right) \right] \end{cases}$$

$$\Rightarrow a_E + a_W = \boxed{(D_e + D_w)} - \frac{1}{2} \underbrace{(F_e - F_w)}_{=0,\text{continuity}}$$



$$(\rho u)_w = F_w = (\rho u)_e = F_e$$

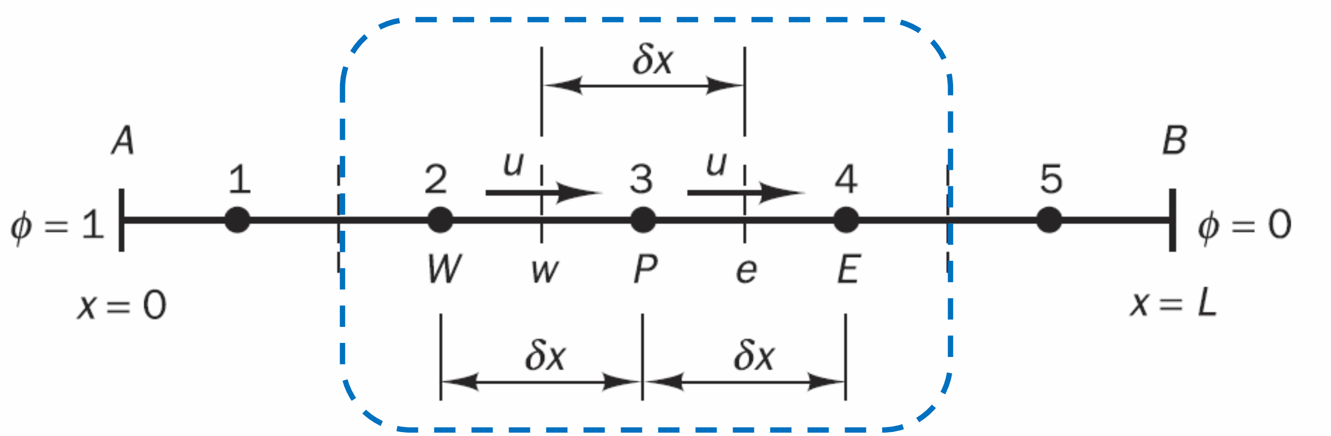
Therefore,

$$a_P = a_E + a_W = \sum_{nb} a_{nb} = \boxed{(D_e + D_w)}$$

Example 5.1
CVs #2,3,4

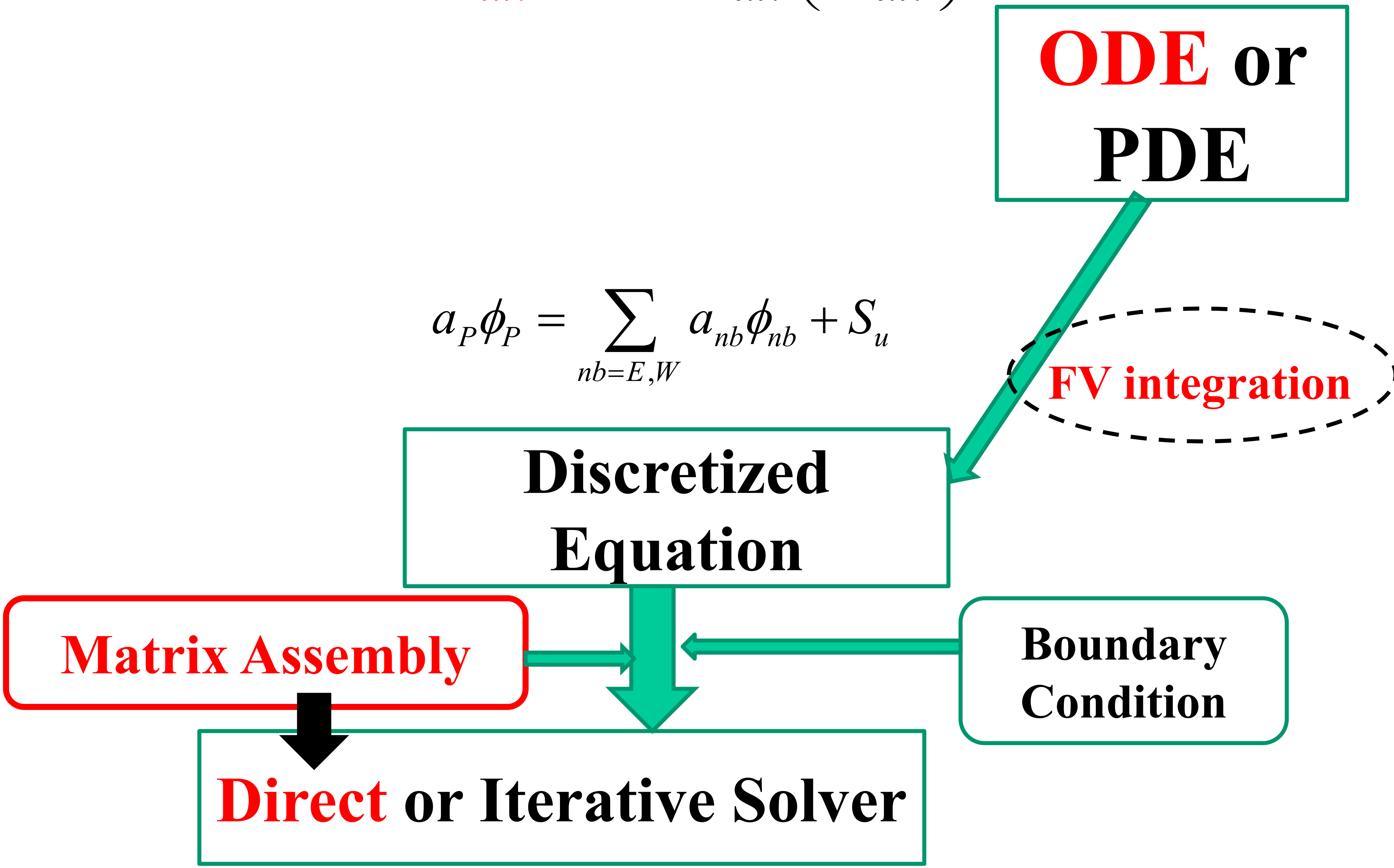
$$\underbrace{\begin{bmatrix} -0.55 & 1.0 & -0.45 \end{bmatrix}}_{\text{row vector}} \bullet \underbrace{\begin{bmatrix} \phi_W \\ \phi_P \\ \phi_E \end{bmatrix}}_{\text{column vector}} = 0$$

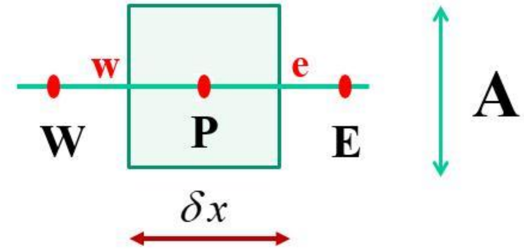
$$\begin{bmatrix} & & & & \\ -0.55 & 1.0 & -0.45 & 0 & 0 \\ 0 & -0.55 & 1.0 & -0.45 & 0 \\ 0 & 0 & -0.55 & 1.0 & -0.45 \\ & & & & \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \\ 0 \\ \\ \end{bmatrix}$$



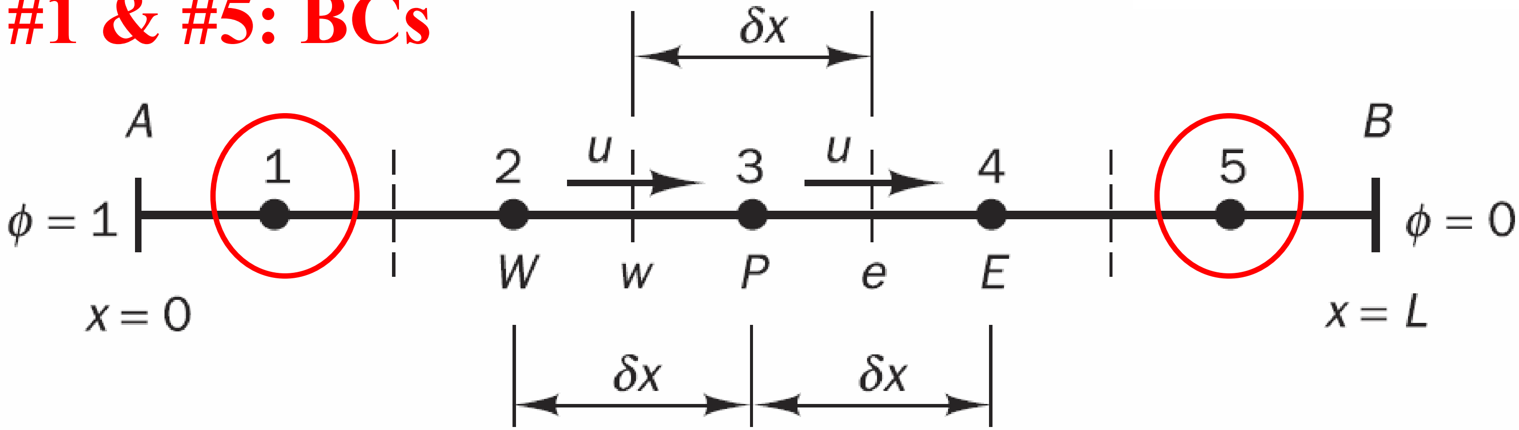
$$\frac{d}{dx}(\rho u \phi) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) = 0$$

$$a_P \phi_P = \sum_{nb=E,W} a_{nb}\phi_{nb} + S_u$$





CV #1 & #5: BCs

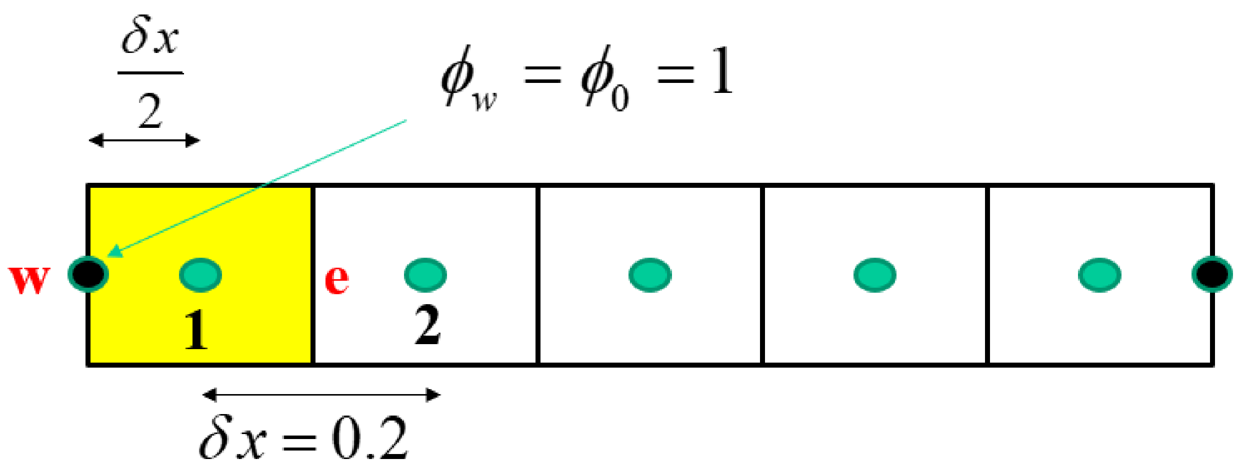


$$\int_w^e \left[\frac{d}{dx}(\rho u \phi) - \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx} \right) \right] dx A = 0$$

$$\Rightarrow \left[F_e \phi_e - F_w \phi_w \right] \cancel{A} = \left[\Gamma \left(\frac{d\phi}{dx} \right)_e - \Gamma \left(\frac{d\phi}{dx} \right)_w \right] \cancel{A}$$

Convection, CDS, EX 5.1

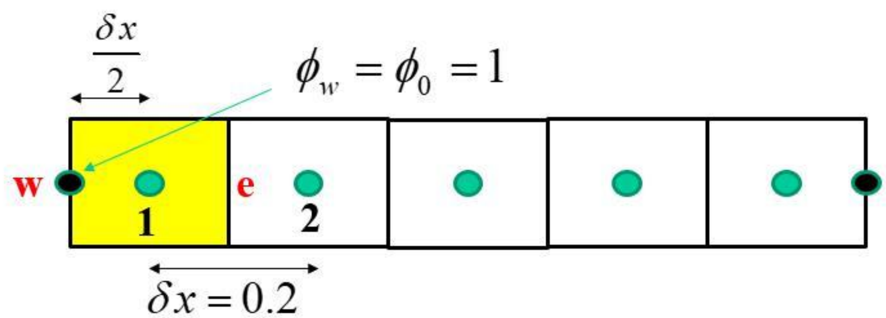
Dirichlet condition: ϕ specified



$$\begin{cases} \left(\frac{d\phi}{dx}\right)_w = \dfrac{\phi_1 - \phi_0}{\frac{\delta x}{2}} = 2\left(\dfrac{\phi_1 - 1}{\delta x}\right) \\ \left(\frac{d\phi}{dx}\right)_e = \dfrac{\phi_2 - \phi_1}{\delta x} \end{cases}$$

diffusion

$$\left[F_e\phi_e - F_w\phi_w\right] = \left[\Gamma \underbrace{\left(\frac{d\phi}{dx}\right)_e}_{\frac{\phi_2 - \phi_1}{\delta x}} - \Gamma \underbrace{\left(\frac{d\phi}{dx}\right)_w}_{2\left(\frac{\phi_1 - 1}{\delta x}\right)}\right]$$



CDS

$$\left[\overbrace{F_e \underbrace{\phi_e}_{\frac{1}{2}(\phi_2+\phi_1)} - F_w \underbrace{\phi_w}_{=1}}^{\text{convection}}\right] = \left[\overbrace{\Gamma \underbrace{\left(\frac{d\phi}{dx}\right)_e}_{\frac{\phi_2-\phi_1}{\delta x}} - \Gamma \underbrace{\left(\frac{d\phi}{dx}\right)_w}_{2\left(\frac{\phi_1-1}{\delta x}\right)}}^{\text{diffusion}}\right]$$

EX 5.1

$$\left.\begin{aligned} &\Gamma = 0.1 \\ &F_e = F_w = 0.1 \\ &\delta x = 0.2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow 0.05(\phi_1 + \phi_2) - 0.1 = (0.1)\left(\frac{\phi_2 - \phi_1}{0.2}\right) - (0.2)\left(\frac{\phi_1 - 1}{0.2}\right)$$

$$\Rightarrow \underbrace{0.05(\phi_1 + \phi_2) - 0.1}_{\text{convection}} = \underbrace{0.5(\phi_2 - \phi_1) - (\phi_1 - 1)}_{\text{diffusion}}$$

$$\Rightarrow 1.55\phi_1 - 0.45\phi_2 = 1.1$$

$$\begin{cases} \phi_P = \phi_1 \\ \phi_E = \phi_2 \\ \phi_W = 1 \ (\text{B.C.}) \end{cases}$$

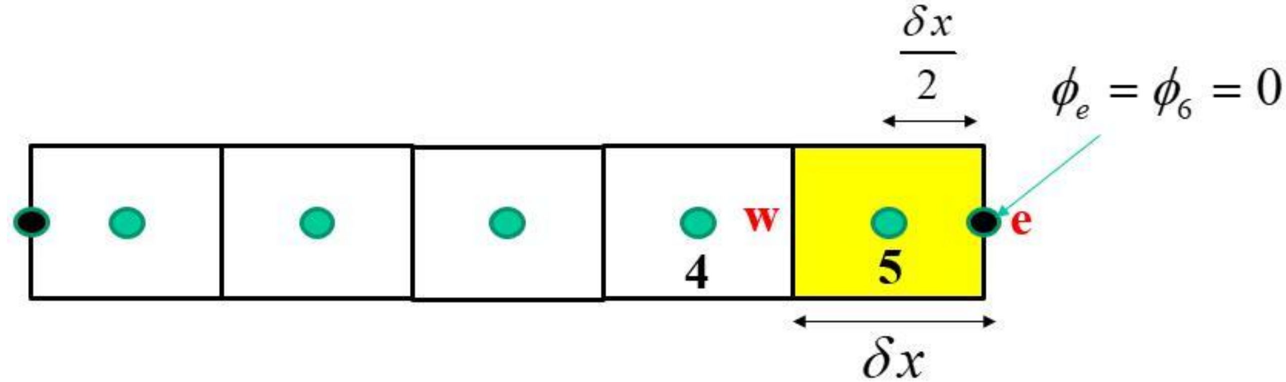
matrix assembly ← discretized eq. (CV #1)
vector dot product

$$1.55\phi_1 - 0.45\phi_2 = 1.1$$

$$\Rightarrow \underbrace{\begin{bmatrix} 1.55 & -0.45 \end{bmatrix}}_{\text{row vector}} \bullet \underbrace{\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}}_{\text{column vector}} = 1.1$$

$$\begin{array}{ll} \textbf{row 1} & \begin{bmatrix} 1.55 & -0.45 & 0 & 0 & 0 \\ & & & & \\ & & & & \end{bmatrix} \end{array} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \\ \end{bmatrix} = \begin{bmatrix} 1.1 \\ \\ \\ \end{bmatrix}$$

CV #5

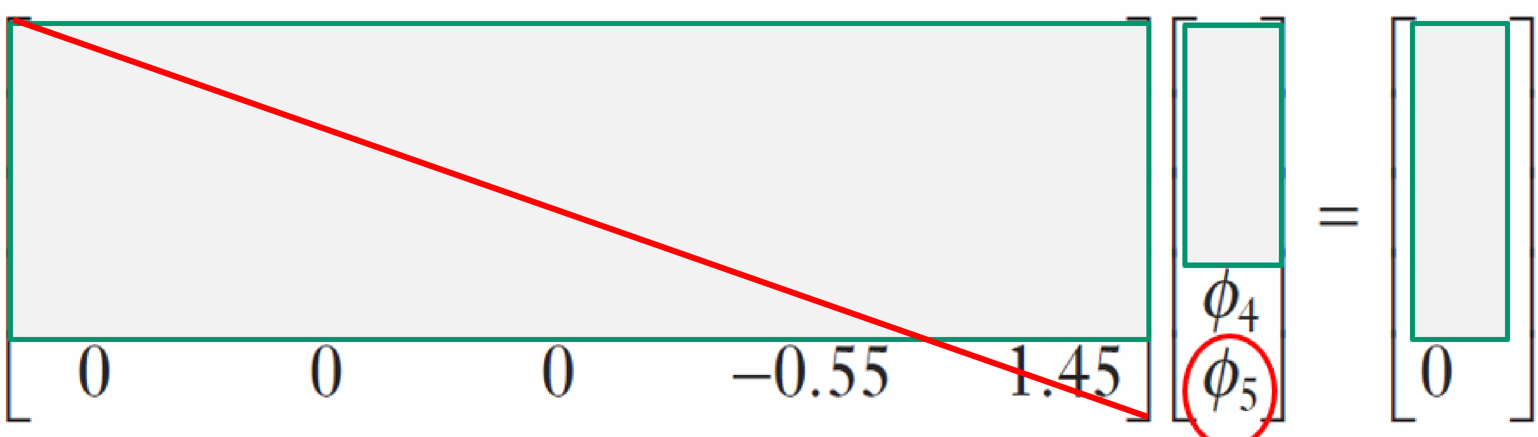


$$-0.55\phi_4 + 1.45\phi_5 = 0$$

$$\Rightarrow \underbrace{\begin{bmatrix} -0.55 & 1.45 \end{bmatrix}}_{\text{row vector}} \bullet \underbrace{\begin{bmatrix} \phi_4 \\ \phi_5 \end{bmatrix}}_{\text{column vector}} = 0$$

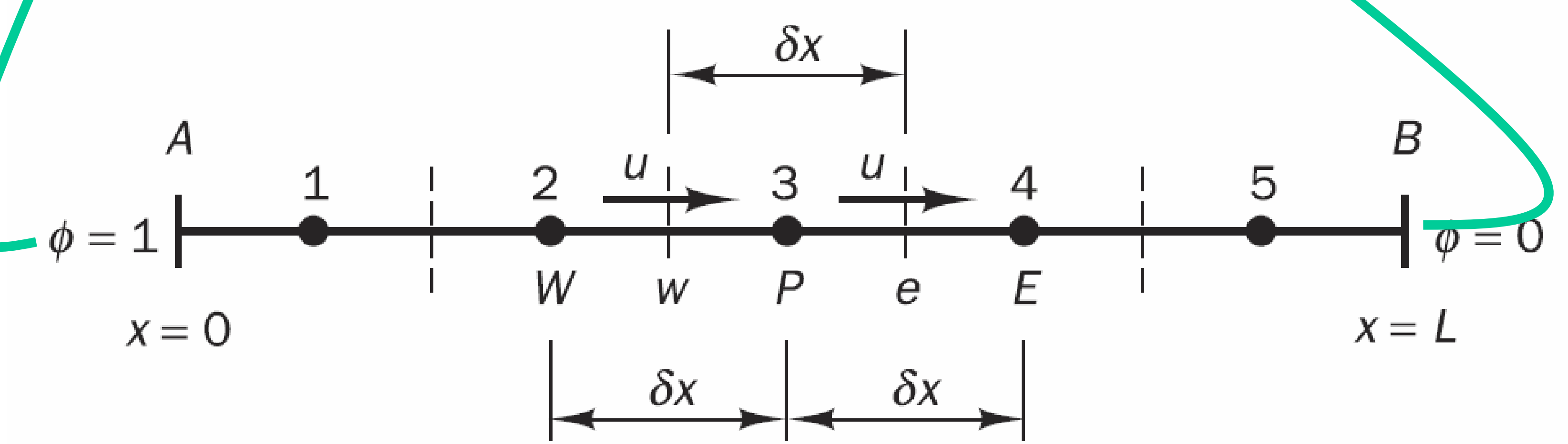
vector dot product

row 5



Example 5.1

$$\begin{bmatrix} 1.55 & -0.45 & 0 & 0 & 0 \\ & & & & \\ & & & & \\ & & & & \\ 0 & 0 & 0 & -0.55 & 1.45 \end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\\ \phi_5\end{bmatrix} = \begin{bmatrix}1.1\\ \\ \\ \\ 0\end{bmatrix}$$



Equation (5.19)

Tri-Diagonal Matrix

$$\underbrace{\begin{bmatrix} 1.55 & -0.45 & 0 & 0 & 0 \\ -0.55 & 1.0 & -0.45 & 0 & 0 \\ 0 & -0.55 & 1.0 & -0.45 & 0 \\ 0 & 0 & -0.55 & 1.0 & -0.45 \\ 0 & 0 & 0 & -0.55 & 1.45 \end{bmatrix}}_{[a]_{5\times 5}} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solved by TDMA
(Tri-Diagonal Matrix Algorithm)
in Chap 7

CDS

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 1.55 & -0.45 & 0 & 0 & 0 \\ -0.55 & 1.0 & -0.45 & 0 & 0 \\ 0 & -0.55 & 1.0 & -0.45 & 0 \\ 0 & 0 & -0.55 & 1.0 & -0.45 \\ & & & -0.55 & 1.45 \end{bmatrix}^{-1} \begin{bmatrix} 1.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow [\phi]_{5\times 1} = [a]_{5\times 5}^{-1} [S_u]_{5\times 1}$$