Decomposing Differences in Portfolio Returns Between North America and Europe

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Abstract

This paper uses the Fama and French (2017) dataset to examine some patterns on the quantiles, interquantile ranges and standard deviations of returns on portfolios of North America and Europe. We also study the patterns on the differences in these variables. We decompose the mean and quantile differences in the returns of the matching portfolios of North America and Europe into the five factors of Fama and French (2015) five-factor asset pricing model using Oaxaca-Blinder decomposition and distribution regressions. We find that market factor positively and significantly contributes to the mean, 5th and 95th quantile differences. The size factor positively contributes to the mean difference but contributes a large and negative influence on the 5th and 95th quantile differences. The value, profitability and investment factors do not seem to play a significant role in the mean differences. However, the investment factor does negatively and significantly contribute to the 5th and 95th quantile differences. The profitability factor plays a strong and negative role in the 5th quantile differences but weakly contributes to the 95th quantile differences.

Key words: Distribution Regression, Oaxaca-Blinder Decomposition, Decomposition Analysis, Five-factor Asset Pricing Model

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1 Introduction

There are three popular rational and theoretical asset pricing models: the capital asset pricing model (CAMP) of Sharpe (1964), Lintner (1965) and Black (1972); the consumption based CAMP (CCAMP) of Lucas Jr (1978) and Breeden (1979); and the intertemporal CAMP (ICAMP) of Merton (1973). The results of research on asset pricing in the last 50 years show that it is difficult to have a theoretical model that captures the salient features of expected returns. For example, the literature acknowledges the failure of the CAMP of explaining the expected returns. The CCAMP seems to be neglected empirically. Though the ICAMP can accommodate factor-based empirical asset pricing models, it is difficult to identify and empirically validate the model's state variables.

Motivated by the evidence of Novy-Marx (2013) and Titman, Wei, and Xie (2004) and the dividend discount model, Fama and French (2015) (henceforth FF(2015)) establish the five-factor model by adding profitability and investment factors into the Fama and French (1993) (henceforth FF(1993)) three-factor model. The five-factor model is an empirical asset pricing model, built on insubstantial theoretical underpinnings and designed to capture prominent patterns in average returns. Even though FF(2015) five-factor model directed at capturing the patterns is rejected by the GRS statistic of Gibbons, Ross, and Shanken (1989), the model is able to explain between 71% and 94% of the cross-section variance of expected returns for the size, value, profitability and investment portfolios they examine in US capital market. Using international data, Fama and French (2017) (henceforth FF(2017)) show the empirical robustness of the regional five-factor model in explaining the monthly excess portfolio returns, especially in North America and European capital markets. All these results show that the FF(2015) five-factor model empirically captures well the excess returns on portfolios.

The motivations for performing the decomposition analysis in this paper are as follows. Firstly, although decomposition analysis (Oaxaca (1973) and Blinder (1973)) is widely used in labor economics, to our knowledge, it is new to apply decomposition approach to the asset pricing literature.[1] Thus, the paper extends the literature in decomposition to financial economics.[2] Secondly,

[1] Fortin, Lemieux, and Firpo (2011) provide a comprehensive review on the decomposition methods in economics.

[2] Though not related to our questions of interest, we are aware of a few papers related to decomposition in financial economics. See Campbell and Mei (1993) and Cornell (1999) and Tuomo and John (2004)

we find that for practical purposes there exists large differences in the returns on the matching portfolios of North America and Europe; although most of differences are not significant in statistical terms due to the large standard deviations (see Panel a of Table 4). Thirdly, when we are exploring the differences between factor returns in North America and Europe, we find that the mean differences of factor returns is large in practical terms. What's more, the correlations between the factors of North America are quite different from the ones of Europe (See Panel a and b of Table 1). Fourthly, we find that the factor loadings are different in the asset pricing regressions of North America and Europe (not shown). FF(2017) show that the global five-factor models perform poorly in tests on regional portfolios. That is, the global factor models can not capture well the patterns in average returns. These findings imply that there possibly exists various factor distributions and structures in the two separate capital markets. The structures of capital markets in North America and Europe are likely to be different, that is, the distributions of factors and the "returns" to matching factors (factor exposures/loadings) are likely different. These findings make the decomposition analysis interesting.

In this paper, we use decomposition methods to study the differences in the portfolio returns Fortin, Lemieux, and Firpo (2011). A traditional Oaxaca-Blinder decomposition method is used to decompose the mean differences into five factors and distribution regressions are applied to perform the decomposition of the quantile differences. The decomposition helps explain to what degree the differences are contributed to the structure and composition effects.

In this paper, we use the monthly excess returns on a variety of diversified portfolios obtained by sorting on various variables described in detail in FF(2015) and FF(2017) to represent the returns of the capital market in North America and Europe. We first offer the summary statistics for these portfolio returns and five factor returns. We then examine the patterns in the quantiles of returns on the portfolios. Lastly, we decompose the mean, 5th and 95th quantile differences in the returns of the matching portfolios of North America and Europe to the FF(2015) five factors. Aggregate and detailed decomposition approaches are used to study the main question of what components most contribute to these differences.

We are the first to discover patterns on the quantiles, standard deviations and interquantile ranges of portfolio returns. We also present some patterns on the differences of these above variables between North America and European markets are also presented. We find that the market factor positively and significantly contributes to the mean, 5th and 95th quantile differences. The size factor positively contributes to the mean differences but contributes a large and negative influence on the 5th and 95th quantile differences. The value, profitability and investment factors do not seem to play a significant role in the mean differences. However, the investment factor negatively and significantly contributes to the 5th and 95th quantile differences. The profitability factor plays a strong and negative role in the 5th quantile differences but relatively attributes weakly to the 95th quantile differences.

We organize the rest of the paper as follows. Section 2 describes the FF(2015) five-factor model and decomposition. In Section 3, we present the approaches used to decompose the mean and quantile differences in the returns on the portfolios of North America and Europe. The summary statistics for the five factors and portfolios and the main results are reported in Section 4. We conclude in Section 5.

2 Five-factor model and questions of interest

This section offers a brief review of FF(2015) five-factor model and present the decomposition process using five-factor model.

2.1 Five-factor model

There is much evidence that average stock returns are related to the overall market (Sharpe (1964), Lintner (1965), and Breeden (1979)), firm size (Banz (1981)), fundamental value (Basu (1983) and Rosenberg, Reid, and Lanstein (1985)), profitability (Novy-Marx (2013)) and investment (Aharoni, Grundy, and Zeng (2013)). [3] FF(1993) three-factor model includes a market, size and value factors. Motivated by the dividend discount model, FF(2015) created the five-factor model by adding the profitability and investment factors into the FF(1993) three-factor model.

Before introducing five-factor model, we briefly review the dividend discount model. The dividend discount model is used to explain why these variables are related to average returns.The

[3] There are more research including Breeden, Gibbons, and Litzenberger (1989), Reinganum (1981), Haugen and Baker (1996), Cohen, Gompers, and Vuolteenaho (2002), Fairfield, Whisenant, and Yohn (2003), Titman, Wei, and Xie (2004), Fama and French (2006), Fama and French (2008), Hou, Xue, and Zhang (2015), and Fama and French (2016) and among others.

model says the market value of a share of stock is the discounted value of expected dividends per share,

$$m_t = \sum_{\tau=1}^{\infty} E(d_{t+\tau}/(1+r)^{\tau}). \tag{2.1}$$

where m_t is the share price at time t, $E(d_{t+\tau})$ is the expected dividend per share for period $t+\tau$, and r is (approximately) the long-term average expected stock return or, more precisely, the internal rate of return[4] on expected dividends. Miller and Modigliani 1961 show that the time t total market value of the firm's stock implied by equation 2.1 is

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}. \tag{2.2}$$

where $Y_{t+\tau}$ is the total equity earnings for period $t+\tau$ and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity. Dividing Equation 2.2 by book equity B_t gives the following equation

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}. \tag{2.3}$$

From equation 2.3, we can see, (a) holding everything constant except the current value (market capitalization), M_t (size), and the expected return, r, then a lower value of M_t, or equivalently a higher book-to-market equity ratio, B_t/M_t (value), implies a higher expected return; (b) holding everything constant except the expected future earnings, $Y_{t+\tau}$ (profitability), and the expected stock return, then higher expected earnings imply a higher expected return. (c) holding everything constant except the growth in book equity, $dB_{t+\tau}$ (investment), and the expected return, then higher expected growth in investment implies a lower expected return.

The specification of the FF(2015) five-factor model is as follows,

$$R_{it} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \tag{2.4}$$

where e_{it} is a zero-mean residual and

- R_{it} is the return on portfolio i for month t in excess of riskfree rate (the one-month US Treasury

[4] Internal rate of return (IRR) is the interest rate at which the net present value of all the cash flows (both positive and negative) from a project or investment equal zero. IRR is used to evaluate the attractiveness of a project or investment. If the IRR of a new project exceeds a company's required rate of return, that project is desirable. If IRR falls below the required rate of return, the project should be rejected.

bill rate)

- Mkt_t is the excess return on the value-weighted market portfolio for month t
- SMB_t (size factor) is the difference between the returns on a diversified portfolio of small and big stocks
- HML_t (value factor) is the difference between the returns of high and low B/M stocks
- RMW_t (profitability factor) is the difference between the returns on diversified portfolios of stocks with robust and weak profitability
- CMA_t (investment factor) is the difference between the returns on diversified portfolios of stocks of low and high (conservative and aggressive) investment firms

For the purpose of decomposition and concision, we simplify the notation by dropping the subscript i indexed for portfolio and t denoting the time period and adding the subscript $g = 0, 1$ which denotes the North America or Europe, respectively. That is as follows

$$Y_g = X_g'\beta_g + e_g \tag{2.5}$$

where Y_g denotes the excess portfolio return on a portfolio in region g, X_g is a vector of a constant and five factors in region g and e_g is error term. Hereafter, we call Y_g portfolio return instead of excess portfolio return. The distinction will be made whenever emphasis is necessary.

One result of FF(2015) is that inferences about the asset pricing models do not seem to be sensitive to the way factors are defined. Thus, in the paper we use factors constructed by 2×3 sorts. Here we briefly describe how the portfolios and five factors are constructed and the details can be found in FF(2017). To construct the factors based on 2×3 sorts, they sort stocks in 2×3 sorts on Size and B/M, or Size and Operating profitability (OP), or Size and investment(Inv). Specifically, stocks are allocated independently to Size (Small to Big), B/M (Low to High), OP (Robust to Weak) and Inv (Conservative to Aggressive) groups using market capitalization, B/M, OP and Inv breakpoints respectively at the end of each June.[5]

[5]In the sort for June of year t, the book equity B is measured at the end of the fiscal year ending in year $t-1$ and market capitalization M is measured at the end of December of year $t-1$, adjusted for changes in shares outstanding between the measurement of B and the end of December. OP is measured with accounting data for the

Fama and French construct the factors by the value weighted (VW) portfolios defined by the intersections of the groups.[6] The portfolio returns we are going to examine are from finer versions of the sorts that produce the factors. In this paper, we choose portfolios based on 5×5 sorts on size and B/M, size and OP, and size and Inv. Thus, we have 150 portfolios of North America and Europe, 75 for each continent.[7]

In all, factors are meant to mimic the underlying risk factors in returns related to the variable sorted. For example, the value factor mimics the risk factor in returns related to book-to-market equity ratio. The value, profitability, and investment factors are different mixes of size, value, profitability, and investment effects in returns because the correlations between the size, value, profitability, and investment variables used to construct factors.[8]

2.2 Counterfactuals and decomposition

We first define the counterfactual distributions which is used to compute the decomposition components. In order to decompose the quantile portfolio return differences, we need to estimate a series of counterfactual distributions which would then be inverted to obtain the corresponding quantiles.

2.2.1 Counterfactual distributions

Under the law of iterated probabilities, the distribution of the returns on a portfolio in region g, $F_{Y\langle g|g\rangle}(y)$, is obtained by integrating over the observed characteristics respectively as follows

$$F_{Y\langle g|g\rangle}(y) = \int F_{Y_g|X_g}(y|x)dF_{X_g}(x) \tag{2.6}$$

fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Inv is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets.

[6]Fama and French use independent sorts to assign stocks to two size groups, and three B/M, OP, and Inv groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. They label these portfolios with two or four letters. The first always describes the Size group, small (S) or big (B). In the 2 x 3 sorts, the second describes the B/M group, high (H), neutral (N), or low (L), the OP group, robust (R), neutral (N), or weak (W), or the Inv group, conservative (C), neutral (N), or aggressive (A). The factors are SMB (small minus big), HML (high minus low B/M), RMW (robust minus weak OP), and CMA (conservative minus aggressive Inv).

[7]One concern about the factor model is that the role of size and book-to-market factors in the three-factor model is spurious, arising only because size and book-to-market portfolios are used for both our dependent and explanatory returns. Fama and French (1993) split-sample tests address this concern.

[8]High B/M value stocks, for example, tend to have low profitability and investment, and low B/M growth stocks, especially large low B/M stocks, tend to be profitable and invest aggressively. (Fama and French (1995))

where $F_{Y_g|X_g}(y|x)$ is the conditional distribution of the returns on a portfolio in region g. Specifically, the conditional distribution functions $F_{Y_0|X_0}(y|x)$ and $F_{Y_1|X_1}(y|x)$ describe the portfolio returns with factors x, for North America and Europe, respectively. $F_{Y\langle 0|0\rangle}(y)$ and $F_{Y\langle 1|1\rangle}(y)$ represent the observed distribution function of portfolio returns for North America and Europe.

We obtain a counterfactual distribution as we change the distribution of X_g or the conditional distribution function $F_{Y_g|X_g}(y|x)$. Let $F_{Y\langle 0|1\rangle}(y)$ represent the counterfactual distribution of portfolio returns that would have prevailed for Europe had it faced the North America's portfolio returns structure. We obtain $F_{Y\langle 0|1\rangle}(y)$ by integrating the conditional distribution of portfolio returns for North America with respect to the distribution of factors for Europe as follows

$$F_{Y\langle 0|1\rangle}(y) := \int F_{Y_0|X_0}(y|x)dF_{X_1}(x). \tag{2.7}$$

In this paper, we decompose both mean and quantile portfolio return differences. In quantile cases, we need to sequentially estimate the detailed components which is path dependent. Thus, we first select the order of decomposition. In this paper, we calculate the detailed components in this order: the constant, market, size, value, profitability and investment factors. That is, we five components in detailed composition effects and six components in detailed structure effects.[9] To obtain the detailed decomposition of differences between quantile portfolio returns, we define the following counterfactual distributions.

Let $X_{(0,1,j)}, j = 1 \cdots 6$ denotes a vector of counterfacutal factors in which the first j factors of X_0 in North America have been replaced with the first s factors of X_1 in Europe, holding the last $6-j$ factors of X_0 constant. Note that $X_{(0,1,6)} = X_1$ and $X_{(1,0,6)} = X_0$. For instance, $X_{(0,1,3)}$ denotes a vector of factors in which the constant term, market factor, and size factor is from Europe and the value, profitability and investment factors from North America. Let $F_{X_{(0,1,j)}}(x)$ represent the joint distribution of factors $X_{(0,1,j)}$.

Let $F_{Y\langle 0|(0,1,j)\rangle}(y)$ represent the counterfactual distribution of portfolio returns that would have prevailed for North America had it had the factors $X_{(0,1,j)}$:

$$F_{Y\langle 0|(0,1,j)\rangle}(y) := \int F_{Y_0|X_0}(y|x)dF_{X_{(0,1,j)}}(x) \tag{2.8}$$

[9]Note that he detailed composition effect linked to the constant term is zero.

Let $F_{Y_{(0,1,j)}|X_0}(y|x)$ denote the counterfactual conditional distribution of portfolio returns that has the structure of the first j factors in North America replaced by the structure of the first j factors in Europe, holding constant the structure of the last $6-j$ factors in North America. Note that $F_{Y_{(0,1,6)}|X_0}(y|x) = F_{Y_1|X_1}(y|x)$ and $F_{Y_{(1,0,6)}|X_1}(y|x) = F_{Y_0|X_0}(y|x)$. For instance, $F_{Y_{(0,1,3)}|X_0}(y|x)$ denotes the conterfactual conditional distribution of portfolio returns that has the structure linked to the constant term, market factor and size factor in North America replaced by the structure linked to the ones in Europe, holding constant the structure of the value, profitability and investment factors in North America.

Let $F_{Y\langle(0,1,j)|1\rangle}(y)$ represent the counterfactual distribution of portfolio returns for Europe that would prevail had the conditional distribution $F_{Y_1|X_1}(y|x)$ replaced by $F_{Y_{(0,1,j)}|X_0}(y|x)$:

$$F_{Y\langle(0,1,j)|1\rangle}(y) := \int F_{Y_{(0,1,j)}|X_0}(y|x)dF_{X_1}(x) \tag{2.9}$$

2.2.2 Decomposition

In the paper, we are interested in decomposing the mean or quantile portfolio return differences between North America and Europe into the FF(2015) five factors. Using the language of labor economics, the differences are attributed to two parts, one of which is the aggregate composition effect and the other of which is the structure effect. The aggregate composition effect is due to the differences in the distributions of the factor returns X of North America and Europe. The structure effect is the component which is contributed to the differences in the factor loadings between North America and Europe.

The overall difference in two distributional statistics of portfolio returns between North America and Europe is defined as follows:

$$
\begin{aligned}
\Delta_O^\nu &\equiv \nu\left(F_{Y\langle 0|0\rangle}(y)\right) - \nu\left(F_{Y\langle 1|1\rangle}(y)\right) \\
&= \left(\nu\left(F_{Y\langle 0|0\rangle}(y)\right) - \nu\left(F_{Y\langle 0|1\rangle}(y)\right)\right) + \left(\nu\left(F_{Y\langle 0|1\rangle}(y)\right) - \nu\left(F_{Y\langle 1|1\rangle}(y)\right)\right) \\
&\equiv \Delta_C^\nu + \Delta_S^\nu \\
&= \sum_{j=1}^{6}\left(\nu\left(F_{Y\langle 0|(0,1,j-1)\rangle}(y)\right) - \nu\left(F_{Y\langle 0|(0,1,j)\rangle}(y)\right)\right) + \sum_{j=1}^{6}\left(\nu\left(F_{Y\langle (0,1,j-1)|1\rangle}(y)\right) - \nu\left(F_{Y\langle (0,1,j)|1\rangle}(y)\right)\right) \\
&\equiv \sum_{j=1}^{6}\Delta_{C_j}^\nu + \sum_{j=1}^{6}\Delta_{S_j}^\nu
\end{aligned}
\tag{2.10}
$$

where

- $\nu\left(F_{Y\langle g|g\rangle}(y)\right)$ is any distributional statistics of the observed distribution of the returns on the portfolios in region g
- Δ_C^ν is the aggregate composition effect, which is equal to $\nu\left(F_{Y\langle 0|0\rangle}(y)\right) - \nu\left(F_{Y\langle 0|1\rangle}(y)\right)$
- Δ_S^ν is the aggregate structure effect, which is equal to $\nu\left(F_{Y\langle 0|1\rangle}(y)\right) - \nu\left(F_{Y\langle 1|1\rangle}(y)\right)$
- $\Delta_{C_j}^\nu$ is the detailed composition effect linked to the jth factor, which is equal to $\nu\left(F_{Y\langle 0|(0,1,j-1)\rangle}(y)\right) - \nu\left(F_{Y\langle 0|(0,1,j)\rangle}(y)\right)$
- $\Delta_{S_j}^\nu$ is the detailed structure effect linked to the sth factor, which is equal to $\nu\left(F_{Y\langle (0,1,j-1)|1\rangle}(y)\right) - \nu\left(F_{Y\langle (0,1,j)|1\rangle}(y)\right)$.

We note that:

- $\Delta_{C_1}^\nu$ is always equal to zero because $F_{Y\langle 0|(0,1,0)\rangle}(y) = F_{Y\langle 0|(0,1,1)\rangle}(y)$ for the first element in the factor vector is a constant term.
- $\nu\left(F_{Y\langle 0|(0,1,0)\rangle}(y)\right) = \nu\left(F_{Y\langle 0|0\rangle}(y)\right)$ because $F_{Y\langle 0|(0,1,0)\rangle}(y) = F_{Y\langle 0|0\rangle}(y)$.
- $\nu\left(F_{Y\langle 0|1\rangle}(y)\right) = \nu\left(F_{Y\langle 0|(0,1,6)\rangle}(y)\right) = \nu\left(F_{Y\langle (0,1,0)|1\rangle}(y)\right)$ because $F_{Y\langle 0|1\rangle}(y) = F_{Y\langle 0|(0,1,6)\rangle}(y) = F_{Y\langle (0,1,0)|1\rangle}(y)$.

Remark 1. *In the literature, there are two main approaches to estimate a distribution which are distribution regression and quantile regression.*[10] *In this paper, we use distribution regression but not quantile regression simply because the results would be similar had we used quantile regression.*

Remark 2. *To compute the aggregate decomposition effects, we select the structure of North America and distribution of factors in Europe to construct the counterfactual distribution. There is alternative, that is to select the structure of Europe and distribution of factors in North America to construct the counterfactual distribution and compute the aggregate effects. The decomposition results are possibly different. In this paper, we show the main results using the former construction and the results using the latter construction is presented in the appendix. The two results seem* $\cdots$

Remark 3. *We compute the detailed decomposition effects in the spirit of Gomulka and Stern (1990) and Fairlie (2005). Specifically, we estimate a series of counterfactual distribution. This leads us to the inevitable path dependent problem. That is, the decomposition results are likely various depending on the order of computing the detailed effects. To obtain the suboptimal results, we calculate the detailed effects following the order of the effect linked to the constant term, market, size, value, profitability and investment factor. The order we choose is consistent with the order of the factors being added into the FF(2015) five-factor model and being studied in the literature of financial economics. In the empirical asset pricing literature, reseachers first study the market factor in CAPM. Fama and French (1993) introduce the size and value factors and FF(2015) add the profitability and investment factors. We also show the results of switching the order of size and value factors, and the order of profitability and investment factors.*

Remark 4. *We are limited to decompose the quantile return differences, that is* ν *is the quantile function. To limit the size of the paper, we do not decompose the differences in the portfolio return variance between North America and Europe although it is straightforward to decompose the differences in any distributional statistics of the distribution of portfolio returns between two regions. In the paper, we show the decomposition results of mean differences in portfolio returns using the traditional Oaxaca-Blinder decomposition method, which is path independent. Also, we use distribution regression to decompose the mean differences. Note that the results from distribution regression is path dependent. We compare both results.*

[10] See Foresi and Peracchi (1995), Melly (2005) Chernozhukov, Fernández-Val, and Melly (2013), Koenker and Bassett Jr (1978), and Koenker (2005).

2.2.3 Shares of decomposition components

To show the relative importance of different decomposition components and thus the relative importance of different factors in the contribution to the estimated overall mean or quantile difference in the portfolio returns between North America and Europe, we compute the following shares of the various decomposition components in percentage:

- the share of aggregate composition (structure) effect in the estimated overall difference in portfolio returns for each of 75 portfolios
- the share of the average aggregate composition (structure) effect in the average estimated overall difference in portfolio returns for all portfolios[11]
- the share of detailed composition (structure) effect in the aggregate composition (structure) effect for each of 75 portfolios
- the share of the average detailed composition (structure) effect in the average aggregate composition (structure) effect for all portfolios
- the share of detailed composition (structure) effect in the aggregate effect for each portfolio
- the share of the average detailed composition (structure) effect in the average aggregate effect for all portfolios

3 Estimation and inference

3.1 Decomposing mean differences using Oaxaca-Blinder decomposition

This section shows how to decompose the mean difference in portfolio returns between North America and Europe using the usual Oaxaca-Blinder decomposition method. We run the FF(2015) five-factor model as model 2.5 for each portfolio of North America and Europe separately. With

[11] This share is computed by two steps. In step 1, we average the estimated overall differences and the aggregate composition (structure) effect for all portfolios. Step 2 involves dividing the latter average with the former average to attain the share value. All the average values below are computed by exactly the same way.

the estimates $\hat{\beta}_g$ for each portfolio, the decomposition directly follows

$$\begin{aligned}
\hat{\Delta}_O &= \hat{E}(Y_0) - \hat{E}(Y_1) \\
&= \bar{X}_0'\hat{\beta}_0 - \bar{X}_1\hat{\beta}_1 \\
&= (\bar{X}_0' - \bar{X}_1')\hat{\beta}_0 + \bar{X}_1'(\hat{\beta}_0 - \hat{\beta}_1) \\
&\equiv \hat{\Delta}_C + \hat{\Delta}_S \\
&= \sum_{j=1}^{6}(\bar{X}_{0j} - \bar{X}_{1j})\hat{\beta}_{0j} + \sum_{j=1}^{6}\bar{X}_{1j}(\hat{\beta}_{0j} - \hat{\beta}_{1j}) \\
&\equiv \sum_{j=1}^{6}\hat{\Delta}_{C_j} + \sum_{j=1}^{6}\hat{\Delta}_{S_j}
\end{aligned}$$

where

- $\hat{\Delta}_O$ is the estimated overall mean difference in the portfolio returns between North America and Europe
- $\bar{X}_g$ is the average factor returns in region g
- $\hat{\Delta}_C = (\bar{X}_0' - \bar{X}_1')\hat{\beta}_0$ and $\hat{\Delta}_S = \bar{X}_1'(\hat{\beta}_0 - \hat{\beta}_1)$ are the aggregate composition and structure effects, respectively
- $\hat{\Delta}_{C_j} = (\bar{X}_{0j} - \bar{X}_{1j})\hat{\beta}_{0j}$ and $\hat{\Delta}_{S_j} = \bar{X}_{1j}(\hat{\beta}_{0j} - \hat{\beta}_{1j})$ are the detailed composition and structure effect linked to the jth factor, respectively.

We note that $\hat{\Delta}_{C_1}$ is equal to zero because the first element in each factor vector X_g is a constant. The Oaxaca-Blinder decomposition results of the mean difference is path independent, that is, the detailed decomposition results are not affected by the decomposing order. We also decompose the mean differences using distribution regression below and compare both results using both approaches. The decomposition results from distribution regression is path dependent.

3.2 Decomposing differences using distribution regression

Foresi and Peracchi (1995) are the first ones who use distribution regressions to study the equity returns. Chernozhukov, Fernández-Val, and Melly (2013) use distribution regressions to estimate the counterfactual distributions. Fairlie (2005) extends the Blinder-Oaxaca decomposition technique to logit and probit models. We introduce distribution regression version of Blinder-Oaxaca decomposition to decompose the difference in the distributional statistics of distributions of portfolio returns between North America and Europe.

Estimation proceeds in four steps. In the step 1, we estimate conditional distributions of portfolio returns in both North America and Europe using distribution regressions. With step 1 complete, step 2 uses the plug-in approach to estimate a series of counterfactual (observed) distributions. Step 3 involves inverting the counterfactual (observed) distributions to obtain the corresponding quantiles. In the final step, decomposition results can be computed using equation 2.10.

3.2.1 Distribution regression

In the paper, we use distribution regression method to estimate the entire conditional distribution of portfolio returns, use plug-in to compute a series of counterfactual (observed) distributions and then invert the distributions to obtain the quantiles.[12]

The main idea of distribution regression is to estimate a series of binary response models using $\mathbb{1}\{Y \leq y\}$ as the dependent variable while varying y. To implement the distribution regression estimator, we estimate a series of logit models over a fine grid of possible values for y,[13] that is

$$\begin{aligned} F_{Y_g|X_g}(y|x) =& E[\mathbb{1}\{Y_g \leq y\}|X_g = x] \\ =& \Lambda(x'\beta(y)) \end{aligned}$$

where Λ is a known link function – we use the logistic link function though one could make some other choice here. $\beta(y)$ are unknown parameters corresponding to each y, i.e., the parameters $\beta(y)$ change as y changes. $\mathbb{1}\{Y_g \leq y\}$ is an indicator function that equals one if $Y_g \leq y$ is true and

[12] Quantile regression (Koenker and Bassett Jr (1978) and Koenker (2005)) could be a reasonable alternative approach. In the first step, obtain estimates of conditional quantiles. Step 2 uses plug-in to compute a series of conditional and unconditional counterfactual quantiles. Step 3 inverts them to obtain distributions. From there, decomposition for differences in other distributional statistics would be exactly the same. For decomposing quantile difference, one can skip step 3 and decompose the quantile differences straightly with step 2 complete. Our approach models the conditional distributions using distribution regression and involves inverting the distributions.

[13] In the paper, we evenly choose 100 values between the 1th and 99th percentile of the distribution of the portfolio returns.

zero otherwise. The estimated conditional distribution is

$$\hat{F}_{Y_g|X_g}(y|x) = \Lambda(x'\hat{\beta}(y)) \tag{3.1}$$

For fixed y, estimating $F_{Y\langle g|g\rangle}(y)$ amounts to average over X_g. That is,

$$\hat{F}_{Y\langle g|g\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{Y_g|X_g}(y|X_{gi}) \tag{3.2}$$

which is the same as replacing the population distribution function in Equation 2.6 with the sample distribution function.

Since the estimated distribution obtained may be nonmonotonic in y_0, we apply the monotonization method of Chernozhukov, Fernández-Val, and Galichon (2010) based on rearrangement.[14] The rearranged distribution is

$$\hat{F}^r_{Y\langle g|g\rangle}(y) = \inf\left\{u : \int_0^\infty \mathbb{1}\{\hat{F}_{Y\langle g|g\rangle}(y) \leq u\}dv \geq y\right\}.$$

We invert the rearranged distribution $\hat{F}^r_{Y\langle g|g\rangle}$ to obtain the estimate of τ-quantile $Q(\tau)$ by the following equation

$$\hat{Q}(\tau) = \inf\{Q : \hat{F}^r_{Y\langle g|g\rangle}(Q) \geq \tau\}, \quad \tau \in (0,1). \tag{3.3}$$

3.2.2 Estimating counterfactual distributions

With the estimates $\hat{\beta}(y)$ from Equation 3.1, estimating $F_{Y\langle 0|(0,1,j)\rangle}(y)$ amounts to average $F_{Y_0|X_0}(y|X)$ over the constructed $X_{(0,1,j)}$. That is,

$$\hat{F}_{Y\langle 0|(0,1,j)\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{Y_0|X_0}(y|X_{(0,1,j)i}) \tag{3.4}$$

which is the same as replacing the population distribution function in Equation 2.8 with the sample distribution function.

Similarly, with the estimates $\hat{\beta}(y)$ from Equation 3.1 and thus the constructed $F_{Y_{(0,1,j)}|X_0}(y|x)$,

[14] For practical and computational purposes, it is helpful to think of the rearrangement as sorting (Chernozhukov, Fernández-Val, and Galichon (2010), p. 1098). In practice, we use rearranged estimators of the distributions for all the results, but we omit this discussion throughout the rest of this section for the sake of clarity.

estimating $F_{Y\langle(0,1,j)|1\rangle}(y)$ amounts to average $F_{Y_{(0,1,j)}|X_0}(y|x)$ over X_1. That is,

$$\hat{F}_{Y\langle 0|(0,1,j)\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n}\hat{F}_{Y_{(0,1,j)}|X_0}(y|X_{1i}) \tag{3.5}$$

which is the same as replacing the population distribution function in Equation 2.9 with the sample distribution function.

3.2.3 Decomposing quantile differences

With the observed and counterfactual distributions from Equations 3.2, 3.4 and 3.5, one can compute any distributional statistics and decomposition follows straightforward as Equation 2.10. In the paper, we decompose the τ-quantile portfolio return differences. We compute each τ-quantile for all observed and counterfactual distributions shown in Equation 2.10 using Equation 3.3. Then, the decomposition of differences in quantile portfolio returns between North America and Europe follows directly as in Equation 2.10.

3.2.4 Decomposing mean differences using distribution regression

To decompose the mean difference in portfolio returns between North America and Europe using distribution regression, we first estimate the $E(Y_g)$, $E(Y\langle 0|(0,1,j)\rangle)$ and $E(Y\langle(0,1,j)|1\rangle)$.

Consider a grid of equally spaced values of τ given by $0 < \tau_1 < \tau_2 < \cdots < \tau_M < 1$. Then, estimate $E(Y)$ by

$$\hat{E}(Y) = \frac{1}{M}\sum_{s=1}^{M}\hat{Q}(\tau_m)$$

For each observed and counterfactual distributions in Equation 2.10, we compute the corresponding $\hat{E}(Y_g)$, $\hat{E}(Y\langle 0|(0,1,j)\rangle)$ and $\hat{E}(Y\langle(0,1,j)|1\rangle)$ and the decomposition follows straightforward as Equation 2.10. We compare the results from using the usual and distribution regression version of Blinder-Oaxaca decomposition method.

3.2.5 Computing shares of decomposition components

With the decomposition components from the sections above, it is straightforward to compute the shares of decomposition components stated in section 2.2.3. We note that some share in percentage can go to infinite for some aggregate effect is zero even round off to three decimal digits.

3.3 Inference

Chernozhukov, Fernández-Val, and Melly (2013) show that bootstrap is valid for estimating the limit laws of the estimators of the counterfactual functionals. We use bootstrap to test the significance of the decomposition components. More specifically, we test whether the components are significantly different from zero. We also test whether these shares in percentage is significantly different from zero.

Note: read Chernozhukov, Fernández-Val, and Melly (2013) again to write better the first sentence of the above paragraph.

4 Data and results

In the paper, North America includes United States and Canada, and Europe contains Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. We only decompose the differences between returns on portfolios of these two areas but not Asia Pacific and Japan because market integration is a plausible assumption for North America and Europe and the market shares of these two areas are also large.

We use the FF(2017) dataset which can be downloaded from Kenneth R. French's personal website. The dataset includes the five factor returns and the monthly excess returns on the 5×5 Size-B/M , Size-OP, and Size-Inv portfolios of North America and Europe, ranging from July 1990 to November 2017 (329 months). Thus, we have 75 portfolios each for North America and Europe respectively.[15]

[15] At the end of June each year, stocks are allocated to five Size groups (Small to Big) using as breakpoints the 3rd, 7th, 13th, and 25th percentiles of the region's aggregate market capitalization. Stocks are allocated independently to five B/M groups (Low B/M to High B/M) by the quintile of B/M for the big stocks of the

We choose the five factors based on 2×3 sorts.[16] We do not include the other common factors, e.g., the momentum factor of Carhart (1997) and liquidity factor of Pástor and Stambaugh (2003) because FF(2015) show that the two above factors have regression slopes close to zero so produce trivial changes in model performance when the portfolios examined here are formed on size and B/M, profitability or investment.

4.1 Summary statistics for factor returns

Table 1: Summary statistics for factor returns

(a) Mean and standard error

	North America					Europe					Difference				
	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA
Mean	0.67	0.17	0.20	0.34	0.26	0.51	0.07	0.34	0.40	0.21	0.16	0.10	-0.14	-0.06	0.05
SE	0.23	0.15	0.18	0.13	0.15	0.27	0.12	0.13	0.08	0.10	0.16	0.16	0.14	0.14	0.12

(b) Correlations of factor returns in and across North America and Europe

	North America					Europe					Between				
	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA
Mkt.RF	1.00	0.20	-0.23	-0.37	-0.44	1.00	-0.17	0.18	-0.26	-0.30	0.80	-0.26	0.04	-0.22	-0.35
SMB	0.20	1.00	-0.10	-0.42	-0.14	-0.17	1.00	0.01	-0.05	0.02	-0.26	0.31	0.03	-0.03	-0.12
HML	-0.23	-0.10	1.00	0.38	0.78	0.18	0.01	1.00	-0.54	0.54	0.04	0.03	0.60	-0.15	0.52
RMW	-0.37	-0.42	0.38	1.00	0.35	-0.26	-0.05	-0.54	1.00	-0.18	-0.22	-0.03	-0.15	0.22	0.38
CMA	-0.44	-0.14	0.78	0.35	1.00	-0.30	0.02	0.54	-0.18	1.00	-0.35	-0.12	0.52	0.38	0.57

(c) Kolmogorov–Smirnov test

	North America					Europe					Between				
	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA	Mkt.RF	SMB	HML	RMW	CMA
Mkt.RF	1	0.000	0.000	0.000	0.000	1	0.000	0.000	0.000	0.000	0.345	0.000	0.000	0.000	0.000
SMB	0	1.000	0.643	0.025	0.184	0	1.000	0.345	0.002	0.025	0.000	0.218	0.155	0.000	0.000
HML	0	0.643	1.000	0.025	0.298	0	0.345	1.000	0.130	0.089	0.000	0.155	0.155	0.000	0.001
RMW	0	0.025	0.025	1.000	0.031	0	0.002	0.130	1.000	0.012	0.000	0.000	0.000	0.218	0.073
CMA	0	0.184	0.298	0.031	1.000	0	0.025	0.089	0.012	1.000	0.000	0.000	0.001	0.073	0.031

We report the summary statistics for five factor returns in Table 1. For market, size and value factors, there exists big differences in the mean factor returns of North America and Europe, 0.67% versus 0.51% for market, 0.34% versus 0.07% for size, and 0.2% versus 0.34% for value. These differences are large in practical terms though not statistically significantly different. For example, the difference between Mkt.RF is 0.16% per month, which is 1.92% per annum. The matching

region. The intersections of the two sorts produce 25 Size-B/M portfolios. The 25 Size-Inv or Size-OP portfolios are constructed in the same way as in the Size-B/M portfolios except the second sort is on either profitability (robust minus weak) or investment (conservative minus aggressive).

[16]We choose 2×3 factors instead of 2×2 or $2 \times 2 \times 2 \times 2$ factors because Fama and French 2015 show 2×3 factors perform as well as 2×2 and $2 \times 2 \times 2 \times 2$ factors in the tests of asset pricing models. These factors are constructed by using independent 2×3 sorts on Size and each of B/M, OP, and Inv.

mean returns on RMW and CMA for North America and Europe are very close (Table 1a).

We replicate the factors correlation table of FF(2017) in Table 1b. The correlation between Mkt.RF and RMW or CMA is negative for both North America and Europe. The Mkt.RF is, however, negatively correlated with HML for North America(-0.23) but positively for Europe (0.18). Yet, the opposite applies to the correlation between Mkt.RF and size factor SMB, that is, Mkt.RF is positively correlated with SMB for North America but negatively correlated with SMB for Europe. For North America, the SMB is negatively correlated with HML, RMW and CMA. However, the correlation between SMB and HML, RMW or CMA is very weak for Europe.

We note that HML is highly and positively correlated with RMW and CMA for North America, but highly and negatively correlated with RMW though still highly and positively correlated with CMA for Europe. The correlation between RMW and CMA is 0.35 for North America versus -0.18 for Europe. These differences in correlations between factors for North America and Europe imply that the contributions of factors could be different in the decomposition. We also report the correlations of factors between North America and Europe in the rightmost block of Table 1b. The Mkt.RF, HML, and CMA are highly and positively correlated, 0.8, 0.6 and 0.57 respectively. The correlation of Mkt.RF and RMW or CMA between North America and Europe is also negative as the matching one in North America and Europe. HML is positively correlated with CMA (0.52), which is positively correlated with RMW (0.38).

To check whether the distributions of the factors of North America and Europe are significantly different, we use the Kolmogorov–Smirnov (KS) test.[17] The results of KS tests are shown in Table 1c. Though we also show the test results for factors within North America and Europe, we are more interested in the results for the factors between North America and Europe. We focus on the rightmost block which reports that the distributions of factor returns are not significantly different between North America and Europe for Mkt.RF, SMB, HML, and RMW. The exception is CMA, having a p-value of 0.031. For North America, only the distributions of SMB and HML, and HML and CMA are not statistically different at the traditional significant level. For Europe, only the distributions of SMB and HML, and HML and RMW, and HML and CMA are not significantly different. The differences in the factors means, distributions and the correlations between factors

[17]See Massey Jr (1951), Lilliefors (1967), Lilliefors (1969), Friedman and Rafsky (1979), and Stephens (1974) for more about Kolmogorov–Smirnov test.

make the decomposition interesting.

4.2 Summary statistics for portfolios

Average excess returns on portfolios

Note: A table of risk-adjusted returns as Basu (1983) Table 3 and 4, p. 136-137

Note: decompose the differences in the risk-adjusted portfolio returns ?

Note: move the unimportant patterns to the appendix

Note: need a table of Kolmogorov–Smirnov Test (p-values) For Factor Returns as Table 7

The summary statistics of portfolio returns are reported in Table 4. The average excess returns on all portfolios of North America and Europe market are 0.79% and 0.56% per month respectively with difference 0.23%. The Panel a shows the mean returns on portfolios of North America and Europe and mean differences between North America and Europe. Though the standard deviations of the mean differences shown in Table 8 and the results of Wilcoxon tests in Table 5 show that very few matching portfolios of North America and Europe have a mean difference statistically different from zero, we should note that the mean differences are large in the practical terms (see the rightmost block of Table 4a). The average mean difference for all portfolio returns is 0.23% per month, which is roughly equivalent to 2.73% per annum. Graphically, Panel a of Figure 2 shows the mean differences in the returns on portfolios of North America and Europe.

To check the normality of the portfolio returns graphically, we plot the probability density function of the returns (not shown). Graphically, they all do not seem to be normally distributed. We also use a KS test to statistically check the normality of returns on the portfolios of North America and Europe and find that the null hypotheses of normality of these portfolios are all rejected with p-values equal to zero to at least three decimal places (See Table 6). Because of non-normality, the t-test for the mean difference of the returns on the matching portfolios of North America and Europe is not a good choice. In the paper, we use a nonparametric test, the Wilcoxon Test.[18] We show the results of Wilcoxon Tests for the returns on the matching portfolio returns of North America and Europe in Table 5. In the tests, the null hypothesis is that the distributions of the returns on the matching portfolios of North America and Europe differ by a location shift

[18] Wilcoxon test is a nonparametric test which can be used as an alternative to t-test for samples when the population distribution cannot be assumed to be a normal distribution. (Wilcoxon (1945) and Siegal (1956))

of 0 and the alternative is that the returns on portfolios of North America shift to the right of the matching ones of Europe. We use one-tailed test because the mean returns on portfolios of North America seem to be greater than the matching ones of Europe (see Table 4a). Table 5 shows only the p-values for the portfolios in the lowest size quintile and the lowest Inv or OP quintile are less than 0.05 at the confidence level of 95%. There are also only six portfolios with Wilcoxon test p-value less than 0.1. Thus, for most of the time we cannot reject the difference between the mean returns on the matching portfolios of North America and Europe are significantly different from 0. This result is not surprising due to the high standard deviations of portfolio returns (see Table 4b).

To check whether the distributions of returns on the matching portfolios of North America and Europe are significantly different, we report the results of KS tests on the portfolios in Table 7. The table shows the p-values for KS tests for the returns on the matching portfolios of North America and Europe. 9 of 75 matching portfolios have p-values less than 0.05, and 17 portfolios have p-values less than 0.1.

One interesting question that has not been addressed in FF(2015, 2017) is whether the higher returns have unconditional higher risks. In the paper, we use standard deviation and interquantile range (95th minus 5th quantile) of the distribution of the portfolio returns as measures for risks. The rightmost block of Table 4a shows that the mean returns on portfolios of North America are greater than the ones on the matching portfolios of Europe. The rightmost block of Panel b shows the difference between the standard deviations of returns on portfolios of North America and Europe are not all positive as expected. Actually, 40 out of 75 are negative. The rightmost block of Panel e shows the differences between the interquantile ranges of North America and Europe. We can see 45 out of 75 are negative.

Thus, we conclude that the higher returns on portfolios of North America are not necessarily accompanied by higher risks measured by standard deviation and interquantile range. Figure 1 shows relations between the mean returns and risk measures. Panel a and d show that for the portfolios of North America, the mean returns do not seem to be positively correlated with standard deviations and interquantile ranges. Rather, it seems to be negatively correlated. The same applies to the portfolios of Europe (Panel b and e). Panel c and f show that mean differences between the returns on portfolios of North America and Europe seem to be positively correlated with the differences between the risk measures, although we also see the negative values of the differences in risk measures.

The patterns on the average returns can be found in Table 4a and graphically in Figure 3. FF(2015, 2017) have examined the patterns on the average returns. Here we replicate the results in both tables and figures. With figures, it is easier to see the patterns. We can see that for all size quintiles in both North America and European markets, the average returns tend to increase with the value factor, B/M, and profitability factor, OP, but decrease with the investment factor, Inv (see Panel a, b, c, g, h and i of Figure 3). For portfolios sorted on size and B/M, Panel d and j show the average returns on portfolios in the highest B/M quintile decrease with size for both North America and Europe. In the North America, for the portfolios sorted on size and Inv, Panel e shows that average returns increase with size in all investment quintiles. In Europe, however, we only see these trends for the portfolios in the first, third and fourth Inv quintiles but not in the second and highest Inv quintiles (Panel e and k). Panel f and l show for the portfolios sorted on size and OP in both North America and Europe market, the average returns are negatively related to size in all OP quintiles except the one in the lowest OP quintile in Europe.

Quantile excess returns on portfolios

The summary statistics for the quantile excess returns on portfolios are represented in Table 4c and 4d. The average 5th quantile returns on all portfolios of North America and Europe are -7.96% and -8.63% per month respectively. The average 95th quantile returns on all portfolios of North America and Europe are 8.46% and 8.22% per month respectively. The 5th and 95th quantile differences between North America and Europe are 0.67% and 0.23% per month respectively. Not all the quintile differences between two areas are positive. 22 out of 75 portfolios have negative 5th quantile return differences. 37 portfolios have negative 95th quantile return differences. We plot these differences in Figure 2. Comparing with the mean differences having all values positive, we suspect we would find the same patterns on the quantile returns. We know if the distributions of portfolio returns are normally distributed, we should find the same patterns on the quantile returns as the ones found on the mean returns. In the paper, however, we examine if there are any patterns on the quantile returns.

Graphically, we show the patterns on the 5th quantile returns in Figure 4. For portfolios sorted on size and B/M in North America, Panel a and d show that the 5th quantile returns tend to increase with B/M in all size quintiles and increase with size in all B/M quintiles. However, in Europe, the 5th quantile returns on the portfolios in the highest size quintile decrease with B/M and the returns on the portfolios in the highest B/M quintile decrease with size. The quantile returns in the lowest size quintile increase with B/M in Europe (Panel g and j).

For portfolios sorted on size and Inv in North America, the returns on portfolios in all size quintiles decrease with Inv (except the ones in the lowest Inv quintile) and the returns on portfolios in all Inv quintiles tend to weakly increase with size. In Europe, however, the returns on portfolios in all Inv quintiles tend to decrease with size. The portfolios in the highest size quintile do not have return patterns in Europe. In Europe, the portfolios in the lower four size quintiles tend to have decreasing returns as the Inv increases except the ones in the lowest Inv quintile.

For portfolios sorted on the size and OP, the returns on portfolios in all size quintiles tend to increase with OP in North America. However, the same pattern can not be found in Europe. We note the especially low returns on the portfolios in the lowest OP quintile (Panel f) and on the portfolios in the highest size quintile (Panel i). Panel l shows that except for the portfolios in the highest OP quintile, the returns tend to decrease with size. We also note that the 5th quantile returns on portfolios in the highest size quintile and in the highest Inv or Op quintile are very low (see Panel k and l).

The summary statistics for the 95th quantile excess returns are presented in Table 4d. We explore the patterns on the 95th quantile returns graphically in Figure 5. For portfolios sorted on size and B/M in North America, we can see that the returns on portfolios in all size quintiles (except the ones in the highest size quintile) tend to decrease with B/M and the returns on portfolios in all B/M quintiles tend to decrease as the size increases (Panel a and d). In Europe, we can see the returns on portfolios in the two highest size quintiles increase with B/M in panel g. Panel j shows the portfolios in the highest B/M quintile have increasing 95th quantile returns as the size increases.

For portfolios sorted on size and Inv in North America, Panel b shows the the returns on portfolios in all size quintiles increase with Inv (except the ones in the lowest Inv quintile). The returns on portfolios in all Inv quintiles tend to decrease with size in North America (Panel e). We do not find any patterns in Europe. We note in Panel k that the 95th quantile returns on portfolios in the highest Inv quintile strongly increase with size.

For portfolios sorted on size and OP in North America, Panel c shows that the 95th quantile returns on portfolios in the two highest size quintile tend to decrease with OP. For portfolios in the four higher OP quintiles in North America, the returns tend to decease with size (Panel f). In Europe, we do not find any patterns for portfolios on this sort except we note that the portfolios in the highest size quintile have decreasing 95th quantile returns as OP increases.

In sum, the results shown above are saying that we do not find the same patterns on the quantile returns as the patterns found on the mean returns. These findings are consistent with the non-normality of portfolio returns tested above.

It is interesting to explore id there are any patterns on the average and quantile return differences. The return differences are shown in Table 4. The patterns can be studied graphically in Figure 8, 9 and 10. We do not find many patterns in the mean differences. Panel a of Figure 8 shows that the mean differences in returns on portfolios in the highest size quintile tend to decrease with B/M. For portfolios in the lowest size quintile, the mean differences tends to decrease with Inv or OP in the three lower Inv or OP quintiles (Panel b and c of Figure 8). Relatively, there exists some obvious patterns in the 5th quantile return differences. Panel a of Figure 9 shows that the 5th quantile differences tends to increase with B/M in all size quintiles for portfolios sorted on size and B/M.

For portfolios sorted on size and Inv, the 5th quantile differences have decreasing trends for portfolios in the four higher size and Inv quintiles (Panel b of 9). For portfolios sorted on size and OP and in the lower OP quintiles, the 5th quantile differences tends to increase with OP. The most obvious pattern is that for all portfolios sorted on size or B/M, Inv and OP, the 5th quantile differences tend to increase with size (Panel d, e and f of Figure 9). We also note that the portfolios sorted on size and OP in the lowest size quintile have very low 5th quantile differences (Panel f). We find another important pattern on the 95th quantile differences, that is, the differences tend to decease with size for all portfolios on all sorts (Panel d, e and f of Figure 10). This is the opposite of the 5th quantile differences. Panel a of Figure 10 shows that for portfolios sorted on size and B/M, the 95th quantile differences tend to decrease with B/M in all size quintiles. For portfolios in higher size and Inv quintiles, the 95th quantile differences tend to increase with Inv. The 95th quantile differences of returns on portfolios in the higher size quintile decrease with OP.

Patterns on interquantile ranges (IR) and standard deviations (SD)

Table 4e and Figure 6 present the patterns on the 95-5th interquantile ranges. For portfolios sorted on size and B/M in North America, the interquantile returns tend to decrease with B/M in the four lower size quintiles and decrease with size in the four lower size B/M quintiles (Panel a and d of 6). These patterns are not found in the European market. We note that in Panel g, the portfolios in the highest size quintile have increasing interquantile ranges as B/M increase and the interquantile ranges of returns on portfolios in the lowest size quintile tend to decrease with B/M.

For portfolios sorted on size and Inv in North America, Panel b shows that the interquantile ranges tend to increase with Inv in the higher Inv quintiles. The similar patterns can be found in Europe except for the portfolios in the highest size quintile (Panel h). For portfolios sorted on size and Inv, the interquantile ranges of portfolio returns decrease as size increases in North America but it seems that the opposite can be found in Europe, especially for the portfolios in the lowest Inv quintile.

For portfolios sorted on size and OP, we find that the interquantile ranges of returns on portfolios in the highest size quintile tend to decrease with OP in both North America and Europe. In North America, the interquantile ranges of portfolio returns in the higher OP quintiles tends to decrease with size.

Table 4e and Figure 11 originally show the patterns on the differences between 95-5th interquantile ranges. It is notable that the differences tend to decrease with market capitalization (Panel d, e and f). For portfolios sorted on size and B/M, the differences tend to decrease with B/M for portfolios in all size quintiles (Panel a). There are increasing trends for portfolios in the three higher size quintiles and in the four higher Inv quintiles (Panel b). For portfolios sorted on size and OP, the differences tend to decrease with OP in the lower OP quintiles and increase with OP in the higher OP quintiles.

The patterns on standard deviations (SD) of returns on LHS portfolios are shown in the Table 4b, Figure 7 and 12. Panel a and d of Figure 7 show that the SD of portfolio returns in the four lower size or B/M quintiles tend to decrease with B/M or size in North America. In Panel g and j of Figure 7, we see the SD of returns on portfolios in the highest size quintile tend to increase with B/M but the SD in the lowest size quintile tend to decrease with B/M in Europe. The SD in the two highest B/M quintiles tend to increase with size in Europe. Panel b and h of 7 show that in both North America and Europe, the SD of portfolio returns in the four higher Inv quintiles tend to increase with Inv. However, the SD of portfolio returns in North America tend to decrease with size in all Inv quintiles but tend to increase with size, especially for the ones in the lowest Inv quintile in Europe (Panel e and k).

Panel c clearly shows that the SD of returns on portfolios in the highest size quintile decrease with OP. Panel f shows the SD in all OP quintiles tend to decrease with size in North America. In Europe, the SD of returns on portfolios in the highest size quintile decrease with OP and the SD of returns on portfolios in the two lower OP quintiles tend to increase with size (Panel i and l of 7). Figure 12 clearly shows that the SD differences in returns on portfolios in all sorts tends to decrease with size. The SD differences of returns on portfolios sorted on size and B/M tend to decrease as B/M. For portfolios sorted on size and Inv, the differences tend to increase with Inv in the four higher Inv quintiles.

Relations between patterns on mean returns and risk measures (IR and SD)

One interesting question to ask is that whether the patterns on mean returns are the same as the ones on IR or SD because higher mean returns should have higher risks. Thus, we compare these patterns. Some results are quite surprising. The matching panels of Figure 3 and 7 show these surprising results. The mean returns on portfolios sorted on size and B/M increase with B/M but its SD decrease with B/M in North America (Panel a). The mean returns on portfolios sorted on size and Inv tend to decrease with Inv but their SD increase with Inv in North America (Panel b Figure 3 and 7). The portfolios in the size and OP sorts tend to have increasing mean returns as OP but decreasing SD in North America (Panel c Figure 3 and 7), especially for the ones in the highest size quintile.

Panel k shows another surprising result, that is, the mean returns on portfolios in the lowest Inv quintile tend to decrease with size but their SD increase with size. Similar surprising relations can be found in the comparison of patterns on mean return and patterns of IR (See Figure 3 and 6) In sum, these surprising findings are likely to be found for portfolios in the extreme quintiles. This implies that higher risks do not necessarily come with higher returns, especially for portfolios in the extreme quintiles.

4.3 Decomposition results

The average mean difference in returns on portfolios of North America and Europe is 0.23% per month, roughly 2.73% per annum. The results of decomposing the mean return differences are presented in Table 9, 10 and 11. We also show these results graphically in Figure 13. Graphically, we can see that mean differences are mostly due to aggregate effects (Panel a of 13). Table 3 shows the percent average aggregate effects and detailed effects linked to the five factors.[19] On average, the aggregate composition and structure effect is about 89.5% and 10.5% of overall mean differences respectively. The aggregate composition effect is most due to the market and size factors, 71.4% and 23% respectively. The composition effect linked to the value factor is on average -4.3% but the structure effect linked to the value factor is positively 3.8%. In Table 2, we note that there are zero out of 75 portfolios with negative aggregate composition effects but 30 with negative aggregate structure effects. Zero portfolios have negative composition effects linked to market factor but 15 portfolios have negative composition effects linked to SMB. Thus, we conclude that the differences in the distributions of the market and size factors of North America and Europe play a great role in the mean differences in returns on portfolios of North America and Europe.

The average overall 5th quantile difference in returns on portfolios of North America and Europe is 0.66% per month, 7.99% per annum. Table 12a, 13 and 14 show the decomposition results of 5th quantile differences. The results are also shown graphically in Figure 14. Table 3 shows the percent average aggregate effects and detailed effects linked to the five factors. The percent average aggregate composition and structure effects are 137.7% and -37.7% respectively. The huge positive composition effect is due to the difference in the market factor, 401.5%. The negative structure effect is contributed mostly by the differences in the constant term and the loadings on the investment factor, -103.8% and -132.3% respectively.

We note that both differences in the distributions of the size and profitability factors of North America and Europe negatively contribute to the composition effects, -205.4% and -70%. However,

[19] We calculate the percent average effects by first summing all the (aggregate or detailed) effects for all portfolios and then dividing with the sum of the overall differences for all portfolios.

the difference in the market factor loadings positively contributes to the structure effect, 319.2%. In Table 2, we note that 24 out of 74 portfolios have negative values in the 5th quantile differences. 18 and 37 portfolios have negative composition and structure effects respectively. We conclude that the differences in the distribution of the market factors of North America and Europe and in the loadings linked to this factor contribute significantly and positively to the 5th quantile differences.

The average overall 95th quantile difference in returns on portfolios of North America and Europe is 0.24% per month, 2.84% per annum. Table 12b, 15 and 16 show the detailed decomposition results of 95th quantile differences. We graphically show the the same results in Figure 15. Table 3 show the summary results for all portfolios. On average, the percent aggregate composition and structure effects are 50% and 50% respectively. However, this does not mean that the same proportion applies to the factors. Again, the market factor positively and greatly contributes to the 95th quantile difference as it does to the 5th quantile difference, 462.5% in composition effects and 740.6% in structure effects.

The difference in the distribution of the size factor negatively attributes to the difference, -415.6%. The differences in the loadings linked to SMB, HML and CMA are also negatively contributing to the differences, -62.7%, -171.9% and -506.3% respectively. In Table 2, we note there are 37 out of 75 portfolios having negative overall differences in 95th quantile differences. 35 and 32 portfolios have negative aggregate composition and structure effects respectively.

Table 2: Number of portfolios with negative effects

	Overall	Agg_C	Agg_S	Mkt.RF_C	SMB_C	HML_C	RMW_C	CMA_C	Constant_S	Mkt.RF_S	SMB_S	HML_S	RMW_S	CMA_S
Mean	1	0	30	0	15	50	42	44	37	30	41	34	35	47
5th	24	18	37	0	57	12	23	15	44	13	40	24	28	50
95th	37	35	32	57	12	23	13	6	34	54	18	14	20	7

Table 3: Average effects of factors (in percentage)

	Agg_C	Agg_S	Mkt.RF_C	SMB_C	HML_C	RMW_C	CMA_C	Constant_S	Mkt.RF_S	SMB_S	HML_S	RMW_S	CMA_S
Mean	89.5	10.5	71.4	23.0	-4.3	0.3	-0.9	4.4	2.2	-0.2	3.8	3.2	-2.8
5th	137.7	-37.7	401.5	-205.4	1.5	-70.0	10.0	-103.8	319.2	-85.4	-14.6	-20.8	-132.3
95th	50.0	50.0	462.5	-415.6	34.4	-21.9	-9.4	31.2	740.6	-62.5	-171.9	18.7	-506.3

5 Conclusion

In this paper, we originally find patterns on the quantiles, standard deviations, interquantile ranges of returns on portfolios of North America and Europe. Patterns on the differences between these above variables for North America and Europe are also examined. In North America, the 5th quantile returns on portfolios in the size and B/M sorts tend to increase with size and B/M. In North America, the 95th quantile returns on portfolios sorted on size and B/M tend to decrease with size and the returns on portfolios in size and Inv sorts tend to decrease with size too. The interquantile ranges tend to decrease with size in both size and B/M, and size and Inv sorts in North America market. The standard deviations tend to decrease with size in all sorts in North America but we do not see the same patterns in Europe. The SD of returns on portfolios in the highest B/M or lowest Inv quintile increase strongly with size in Europe. The 5th quantile differences tend to increase with size for portfolios in all sorts. The 95th quantile differences in returns on portfolios in all sorts have decreasing trends as the size increases. The 95-5th interquantile differences and SD differences in all sorts tend to decrease with size. Therefore, we conclude that the portfolios with higher risks do not necessarily bring higher returns.

By applying traditional Oaxaca-Blind decomposition and distribution regressions, we decompose the mean and quantile differences in returns on portfolios of North America and Europe. We find that the market factor positively and significantly contributes to the mean, 5th and 95th quantile differences. The size factor positively contributes to the mean difference but largely and negatively contributes to the 5th and 95th quantile differences. The value, profitability and investment factors do not seem to play a great role in the mean differences. However, the investment factor does negatively and largely contribute to the 5th and 95th quantile differences. The profitability plays a large and negative role in the 5th quantile difference but weakly contributes to the 95th quantile difference.

Although the decomposition results for the mean differences are path independent, the main concern in the paper is that the decomposition results for the quantile differences are path dependent since the different counterfactual elements of the detailed decompositions have to be computed sequentially. We choose the decomposition order of the market, size, value, profitability and investment factors, which is consistent with the order of the factors being added into the

FF(2015) five-factor model and being studied in the literature of financial economics. However, the results could be different if we decompose the quantile differences in other orders. Another concern is that we use the loadings on the factors of North America as a reference group to calculate the composition effects and the factors of Europe to estimate the structure effects. The results might not be robust if we change the reference group, that is, using the Europe's factor loadings as the reference group.

Our results raise the obvious question of what the economic or behavioral forces are underlying. Unfortunately, in the paper the decomposition results using FF(2015) five-factor model have little, if any, economic or behavioral interpretation and the paper is more likely a statistical exercise in this sense. The reason is that FF(2015) five-factor model is an example of empirical asset pricing models. That is, it tries to explain the cross-section of expected returns without specifying the underlying economic model that governs asset pricing.[20] We believe a decomposition method based on a structural model is necessary if one tries to shed further light on the economic or behavioral explanations.[21]

[20]The profitability and investment factors of FF(2015) five-factor model is motivated by the dividend discount model. Although the dividend discount model is useful for suggesting variables related to differences in expected asset returns, but it is silent on economic or behavioral explanations of the differences. Thus, we suspect FF(2015) five-factor model is silent on economic or behavioral explanations too. Fama and French (2017) offer two interpretations of FF(2015) five-factor model. One interpretation is that "it is the regression equation for a multifactor version of Merton (1973) intertemporal capital asset pricing model". Another interpretation is that "it is the regression equation of an empirical asset pricing model designed to span the mean-variance efficient tangency portfolio and so capture expected asset returns."

[21]For instance, the decomposition results using Merton (1973) ICAPM would have economic explanations had one be able to obtain the state variables themselves and decompose the differences into these variables.

Other papers might be cited:

Banz (1981): "It is not known whether size per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size."

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A Tables and Figures

B More about patterns on returns

B.1 Patterns on mean returns

B.2 Patterns on quantile returns

B.3 Patterns on interquantile return differences

Table 4: Summary statistics for portfolios returns

(a) Mean

Portfolios	North America					Europe					Difference				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.43	0.40	0.79	0.82	0.64	-0.08	0.29	0.34	0.49	0.34	0.52	0.11	0.45	0.33	0.30
2	0.60	0.62	0.68	0.70	0.66	0.37	0.49	0.56	0.53	0.53	0.23	0.14	0.12	0.17	0.13
3	0.91	0.79	0.78	0.81	0.63	0.45	0.55	0.54	0.59	0.57	0.45	0.24	0.24	0.22	0.06
4	0.88	0.80	0.78	0.77	0.65	0.59	0.71	0.54	0.55	0.64	0.29	0.09	0.23	0.21	0.00
High B/M	1.16	0.86	0.89	0.86	0.57	0.75	0.76	0.74	0.65	0.54	0.41	0.10	0.15	0.21	0.04
Low Inv	1.21	0.88	0.91	0.90	0.74	0.55	0.59	0.64	0.64	0.57	0.66	0.29	0.26	0.26	0.17
2	1.12	0.90	0.90	0.94	0.65	0.71	0.77	0.67	0.63	0.58	0.41	0.13	0.22	0.31	0.06
3	0.99	0.88	0.90	0.85	0.65	0.71	0.76	0.70	0.64	0.48	0.28	0.12	0.20	0.21	0.17
4	0.96	0.87	0.79	0.88	0.62	0.65	0.64	0.48	0.63	0.45	0.31	0.23	0.32	0.24	0.17
High Inv	0.54	0.36	0.54	0.51	0.55	0.16	0.37	0.30	0.39	0.45	0.38	0.00	0.24	0.12	0.10
Low OP	0.84	0.44	0.59	0.57	0.25	0.15	0.25	0.26	0.20	0.16	0.69	0.19	0.33	0.36	0.09
2	1.06	0.78	0.78	0.79	0.57	0.67	0.57	0.56	0.55	0.55	0.39	0.21	0.23	0.23	0.02
3	1.05	0.95	0.81	0.93	0.63	0.75	0.70	0.73	0.71	0.56	0.30	0.25	0.08	0.22	0.06
4	1.07	1.01	0.90	0.78	0.71	0.90	0.75	0.63	0.69	0.47	0.16	0.26	0.27	0.09	0.25
High OP	1.09	1.14	1.06	0.93	0.72	0.76	0.96	0.81	0.70	0.60	0.32	0.18	0.25	0.23	0.11

(b) Standard Deviation

Portfolios	North America					Europe					Difference				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	7.96	7.31	6.88	6.44	4.53	5.49	5.63	5.71	5.39	4.84	2.46	1.69	1.17	1.05	-0.31
2	6.82	6.49	5.70	5.06	4.10	5.27	5.25	5.23	4.99	4.77	1.55	1.24	0.47	0.07	-0.67
3	6.13	5.47	5.02	4.57	4.21	4.95	5.02	5.14	5.01	5.21	1.17	0.45	-0.12	-0.45	-1.00
4	5.39	4.92	4.72	4.58	4.12	4.84	5.06	5.18	5.35	5.42	0.54	-0.14	-0.46	-0.77	-1.29
High B/M	5.28	5.16	4.88	4.76	5.23	4.83	5.30	5.59	5.80	6.34	0.45	-0.15	-0.71	-1.05	-1.11
Low Inv	6.42	5.68	5.09	4.76	4.05	4.98	5.22	5.46	5.20	4.85	1.44	0.46	-0.37	-0.44	-0.80
2	4.97	4.76	4.36	4.21	3.69	4.46	4.82	5.04	5.00	4.81	0.50	-0.07	-0.67	-0.79	-1.12
3	4.92	4.78	4.65	4.38	4.05	4.56	4.80	4.89	4.85	5.17	0.35	-0.03	-0.24	-0.47	-1.12
4	5.29	5.42	5.23	4.87	4.77	4.76	5.14	5.17	5.23	5.45	0.53	0.28	0.06	-0.37	-0.67
High Inv	6.80	6.92	7.26	6.64	5.82	5.69	5.74	5.94	6.07	5.35	1.11	1.19	1.32	0.57	0.47
Low OP	6.68	6.65	6.77	6.26	5.60	5.18	5.43	5.56	5.52	6.09	1.50	1.22	1.21	0.74	-0.49
2	4.91	4.94	4.87	4.69	4.89	4.67	5.00	5.09	5.14	5.53	0.24	-0.05	-0.22	-0.45	-0.64
3	4.98	4.99	4.60	4.40	4.31	4.81	5.02	5.07	5.25	5.08	0.17	-0.04	-0.47	-0.85	-0.78
4	5.34	5.27	4.93	4.45	4.07	4.79	5.09	5.17	5.08	4.99	0.55	0.19	-0.24	-0.63	-0.92
High OP	5.66	5.44	5.40	4.83	3.96	4.94	5.32	5.33	5.28	4.76	0.72	0.12	0.07	-0.45	-0.81

(c) 5th Quantiles

Portfolios	North America					Europe					Difference				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-12.67	-11.32	-10.19	-9.63	-6.97	-9.29	-10.42	-10.08	-10.22	-8.13	-3.39	-0.91	-0.12	0.59	1.16
2	-10.71	-9.65	-9.01	-7.55	-6.71	-9.10	-9.96	-9.44	-7.91	-8.67	-1.61	0.31	0.43	0.36	1.96
3	-8.73	-8.12	-7.50	-6.30	-6.49	-8.49	-8.46	-7.42	-7.98	-9.27	-0.24	0.34	-0.08	1.68	2.78
4	-8.00	-7.49	-6.99	-6.55	-6.16	-8.62	-7.32	-8.17	-8.41	-10.02	0.61	-0.16	1.18	1.86	3.86
High B/M	-7.75	-8.31	-7.44	-7.00	-8.28	-7.37	-8.46	-8.32	-10.22	-10.44	-0.38	0.16	0.89	3.22	2.16
Low Inv	-9.24	-9.18	-7.84	-6.98	-6.61	-7.89	-8.64	-9.20	-8.40	-8.56	-1.35	-0.53	1.36	1.42	1.95
2	-7.39	-7.05	-6.39	-6.10	-5.34	-7.26	-7.50	-7.50	-9.05	-8.81	-0.14	0.45	1.11	2.96	3.46
3	-6.89	-6.98	-6.98	-6.48	-6.68	-7.64	-7.51	-7.45	-7.90	-9.45	0.75	0.53	0.47	1.42	2.77
4	-7.20	-7.79	-7.69	-6.86	-8.18	-8.06	-7.76	-7.98	-8.19	-8.98	0.86	-0.03	0.29	1.33	0.79
High Inv	-10.97	-11.28	-11.59	-10.67	-9.47	-10.30	-10.51	-9.85	-9.70	-8.17	-0.67	-0.77	-1.73	-0.97	-1.31
Low OP	-10.55	-11.04	-10.70	-9.25	-9.69	-8.36	-9.11	-9.17	-8.68	-10.69	-2.18	-1.93	-1.54	-0.57	1.00
2	-7.18	-7.25	-8.03	-7.07	-8.26	-7.19	-7.99	-7.83	-8.35	-10.37	0.01	0.74	-0.21	1.29	2.11
3	-7.56	-7.02	-6.15	-5.62	-6.74	-8.06	-8.17	-8.30	-8.37	-9.45	0.49	1.15	2.15	2.76	2.71
4	-7.43	-7.62	-7.25	-6.50	-6.15	-7.85	-8.28	-8.74	-8.20	-8.99	0.42	0.66	1.50	1.70	2.84
High OP	-8.12	-7.94	-7.52	-7.52	-5.85	-8.14	-8.78	-8.01	-8.08	-7.66	0.02	0.84	0.49	0.57	1.81

(d) 95th Quantiles

Portfolios	North America					Europe					Difference				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	11.58	11.20	11.06	9.46	7.36	8.16	8.19	9.10	7.82	7.40	3.42	3.01	1.96	1.64	-0.04
2	10.42	8.80	8.38	8.52	6.74	7.92	7.76	7.95	7.97	7.43	2.50	1.04	0.43	0.55	-0.68
3	9.52	8.77	8.06	7.56	7.03	7.44	7.78	7.69	7.97	8.14	2.08	0.99	0.38	-0.41	-1.10
4	8.63	7.70	7.90	7.77	7.26	7.35	8.08	8.88	8.75	8.20	1.29	-0.38	-0.98	-0.99	-0.93
High B/M	8.86	8.81	8.07	8.14	7.81	8.00	8.78	9.34	9.86	10.36	0.85	0.04	-1.27	-1.72	-2.54
Low Inv	10.29	9.77	8.71	7.98	7.37	8.30	8.49	9.29	8.43	7.80	1.98	1.28	-0.58	-0.45	-0.44
2	8.51	7.94	7.41	7.56	6.20	7.19	7.97	8.72	8.32	7.62	1.32	-0.03	-1.32	-0.77	-1.42
3	7.67	7.92	7.88	7.44	6.85	7.26	7.90	7.83	8.03	7.86	0.41	0.02	0.05	-0.59	-1.01
4	8.86	8.97	8.09	8.08	7.89	7.33	7.98	8.29	8.10	7.83	1.53	0.99	-0.20	-0.02	0.05
High Inv	10.34	9.72	10.15	9.88	9.41	8.73	8.58	9.39	9.18	8.65	1.61	1.14	0.76	0.70	0.76
Low OP	9.85	9.96	10.85	9.39	8.60	7.85	8.41	9.01	8.61	9.26	2.00	1.55	1.83	0.78	-0.66
2	8.01	7.82	7.90	7.81	7.78	7.31	7.70	8.44	7.93	8.39	0.70	0.12	-0.54	-0.12	-0.61
3	8.16	8.02	7.42	7.71	7.41	7.69	8.31	8.27	8.32	8.16	0.46	-0.29	-0.86	-0.61	-0.75
4	8.24	8.89	7.83	7.14	6.94	8.25	7.96	8.24	8.06	8.27	-0.02	0.93	-0.41	-0.93	-1.33
High OP	9.98	9.04	8.68	7.87	6.88	7.78	8.51	8.89	8.14	7.54	2.20	0.53	-0.21	-0.27	-0.67

(e) 95-5th interquantiles

Portfolios	North America					Europe					Difference				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	24.25	22.52	21.25	19.09	14.33	17.44	18.60	19.17	18.04	15.53	6.81	3.92	2.08	1.05	-1.20
2	21.13	18.46	17.39	16.07	13.46	17.02	17.72	17.39	15.88	16.10	4.11	0.74	0.00	0.19	-2.64
3	18.25	16.89	15.57	13.86	13.52	15.93	16.24	15.11	15.95	17.40	2.32	0.65	0.46	-2.08	-3.88
4	16.64	15.19	14.89	14.31	13.43	15.96	15.41	17.05	17.16	18.22	0.67	-0.22	-2.16	-2.85	-4.79
High B/M	16.61	17.12	15.51	15.14	16.10	15.38	17.24	17.66	20.08	20.80	1.23	-0.12	-2.16	-4.94	-4.70
Low Inv	19.53	18.95	16.55	14.96	13.98	16.20	17.13	18.49	16.83	16.37	3.33	1.82	-1.95	-1.87	-2.39
2	15.90	15.00	13.80	13.66	11.54	14.45	15.47	16.22	17.38	16.42	1.45	-0.48	-2.43	-3.72	-4.88
3	14.56	14.89	14.86	13.92	13.53	14.91	15.41	15.27	15.92	17.31	-0.34	-0.52	-0.41	-2.01	-3.78
4	16.06	16.76	15.78	14.95	16.07	15.38	15.74	16.27	16.29	16.81	0.67	1.02	-0.49	-1.35	-0.74
High Inv	21.31	21.00	21.74	20.55	18.89	19.03	19.09	19.24	18.88	16.82	2.28	1.91	2.49	1.67	2.07
Low OP	20.40	21.00	21.55	18.64	18.30	16.22	17.52	18.18	17.29	19.95	4.18	3.48	3.37	1.35	-1.66
2	15.19	15.07	15.93	14.88	16.04	14.49	15.69	16.26	16.28	18.76	0.69	-0.62	-0.33	-1.40	-2.72
3	15.72	15.04	13.57	13.32	14.15	15.75	16.48	16.58	16.69	17.61	-0.03	-1.44	-3.01	-3.37	-3.46
4	15.67	16.51	15.08	13.63	13.10	16.11	16.24	16.98	16.26	17.26	-0.44	0.27	-1.91	-2.63	-4.16
High OP	18.10	16.98	16.20	15.39	12.73	15.91	17.29	16.90	16.23	15.20	2.18	-0.32	-0.70	-0.84	-2.48

Table 5: Wilcoxon Test (p-values)

	Small	2	3	4	Big
Low B/M	0.08*	0.26	0.14	0.18	0.21
2	0.21	0.24	0.30	0.30	0.40
3	0.10*	0.25	0.21	0.21	0.52
4	0.25	0.32	0.21	0.32	0.66
High B/M	0.08*	0.20	0.30	0.30	0.44
Low Inv	0.04**	0.10*	0.18	0.20	0.30
2	0.09*	0.26	0.20	0.20	0.56
3	0.17	0.31	0.24	0.25	0.48
4	0.17	0.18	0.14	0.27	0.34
High Inv	0.11	0.35	0.21	0.27	0.31
Low OP	0.03**	0.13	0.10*	0.09*	0.31
2	0.10	0.18	0.20	0.23	0.44
3	0.18	0.22	0.50	0.27	0.61
4	0.19	0.20	0.22	0.48	0.26
High OP	0.15	0.28	0.22	0.22	0.41

The table shows the p-values (at the confidence level of 95%) for Wilcoxon Test for the matching portfolios returns between North America and Europe

Table 6: Kolmogorov–Smirnov Test (p-values) For Normality of Portfolio Returns

Portfolios	North America					Europe				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
High B/M	0	0	0	0	0	0	0	0	0	0
Low Inv	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
High Inv	0	0	0	0	0	0	0	0	0	0
Low OP	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
High OP	0	0	0	0	0	0	0	0	0	0

Table 7: Kolmogorov–Smirnov Test (p-values) For Matching Portfolio Returns

	Small	2	3	4	Big
Low B/M	0.00***	0.11	0.09*	0.22	0.71
2	0.03**	0.16	0.40	0.77	0.71
3	0.02**	0.58	0.40	0.18	0.34
4	0.30	0.40	0.16	0.30	0.07*
High B/M	0.13	0.34	0.64	0.30	0.64
Low Inv	0.06*	0.02**	0.34	0.30	0.34
2	0.07*	0.40	0.22	0.18	0.03**
3	0.45	0.40	0.51	0.40	0.34
4	0.18	0.05**	0.09*	0.51	0.45
High Inv	0.01***	0.16	0.07*	0.71	0.83
Low OP	0.02**	0.16	0.06*	0.26	0.26
2	0.34	0.02**	0.16	0.58	0.58
3	0.58	0.51	0.93	0.30	0.40
4	0.06*	0.18	0.83	0.64	0.13
High OP	0.11	0.58	0.22	0.58	0.58

The table shows the p-values for Kolmogorov–Smirnov test for the matching portfolios returns between North America and Europe. 9 of 75 portfolios are less than 0.05, and 17 portfolios are less than 0.1.

Table 8: Mean Differences and Standard Deviations Between North America and Europe (Differencing first)

Portfolios	Mean Difference					Standard Deviation				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.52	0.11	0.45	0.33	0.30	5.83	5.60	4.91	4.41	3.27
2	0.23	0.14	0.12	0.17	0.13	5.28	5.22	4.44	3.67	3.30
3	0.45	0.24	0.24	0.22	0.06	4.68	4.41	4.06	3.45	3.18
4	0.29	0.09	0.23	0.21	0.00	4.48	4.13	3.85	3.70	3.62
High B/M	0.41	0.10	0.15	0.21	0.04	4.35	4.29	4.22	4.10	4.37
Low Inv	0.66	0.29	0.26	0.26	0.17	5.11	4.35	4.16	3.54	3.25
2	0.41	0.13	0.22	0.31	0.06	4.26	4.13	3.83	3.48	3.14
3	0.28	0.12	0.20	0.21	0.17	4.26	4.28	3.78	3.58	3.26
4	0.31	0.23	0.32	0.24	0.17	4.35	4.38	4.08	3.75	3.76
High Inv	0.38	0.00	0.24	0.12	0.10	5.01	5.12	5.08	4.40	4.55
Low OP	0.69	0.19	0.33	0.36	0.09	5.10	5.02	5.14	4.70	4.36
2	0.39	0.21	0.23	0.23	0.02	4.40	4.31	3.99	3.78	3.74
3	0.30	0.25	0.08	0.22	0.06	4.36	4.20	3.79	3.47	3.17
4	0.16	0.26	0.27	0.09	0.25	4.43	4.47	4.16	3.56	3.36
High OP	0.32	0.18	0.25	0.23	0.11	4.49	4.61	4.27	3.72	3.30

Table 9: Aggregate decomposition of mean difference

Portfolios	Overall Difference				
	Small	2	3	4	Big
Low B/M	0.52	0.11	0.45	0.33	0.30
2	0.23	0.14	0.12	0.17	0.13
3	0.45	0.24	0.24	0.22	0.06
4	0.29	0.09	0.23	0.21	0.00
High B/M	0.41	0.10	0.15	0.21	0.04
Low Inv	0.66	0.29	0.26	0.26	0.17
2	0.41	0.13	0.22	0.31	0.06
3	0.28	0.12	0.20	0.21	0.17
4	0.31	0.23	0.32	0.24	0.17
High Inv	0.38	0.00	0.24	0.12	0.10
Low OP	0.69	0.19	0.33	0.36	0.09
2	0.39	0.21	0.23	0.23	0.02
3	0.30	0.25	0.08	0.22	0.06
4	0.16	0.26	0.27	0.09	0.25
High OP	0.32	0.18	0.25	0.23	0.11

Portfolios	Composition effects					Structure effects				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.4(77%)	0.34(309%)	0.35(78%)	0.31(94%)	0.18(60%)	0.11(21%)	-0.23(-209%)	0.1(22%)	0.02(6%)	0.11(37%)
2	0.32(139%)	0.32(229%)	0.23(192%)	0.2(118%)	0.16(123%)	-0.09(-39%)	-0.19(-136%)	-0.11(-92%)	-0.02(-12%)	-0.03(-23%)
3	0.28(62%)	0.27(113%)	0.19(79%)	0.17(77%)	0.12(200%)	0.17(38%)	-0.03(-12%)	0.05(21%)	0.04(18%)	-0.06(-100%)
4	0.24(83%)	0.19(211%)	0.16(70%)	0.14(67%)	0.08(Inf%)	0.05(17%)	-0.09(-100%)	0.07(30%)	0.08(38%)	-0.07(-Inf%)
High B/M	0.19(46%)	0.18(180%)	0.16(107%)	0.12(57%)	0.05(125%)	0.22(54%)	-0.08(-80%)	0(0%)	0.09(43%)	-0.02(-50%)
Low Inv	0.33(50%)	0.3(103%)	0.22(85%)	0.21(81%)	0.19(112%)	0.33(50%)	-0.01(-3%)	0.05(19%)	0.05(19%)	-0.02(-12%)
2	0.23(56%)	0.24(185%)	0.18(82%)	0.16(52%)	0.15(250%)	0.19(46%)	-0.11(-85%)	0.04(18%)	0.14(45%)	-0.09(-150%)
3	0.22(79%)	0.22(183%)	0.18(90%)	0.15(71%)	0.14(82%)	0.05(18%)	-0.1(-83%)	0.02(10%)	0.06(29%)	0.03(18%)
4	0.21(68%)	0.21(91%)	0.19(59%)	0.18(75%)	0.12(71%)	0.1(32%)	0.02(9%)	0.13(41%)	0.06(25%)	0.04(24%)
High Inv	0.25(66%)	0.25(-Inf%)	0.26(108%)	0.23(192%)	0.1(100%)	0.12(32%)	-0.25(Inf%)	-0.02(-8%)	-0.11(-92%)	0(0%)
Low OP	0.32(46%)	0.32(168%)	0.3(91%)	0.27(75%)	0.13(144%)	0.37(54%)	-0.13(-68%)	0.03(9%)	0.1(28%)	-0.04(-44%)
2	0.18(46%)	0.2(95%)	0.21(91%)	0.16(70%)	0.15(750%)	0.22(56%)	0.01(5%)	0.01(4%)	0.07(30%)	-0.13(-650%)
3	0.19(63%)	0.21(84%)	0.15(188%)	0.17(77%)	0.12(200%)	0.1(33%)	0.03(12%)	-0.07(-88%)	0.05(23%)	-0.06(-100%)
4	0.18(112%)	0.19(73%)	0.16(59%)	0.17(189%)	0.12(48%)	-0.02(-12%)	0.07(27%)	0.1(37%)	-0.08(-89%)	0.12(48%)
High OP	0.18(56%)	0.19(106%)	0.18(72%)	0.17(74%)	0.16(145%)	0.14(44%)	-0.01(-6%)	0.07(28%)	0.05(22%)	-0.04(-36%)

The percentage of composition effects is in the parentheses. Greater than 100% means negatvie structure effects. All aggregate composition effects are positeve, but 38.7% of portfolios has negative structrue effects. 6.7% of portfolios have the percentage of composition effects less than 50%. On average, the percentage of composition effects is 115.2% and the one of structure effects is -15.6%.

Table 10: Detailed Decomposition of mean difference (Composition)

Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.16(31%)	0.17(155%)	0.17(38%)	0.17(52%)	0.16(53%)	0.12(23%)	0.09(82%)	0.08(18%)	0.04(12%)	-0.03(-10%)
2	0.16(70%)	0.16(114%)	0.17(142%)	0.16(94%)	0.16(123%)	0.11(48%)	0.1(71%)	0.07(58%)	0.04(24%)	-0.02(-15%)
3	0.15(33%)	0.16(67%)	0.17(71%)	0.17(77%)	0.16(267%)	0.11(24%)	0.09(38%)	0.06(25%)	0.03(14%)	-0.01(-17%)
4	0.14(48%)	0.16(178%)	0.17(74%)	0.17(81%)	0.15(Inf%)	0.1(34%)	0.07(78%)	0.05(22%)	0.02(10%)	-0.02(-Inf%)
High B/M	0.15(37%)	0.17(170%)	0.17(113%)	0.17(81%)	0.18(450%)	0.09(22%)	0.08(80%)	0.05(33%)	0.03(14%)	-0.02(-50%)
Low Inv	0.16(24%)	0.18(62%)	0.18(69%)	0.18(69%)	0.16(94%)	0.12(18%)	0.09(31%)	0.05(19%)	0.03(12%)	-0.01(-6%)
2	0.14(34%)	0.15(115%)	0.16(73%)	0.17(55%)	0.15(250%)	0.09(22%)	0.08(62%)	0.05(23%)	0.02(6%)	-0.01(-17%)
3	0.14(50%)	0.15(125%)	0.16(80%)	0.16(76%)	0.16(94%)	0.09(32%)	0.08(67%)	0.06(30%)	0.03(14%)	-0.02(-12%)
4	0.14(45%)	0.16(70%)	0.17(53%)	0.17(71%)	0.16(94%)	0.09(29%)	0.09(39%)	0.07(22%)	0.04(17%)	-0.03(-18%)
High Inv	0.15(39%)	0.17(-Inf%)	0.17(71%)	0.18(150%)	0.16(160%)	0.11(29%)	0.09(-Inf%)	0.08(33%)	0.05(42%)	-0.02(-20%)
Low OP	0.15(22%)	0.17(89%)	0.18(55%)	0.18(50%)	0.17(189%)	0.11(16%)	0.09(47%)	0.06(18%)	0.03(8%)	-0.03(-33%)
2	0.14(36%)	0.16(76%)	0.16(70%)	0.17(74%)	0.17(850%)	0.09(23%)	0.08(38%)	0.06(26%)	0.03(13%)	-0.02(-100%)
3	0.15(50%)	0.16(64%)	0.16(200%)	0.16(73%)	0.16(267%)	0.09(30%)	0.09(36%)	0.05(62%)	0.04(18%)	-0.02(-33%)
4	0.16(100%)	0.17(65%)	0.17(63%)	0.17(189%)	0.15(60%)	0.1(62%)	0.09(35%)	0.06(22%)	0.04(44%)	-0.02(-8%)
High OP	0.16(50%)	0.18(100%)	0.18(72%)	0.17(74%)	0.16(145%)	0.1(31%)	0.09(50%)	0.08(32%)	0.05(22%)	-0.02(-18%)

Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.08(15%)	0.08(73%)	0.09(20%)	0.09(27%)	0.06(20%)	0.04(8%)	0.02(18%)	0.01(2%)	0.02(6%)	-0.01(-3%)
2	0.04(17%)	0.05(36%)	0.01(8%)	0(0%)	0.02(15%)	0.02(9%)	0.01(7%)	0(0%)	0(0%)	-0.01(-8%)
3	0.01(2%)	0.01(4%)	-0.02(-8%)	-0.02(-9%)	-0.03(-50%)	0.02(4%)	0(0%)	-0.01(-4%)	-0.01(-5%)	0(0%)
4	-0.01(-3%)	-0.04(-44%)	-0.05(-22%)	-0.05(-24%)	-0.06(-Inf%)	0.01(3%)	0(0%)	-0.01(-4%)	-0.02(-10%)	0(NaN%)
High B/M	-0.06(-15%)	-0.07(-70%)	-0.07(-47%)	-0.08(-38%)	-0.11(-275%)	0.01(2%)	0(0%)	0(0%)	-0.01(-5%)	0.01(25%)
Low Inv	0.01(2%)	0(0%)	-0.02(-8%)	-0.02(-8%)	0.01(6%)	0.02(3%)	0.01(3%)	-0.01(-4%)	0(0%)	0.01(6%)
2	-0.02(-5%)	-0.01(-8%)	-0.02(-9%)	-0.02(-6%)	0(0%)	0.01(2%)	0(0%)	-0.01(-5%)	-0.01(-3%)	-0.01(-17%)
3	-0.01(-4%)	-0.02(-17%)	-0.02(-10%)	-0.03(-14%)	0(0%)	0(0%)	0(0%)	-0.01(-5%)	-0.01(-5%)	0(0%)
4	-0.03(-10%)	-0.02(-9%)	-0.02(-6%)	-0.01(-4%)	0(0%)	0.01(3%)	-0.01(-4%)	-0.01(-3%)	-0.01(-4%)	0(0%)
High Inv	0(0%)	0.01(-Inf%)	0.02(8%)	0.02(17%)	0(0%)	0.02(5%)	0.02(-Inf%)	0.02(8%)	0.02(17%)	0(0%)
Low OP	0.02(3%)	0.03(16%)	0.03(9%)	0.02(6%)	-0.04(-44%)	0.04(6%)	0.04(21%)	0.04(12%)	0.04(11%)	0.05(56%)
2	-0.05(-13%)	-0.03(-14%)	-0.01(-4%)	-0.03(-13%)	-0.02(-100%)	0(0%)	-0.01(-5%)	0(0%)	0(0%)	0.03(150%)
3	-0.03(-10%)	-0.02(-8%)	-0.03(-38%)	-0.02(-9%)	-0.02(-33%)	-0.01(-3%)	-0.02(-8%)	-0.02(-25%)	-0.01(-5%)	0(0%)
4	-0.04(-25%)	-0.03(-12%)	-0.03(-11%)	-0.02(-22%)	0(0%)	-0.02(-12%)	-0.03(-12%)	-0.03(-11%)	-0.02(-22%)	0(0%)
High OP	-0.05(-16%)	-0.03(-17%)	-0.03(-12%)	-0.01(-4%)	0.04(36%)	-0.02(-6%)	-0.04(-22%)	-0.04(-16%)	-0.03(-13%)	-0.03(-27%)

Portfolios	CMA				
	Small	2	3	4	Big
Low B/M	0(0%)	-0.02(-18%)	0(0%)	0(0%)	0(0%)
2	-0.01(-4%)	0(0%)	-0.01(-8%)	-0.01(-6%)	0.01(8%)
3	0(0%)	0.01(4%)	-0.01(-4%)	0(0%)	0(0%)
4	0(0%)	0(0%)	0(0%)	0(0%)	0(NaN%)
High B/M	0(0%)	0.01(10%)	0(0%)	0(0%)	-0.01(-25%)
Low Inv	0.02(3%)	0.02(7%)	0.01(4%)	0.02(8%)	0.02(12%)
2	0.01(2%)	0.02(15%)	0.01(5%)	0.01(3%)	0.02(33%)
3	0.01(4%)	0.01(8%)	0(0%)	0(0%)	0.01(6%)
4	-0.01(-3%)	-0.01(-4%)	-0.01(-3%)	-0.01(-4%)	-0.02(-12%)
High Inv	-0.03(-8%)	-0.03(Inf%)	-0.04(-17%)	-0.03(-25%)	-0.04(-40%)
Low OP	0(0%)	0(0%)	-0.01(-3%)	0(0%)	-0.02(-22%)
2	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
3	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
4	-0.01(-6%)	-0.01(-4%)	-0.01(-4%)	0.01(11%)	-0.01(-4%)
High OP	-0.02(-6%)	-0.01(-6%)	-0.01(-4%)	0(0%)	0.01(9%)

Table 11: Detailed Decomposition of mean difference (Structure)

Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.02(4%)	0.01(9%)	0(0%)	0.02(6%)	0.03(10%)	0.02(4%)	-0.01(-9%)	0.01(2%)	0.01(3%)	0(0%)
2	0(0%)	-0.02(-14%)	0(0%)	0(0%)	-0.01(-8%)	0.01(4%)	0.01(7%)	0(0%)	0(0%)	0(0%)
3	0.01(2%)	0(0%)	0.01(4%)	0.01(5%)	-0.02(-33%)	0.01(2%)	0(0%)	-0.01(-4%)	0(0%)	0.01(17%)
4	-0.04(-14%)	0.01(11%)	0.02(9%)	0.01(5%)	-0.03(-Inf%)	0(0%)	-0.01(-11%)	-0.01(-4%)	-0.01(-5%)	0(NaN%)
High B/M	0(0%)	0.02(20%)	0(0%)	0.01(5%)	0.06(150%)	0(0%)	-0.01(-10%)	-0.01(-7%)	-0.01(-5%)	0.01(25%)
Low Inv	0.02(3%)	0.04(14%)	0.02(8%)	0.03(12%)	0.01(6%)	0.01(2%)	0(0%)	-0.01(-4%)	-0.01(-4%)	0.01(6%)
2	-0.01(-2%)	-0.01(-8%)	0(0%)	0(0%)	-0.02(-33%)	0.01(2%)	0(0%)	-0.01(-5%)	-0.01(-3%)	0(0%)
3	0(0%)	0(0%)	0.02(10%)	0.01(5%)	-0.02(-12%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
4	-0.02(-6%)	0.01(4%)	0.01(3%)	0.01(4%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
High Inv	-0.03(-8%)	0(NaN%)	-0.03(-12%)	-0.01(-8%)	0.04(40%)	0(0%)	0(NaN%)	0(0%)	0(0%)	0(0%)
Low OP	-0.01(-1%)	0.01(5%)	0.01(3%)	0.05(14%)	0.02(22%)	0.01(1%)	0(0%)	0(0%)	-0.01(-3%)	-0.01(-11%)
2	-0.01(-3%)	0.01(5%)	0(0%)	0.02(9%)	0.01(50%)	0(0%)	0(0%)	0(0%)	-0.01(-4%)	0(0%)
3	-0.01(-3%)	-0.01(-4%)	0(0%)	-0.03(-14%)	0.01(17%)	0(0%)	0(0%)	-0.01(-12%)	0(0%)	0(0%)
4	0.01(6%)	0.02(8%)	0.01(4%)	0(0%)	-0.04(-16%)	0.01(6%)	0.01(4%)	0(0%)	0(0%)	0(0%)
High OP	0.03(9%)	0.03(17%)	0.04(16%)	0.01(4%)	0.01(9%)	0.01(3%)	0(0%)	0(0%)	0(0%)	0(0%)

Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-0.01(-2%)	-0.01(-9%)	-0.01(-2%)	-0.05(-15%)	0.06(20%)	-0.04(-8%)	0.01(9%)	0.06(13%)	-0.01(-3%)	0.07(23%)
2	0.02(9%)	-0.05(-36%)	0.03(25%)	0.02(12%)	0.03(23%)	-0.01(-4%)	-0.04(-29%)	0.02(17%)	0(0%)	-0.09(-69%)
3	0.04(9%)	-0.03(-12%)	-0.01(-4%)	-0.01(-5%)	0.07(117%)	-0.01(-2%)	-0.01(-4%)	0(0%)	-0.01(-5%)	-0.01(-17%)
4	0.01(3%)	-0.01(-11%)	-0.01(-4%)	-0.01(-5%)	0.04(Inf%)	-0.08(-28%)	-0.03(-33%)	0.01(4%)	0.06(29%)	-0.01(-Inf%)
High B/M	0.05(12%)	0.01(10%)	-0.02(-13%)	-0.05(-24%)	0(0%)	-0.05(-12%)	0.03(30%)	0.01(7%)	0.03(14%)	0.09(225%)
Low Inv	0(0%)	-0.04(-14%)	0(0%)	-0.02(-8%)	0.03(18%)	-0.07(-11%)	-0.04(-14%)	0(0%)	-0.02(-8%)	0.02(12%)
2	0.01(2%)	-0.03(-23%)	-0.02(-9%)	0.02(6%)	-0.01(-17%)	-0.04(-10%)	-0.04(-31%)	0.06(27%)	0.09(29%)	0(0%)
3	-0.02(-7%)	-0.01(-8%)	-0.03(-15%)	0.01(5%)	0.04(24%)	-0.07(-25%)	0(0%)	0.03(15%)	0.04(19%)	0.02(12%)
4	0.07(23%)	0(0%)	0.01(3%)	-0.06(-25%)	0.01(6%)	-0.05(-16%)	0.01(4%)	0.05(16%)	0.01(4%)	-0.02(-12%)
High Inv	0.11(29%)	0.03(-Inf%)	0.02(8%)	0(0%)	-0.06(-60%)	0.02(5%)	0.01(-Inf%)	-0.08(-33%)	-0.05(-42%)	-0.08(-80%)
Low OP	0.02(3%)	-0.04(-21%)	-0.04(-12%)	-0.05(-14%)	0.06(67%)	-0.05(-7%)	-0.08(-42%)	-0.11(-33%)	0(0%)	0.16(178%)
2	0.09(23%)	0.02(10%)	-0.01(-4%)	-0.02(-9%)	0.02(100%)	0.04(10%)	0.08(38%)	0.03(13%)	0(0%)	0.02(100%)
3	0.09(30%)	0(0%)	0.01(12%)	-0.02(-9%)	0.05(83%)	0.09(30%)	0.02(8%)	0.04(50%)	0.01(5%)	-0.01(-17%)
4	0.08(50%)	0.03(12%)	0.03(11%)	-0.01(-11%)	-0.03(-12%)	0.05(31%)	0.1(38%)	0.11(41%)	0.02(22%)	-0.15(-60%)
High OP	0.12(38%)	0.07(39%)	0.03(12%)	0.03(13%)	-0.02(-18%)	0.05(16%)	0.17(94%)	0.11(44%)	0.09(39%)	-0.04(-36%)

Portfolios	CMA					Constant				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.06(12%)	0(0%)	0.06(13%)	0.04(12%)	-0.01(-3%)	0.07(13%)	-0.23(-209%)	-0.02(-4%)	0.02(6%)	-0.04(-13%)
2	0.01(4%)	0(0%)	-0.05(-42%)	-0.06(-35%)	-0.04(-31%)	-0.13(-57%)	-0.1(-71%)	-0.1(-83%)	0.02(12%)	0.08(62%)
3	0.01(2%)	0.01(4%)	-0.04(-17%)	-0.01(-5%)	-0.01(-17%)	0.11(24%)	0(0%)	0.09(38%)	0.07(32%)	-0.09(-150%)
4	-0.01(-3%)	-0.01(-11%)	0.01(4%)	-0.01(-5%)	0(NaN%)	0.17(59%)	-0.04(-44%)	0.06(26%)	0.03(14%)	-0.07(-Inf%)
High B/M	-0.05(-12%)	-0.02(-20%)	-0.01(-7%)	0.01(5%)	0.04(100%)	0.27(66%)	-0.12(-120%)	0.03(20%)	0.1(48%)	-0.21(-525%)
Low Inv	0(0%)	0.02(7%)	0(0%)	0(0%)	-0.04(-24%)	0.37(56%)	0.01(3%)	0.04(15%)	0.06(23%)	-0.04(-24%)
2	-0.02(-5%)	0.02(15%)	0(0%)	-0.02(-6%)	-0.01(-17%)	0.23(56%)	-0.04(-31%)	0.01(5%)	0.06(19%)	-0.06(-100%)
3	0.03(11%)	0.02(17%)	0(0%)	-0.01(-5%)	0(0%)	0.11(39%)	-0.11(-92%)	0(0%)	0.01(5%)	0.01(6%)
4	-0.03(-10%)	-0.01(-4%)	-0.02(-6%)	0.01(4%)	0.03(18%)	0.13(42%)	0(0%)	0.07(22%)	0.08(33%)	0.04(24%)
High Inv	-0.01(-3%)	-0.03(Inf%)	-0.03(-12%)	0.02(17%)	-0.04(-40%)	0.03(8%)	-0.26(Inf%)	0.1(42%)	-0.07(-58%)	0.15(150%)
Low OP	0.02(3%)	0.02(11%)	-0.03(-9%)	-0.01(-3%)	-0.03(-33%)	0.39(57%)	-0.03(-16%)	0.21(64%)	0.12(33%)	-0.25(-278%)
2	-0.03(-8%)	-0.01(-5%)	0.01(4%)	-0.02(-9%)	-0.02(-100%)	0.13(33%)	-0.09(-43%)	-0.01(-4%)	0.1(43%)	-0.16(-800%)
3	-0.02(-7%)	-0.01(-4%)	-0.01(-12%)	0.01(5%)	-0.03(-50%)	-0.05(-17%)	0.04(16%)	-0.1(-125%)	0.08(36%)	-0.09(-150%)
4	-0.03(-19%)	-0.02(-8%)	-0.02(-7%)	0.02(22%)	-0.03(-12%)	-0.14(-88%)	-0.07(-27%)	-0.03(-11%)	-0.11(-122%)	0.36(144%)
High OP	-0.05(-16%)	-0.01(-6%)	-0.01(-4%)	0.01(4%)	0.03(27%)	-0.02(-6%)	-0.26(-144%)	-0.12(-48%)	-0.09(-39%)	-0.02(-18%)

Table 12: Aggregate Decomposition of quantile difference

(a) Aggregate Decomposition of 5th quantile difference

Portfolios	Overall Difference					Composition Effects					Structure Effects				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-4.15	-1.11	-0.83	0.55	0.55	-2.76(67%)	-1.11(100%)	-2.21(266%)	0(0%)	1.38(251%)	-1.38(33%)	0(0%)	1.38(-166%)	0.55(100%)	-0.83(-151%)
2	-1.93	-1.38	0.28	0.55	3.04	-0.28(15%)	-1.66(120%)	-0.28(-100%)	0(0%)	1.11(37%)	-1.66(86%)	0.28(-20%)	0.55(196%)	0.55(100%)	1.93(63%)
3	0	0	0.55	1.66	3.87	-0.55(-Inf%)	0(NaN%)	2.21(402%)	1.11(67%)	1.66(43%)	0.55(Inf%)	0(NaN%)	-1.66(-302%)	0.55(33%)	2.21(57%)
4	0	0	0.83	2.76	3.32	-0.28(-Inf%)	0.28(Inf%)	1.11(134%)	4.97(180%)	1.38(42%)	0.28(Inf%)	-0.28(-Inf%)	-0.28(-34%)	-2.21(-80%)	1.93(58%)
High B/M	-0.83	1.11	0	2.76	3.04	1.38(-166%)	0.83(75%)	0.55(Inf%)	2.76(100%)	1.66(55%)	-2.21(266%)	0.28(25%)	-0.55(-Inf%)	0(0%)	1.38(45%)
Low Inv	-2.21	-0.55	1.66	1.38	0.83	-0.28(13%)	0.28(-51%)	1.11(67%)	1.66(120%)	1.93(233%)	-1.93(87%)	-0.83(151%)	0.55(33%)	-0.28(-20%)	-1.11(-134%)
2	-2.21	0.55	0.83	3.32	3.32	1.11(-50%)	0.28(51%)	1.38(166%)	1.66(50%)	3.87(117%)	-3.32(150%)	0.28(51%)	-0.55(-66%)	1.66(50%)	-0.55(-17%)
3	0	0.28	0.55	0.28	3.04	0.55(Inf%)	0.28(100%)	1.38(251%)	1.11(396%)	3.59(118%)	-0.55(-Inf%)	0(0%)	-0.83(-151%)	-0.83(-296%)	-0.55(-18%)
4	0.83	-0.83	0	1.38	0.83	-0.28(-34%)	0(0%)	0(NaN%)	1.11(80%)	1.38(166%)	1.11(134%)	-0.83(100%)	0(NaN%)	0.28(20%)	-0.55(-66%)
High Inv	-0.55	-1.38	-1.66	-0.28	-0.55	-1.66(302%)	-1.11(80%)	-0.83(50%)	-0.83(296%)	0(0%)	1.11(-202%)	-0.28(20%)	-0.83(50%)	0.55(-196%)	-0.55(100%)
Low OP	-3.04	-1.93	-0.55	-1.38	0.83	-1.38(45%)	-1.66(86%)	-1.11(202%)	-1.93(140%)	0.83(100%)	-1.66(55%)	-0.28(15%)	0.55(-100%)	0.55(-40%)	0(0%)
2	-0.83	1.11	-0.28	0	0.55	0.83(-100%)	1.66(150%)	0(0%)	1.93(Inf%)	1.11(202%)	-1.66(200%)	-0.55(-50%)	-0.28(100%)	-1.93(-Inf%)	-0.55(-100%)
3	1.11	-0.55	2.21	3.32	2.76	0.83(75%)	0.83(-151%)	1.93(87%)	2.49(75%)	2.21(80%)	0.28(25%)	-1.38(251%)	0.28(13%)	0.83(25%)	0.55(20%)
4	-1.38	-0.28	1.93	1.38	2.21	0.28(-20%)	0(0%)	1.38(72%)	1.66(120%)	1.93(87%)	-1.66(120%)	-0.28(100%)	0.55(28%)	-0.28(-20%)	0.28(13%)
High OP	0	0.55	1.66	1.66	1.38	0(NaN%)	0.55(100%)	0.28(17%)	0.83(50%)	3.04(220%)	0(NaN%)	0(0%)	1.38(83%)	0.83(50%)	-1.66(-120%)

(b) Aggregate Decomposition of 95th quantile difference

Portfolios	Overall Difference					Composition Effects					Structure Effects				
	Small	2	3	4	Big	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	2.21	-0.55	0.55	-0.28	-0.83	2.76(125%)	1.11(-202%)	0.28(51%)	0.55(-196%)	-0.55(66%)	-0.55(-25%)	-1.66(302%)	0.28(51%)	-0.83(296%)	-0.28(34%)
2	-0.28	1.11	0.55	0.28	-1.11	1.66(-593%)	0.83(75%)	0.28(51%)	1.38(493%)	-1.66(150%)	-1.93(689%)	0.28(25%)	0.28(51%)	-1.11(-396%)	0.55(-50%)
3	2.21	0.83	0.28	-1.11	-0.55	1.66(75%)	0.55(66%)	-0.83(-296%)	-0.83(75%)	-1.38(251%)	0.55(25%)	0.28(34%)	1.11(396%)	-0.28(25%)	0.83(-151%)
4	-1.66	-0.28	-1.11	-0.55	-0.55	0.83(-50%)	0(0%)	-1.66(150%)	-0.55(100%)	-1.66(302%)	-2.49(150%)	-0.28(100%)	0.55(-50%)	0(0%)	1.11(-202%)
High B/M	0.83	0.55	-1.11	-1.66	-1.93	1.11(134%)	0.55(100%)	-1.66(150%)	-1.38(83%)	-1.11(58%)	-0.28(-34%)	0(0%)	0.55(-50%)	-0.28(17%)	-0.83(43%)
Low Inv	0.55	0.55	0	-0.28	-0.28	0.55(100%)	0.28(51%)	-0.55(-Inf%)	-1.66(593%)	-0.55(196%)	0(0%)	0.28(51%)	0.55(Inf%)	1.38(-493%)	0.28(-100%)
2	1.11	0.28	-1.11	-1.11	-1.11	0.83(75%)	0(0%)	-0.55(50%)	-1.11(100%)	-1.11(100%)	0.28(25%)	0.28(100%)	-0.55(50%)	0(0%)	0(0%)
3	0.28	-1.11	0	-0.55	-1.38	0.55(196%)	0(0%)	-0.28(-Inf%)	-0.55(100%)	-1.38(100%)	-0.28(-100%)	-1.11(100%)	0.28(Inf%)	0(0%)	0(0%)
4	-0.55	0.83	-1.38	0	1.11	0.83(-151%)	0.55(66%)	0(0%)	0(NaN%)	-0.55(-50%)	-1.38(251%)	0.28(34%)	-1.38(100%)	0(NaN%)	1.66(150%)
High Inv	1.38	1.11	-1.38	-0.28	0.83	1.66(120%)	1.38(124%)	0.28(-20%)	0.55(-196%)	0(0%)	-0.28(-20%)	-0.28(-25%)	-1.66(120%)	-0.83(296%)	0.83(100%)
Low OP	0	1.38	1.66	0.83	-0.83	1.11(Inf%)	1.11(80%)	1.11(67%)	0.55(66%)	0(0%)	-1.11(-Inf%)	0.28(20%)	0.55(33%)	0.28(34%)	-0.83(100%)
2	-0.28	0.28	0	0	-1.38	0.83(-296%)	-0.55(-196%)	-0.28(-Inf%)	-0.28(-Inf%)	-0.83(60%)	-1.11(396%)	0.83(296%)	0.28(Inf%)	0.28(Inf%)	-0.55(40%)
3	-0.28	0.28	0.55	-1.38	0	0(0%)	0(0%)	-1.11(-202%)	-0.28(20%)	-1.93(-Inf%)	-0.28(100%)	0.28(100%)	1.66(302%)	-1.11(80%)	1.93(Inf%)
4	-0.28	0.55	0	-1.11	-1.66	0(0%)	0.55(100%)	-0.83(-Inf%)	-0.83(75%)	-0.55(33%)	-0.28(100%)	0(0%)	0.83(Inf%)	-0.28(25%)	-1.11(67%)
High OP	2.49	0.55	0	-1.11	-2.49	1.11(45%)	-0.55(-100%)	-0.83(-Inf%)	-0.55(50%)	-0.83(33%)	1.38(55%)	1.11(202%)	0.83(Inf%)	-0.55(50%)	-1.66(67%)

Table 13: Detailed Decomposition of 5th quantile difference (Composition)

Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.83(-20%)	1.93(-174%)	1.93(-233%)	1.38(251%)	1.38(251%)	-2.76(67%)	-2.49(224%)	-1.93(233%)	0.55(100%)	0(0%)
2	0.83(-43%)	1.11(-80%)	1.93(689%)	3.32(604%)	1.11(37%)	-0.83(43%)	-1.11(80%)	-1.93(-689%)	-3.04(-553%)	0(0%)
3	1.93(Inf%)	0.83(Inf%)	2.49(453%)	2.21(133%)	1.66(43%)	-1.93(-Inf%)	-0.83(-Inf%)	-0.28(-51%)	-1.38(-83%)	0(0%)
4	1.38(Inf%)	3.32(Inf%)	1.66(200%)	4.42(160%)	1.38(42%)	-1.66(-Inf%)	-3.04(-Inf%)	-0.55(-66%)	-1.38(-50%)	0(0%)
High B/M	1.38(-166%)	1.11(100%)	1.93(Inf%)	2.49(90%)	1.66(55%)	-0.55(66%)	-0.55(-50%)	-1.93(-Inf%)	0(0%)	0(0%)
Low Inv	1.38(-62%)	2.21(-402%)	1.93(116%)	2.21(160%)	1.38(166%)	-1.66(75%)	-2.21(402%)	-0.83(-50%)	-0.55(-40%)	0(0%)
2	1.11(-50%)	2.49(453%)	1.93(233%)	1.66(50%)	1.93(58%)	-0.55(25%)	-2.49(-453%)	-1.11(-134%)	-0.55(-17%)	0.83(25%)
3	1.93(Inf%)	2.49(889%)	2.21(402%)	1.38(493%)	3.04(100%)	-0.28(-Inf%)	-1.66(-593%)	-1.11(-202%)	-0.28(-100%)	0(0%)
4	1.93(233%)	1.38(-166%)	1.38(Inf%)	2.21(160%)	1.38(166%)	-0.28(-34%)	-1.38(166%)	-1.38(-Inf%)	-1.11(-80%)	0(0%)
High Inv	1.93(-351%)	2.76(-200%)	2.76(-166%)	3.04(-1086%)	0.83(-151%)	-1.38(251%)	-2.76(200%)	-0.83(50%)	0.28(-100%)	2.76(-502%)
Low OP	0.28(-9%)	0.55(-28%)	2.49(-453%)	2.76(-200%)	0.83(100%)	-0.28(9%)	-0.55(28%)	-1.38(251%)	-0.28(20%)	0.83(100%)
2	1.93(-233%)	2.21(199%)	0.83(-296%)	1.11(Inf%)	2.76(502%)	-1.66(200%)	-1.38(-124%)	-0.83(296%)	-0.28(-Inf%)	0(0%)
3	1.66(150%)	1.11(-202%)	3.59(162%)	3.32(100%)	1.93(70%)	-1.11(-100%)	-0.55(100%)	-2.76(-125%)	-1.66(-50%)	0.28(10%)
4	2.21(-160%)	2.21(-789%)	2.76(143%)	3.04(220%)	1.93(87%)	-2.21(160%)	-2.21(789%)	-2.49(-129%)	-1.66(-120%)	0(0%)
High OP	1.66(Inf%)	2.76(502%)	2.21(133%)	1.11(67%)	2.49(180%)	-1.66(-Inf%)	-3.32(-604%)	-2.21(-133%)	-0.28(-17%)	0(0%)

Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-0.28(7%)	0(0%)	-1.38(166%)	0(0%)	0(0%)	-0.55(13%)	0(0%)	-0.83(100%)	-1.93(-351%)	0(0%)
2	-0.28(15%)	-1.38(100%)	0(0%)	-0.28(-51%)	0(0%)	0(0%)	-0.55(40%)	0.28(100%)	0(0%)	0(0%)
3	0(NaN%)	0(NaN%)	0(0%)	0(0%)	0(0%)	-0.55(-Inf%)	0(NaN%)	0(0%)	0(0%)	0(0%)
4	0(NaN%)	0(NaN%)	0(0%)	0(0%)	0.28(8%)	0(NaN%)	0(NaN%)	0(0%)	0.83(30%)	-0.28(-8%)
High B/M	0(0%)	0(0%)	0.55(Inf%)	0.28(10%)	0(0%)	0(0%)	0(0%)	0(NaN%)	-0.28(-10%)	0(0%)
Low Inv	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)	-0.83(38%)	0(0%)	0(0%)	0(0%)	0(0%)
2	0.83(-38%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)	-0.55(-100%)	0(0%)	0(0%)	0(0%)
3	0(NaN%)	0.55(196%)	0.28(51%)	0(0%)	0(0%)	-1.38(-Inf%)	-1.93(-689%)	0(0%)	0(0%)	0(0%)
4	0(0%)	0(0%)	0(NaN%)	0(0%)	0.55(66%)	-1.66(-200%)	0(0%)	0(NaN%)	0(0%)	0(0%)
High Inv	0(0%)	-1.38(100%)	-0.83(50%)	-0.55(196%)	0(0%)	-0.83(151%)	0.28(-20%)	-1.38(83%)	-3.04(1086%)	-3.32(604%)
Low OP	-0.28(9%)	-0.28(15%)	0(0%)	-0.28(20%)	0.55(66%)	-1.11(37%)	-1.11(58%)	-2.21(402%)	-4.15(301%)	-0.55(-66%)
2	0(0%)	1.66(150%)	0(0%)	0(NaN%)	0(0%)	0(0%)	0(0%)	0(0%)	0.28(Inf%)	-0.55(-100%)
3	0(0%)	0(0%)	0(0%)	0(0%)	1.66(60%)	0.28(25%)	0.28(-51%)	0.55(25%)	0(0%)	-0.28(-10%)
4	0.28(-20%)	0(0%)	-0.28(-15%)	0(0%)	0(0%)	0(0%)	0(0%)	0.28(15%)	0(0%)	0(0%)
High OP	0(NaN%)	0.28(51%)	0.28(17%)	0(0%)	0(0%)	0(NaN%)	0.83(151%)	0(0%)	0.28(17%)	0.55(40%)

Portfolios	CMA				
	Small	2	3	4	Big
Low B/M	0(0%)	-0.55(50%)	0(0%)	0(0%)	0(0%)
2	0(0%)	0.28(-20%)	-0.55(-196%)	0(0%)	0(0%)
3	0(NaN%)	0(NaN%)	0(0%)	0.28(17%)	0(0%)
4	0(NaN%)	0(NaN%)	0(0%)	1.11(40%)	0(0%)
High B/M	0.55(-66%)	0.28(25%)	0(NaN%)	0.28(10%)	0(0%)
Low Inv	0.83(-38%)	0.28(-51%)	0(0%)	0(0%)	0.55(66%)
2	-0.28(13%)	0.83(151%)	0.55(66%)	0.55(17%)	1.11(33%)
3	0.28(Inf%)	0.83(296%)	0(0%)	0(0%)	0.55(18%)
4	-0.28(-34%)	0(0%)	0(NaN%)	0(0%)	-0.55(-66%)
High Inv	-1.38(251%)	0(0%)	-0.55(33%)	-0.55(196%)	-0.28(51%)
Low OP	0(0%)	-0.28(15%)	0(0%)	0(0%)	-0.83(-100%)
2	0.55(-66%)	-0.83(-75%)	0(0%)	0.83(Inf%)	-1.11(-202%)
3	0(0%)	0(0%)	0.55(25%)	0.83(25%)	-1.38(-50%)
4	0(0%)	0(0%)	1.11(58%)	0.28(20%)	0(0%)
High OP	0(NaN%)	0(0%)	0(0%)	-0.28(-17%)	0(0%)

Table 14: Detailed Decomposition of 5th quantile difference (Structure)

Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0(0%)	3.32(-299%)	2.49(-300%)	3.59(653%)	1.66(302%)	0(0%)	-1.38(124%)	-0.28(34%)	-1.11(-202%)	-0.83(-151%)
2	3.87(-201%)	3.59(-260%)	3.59(1282%)	3.59(653%)	2.49(82%)	-0.55(28%)	-0.55(40%)	0(0%)	0(0%)	0(0%)
3	3.32(Inf%)	2.49(Inf%)	3.04(553%)	2.21(133%)	0.83(21%)	-0.83(-Inf%)	-1.11(-Inf%)	-1.11(-202%)	0(0%)	0(0%)
4	2.76(Inf%)	-0.83(-Inf%)	0.83(100%)	-0.83(-30%)	-0.28(-8%)	-0.83(-Inf%)	-0.28(-Inf%)	0(0%)	-0.28(-10%)	0(0%)
High B/M	1.38(-166%)	-1.38(-124%)	1.38(Inf%)	0(0%)	3.32(109%)	-0.55(66%)	0(0%)	0(NaN%)	0(0%)	0.55(18%)
Low Inv	0.83(-38%)	-1.66(302%)	1.38(83%)	-0.55(-40%)	1.11(134%)	-0.55(25%)	0.28(-51%)	0(0%)	0(0%)	0(0%)
2	1.93(-87%)	1.11(202%)	1.93(233%)	2.49(75%)	-0.55(-17%)	-1.11(50%)	-0.83(-151%)	-2.21(-266%)	0(0%)	0(0%)
3	-1.11(-Inf%)	1.66(593%)	1.11(202%)	2.49(889%)	1.66(55%)	0(NaN%)	0(0%)	0(0%)	-0.55(-196%)	-0.28(-9%)
4	1.38(166%)	1.11(-134%)	1.66(Inf%)	1.66(120%)	4.15(500%)	-0.55(-66%)	-1.38(166%)	-1.38(-Inf%)	-0.28(-20%)	-0.55(-66%)
High Inv	3.32(-604%)	4.42(-320%)	0(0%)	4.7(-1679%)	1.11(-202%)	0(0%)	0(0%)	0(0%)	-1.11(396%)	0.83(-151%)
Low OP	-0.28(9%)	2.21(-115%)	2.21(-402%)	0.55(-40%)	2.49(300%)	0(0%)	-1.38(72%)	-0.28(51%)	0(0%)	-0.28(-34%)
2	1.38(-166%)	1.66(150%)	1.38(-493%)	-0.55(-Inf%)	0.28(51%)	0.28(-34%)	-0.55(-50%)	-1.38(493%)	0.28(Inf%)	1.38(251%)
3	-0.55(-50%)	0.83(-151%)	3.59(162%)	3.87(117%)	-2.21(-80%)	-1.38(-124%)	-0.28(51%)	0(0%)	0(0%)	-0.28(-10%)
4	0.28(-20%)	2.49(-889%)	2.76(143%)	0.55(40%)	2.21(100%)	-1.11(80%)	-2.21(789%)	-0.55(-28%)	-0.28(-20%)	0(0%)
High OP	0.83(Inf%)	2.21(402%)	2.21(133%)	0.83(50%)	-0.28(-20%)	-1.66(-Inf%)	-1.66(-302%)	-1.11(-67%)	0(0%)	0.55(40%)

Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0(0%)	0.83(-75%)	0.28(-34%)	0.28(51%)	-0.55(-100%)	0(0%)	-0.55(50%)	0(0%)	-1.11(-202%)	0(0%)
2	-0.28(15%)	-0.55(40%)	0(0%)	0(0%)	0.28(9%)	0.28(-15%)	-0.28(20%)	0(0%)	-0.83(-151%)	0(0%)
3	0(NaN%)	-0.28(-Inf%)	-0.55(-100%)	0(0%)	0(0%)	0(NaN%)	0.28(Inf%)	0(0%)	-0.28(-17%)	0(0%)
4	0.28(Inf%)	0(NaN%)	0(0%)	0(0%)	0(0%)	-0.55(-Inf%)	0(NaN%)	0.55(66%)	0(0%)	-0.28(-8%)
High B/M	-1.66(200%)	0(0%)	0(NaN%)	-0.28(-10%)	-0.28(-9%)	-0.55(66%)	-0.55(-50%)	-0.83(-Inf%)	-0.28(-10%)	0.55(18%)
Low Inv	-0.83(38%)	-0.28(51%)	-0.28(-17%)	0(0%)	0(0%)	1.66(-75%)	-0.28(51%)	0.28(17%)	0(0%)	0(0%)
2	-0.83(38%)	-0.55(-100%)	0.55(66%)	0.28(8%)	0.28(8%)	-0.28(13%)	0.28(51%)	1.11(134%)	0(0%)	0.83(25%)
3	0(NaN%)	-0.28(-100%)	0(0%)	0(0%)	1.38(45%)	-0.83(-Inf%)	-0.28(-100%)	-0.28(-51%)	0(0%)	0.28(9%)
4	-0.55(-66%)	0(0%)	0.83(Inf%)	0(0%)	-0.28(-34%)	0(0%)	0(0%)	-0.28(-Inf%)	0(0%)	0.55(66%)
High Inv	-0.28(51%)	0(0%)	0(0%)	0(0%)	0.28(-51%)	0.28(-51%)	0.28(-20%)	0(0%)	0(0%)	-1.93(351%)
Low OP	0(0%)	0(0%)	-0.55(100%)	0(0%)	0(0%)	-0.28(9%)	-0.55(28%)	0.28(-51%)	-1.11(80%)	0.28(34%)
2	-0.55(66%)	0(0%)	0(0%)	-0.28(-Inf%)	-1.66(-302%)	-1.66(200%)	0(0%)	-0.28(100%)	0(NaN%)	-0.55(-100%)
3	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)	0.83(75%)	0(0%)	0(0%)	0.55(17%)	0(0%)
4	0(0%)	-0.83(296%)	0.55(28%)	-0.28(-20%)	0(0%)	0(0%)	0.55(-196%)	-0.55(-28%)	0(0%)	0(0%)
High OP	0.55(Inf%)	0.28(51%)	-0.28(-17%)	0.83(50%)	0(0%)	-1.11(-Inf%)	0.28(51%)	0.28(17%)	-0.28(-17%)	-1.11(-80%)

Portfolios	CMA					Constant				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.55(-13%)	0(0%)	1.38(-166%)	0.83(151%)	-1.11(-202%)	-1.93(47%)	-2.21(199%)	-2.49(300%)	-1.93(-351%)	0(0%)
2	-2.21(115%)	-0.55(40%)	-0.83(-296%)	-2.21(-402%)	-0.28(-9%)	-2.76(143%)	-1.38(100%)	-2.21(-789%)	0(0%)	-0.55(-18%)
3	0(NaN%)	0(NaN%)	-0.28(-51%)	-0.83(-50%)	-0.28(-7%)	-1.93(-Inf%)	-1.38(-Inf%)	-2.76(-502%)	-0.55(-33%)	1.66(43%)
4	-0.55(-Inf%)	0(NaN%)	-0.55(-66%)	-1.93(-70%)	0.28(8%)	-0.83(-Inf%)	0.83(Inf%)	-1.11(-134%)	0.83(30%)	2.21(67%)
High B/M	-0.83(100%)	0(0%)	-1.11(-Inf%)	-2.21(-80%)	-0.28(-9%)	0(0%)	2.21(199%)	0(NaN%)	2.76(100%)	-2.49(-82%)
Low Inv	-1.38(62%)	0(0%)	-0.28(-17%)	-1.66(-120%)	-0.55(-66%)	-1.66(75%)	1.11(-202%)	-0.55(-33%)	1.93(140%)	-1.66(-200%)
2	-2.21(100%)	0.28(51%)	-1.66(-200%)	-1.38(-42%)	-2.49(-75%)	-0.83(38%)	0(0%)	-0.28(-34%)	0.28(8%)	1.38(42%)
3	-0.55(-Inf%)	-0.55(-196%)	-0.55(-100%)	0(0%)	-1.93(-63%)	1.93(Inf%)	-0.55(-196%)	-1.11(-202%)	-2.76(-986%)	-1.66(-55%)
4	-2.76(-333%)	-0.55(66%)	-1.38(-Inf%)	-2.21(-160%)	-2.21(-266%)	3.59(433%)	0(0%)	0.55(Inf%)	1.11(80%)	-2.21(-266%)
High Inv	-0.83(151%)	0.28(-20%)	1.66(-100%)	0.28(-100%)	-0.83(151%)	-1.38(251%)	-5.25(380%)	-2.49(150%)	-3.32(1186%)	0(0%)
Low OP	-1.11(37%)	0.55(-28%)	0.55(-100%)	0.28(-20%)	-0.55(-66%)	0(0%)	-1.11(58%)	-1.66(302%)	0.83(-60%)	-1.93(-233%)
2	-2.76(333%)	-0.55(-50%)	0.28(-100%)	-1.11(-Inf%)	-0.83(-151%)	1.66(-200%)	-1.11(-100%)	-0.28(100%)	-0.28(-Inf%)	0.83(151%)
3	-1.38(-124%)	-0.28(51%)	0(0%)	-1.11(-33%)	0.28(10%)	2.76(249%)	-1.66(302%)	-3.32(-150%)	-2.49(-75%)	2.76(100%)
4	-0.28(20%)	0(0%)	-0.55(-28%)	-0.28(-20%)	0.28(13%)	-0.55(40%)	-0.28(100%)	-1.11(-58%)	0(0%)	-2.21(-100%)
High OP	0(NaN%)	-0.55(-100%)	-0.28(-17%)	-1.93(-116%)	0.28(20%)	1.38(Inf%)	-0.55(-100%)	0.55(33%)	1.38(83%)	-1.11(-80%)

Table 15: Detailed Decomposition of 95th quantile difference (Composition)

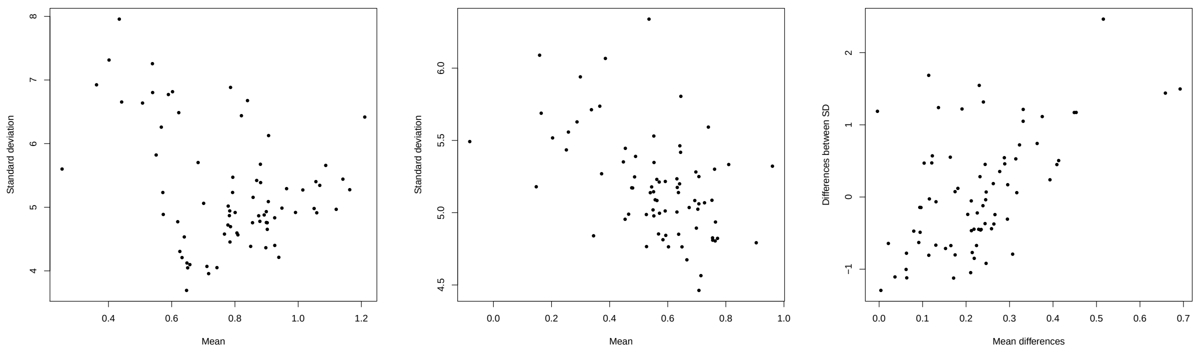
Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0(0%)	0(0%)	-0.55(-100%)	-0.28(100%)	-0.55(66%)	1.38(62%)	1.11(-202%)	0(0%)	0(0%)	-0.28(34%)
2	0.55(-196%)	0.28(25%)	-0.55(-100%)	0.55(196%)	-0.83(75%)	0.83(-296%)	0.55(50%)	0.83(151%)	0.28(100%)	-0.83(75%)
3	0(0%)	-0.28(-34%)	-0.83(-296%)	-0.28(25%)	-0.28(51%)	1.66(75%)	0.83(100%)	0.55(196%)	0(0%)	0(0%)
4	-0.55(33%)	-1.38(493%)	-1.11(100%)	-0.28(51%)	-0.83(151%)	1.11(-67%)	1.38(-493%)	0(0%)	-0.28(51%)	-0.28(51%)
High B/M	-0.55(-66%)	-1.11(-202%)	-0.55(50%)	-1.11(67%)	-0.28(15%)	1.93(233%)	0.83(151%)	0.55(-50%)	0(0%)	-0.83(43%)
Low Inv	0(0%)	-0.83(-151%)	-1.11(-Inf%)	-0.55(196%)	-0.55(196%)	0.55(100%)	1.11(202%)	0.83(Inf%)	0(0%)	0(0%)
2	-0.55(-50%)	-1.11(-396%)	-0.55(50%)	-0.83(75%)	-0.83(75%)	1.93(174%)	0.83(296%)	0.55(-50%)	0(0%)	-0.55(50%)
3	-1.38(-493%)	-0.83(75%)	-0.83(-Inf%)	-0.55(100%)	-0.83(60%)	1.66(593%)	0.83(-75%)	0.55(Inf%)	0(0%)	-0.55(40%)
4	0(0%)	-0.28(-34%)	-0.55(40%)	-0.28(-Inf%)	-0.55(-50%)	0.83(-151%)	0.83(100%)	0.55(-40%)	0.28(Inf%)	-0.28(-25%)
High Inv	0.28(20%)	0(0%)	-0.28(20%)	-1.38(493%)	0(0%)	1.11(80%)	1.38(124%)	0.28(-20%)	0.28(-100%)	0(0%)
Low OP	-0.28(-Inf%)	0(0%)	0(0%)	0(0%)	0(0%)	0.55(Inf%)	1.11(80%)	1.11(67%)	0.28(34%)	0(0%)
2	0(0%)	-3.59(-1282%)	-1.38(-Inf%)	-0.55(-Inf%)	-0.83(60%)	1.11(-396%)	3.04(1086%)	1.11(Inf%)	0(NaN%)	-0.55(40%)
3	-0.28(100%)	-0.83(-296%)	-1.11(-202%)	-0.28(20%)	-0.28(-Inf%)	0.28(-100%)	1.11(396%)	0.55(100%)	0(0%)	-1.38(-Inf%)
4	-0.83(296%)	-1.93(-351%)	-0.55(-Inf%)	-0.55(50%)	0(0%)	1.38(-493%)	1.93(351%)	-0.28(-Inf%)	0.28(-25%)	-0.83(50%)
High OP	0(0%)	-0.55(-100%)	-0.55(-Inf%)	-0.55(50%)	-0.83(33%)	1.66(67%)	0(0%)	0(NaN%)	0(0%)	0(0%)
Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	1.11(50%)	-0.28(51%)	0.83(151%)	0.83(-296%)	0.28(-34%)	0.28(13%)	0.28(-51%)	0(0%)	0(0%)	0(0%)
2	-0.28(100%)	0(0%)	0(0%)	0.55(196%)	0.28(-25%)	0.28(-100%)	0(0%)	0(0%)	0(0%)	-0.28(25%)
3	0(0%)	0(0%)	-0.55(-196%)	-0.55(50%)	-1.11(202%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
4	0.28(-17%)	0(0%)	0(0%)	0(0%)	-0.83(151%)	0(0%)	0(0%)	-0.55(50%)	0(0%)	0.28(-51%)
High B/M	-0.55(-66%)	0.83(151%)	-1.66(150%)	-0.28(17%)	0(0%)	0.28(34%)	0(0%)	0(0%)	0.28(-17%)	0(0%)
Low Inv	0(0%)	0(0%)	-0.28(-Inf%)	-0.83(296%)	-0.83(296%)	0(0%)	0(0%)	0(NaN%)	0.28(-100%)	0.83(-296%)
2	-0.55(-50%)	0(0%)	-0.28(25%)	0(0%)	0(0%)	0(0%)	0(0%)	-0.28(25%)	-0.28(25%)	0(0%)
3	0(0%)	0(0%)	0.28(Inf%)	0(0%)	0(0%)	0(0%)	0(0%)	-0.28(-Inf%)	0(0%)	0(0%)
4	0(0%)	0(0%)	0(0%)	0(NaN%)	0.55(50%)	0(0%)	0(0%)	0(0%)	0(NaN%)	-0.28(-25%)
High Inv	-0.28(-20%)	0(0%)	0.28(-20%)	1.66(-593%)	-0.83(-100%)	0.55(40%)	1.11(100%)	0(0%)	-0.28(100%)	0(0%)
Low OP	0.28(Inf%)	0(0%)	-0.28(-17%)	0.28(34%)	0(0%)	0.28(Inf%)	0(0%)	0.28(17%)	0(0%)	0(0%)
2	-0.55(196%)	0.55(196%)	0(NaN%)	0.28(Inf%)	0(0%)	0.28(-100%)	0(0%)	0(NaN%)	0(NaN%)	0.55(-40%)
3	0(0%)	-0.28(-100%)	-0.55(-100%)	0.28(-20%)	0.28(Inf%)	0(0%)	0(0%)	0(0%)	-0.28(20%)	-0.28(-Inf%)
4	-0.55(196%)	0.55(100%)	0.28(Inf%)	0(0%)	-0.28(17%)	0(0%)	0(0%)	-0.28(-Inf%)	-0.28(25%)	0.28(-17%)
High OP	-1.38(-55%)	0(0%)	0(NaN%)	0(0%)	0.28(-11%)	0(0%)	0(0%)	-0.55(-Inf%)	0(0%)	-0.28(11%)
Portfolios	CMA									
	Small	2	3	4	Big					
Low B/M	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)					
2	0.28(-100%)	0(0%)	0(0%)	0(0%)	0(0%)					
3	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)					
4	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)					
High B/M	0(0%)	0(0%)	0(0%)	-0.28(17%)	0(0%)					
Low Inv	0(0%)	0(0%)	0(NaN%)	-0.55(196%)	0(0%)					
2	0(0%)	0.28(100%)	0(0%)	0(0%)	0.28(-25%)					
3	0.28(100%)	0(0%)	0(NaN%)	0(0%)	0(0%)					
4	0(0%)	0(0%)	0(0%)	0(NaN%)	0(0%)					
High Inv	0(0%)	-1.11(-100%)	0(0%)	0.28(-100%)	0.83(100%)					
Low OP	0.28(Inf%)	0(0%)	0(0%)	0(0%)	0(0%)					
2	0(0%)	-0.55(-196%)	0(NaN%)	0(NaN%)	0(0%)					
3	0(0%)	0(0%)	0(0%)	0(0%)	-0.28(-Inf%)					
4	0(0%)	0(0%)	0(NaN%)	-0.28(25%)	0.28(-17%)					
High OP	0.83(33%)	0(0%)	0.28(Inf%)	0(0%)	0(0%)					

Table 16: Detailed Decomposition of 95th quantile difference (Structure)

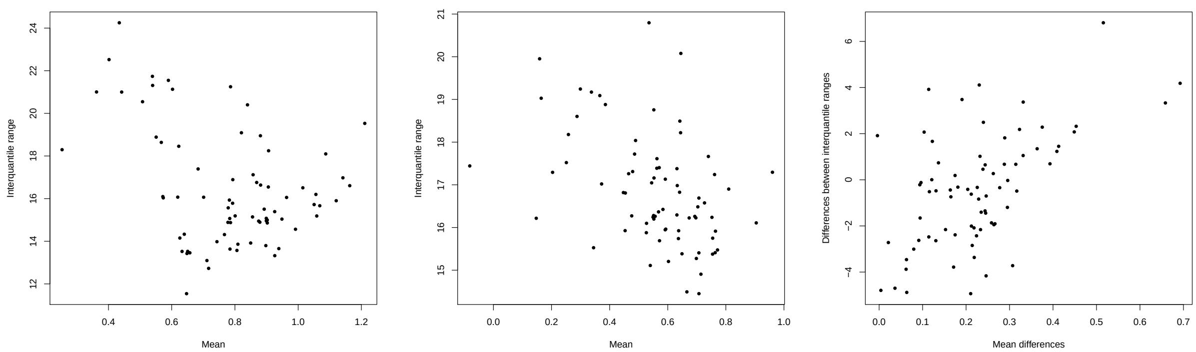
Portfolios	Mkt.RF					SMB				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-3.59(-162%)	-1.66(302%)	-0.55(-100%)	1.66(-593%)	0(0%)	0.83(38%)	0(0%)	0(0%)	0(0%)	-0.55(66%)
2	-3.04(1086%)	-1.66(-150%)	-1.66(-302%)	3.32(1186%)	-0.28(25%)	0(0%)	-0.55(-50%)	0(0%)	0(0%)	-0.83(75%)
3	-1.66(-75%)	-1.93(-233%)	-0.28(-100%)	-0.83(75%)	-0.83(151%)	0.55(25%)	-0.28(-34%)	-0.83(-296%)	0(0%)	-0.28(51%)
4	-2.76(166%)	-1.11(396%)	-1.38(124%)	-0.55(100%)	-0.28(51%)	-0.83(50%)	0.55(-196%)	0(0%)	0.55(-100%)	0(0%)
High B/M	-1.38(-166%)	0.83(151%)	1.11(-100%)	1.11(-67%)	-0.28(15%)	-0.83(-100%)	0(0%)	0(0%)	0(0%)	-0.28(15%)
Low Inv	-2.76(-502%)	-0.28(-51%)	0.28(Inf%)	-0.55(196%)	0.83(-296%)	-0.28(-51%)	0(0%)	0.83(Inf%)	0.55(-196%)	0.28(-100%)
2	0(0%)	0.55(196%)	-1.38(124%)	-0.28(25%)	-0.55(50%)	1.11(100%)	1.11(396%)	0(0%)	0(0%)	0(0%)
3	-1.38(-493%)	-0.55(50%)	0(NaN%)	-2.21(402%)	-2.21(160%)	0.83(296%)	-0.28(25%)	0(NaN%)	0(0%)	0.28(-20%)
4	-0.55(100%)	-0.55(-66%)	-2.76(200%)	-1.11(-Inf%)	-1.11(-100%)	0.55(-100%)	-0.28(-34%)	0.83(-60%)	0(NaN%)	1.66(150%)
High Inv	-0.28(-20%)	-2.21(-199%)	-3.32(241%)	-1.66(593%)	0(0%)	0.28(20%)	0.55(50%)	-0.55(40%)	0.28(-100%)	-0.83(-100%)
Low OP	-2.76(-Inf%)	1.93(140%)	-2.76(-166%)	1.38(166%)	0.28(-34%)	-0.28(-Inf%)	0(0%)	0(0%)	0(0%)	0(0%)
2	-3.32(1186%)	0(0%)	-0.28(-Inf%)	-0.55(-Inf%)	0.55(-40%)	-0.28(100%)	0.28(100%)	0(NaN%)	0(NaN%)	0.28(-20%)
3	-1.93(689%)	-1.38(-493%)	0(0%)	-2.76(200%)	0(NaN%)	-0.55(196%)	0(0%)	0.28(51%)	0.28(-20%)	0.28(Inf%)
4	-3.32(1186%)	-1.38(-251%)	-1.11(-Inf%)	0(0%)	-1.93(116%)	0(0%)	0.28(51%)	0(NaN%)	0(0%)	0.28(-17%)
High OP	-0.55(-22%)	0.83(151%)	-1.38(-Inf%)	-1.38(124%)	-1.93(78%)	0.83(33%)	0(0%)	0(NaN%)	0.28(-25%)	-0.55(22%)
Portfolios	HML					RMW				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0(0%)	-0.28(51%)	-0.28(-51%)	0(0%)	-0.28(34%)	0(0%)	0(0%)	0(0%)	0(0%)	-0.28(34%)
2	0(0%)	0(0%)	0.28(51%)	0(0%)	0.28(-25%)	0(0%)	0(0%)	-0.28(-51%)	-1.38(-493%)	0.28(-25%)
3	0(0%)	0.28(34%)	0.83(296%)	0(0%)	1.11(-202%)	0(0%)	0(0%)	-0.55(-196%)	0(0%)	-0.28(51%)
4	0.28(-17%)	0.83(-296%)	1.11(-100%)	-0.28(51%)	0.55(-100%)	0(0%)	0(0%)	0.28(-25%)	-0.28(51%)	0.83(-151%)
High B/M	0.83(100%)	0.28(51%)	0(0%)	0.55(-33%)	0(0%)	0(0%)	0(0%)	-0.28(25%)	-1.38(83%)	0(0%)
Low Inv	0(0%)	0(0%)	0(NaN%)	0.28(-100%)	0.28(-100%)	0(0%)	0(0%)	0.55(Inf%)	0(0%)	0.28(-100%)
2	0(0%)	0.55(196%)	1.11(-100%)	0(0%)	-0.55(50%)	0(0%)	0(0%)	0(0%)	0(0%)	0(0%)
3	0(0%)	0.28(-25%)	-0.28(-Inf%)	-0.28(51%)	0(0%)	0(0%)	-0.28(25%)	-0.28(-Inf%)	0(0%)	0(0%)
4	-0.28(51%)	0.83(100%)	0(0%)	0(NaN%)	0.28(25%)	0.28(-51%)	0(0%)	0.28(-20%)	-0.28(-Inf%)	0(0%)
High Inv	0(0%)	-0.83(-75%)	-0.28(20%)	-0.28(100%)	0.55(66%)	0(0%)	0.28(25%)	-0.28(20%)	0(0%)	0(0%)
Low OP	0.55(Inf%)	0(0%)	0(0%)	-0.83(-100%)	0.28(-34%)	-0.28(-Inf%)	-0.55(-40%)	1.66(100%)	1.11(134%)	0(0%)
2	1.11(-396%)	1.11(396%)	0(NaN%)	0(NaN%)	0.28(-20%)	0(0%)	1.11(396%)	-0.55(-Inf%)	0(NaN%)	-0.28(20%)
3	0.55(-196%)	-0.28(-100%)	2.21(402%)	0.55(-40%)	0.28(Inf%)	0(0%)	-0.28(-100%)	-0.55(-100%)	0(0%)	0(NaN%)
4	0.28(-100%)	0.28(51%)	0.55(Inf%)	-0.28(25%)	0(0%)	0(0%)	0(0%)	-0.28(-Inf%)	0(0%)	-0.28(17%)
High OP	0(0%)	0(0%)	0.83(Inf%)	0.28(-25%)	0(0%)	0(0%)	0.28(51%)	0(NaN%)	0(0%)	0(0%)
Portfolios	CMA					Constant				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	-0.28(-13%)	-0.28(51%)	-0.28(-51%)	-1.66(593%)	1.11(-134%)	2.49(113%)	0.55(-100%)	1.38(251%)	-0.83(296%)	-0.28(34%)
2	0.55(-196%)	0(0%)	0.55(100%)	0.55(196%)	0.83(-75%)	0.55(-196%)	2.49(224%)	1.38(251%)	-3.59(-1282%)	0.28(-25%)
3	1.38(62%)	0.83(100%)	1.66(593%)	0(0%)	0.83(-151%)	0.28(13%)	1.38(166%)	0.28(100%)	0.55(-50%)	0.28(-51%)
4	0.83(-50%)	0(0%)	1.38(-124%)	2.21(-402%)	0.28(-51%)	0(0%)	-0.55(196%)	-0.83(75%)	-1.66(302%)	-0.28(51%)
High B/M	-0.28(-34%)	0.55(100%)	1.11(-100%)	0.83(-50%)	1.93(-100%)	1.38(166%)	-1.66(-302%)	-1.38(124%)	-1.38(83%)	-2.21(115%)
Low Inv	0(0%)	0.28(51%)	0.28(Inf%)	2.21(-789%)	0.28(-100%)	3.04(553%)	0.28(51%)	-1.38(-Inf%)	-1.11(396%)	-1.66(593%)
2	0.28(25%)	0.55(196%)	1.66(-150%)	0(0%)	3.04(-274%)	-1.11(-100%)	-2.49(-889%)	-1.93(174%)	0.28(-25%)	-1.93(174%)
3	0.83(296%)	1.38(-124%)	0.28(Inf%)	0(0%)	0(0%)	-0.55(-196%)	-1.66(150%)	0.55(Inf%)	2.49(-453%)	1.93(-140%)
4	0.55(-100%)	0(0%)	0.55(-40%)	0(NaN%)	0(0%)	-1.93(351%)	0.28(34%)	-0.28(20%)	1.38(Inf%)	0.83(75%)
High Inv	0.28(20%)	1.66(150%)	-0.83(60%)	-0.28(100%)	0.28(34%)	-0.55(-40%)	0.28(25%)	3.59(-260%)	1.11(-396%)	0.83(100%)
Low OP	0.55(Inf%)	1.66(120%)	0(0%)	0(0%)	1.11(-134%)	1.11(Inf%)	-2.76(-200%)	1.66(100%)	-1.38(-166%)	-2.49(300%)
2	0.28(-100%)	0.28(100%)	1.11(Inf%)	0(NaN%)	0.28(-20%)	1.11(-396%)	-1.93(-689%)	0(NaN%)	0.83(Inf%)	-1.66(120%)
3	0.28(-100%)	0.83(296%)	1.66(302%)	1.11(-80%)	0.55(Inf%)	1.38(-493%)	1.38(493%)	-1.93(-351%)	-0.28(20%)	0.83(Inf%)
4	0.55(-196%)	1.38(251%)	0.28(Inf%)	0(0%)	0(0%)	2.21(-789%)	-0.55(-100%)	1.38(Inf%)	0(0%)	0.83(-50%)
High OP	0(0%)	1.38(251%)	1.11(Inf%)	0.55(-50%)	1.93(-78%)	1.11(45%)	-1.38(-251%)	0.28(Inf%)	-0.28(25%)	-1.11(45%)

Figure 1: Mean returns and risks

(a) North America (b) Europe (c) Between



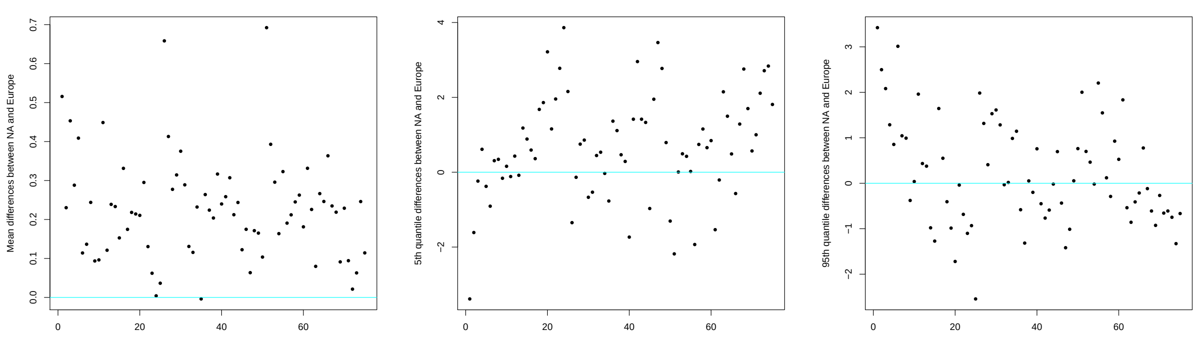
(d) North America (e) Europe (f) Between



Sources: Kenneth R. French's website

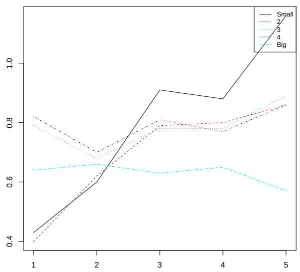
Figure 2: Mean and quantile difference of LHS portfolio returns between North America and Europe

(a) Mean (b) 5th quantile (c) 95th quantile

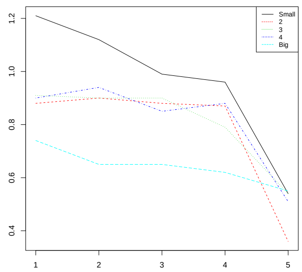


Sources: Kenneth R. French's website

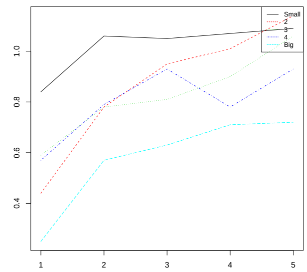
Figure 3: Patterns on Mean Returns



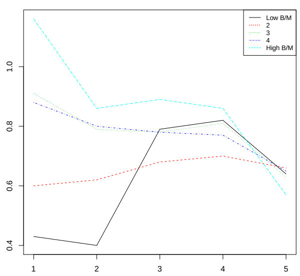
(a) L to H B/M (NA)



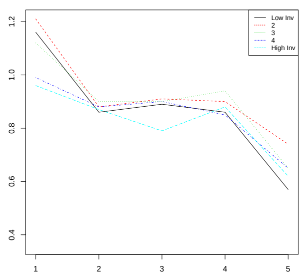
(b) L to H Inv (NA)



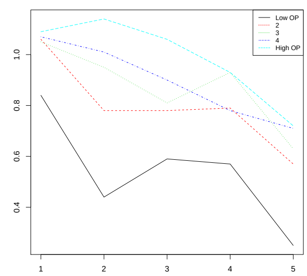
(c) L to H OP (NA)



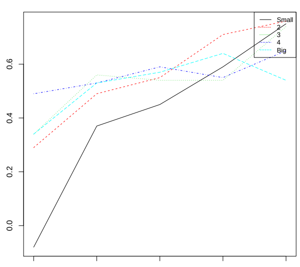
(d) S to B Size (NA)



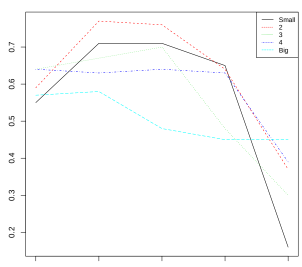
(e) S to B Size (NA)



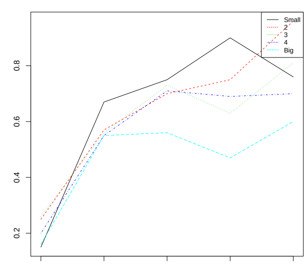
(f) S to B Size (NA)



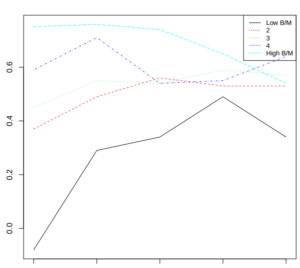
(g) L to H B/M (Europe)



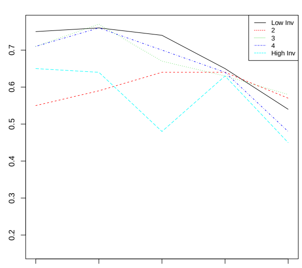
(h) L to H Inv (Europe)



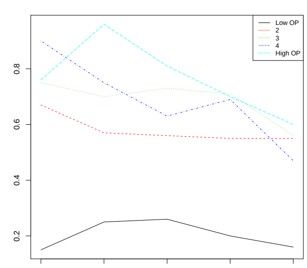
(i) L to H OP (Europe)



(j) S to B Size (Europe)



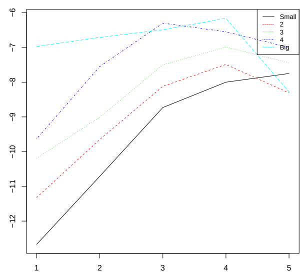
(k) S to B Size (Europe)



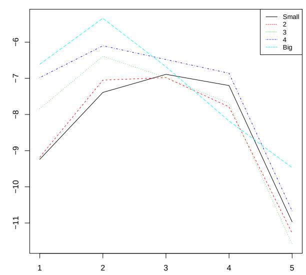
(l) S to B Size (Europe)

Sources: Kenneth R. French's website

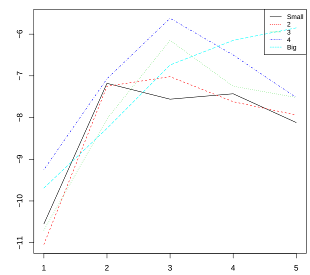
Figure 4: Patterns on 5th Quantile Returns



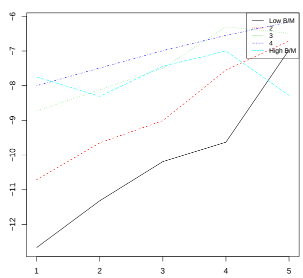
(a) L to H B/M (NA)



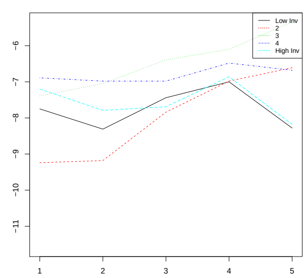
(b) L to H Inv (NA)



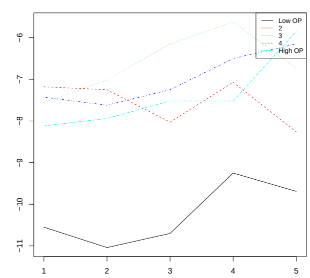
(c) L to H OP (NA)



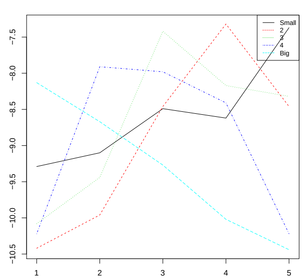
(d) S to B Size (NA)



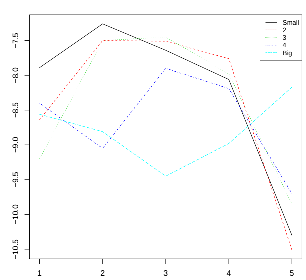
(e) S to B Size (NA)



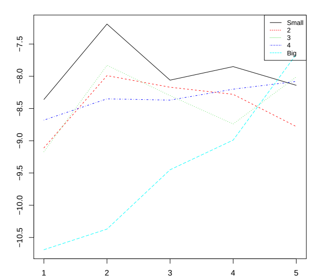
(f) S to B Size (NA)



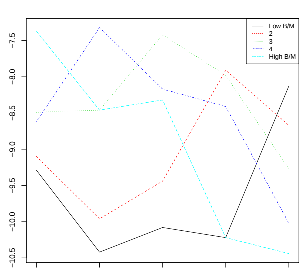
(g) L to H B/M (Europe)



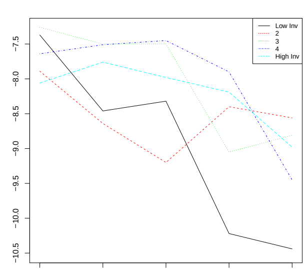
(h) L to H Inv (Europe)



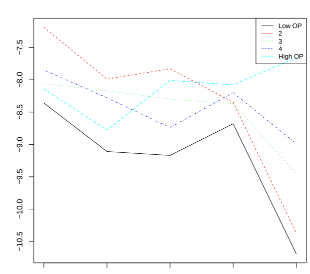
(i) L to H OP (Europe)



(j) S to B Size (Europe)



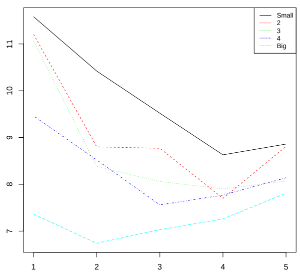
(k) S to B Size (Europe)



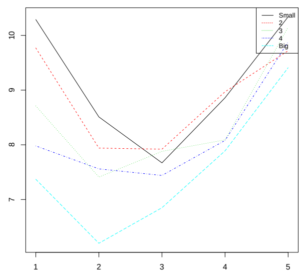
(l) S to B Size (Europe)

Sources: Kenneth R. French's website

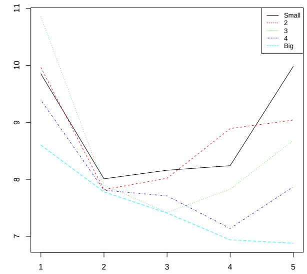
Figure 5: Patterns on 95th Quantile Returns



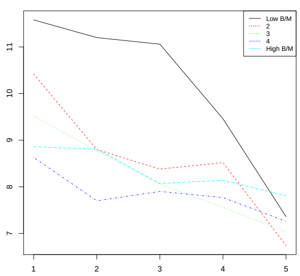
(a) L to H B/M (NA)



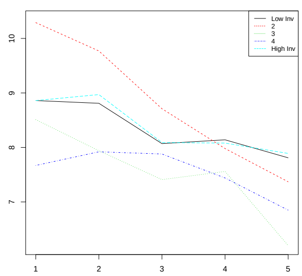
(b) L to H Inv (NA)



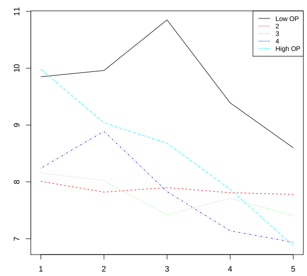
(c) L to H OP (NA)



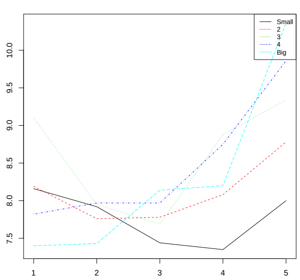
(d) S to B Size (NA)



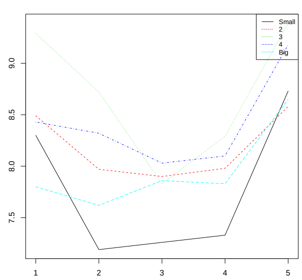
(e) S to B Size (NA)



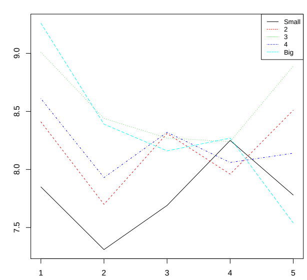
(f) S to B Size (NA)



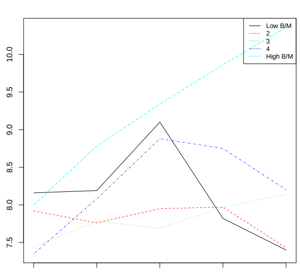
(g) L to H B/M (Europe)



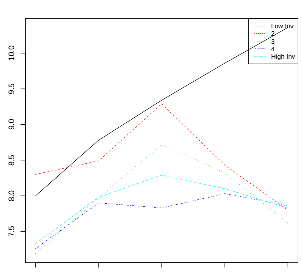
(h) L to H Inv (Europe)



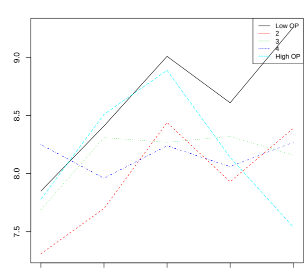
(i) L to H OP (Europe)



(j) S to B Size (Europe)



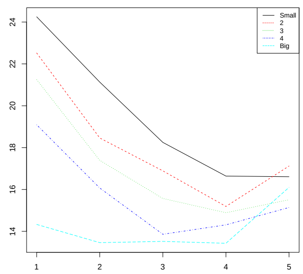
(k) S to B Size (Europe)



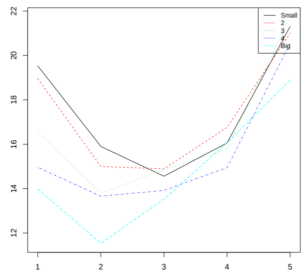
(l) S to B Size (Europe)

Sources: Kenneth R. French's website

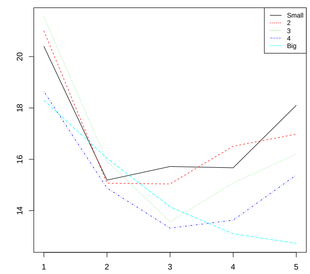
Figure 6: Patterns on 95-5th Interquantile Ranges



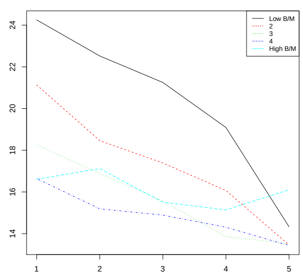
(a) L to H B/M (NA)



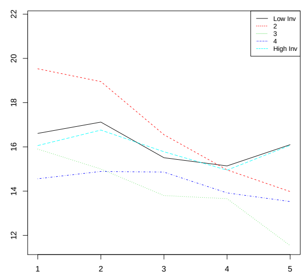
(b) L to H Inv (NA)



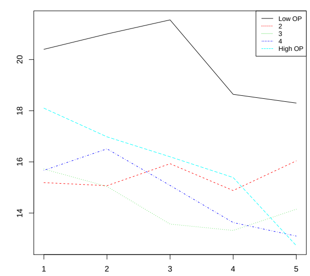
(c) L to H OP (NA)



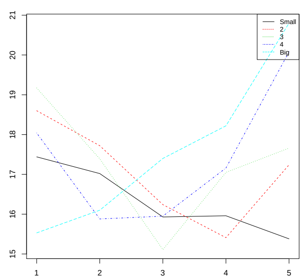
(d) S to B Size (NA)



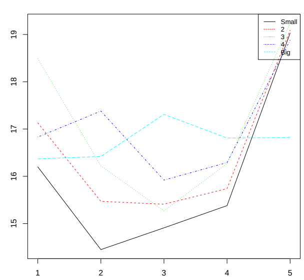
(e) S to B Size (NA)



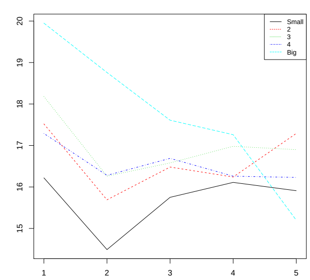
(f) S to B Size (NA)



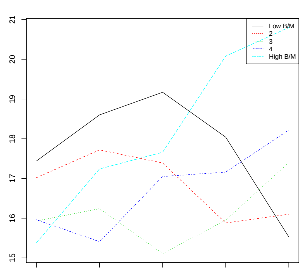
(g) L to H B/M (Europe)



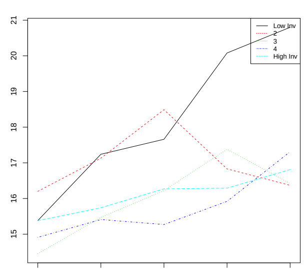
(h) L to H Inv (Europe)



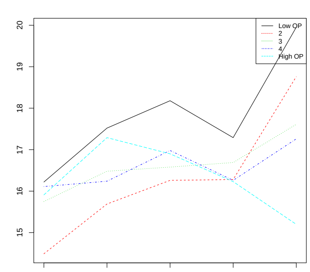
(i) L to H OP (Europe)



(j) S to B Size (Europe)



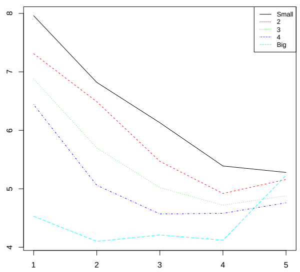
(k) S to B Size (Europe)



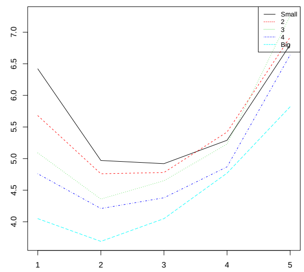
(l) S to B Size (Europe)

Sources: Kenneth R. French's website

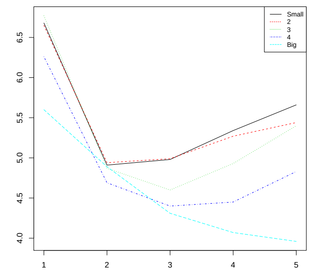
Figure 7: Patterns on Standard Deviation



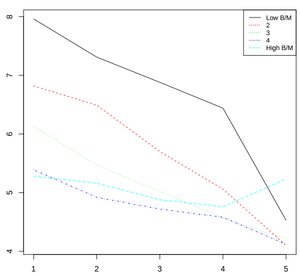
(a) L to H B/M (NA)



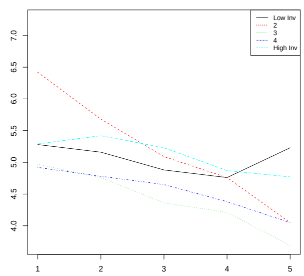
(b) L to H Inv (NA)



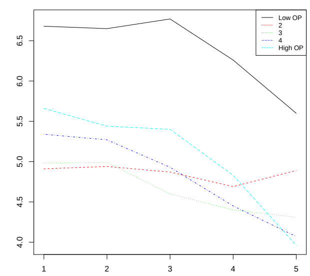
(c) L to H OP (NA)



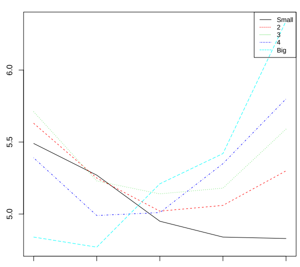
(d) S to B Size (NA)



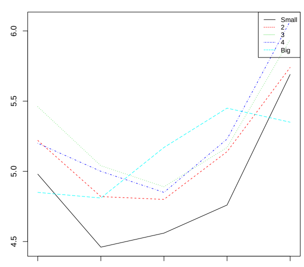
(e) S to B Size (NA)



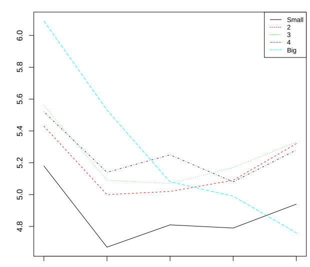
(f) S to B Size (NA)



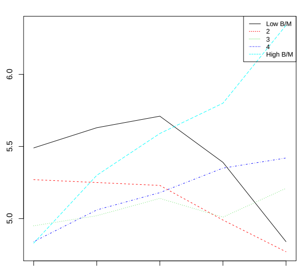
(g) L to H B/M (Europe)



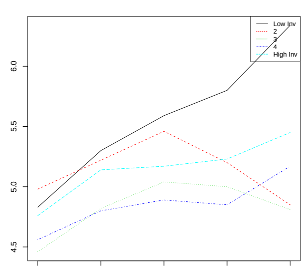
(h) L to H Inv (Europe)



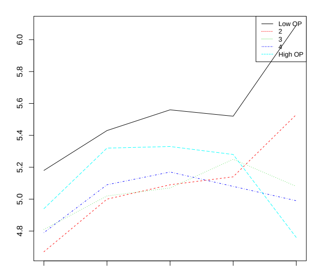
(i) L to H OP (Europe)



(j) S to B Size (Europe)



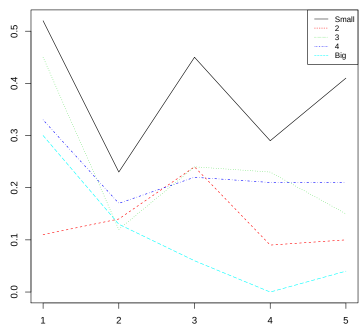
(k) S to B Size (Europe)



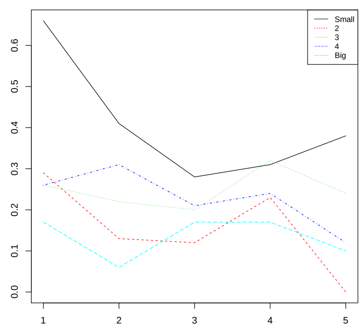
(l) S to B Size (Europe)

Sources: Kenneth R. French's website

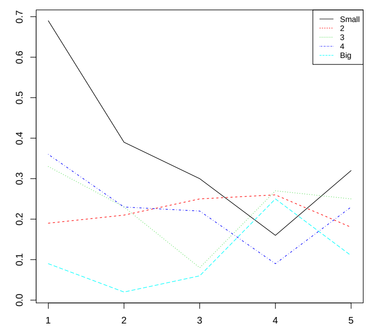
Figure 8: Mean difference patterns



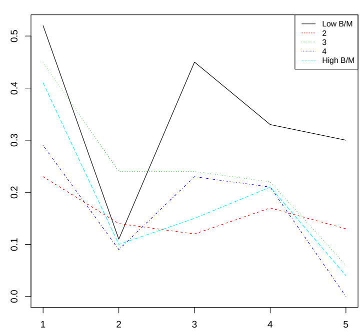
(a) L to H B/M



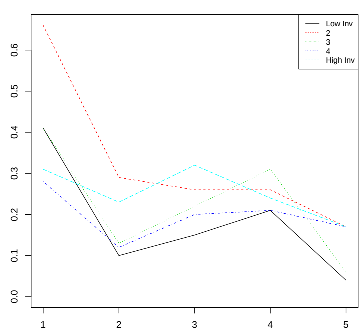
(b) L to H Inv



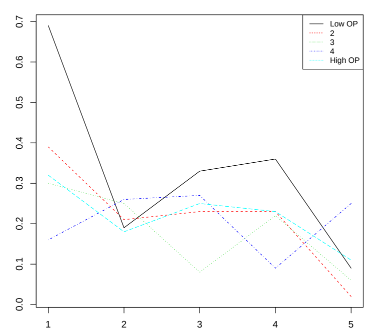
(c) L to H OP



(d) S to B Size



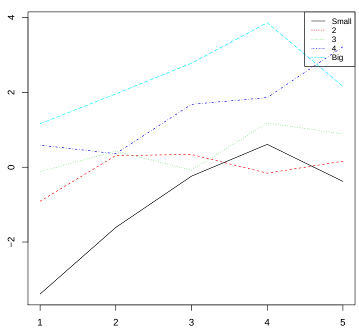
(e) S to B Size



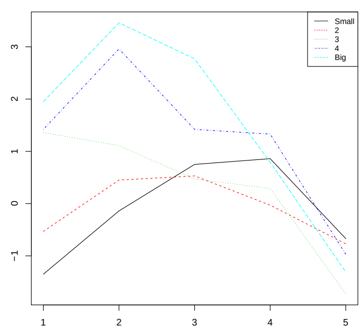
(f) S to B Size

Sources: Kenneth R. French's website

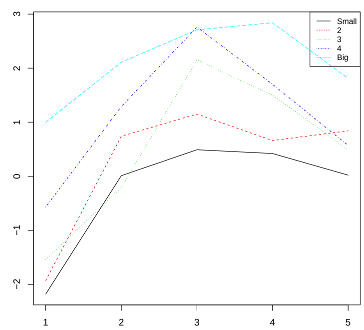
Figure 9: 5th quantile difference patterns



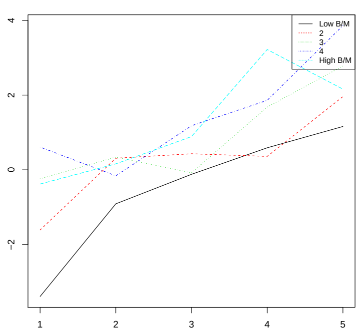
(a) L to H B/M



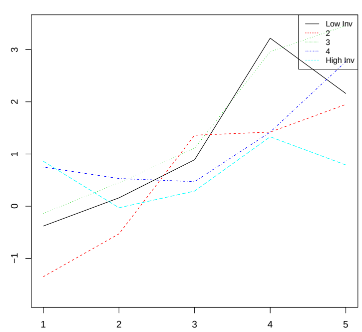
(b) L to H Inv



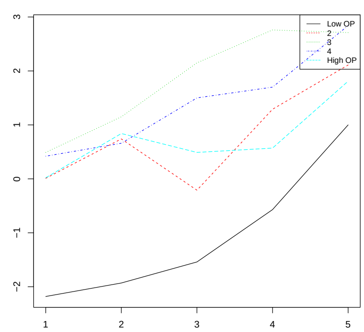
(c) L to H OP



(d) S to B Size



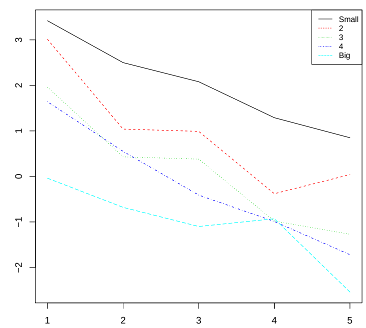
(e) S to B Size



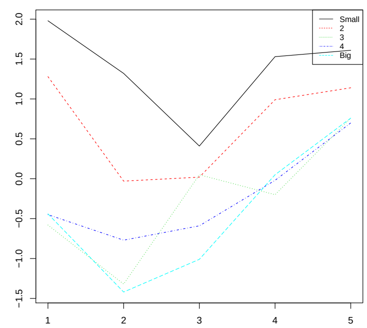
(f) S to B Size

Sources: Kenneth R. French's website

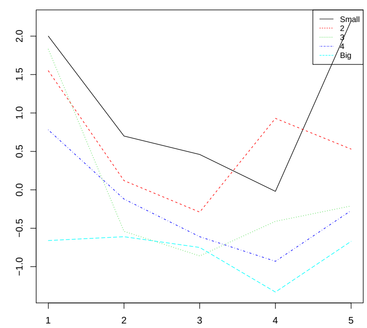
Figure 10: 95th quantile difference patterns



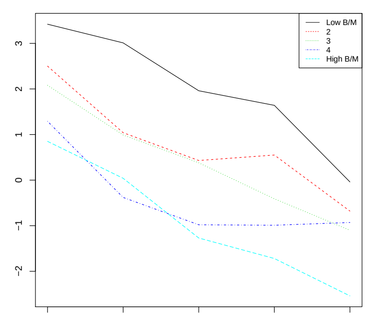
(a) L to H B/M



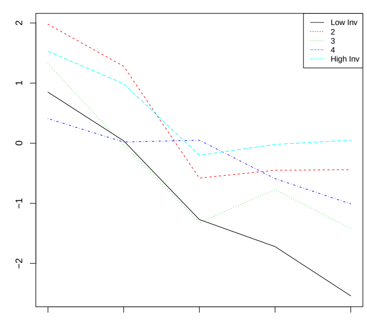
(b) L to H Inv



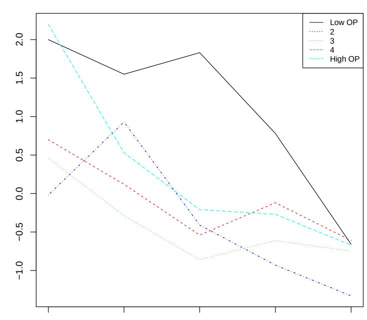
(c) L to H OP



(d) S to B Size



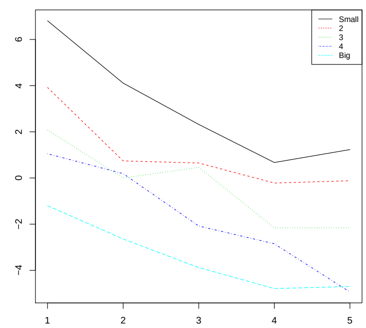
(e) S to B Size



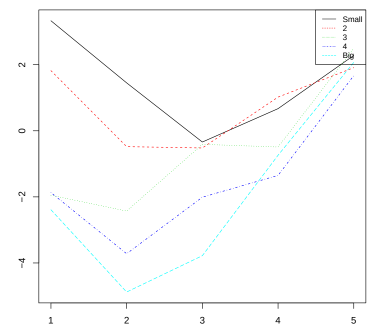
(f) S to B Size

Sources: Kenneth R. French's website

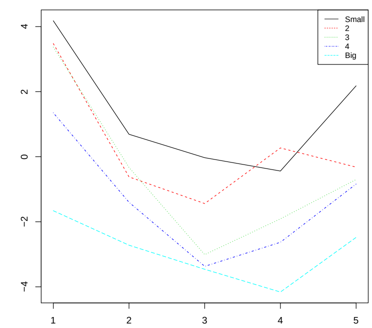
Figure 11: 95-5th interquantile difference patterns



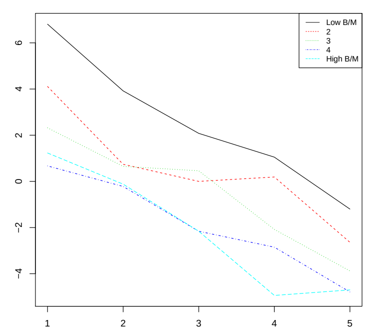
(a) L to H B/M



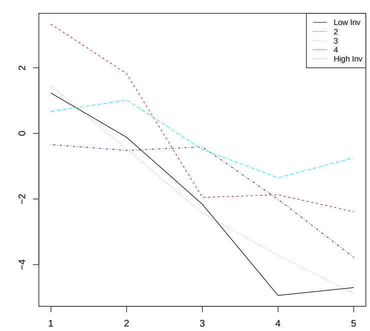
(b) L to H Inv



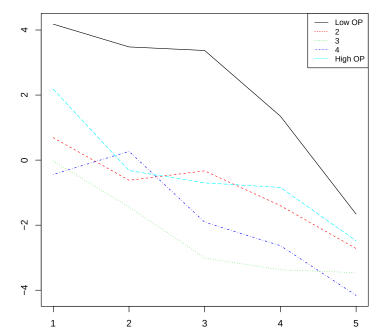
(c) L to H OP



(d) S to B Size



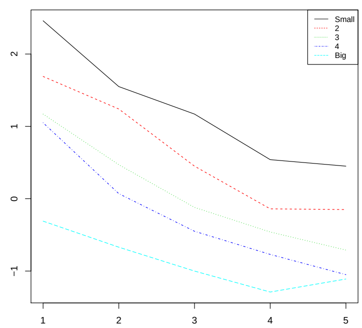
(e) S to B Size



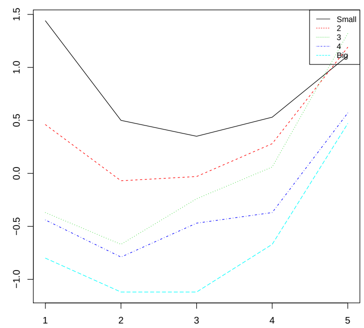
(f) S to B Size

Sources: Kenneth R. French's website

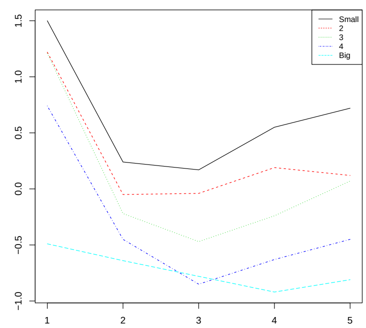
Figure 12: Standard deviation difference patterns



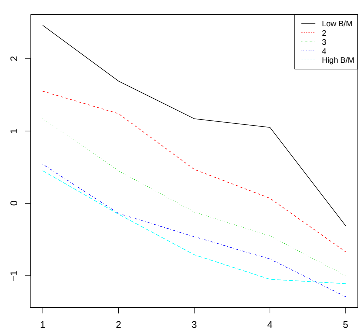
(a) L to H B/M



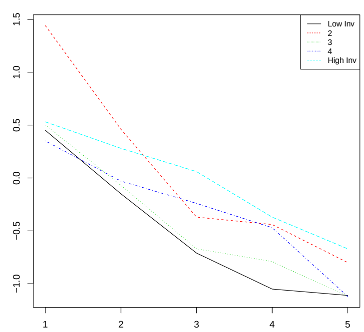
(b) L to H Inv



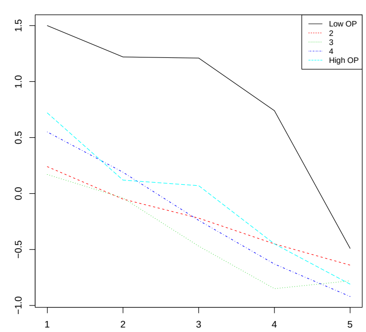
(c) L to H OP



(d) S to B Size



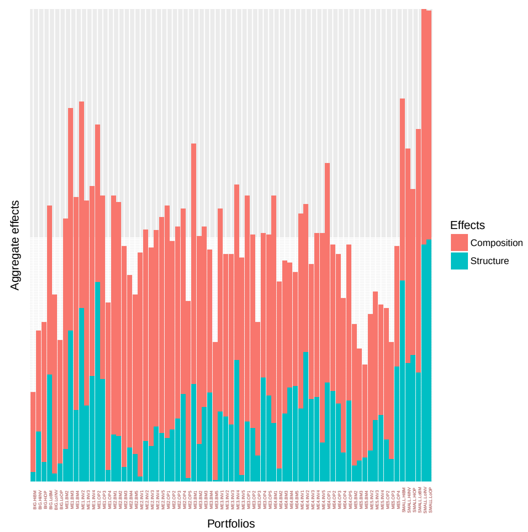
(e) S to B Size



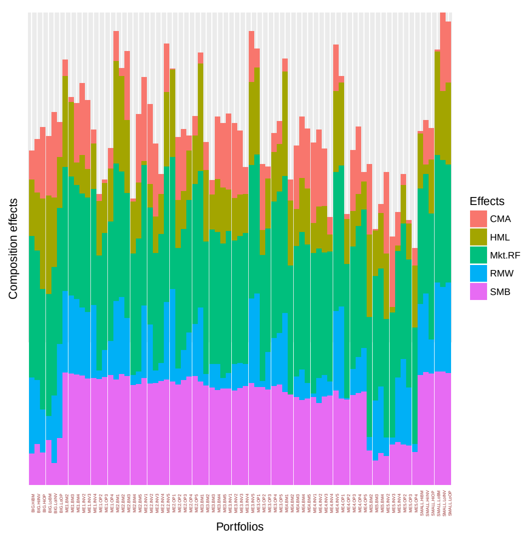
(f) S to B Size

Sources: Kenneth R. French's website

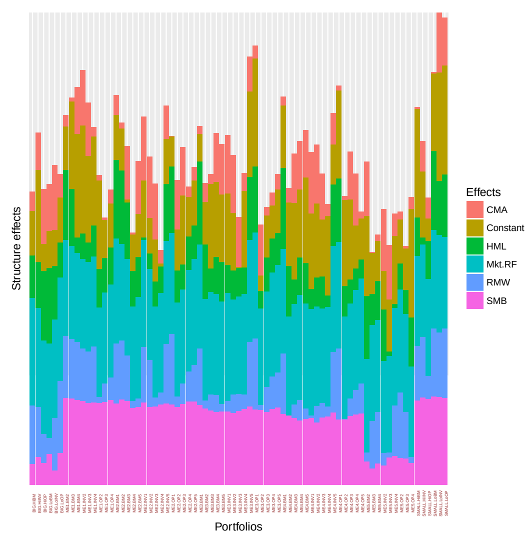
Figure 13: Decomposition of mean differences



(a) Aggregate



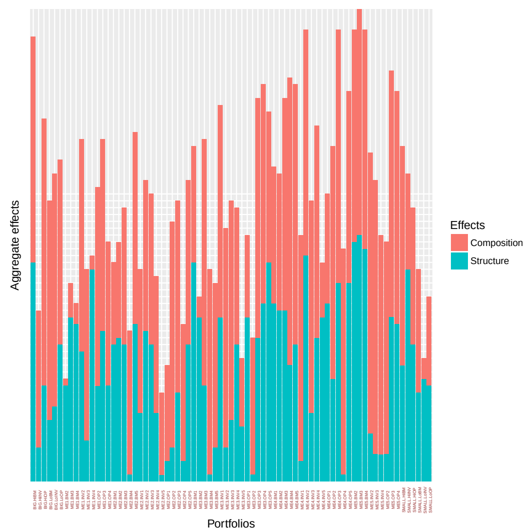
(b) Composition



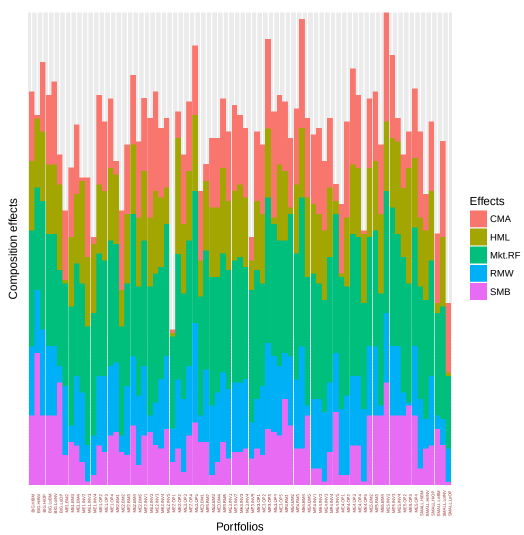
(c) Structure

Sources: Kenneth R. French's website

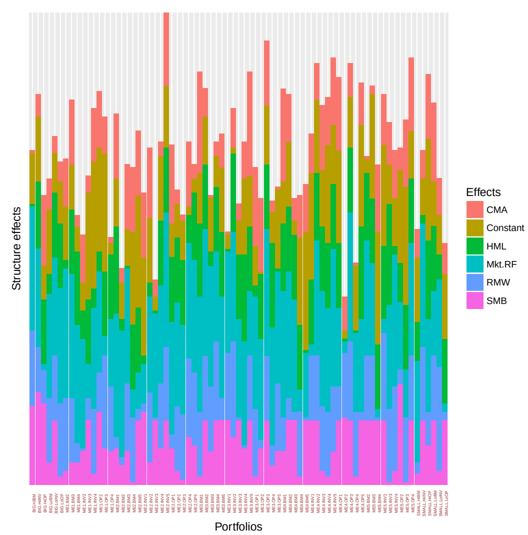
Figure 14: Decomposition of 5th quantile differences



(a) Aggregate



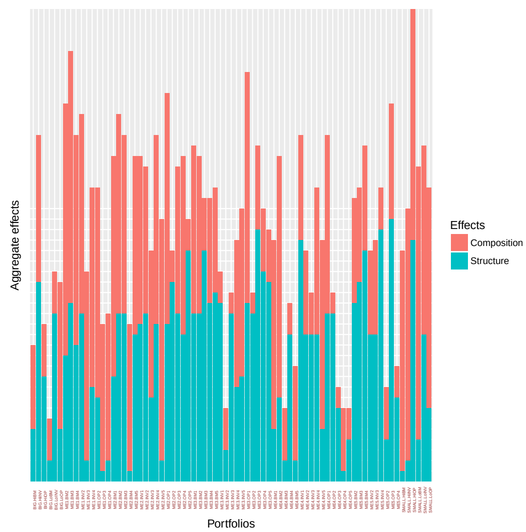
(b) Composition



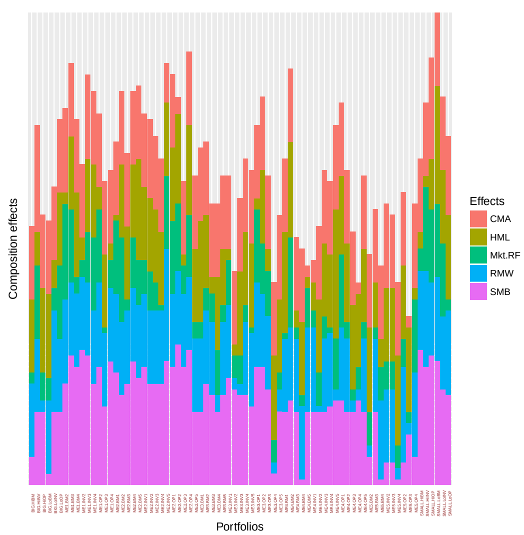
(c) Structure

Sources: Kenneth R. French's website

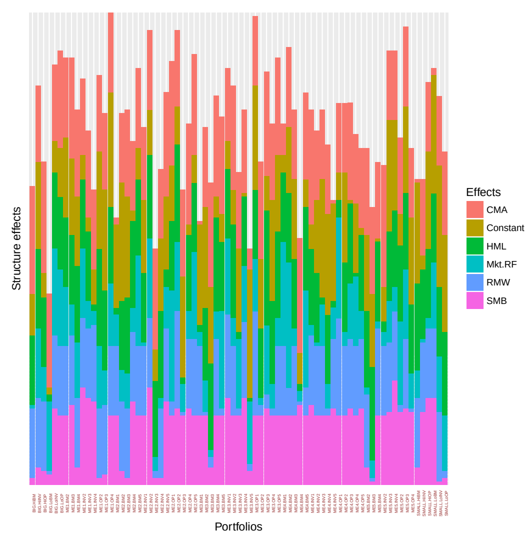
Figure 15: Decomposition of 95th quantile differences



(a) Aggregate



(b) Composition



(c) Structure

Sources: Kenneth R. French's website