Decomposing Differences in Portfolio Returns Between North America and Europe

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Abstract

The paper decomposes differences in mean and a series of quantiles of portfolio returns between North America and Europe into Fama and French's five factors. We show that the differences in risk premia on factors, especially on market and size factors, account for most of the differences and the differences in factor risks seem to play an insignificant role in aggregate. We also show that the roles that the risk premia on market and size factors play are various at different levels of portfolio returns, implying the market and size factor risk premia are various at different levels of portfolio returns. Also, we find that the risks on some factors seem to be various at different levels of portfolio returns. These findings shed further light on empirical asset pricing.

Keywords: Distribution Regression, Blinder-Oaxaca Decomposition, Decomposition Analysis, Five-factor Asset Pricing Model, Counterfactual Distributions, Risk Premium

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1 Introduction

We observe that there exist economically large differences in mean returns on portfolios formed from sorts on size and value, profitability, or investment between North America and Europe (Fama and French (2017)), although the differences are mostly insignificant in statistical terms due to large standard deviations (Table 2). The mean differences in factors between two regions are also large in practical terms and the correlations between factors in North America are quite different from the ones in Europe (Table 1). Applying Fama and French (2015) (FF(2015)) five-factor asset pricing model on the portfolios examined in the paper, we also find that the factor exposures are different between North America and Europe (not shown).

One interesting question raised is that what are the most important explanations accounting for these differences? To explore this question, we decompose these differences into FF(2015) five factors using Blinder-Oaxaca decomposition method (Oaxaca (1973) and Blinder (1973)).[1] More specifically, we decompose these differences into aggregate composition and structure components. Using the language of labor economics, the differences are attributed to two parts, one of which is the aggregate composition component and the other of which is the structure component. The aggregate composition component is due to the differences in the distributions of factors, i.e., factor risk premia, between North America and Europe. The structure component is due to the differences in the structures of factors (factor exposures), i.e., factor risks. Further, the detailed decomposition helps answer the following questions. Which factors are behind most of differences? To what extent have the differences in portfolio returns because of differences in market factor risk premium or in risk on investment factor?

We also observe that the differences in portfolio returns between North America and Europe are not normally distributed. Thus, we decompose the differences in a series of quantile portfolio returns between North America and Europe. The decomposition for a sequence of quantile differences helps explore some interesting questions. For instance, are the roles that the factors

[1] Blinder-Oaxaca decomposition method is widely used in labor economics, see e.g., . Blinder-Oaxaca decomposition can be found in financial economics literature, e.g., Wang and Hanna (2007), Alesina, Lotti, and Mistrulli (2013), Robb, Fairlie, and Robinson (2014), Shin and Hanna (2015), Aristei and Gallo (2016), Brown and Previtero (2014), Kabir and Shakur (2014), Montecino, Epstein, et al. (2015) among others. Füss, Gietzen, and Rindler (2011) decompose the bond spreads over the course of the crisis to study the impact of changes in risk perception. However, we are not aware of any other papers which decompose the equity returns. Thus, we add decomposition analysis to the literature on empirical asset pricing.

play in explaining the differences various at different levels of portfolio returns? More specifically, is the aggregate composition component various at different levels of portfolio returns? Is the composition component associated to market factor various at different levels of portfolio returns? Is the structure component linked to investment factor changing at different levels of portfolio returns? These questions are of great interesting because they are equivalent to asking the following questions. Is the role that the factor risk premia play in explaining differences in quantile portfolio returns various at different levels of portfolio returns? Is the role that the market factor risk premium plays various at different levels of portfolio returns? Is the role that the investment factor risk plays various at different levels of portfolio returns? The answers to these questions could have implications for the question that whether the factor risks or risk premia are various at different levels of portfolio returns. Thus, the examinations on these questions could shed further light on empirical asset pricing by adding to the literature on variations of risks or risk premia.[2]

To decompose the quantiles, one usually needs to deal with nonlinear models. However, in the conventional Blinder-Oaxaca decomposition the dependent variable is linear with the coefficients and thus the detailed decomposition is path independent. Gomulka and Stern (1990), Fairlie (2005) and Bauer and Sinning (2008) extends the Blinder-Oaxaca decomposition technique to nonlinear models like probit and logit. For the nonlinear case, the decomposition could be sensitive to the order of decomposition, namely the results will be path dependent. Decomposing differences in quantile portfolio returns is also related to the literature on decomposing general distributional statistics using flexible methods, see e.g., DiNardo, Fortin, and Lemieux (1995), Gosling, Machin, and Meghir (2000), Donald, Green, and Paarsch (2000), Barsky, Bound, Charles, and Lupton (2002), Machado and Mata (2005), Rothe (2010), and Chernozhukov, Fernández-Val, and Melly (2013) among others. Fortin, Lemieux, and Firpo (2011) offers a comprehensive overview on the decomposition methods in economics.

The components in the decomposition for quantile differences can be computed by a series of counterfactual distributions based on distribution regressions. Distribution regression is a continuum of binary regressions. Chernozhukov, Fernández-Val, and Melly (2013) show that distribution regression provides a flexible model for the entire conditional distribution and also establish the

[2]See, e.g., Gilbert, Hrdlicka, and Kamara (2018), Graham and Harvey (2018) and among others. Gilbert, Hrdlicka, and Kamara (2018) show exposures to SMB and HML vary with firms' earnings announcement month and Graham and Harvey (2018) show risk premia are higher during recessions and higher during periods of uncertainty.

central limit theorems. In the paper, we use distribution regressions to decompose differences in a series of quantile portfolio returns.

Also, in the process of decomposing quantile differences, we estimate a series of counterfactual distributions which are of interesting in their own right. To better predict the returns and describe precisely the risks, one needs to estimate the higher-order multidimensional structure of the portfolio returns (Rothschild and Stiglitz (1971)).[3] With the entire distribution of portfolio returns, one can estimate any higher-order structure. Also, with a series of counterfactual distributions one is able to test functional hypotheses such as no-effect, positive effect, or stochastic dominance had the marginal distribution of the market factor in North America been replaced by the one in Europe. These tests are of interest in their own right. For size limitation, however, we focus on the decomposition analysis in the paper.

Notice that FF(2015) five-factor model is linear, which can not capture the nonlinear relationship between portfolio returns and factors. In the paper, we use distribution regressions to estimate the entire distribution of portfolio returns and then quantiles. Distribution regressions help capture the non-linear relationship between portfolio returns and factors. Distribution regressions estimate well the quantiles of returns on portfolios studied in the paper (not shown). Also, we show that the observed quantile differences in portfolio returns between North America and Europe fit the true quantile differences well, especially in the low quantiles. Thus, we believe distribution regressions do a good job in describing the entire distribution of portfolio returns.

In the paper, we use Blinder-Oaxaca decomposition method to decompose differences in mean portfolio returns. The decomposition components for differences in a series of quantile portfolio returns are computed via the observed distributions and a series of counterfactual distributions based on distribution regressions. We also compute the components in the decomposition for mean differences using distribution regressions. We compare the decomposition results for mean differences from two methods, in which the former are path independent and the later are path dependent.

We find that the differences in mean portfolio returns are mostly due to the differences in risk

[3]The mean-variance analysis based on conditional first and second moments of asset returns are sufficient to inform investors' choice only under special assumptions, such as multivariate normality of asset returns or quadratic utility function of investors. However, the returns on portfolios examined in this paper are not normally distributed (not shown).

premia on factors, especially market and size factors, and the differences in factor risks seem to play an insignificant role in aggregate.[4] The decompositions for differences in a series of quantile portfolio returns also show that the quantile differences are mostly due to the differences in risk premia on factors, especially market and size factors, and the differences in factor risks in aggregate explain little of the quantile differences. We also find that the roles that the market and size factor risk premia play in explaining differences are various at different levels of portfolio returns. The results are robust to the changes in the structures used as reference and the decomposition orders. The roles that the risks on some factors play also seem to be various at different levels of portfolio returns. These findings imply that the factor risks and risk premia on factors are various at different levels of portfolio returns, shedding further light on empirical asset pricing.

We organize the rest of the paper as follows. Section 2 describes FF(2015) five-factor asset pricing model. Section 3 shows the decomposition for mean differences using Blinder-Oaxaca decomposition method. Section 4 describes the decomposition for differences in general distributional statistics. In Section 5, we present the decomposition for differences in quantiles and inference processes. Section 6 describes data and show the main results. Section 7 presents several robustness checks. We conclude in Section 8.

2 Fama and French's five-factor model

There is much evidence that average stock returns are related to overall market performance (Sharpe (1964), Lintner (1965), and Breeden (1979)), firm's size (Banz (1981)), value (Basu (1983) and Rosenberg, Reid, and Lanstein (1985)), profitability (Haugen and Baker (1996) and Novy-Marx (2013)) and investment (Titman, Wei, and Xie (2004), Cooper, Gulen, and Schill (2008), and Aharoni, Grundy, and Zeng (2013)). [5] Motivated by the dividend discount valuation model, FF(2015) test a five-factor model that adds profitability and investment factors into the market, size and value-growth factors of Fama and French (1993) three-factor model. Using international

[4]We note an important limitation of decomposition from Fortin, Lemieux, and Firpo (2011): "A second important limitation is that while decompositions are useful for quantifying the contribution of various factors to a difference or change in outcomes in an accounting sense, they may not necessarily deepen our understanding of the mechanisms underlying the relationship between factors and outcomes."

[5]There are more research including Breeden, Gibbons, and Litzenberger (1989), Reinganum (1981), Cohen, Gompers, and Vuolteenaho (2002), Fairfield, Whisenant, and Yohn (2003), Fama and French (2006), Fama and French (2008), Hou, Xue, and Zhang (2015), and Fama and French (2016) and others.

data, Fama and French (2017) study international markets including North America and Europe, and show the five-factor model largely absorbs the patterns in average returns.

The FF(2015) five-factor time-series regression is

$$R_{it} = a_i + b_i Mkt_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \tag{2.1}$$

where e_{it} is a zero-mean residual and

- R_{it} is the return on portfolio i for month t in excess of risk-free rate (the one-month US Treasury bill rate).

- Mkt_t is the value-weight market portfolio return minus the risk-free rate.

- SMB_t (size factor), HML_t (value factor), RMW_t (profitability factor) and CMA_t (investment factor) are differences between the returns on diversified portfolios of small and big stocks, high and low B/M stocks, stocks with robust and weak profitability, and stocks with low and high (conservative and aggressive) investment, respectively.

- a_i is α for portfolio i and b_i, s_i, h_i, r_i and c_i are factor exposures.

Remark 1. *In empirical asset pricing, expected factor returns are factor risk premia and factor exposures measures factor risks.*

Remark 2. *The factors and the portfolios in the left hand side of the regression can be constructed using various sorts. In the paper, we use the factors from 2×3 sorts on size and B/M, profitability or investment. The 75 portfolios used to decompose are portfolios constructed from 5×5 sorts for each region. The detail will be present in Section 6.*

Remark 3. *One concern about the factor model is that the role of size and book-to-market factors in the three-factor model is spurious, arising only because size and book-to-market portfolios are used for both dependent and explanatory returns. Fama and French (1993) use split-sample tests to address this concern.*

Remark 4. *Factors are meant to mimic the underlying risk factors in returns related to the variable sorted. For example, the value factor mimics the risk factor in returns related to book-to-market equity ratio. The value, profitability, and investment factors are different mixes of size,* *value, profitability, and investment effects in returns because of the correlations between the size, value, profitability, and investment variables used to construct factors.*[6]

For the sake of clarity in decomposition, we simplify the notation by dropping the subscript i and t and adding the subscript g, $g = 0, 1$, in which 0 denotes North America and 1 represents Europe. Then, we have

$$Y_g = X_g'\beta_g + e_g \tag{2.2}$$

where Y_g denotes the excess portfolio returns on a portfolio in region g, X_g is a vector of a constant and five factors in region g and e_g is error term. Hereafter, we mean the excess portfolio returns when the portfolio returns are mentioned. The distinction will be made whenever emphasis is necessary.

3 Blinder-Oaxaca decomposition

This section shows how to decompose the difference in mean portfolio returns between North America and Europe using conventional Blinder-Oaxaca decomposition method. We run regression 2.2 for each portfolio in North America and Europe separately.[7] With estimates $\hat{\beta}_g$ for each portfolio and using the structures in North America, $\hat{\beta}_0$, as reference, the observed overall mean difference in portfolio returns between North America and Europe can be decomposed as follows

$$\begin{aligned}
\hat{\Delta}_O &= \hat{E}(Y_0) - \hat{E}(Y_1) \\
&= \bar{X}_0'\hat{\beta}_0 - \bar{X}_1\hat{\beta}_1 \\
&= \underbrace{(\bar{X}_0' - \bar{X}_1')\hat{\beta}_0}_{\hat{\Delta}_C} + \underbrace{\bar{X}_1'(\hat{\beta}_0 - \hat{\beta}_1)}_{\hat{\Delta}_S} \\
&= \sum_{j=1}^{6} \underbrace{(\bar{X}_{0j} - \bar{X}_{1j})\hat{\beta}_{0j}}_{\hat{\Delta}_{C_j}} + \sum_{j=1}^{6} \underbrace{\bar{X}_{1j}(\hat{\beta}_{0j} - \hat{\beta}_{1j})}_{\hat{\Delta}_{S_j}}
\end{aligned}$$

where

[6] High B/M value stocks tend to have low profitability and investment, and low B/M growth stocks, especially large low B/M stocks, tend to be profitable and invest aggressively. (Fama and French (1995))

[7] We show the decomposition results by averaging out the results for all portfolios.

- $\hat{\Delta}_O$ is the observed overall mean difference in the portfolio returns between North America and Europe.

- $\bar{X}_g$ is the average factor returns in region g.

- $\hat{\Delta}_C = (\bar{X}_0' - \bar{X}_1')\hat{\beta}_0$ and $\hat{\Delta}_S = \bar{X}_1'(\hat{\beta}_0 - \hat{\beta}_1)$ are the aggregate composition and structure components, respectively.

- $\hat{\Delta}_{C_j} = (\bar{X}_{0j} - \bar{X}_{1j})\hat{\beta}_{0j}$ and $\hat{\Delta}_{S_j} = \bar{X}_{1j}(\hat{\beta}_{0j} - \hat{\beta}_{1j})$ are the detailed composition and structure components linked to the jth factor, respectively.

We can see that the aggregate composition component is due to the differences in average factor returns and the aggregate structure component is due to the differences in factor exposures. The detailed composition component associated to jth factor is due to the difference in averages of jth factor returns and the detailed structure component is due to the difference in jth factor exposures. That is, the composition components are due to differences in factor risk premia and the structure components are due to differences in factor risks. We note that $\hat{\Delta}_{C_1}$ is equal to zero because the first element in each factor vector X_g is a constant. Using Blinder-Oaxaca decomposition, the decomposition results for the mean difference are path independent, that is, the detailed decomposition results are not affected by the order of decomposition. We also decompose the mean difference using distribution regression below, in which the results are path dependent. We compare two results.

4 Decomposition for general distributional statistics

In this section, we first introduce several kinds of counterfactul distributions which are useful in decomposition. We then present the processes of decomposing the difference in general distributional statistics including quantile as a special case.

4.1 Counterfactual distributions

We take as the primary building blocks of counterfactuals the conditional distribution of portfolio returns in region g, $F_{Y_g|X_g}(y|x)$, and the distribution of X_g. Notice that the observed distribution of returns on a portfolio in region g is given by

$$F_{Y\langle g|g\rangle}(y) = \int F_{Y_g|X_g}(y|x)dF_{X_g}(x) \tag{4.1}$$

that is, $F_{Y\langle g|g\rangle}(y)$ is the same as integrating the conditional distribution of portfolio returns in region g over the distribution of X_g.

We obtain a counterfactual of $F_{Y\langle g|g\rangle}(y)$ as we change the distribution of X_g or the conditional distribution, $F_{Y_g|X_g}(y|x)$. Let $F_{Y\langle 0|1\rangle}(y)$ represent the counterfactual distribution of returns on a portfolio which has the distribution of X_1 and the conditional distribution, $F_{Y_0|X_0}(y|x)$. We obtain $F_{Y\langle 0|1\rangle}(y)$ by integrating the conditional distribution of portfolio returns in North America with respect to the distribution of factors in Europe as follows

$$F_{Y\langle 0|1\rangle}(y) \equiv \int F_{Y_0|X_0}(y|x)dF_{X_1}(x). \tag{4.2}$$

We can see the difference between this counterfactual distribution, $F_{Y\langle 0|1\rangle}(y)$, and the observed distribution, $F_{Y\langle 1|1\rangle}(y)$, is only on the conditional distributions used to compute the distributions. Or, the difference between this counterfactual distribution, $F_{Y\langle 0|1\rangle}(y)$, and the observed distribution, $F_{Y\langle 0|0\rangle}(y)$, is only on the distributions of X used to compute the distributions.

In this paper, we decompose differences in mean and quantiles of portfolio returns. Especially in the quantile cases, we need to sequentially estimate the detailed components and thus suffer from path dependent problems. Therefore, we first select the order in which the decomposition is performed. For the main results in this paper, we calculate the detailed components in the following order: the constant, market, size, value, profitability and investment factors. Accordingly, we have five composition components and six structure components.[8] To compute detailed decomposition components of differences in quantile portfolio returns, we define the following counterfactual distributions.

Let $X_{(0,1,j)}, j = 1 \cdots 6$ denote a vector of factors in which the first j factors of X_0 have been replaced with the first j factors of X_1, holding the last $6 - j$ factors of X_0 constant. For instance, $X_{(0,1,3)}$ denotes a vector of factors in which the constant term, market factor, and size factor are from Europe and the value, profitability and investment factors from North America. Note that

[8]Note that the first composition component linked to the constant term is zero.

$X_{(0,1,6)} = X_1$ and $X_{(1,0,6)} = X_0$. Let $F_{X_{(0,1,j)}}(x)$ represent the joint distribution of factors $X_{(0,1,j)}$.

Let $F_{Y\langle 0|(0,1,j)\rangle}(y)$ represent the counterfactual distribution of returns on a portfolio which has the conditional distribution, $F_{Y_0|X_0}(y|x)$, and the distribution of $X_{(0,1,j)}$. $F_{Y\langle 0|(0,1,j)\rangle}(y)$ can be estimated by integrating $F_{Y_0|X_0}(y|x)$ over $X_{(0,1,j)}$ as follows

$$F_{Y\langle 0|(0,1,j)\rangle}(y) \equiv \int F_{Y_0|X_0}(y|x)dF_{X_{(0,1,j)}}(x). \tag{4.3}$$

Let $F_{Y_{(0,1,j)}|X}(y|x)$ denote the counterfactual conditional distribution of returns on a portfolio which has the structures associated to the first j factors in $F_{Y_0|X_0}(y|x)$ replaced by the structures associated to the first j factors in $F_{Y_1|X_1}(y|x)$, holding constant the structures associated to the last $6-j$ factors in $F_{Y_0|X_0}(y|x)$. For instance, $F_{Y_{(0,1,3)}|X}(y|x)$ denotes the conterfactual conditional distribution of returns on a portfolio that has the structures linked to the constant term, market and size factors in North America replaced by the structures linked to the ones in Europe, holding constant the structures associated to the value, profitability and investment factors in North America. Note that $F_{Y_{(0,1,6)}|X}(y|x) = F_{Y_1|X_1}(y|x)$ and $F_{Y_{(1,0,6)}|X}(y|x) = F_{Y_0|X_0}(y|x)$.

Let $F_{Y\langle(0,1,j)|1\rangle}(y)$ represent the counterfactual distribution of returns on a portfolio which has the conditional distribution, $F_{Y_{(0,1,j)}|X}(y|x)$, and the distribution of X_1. $F_{Y\langle(0,1,j)|1\rangle}(y)$ can be obtained by integrating $F_{Y_{(0,1,j)}|X}(y|x)$ over X_1 as follows

$$F_{Y\langle(0,1,j)|1\rangle}(y) \equiv \int F_{Y_{(0,1,j)}|X}(y|x)dF_{X_1}(x). \tag{4.4}$$

4.2 Decomposing differences in general distributional statistics

Using the structures in North America as reference, i.e., using $F_{Y\langle 0|1\rangle}(y)$ to compute aggregate components, the overall difference in a distributional statistic of portfolio returns between North America and Europe can be decomposed as follows

$$
\begin{aligned}
\Delta_O^\nu &= \nu\left(F_{Y\langle 0|0\rangle}(y)\right) - \nu\left(F_{Y\langle 1|1\rangle}(y)\right) \\
&= \underbrace{\left(\nu\left(F_{Y\langle 0|0\rangle}(y)\right) - \nu\left(F_{Y\langle 0|1\rangle}(y)\right)\right)}_{\Delta_C^\nu} + \underbrace{\left(\nu\left(F_{Y\langle 0|1\rangle}(y)\right) - \nu\left(F_{Y\langle 1|1\rangle}(y)\right)\right)}_{\Delta_S^\nu} \\
&= \sum_{j=1}^{6} \underbrace{\left(\nu\left(F_{Y\langle 0|(0,1,j-1)\rangle}(y)\right) - \nu\left(F_{Y\langle 0|(0,1,j)\rangle}(y)\right)\right)}_{\Delta_{C_j}^\nu} + \sum_{j=1}^{6} \underbrace{\left(\nu\left(F_{Y\langle (0,1,j-1)|1\rangle}(y)\right) - \nu\left(F_{Y\langle (0,1,j)|1\rangle}(y)\right)\right)}_{\Delta_{S_j}^\nu}
\end{aligned}
\tag{4.5}
$$

where

- $\nu\left(F_{Y\langle\cdot|\cdot\rangle}(y)\right)$ is a distributional statistic of the distribution, $F_{Y\langle\cdot|\cdot\rangle}(y)$.
- Δ_C^ν and Δ_S^ν are aggregate composition and structure components, respectively.
- $\Delta_{C_j}^\nu$ and $\Delta_{S_j}^\nu$ are detailed composition and structure components linked to the jth factor.

Notice that $\Delta_{C_1}^\nu$ is always equal to zero because $F_{Y\langle 0|(0,1,0)\rangle}(y) = F_{Y\langle 0|(0,1,1)\rangle}(y)$ for the first element in the factor vector is a constant term. Also, $\nu\left(F_{Y\langle 0|(0,1,0)\rangle}(y)\right) = \nu\left(F_{Y\langle 0|0\rangle}(y)\right)$ and $\nu\left(F_{Y\langle 0|1\rangle}(y)\right) = \nu\left(F_{Y\langle 0|(0,1,6)\rangle}(y)\right) = \nu\left(F_{Y\langle (0,1,0)|1\rangle}(y)\right)$.

Remark 5. *To compute aggregate decomposition components, we select the structures in North America as reference, that is, we use $F_{Y\langle 0|1\rangle}(y)$ to compute aggregate components. Alternatively, one can select the structures in Europe as reference, i.e., using $F_{Y\langle 1|0\rangle}(y)$ to compute aggregate components. The decomposition results are possibly different. In this paper, we show the main results using the structures in North America as reference and the results using the structures in Europe as reference are shown as a robustness check.*

Remark 6. *We compute detailed components sequentially based on a series of counterfactual distributions. This approach suffers from the inevitable path dependent problem, that is, the decomposition results depend on the order in which the decomposition is performed. In the main results, we calculate detailed components following the order: the constant term, market, size, value, profitability and investment factors. The order chosen is consistent with the order of the factors being added into FF(2015) five-factor model. In the empirical asset pricing literature, researchers first study market factor in CAPM. Fama and French (1993) introduce the size and* *value factors and FF(2015) add the profitability and investment factors. We also show the results of switching the decomposition order as robustness checks.*

Remark 7. *We focus on decomposing the differences in quantile portfolio returns, that is, ν is a quantile function. One can readily extend to decompose the differences in general distributional statistics of the distribution of portfolio returns, such as variance, between two regions. In the paper, we also decompose the differences in mean portfolio returns by distribution regressions. Note that the results from distribution regressions are path dependent.*

5 Decomposition of differences in quantiles and inference

In this section, we firstly introduce distribution regression, which is useful for estimating the entire distribution of an outcome variable. Secondly, we use distribution regressions to estimate a series of observed (counterfactual) distributions shown in Equation 4.5, which are used to compute distributional statistics including quantiles. Thirdly, we present decomposition for the difference in quantile and mean portfolio returns. Lastly, we describe the inference processes.

5.1 Distribution regression

In the literature, there are two main approaches to estimate entire distribution, which are distribution regression and quantile regression. We use distribution regression in this paper.[9] Distribution regression consists of a continuum of binary regressions to the data. Foresi and Peracchi (1995) propose a fixed number of binary regressions to partially describe the conditional distribution of equity excess returns. Chernozhukov, Fernández-Val, and Melly (2013) show that a continuum of binary regressions provide a coherent and flexible model for the entire conditional distribution and also establish the central limit theorems.

[9] To estimate entire distribution, quantile regression (Koenker and Bassett Jr (1978) and Koenker (2005)) could be a reasonable alternative approach. The results in this paper would be similar had we used quantile regression. In the first step, obtain estimates of conditional quantiles. Step 2 uses plug-in to compute a series of conditional (counterfactual) quantiles. Step 3 inverts conditional quantiles to obtain the conditional distributions. From there, decomposition for differences in other distributional statistics would be exactly the same. Our approach models the conditional distributions using distribution regression and involves inverting the distributions. For more about quantile regression and its applications in estimating distribution, see Buchinsky (1994), Gosling, Machin, and Meghir (2000), Machado and Mata (2005), Melly (2005) and among others. See also Chapter 20 by Linton and Xiao in Koenker, Chernozhukov, He, and Peng (2017) for quantile regression applications in finance.

The main idea of distribution regression is to estimate a continuum of binary response models using $\mathbb{1}\{Y \leq y\}$ as the dependent variable while varying y. To implement the distribution regression estimator, we estimate a series of logit models over a fine grid of possible values for y, that is

$$
\begin{aligned}
F_{Y_g|X_g}(y|x) =& E[\mathbb{1}\{Y_g \leq y\}|X_g = x] \\
=& \Lambda(x'\beta_g(y))
\end{aligned}
$$

where Λ is a known link function – we use the logistic link function, $\Lambda(u) = \frac{e^u}{1+e^u}$, though one could make some other choice here. $\beta_g(y)$ are unknown parameters corresponding to each y, i.e., the parameters $\beta_g(y)$ change as y changes. $\mathbb{1}\{Y_g \leq y\}$ is an indicator function that equals one if $Y_g \leq y$ is true and zero otherwise. The estimated conditional distribution is given by

$$
\hat{F}_{Y_g|X_g}(y|x) = \Lambda(x'\hat{\beta}_g(y)) \tag{5.1}
$$

For fixed y, estimating $F_{Y\langle g|g\rangle}(y)$ amounts to average $\hat{F}_{Y_g|X_g}(y|x)$ over X_g. That is,

$$
\hat{F}_{Y\langle g|g\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{Y_g|X_g}(y|X_{gi}) \tag{5.2}
$$

which is the same as replacing the population distribution function in Equation 4.1 with the sample distribution function.

Since the estimated distribution obtained may be nonmonotonic in y, we apply the monotonization method of Chernozhukov, Fernández-Val, and Galichon (2010) based on rearrangement.[10] The rearranged distribution is given by

$$
\hat{F}^r_{Y\langle g|g\rangle}(y) = \inf\left\{u : \int_0^1 \mathbb{1}\{\hat{F}_{Y\langle g|g\rangle}(y) \leq u\}dv \geq y\right\}.
$$

[10]For practical and computational purposes, it is helpful to think of the rearrangement as sorting (Chernozhukov, Fernández-Val, and Galichon (2010), p. 1098). In practice, we use rearranged estimators of the distributions for all the results, but we omit this discussion throughout the rest of this paper for the sake of clarity.

5.2 Estimating counterfactual distributions

With the estimates $\hat{\beta}(y)$ from Equation 5.1, estimating $F_{Y\langle 0|(0,1,j)\rangle}(y)$ amounts to average $F_{Y_0|X_0}(y|X)$ over $X_{(0,1,j)}$. That is,

$$\hat{F}_{Y\langle 0|(0,1,j)\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{Y_0|X_0}(y|X_{(0,1,j)i}) \tag{5.3}$$

which is the same as replacing the population distribution function in Equation 4.3 with the sample distribution function.

Similarly, with the estimates $\hat{\beta}(y)$ from Equation 5.1 and thus the constructed $F_{Y_{(0,1,j)}|X_0}(y|x)$, estimating $F_{Y\langle(0,1,j)|1\rangle}(y)$ amounts to average $F_{Y_{(0,1,j)}|X}(y|x)$ over the distribution of X_1. That is,

$$\hat{F}_{Y\langle 0|(0,1,j)\rangle}(y) = \frac{1}{n}\sum_{i=1}^{n} \hat{F}_{Y_{(0,1,j)}|X}(y|X_{1i}) \tag{5.4}$$

which is the same as replacing the population distribution function in Equation 4.4 with the sample distribution function.

5.3 Decomposing quantile differences

With the observed and counterfactual distributions from Equations 5.2, 5.3 and 5.4, one can compute any distributional statistics. Then, the decomposition components can be computed by plug-in method, that is, plugging the estimates of distributional statistics into Equation 4.5. In the paper, we decompose the difference in τ-quantile of portfolio returns between North America and Europe. We begin by inverting each distribution, $\hat{F}_{Y\langle\cdot|\cdot\rangle}$, to obtain the corresponding estimate of τ-quantile, $Q_{\langle\cdot|\cdot\rangle}(\tau)$, by the following equation

$$\hat{Q}_{\langle\cdot|\cdot\rangle}(\tau) = \inf\{Q : \hat{F}^{r}_{Y\langle\cdot|\cdot\rangle}(Q) \geq \tau\}, \quad \tau \in (0,1). \tag{5.5}$$

Then, the components of decomposition for differences in τ-quantile of portfolio returns can be computed by plugging $\hat{Q}_{\langle\cdot|\cdot\rangle}(\tau)$ into Equation 4.5, where ν is a quantile function: $\nu\left(\hat{F}_{Y\langle\cdot|\cdot\rangle}(y)\right) = \hat{Q}_{\langle\cdot|\cdot\rangle}(\tau)$.

5.4 Decomposing mean difference

To decompose the difference in mean portfolio returns between North America and Europe using distribution regressions, we need to estimate $E(Y\langle\cdot|\cdot\rangle)$.

Consider a grid of equally spaced values of τ given by $0 < \tau_1 < \tau_2 < \cdots < \tau_M < 1$. Then, we estimate $E(Y\langle\cdot|\cdot\rangle)$ by

$$\hat{E}(Y\langle\cdot|\cdot\rangle) = \frac{1}{M}\sum_{m=1}^{M}\hat{Q}_{\langle\cdot|\cdot\rangle}(\tau_m)$$

The decomposition follows straightforward as Equation 4.5 by replacing $\nu\left(\hat{F}_{Y\langle\cdot|\cdot\rangle}(y)\right)$ with $\hat{E}(Y\langle\cdot|\cdot\rangle)$. We compare the results to the ones from Blinder-Oaxaca decomposition.

5.5 Inference

The significance test is previously ignored in most decomposition analyses in economics. Chernozhukov, Fernández-Val, and Melly (2013) provide estimation and inference procedures for the entire marginal counterfactual distribution and its functionals based on regression methods including distribution regression and quantile regression. They also drive a functional central limit theorem and prove the validity of bootstrap for the entire empirical coefficient process of distribution regression and related functionals.

In this paper, we calculate a series of counterfactual distributions and quantiles which are functionals of distributions. In the spirit of Chernozhukov, Fernández-Val, and Melly (2013), we complete the inference processes by bootstrapping.[11]

6 Data and results

We use the Fama and French (2017) dataset which can be downloaded at Kenneth R. French's personal website. We decompose the differences in monthly excess portfolio returns between two regions, North America (United States and Canada), and Europe (Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain,

[11]For inference in decomposing differences in mean portfolio returns using Blinder-Oaxaca decomposition, we also use bootstraps to obtain the standard deviations of decomposition components.

Sweden, Switzerland, and the United Kingdom). The dataset ranges from July 1990 to November 2017 (329 months). Hereafter, we simply call portfolio returns instead of monthly excess portfolio returns. The distinction will be made whenever emphasis is necessary.

The five factors used in the paper are portfolios constructed from 2×3 sorts on market capitalization (*Size*) and book-to-market equity ratio (B/M), operating profitability (OP) or investment (Inv). [12] At the end of each June, stocks are allocated to small and big *Size* groups. Stocks are allocated independently to three B/M groups using B/M breakpoints.[13] The intersections of the two sorts produce 6 value-weight *Size*-B/M portfolios. The value factor, HML, is the average of the value-weight returns on the two high value stock portfolios of the 2×3 sorts minus the average of the value-weight returns on the two low value stock portfolios. The profitability and investment factors, RMW and CMA, are constructed in the same way as HML except the second sort is on either OP or Inv and it is robust minus weak or conservative minus aggressive. The size factor, SMB, is the average of the value-weight returns on the nine small stock portfolios of the three 2×3 sorts minus the average of the value-weight returns on the nine big stock portfolios.[14]

The 75 portfolios used to decompose are 25 *Size*-B/M, 25 *Size*-OP, and 25 *Size*-Inv portfolios constructed from 5×5 sorts for each region. At the end of each June, stocks are allocated to five *Size* groups using *Size* breakpoints. Stocks are allocated independently to five B/M groups using B/M breaks. The intersections of the two sorts produce 25 value-weight *Size*-B/M portfolios. The 25 *Size*-Inv and *Size*-OP portfolios are constructed in the same way as the *Size*-B/M portfolios except the second sort is on either OP or Inv.[15]

[12] We choose factors from 2×3 sorts because the five-factor model's performance is not sensitive to the way its factors are defined. We do not include the other common factors, e.g., the momentum factor of Carhart (1997) and liquidity factor of Pástor and Stambaugh (2003) because these factors have regression slopes close to zero so produce trivial changes in model performance when the portfolios used here are sorted on *Size* and *B/M, OP* or *Inv*. See FF(2015).

[13] In 2×3 sorts, big stocks are those in the top 90% of market capitalization for the region, and small stocks are in the bottom 10%. The *B/M, OP* and *Inv* breakpoints are the 30th and 70th percentiles of *B/M, OP*, and *Inv* for the big stocks of the region, respectively.

[14] B is book equity at the end of the fiscal year ending in year $t-1$ and M is market capitalization at the end of December of year $t-1$, adjusted for changes in shares outstanding between the measurement of B and the end of December. OP is measured with accounting data for the fiscal year ending in year $t-1$ and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by B. Inv is the change in total assets from the fiscal year ending in year $t-2$ to the fiscal year ending in $t-1$, divided by $t-2$ total assets.

[15] The *Size* breakpoints for 5×5 sorts are the 3rd, 7th, 13th, and 25th percentiles of the region's aggregate market capitalization. The *B/M, OP* and *Inv* breaks are the quintile of *B/M, OP*, and *Inv* for the big stocks of the region, respectively.

6.1 Summary statistics for factor returns

Table 1a reports the summary statistics for five factors. For the market, size and value factors, there exist big differences in average factor returns between North America and Europe, which are 0.16%, 0.1% and -0.14% per month, respectively. These differences are large in economic terms though not statistically significantly different from zero. The average returns on RMW and CMA for North America and Europe are very close.

Table 1b shows the correlations in factors within and between two regions. In both North America and Europe, the correlations between Mkt and RMW or CMA are negative. Mkt is negatively correlated with HML in North America (-0.23) but positively in Europe (0.18). However, Mkt is positively correlated with SMB in North America but negatively with SMB in Europe. In North America, SMB is negatively correlated with HML, RMW and CMA. However, the correlation between SMB and HML, RMW or CMA is very small in Europe. HML is highly and positively correlated with RMW and CMA in North America, but highly and negatively correlated with RMW although still highly and positively correlated with CMA in Europe. The correlation between RMW and CMA is 0.35 in North America versus -0.18 in Europe.

The rightmost block of Table 1b reports the correlations in factors between North America and Europe. Mkt, HML, and CMA are highly and positively correlated with correlations 0.8, 0.6 and 0.57, respectively. The correlation between Mkt and SMB, RMW or CMA is negative. SMB is negatively correlated with CMA (-0.12). HML is positively correlated with CMA (0.52) and negatively with RMW (-0.15). RMW is positively correlated with RMW (0.38).

Table 1: Summary statistics for factors

(a) Average and Standard Deviation

	North America					Europe					Difference				
	Mkt	SMB	HML	RMW	CMA	Mkt	SMB	HML	RMW	CMA	Mkt	SMB	HML	RMW	CMA
Average	0.67	0.17	0.20	0.34	0.26	0.51	0.07	0.34	0.40	0.21	0.16	0.10	-0.14	-0.06	0.05
SD	4.20	2.76	3.23	2.41	2.65	4.90	2.17	2.38	1.51	1.82	2.96	2.94	2.61	2.55	2.20

(b) Correlations

	North America					Europe					Between				
	Mkt	SMB	HML	RMW	CMA	Mkt	SMB	HML	RMW	CMA	Mkt	SMB	HML	RMW	CMA
Mkt	1.00	0.20	-0.23	-0.37	-0.44	1.00	-0.17	0.18	-0.26	-0.30	0.80	-0.26	0.04	-0.22	-0.35
SMB	0.20	1.00	-0.10	-0.42	-0.14	-0.17	1.00	0.01	-0.05	0.02	-0.26	0.31	0.03	-0.03	-0.12
HML	-0.23	-0.10	1.00	0.38	0.78	0.18	0.01	1.00	-0.54	0.54	0.04	0.03	0.60	-0.15	0.52
RMW	-0.37	-0.42	0.38	1.00	0.35	-0.26	-0.05	-0.54	1.00	-0.18	-0.22	-0.03	-0.15	0.22	0.38
CMA	-0.44	-0.14	0.78	0.35	1.00	-0.30	0.02	0.54	-0.18	1.00	-0.35	-0.12	0.52	0.38	0.57

Note: The table shows summary statistics and correlations between factors. Panel a shows the summary statistics and Panel b shows the correlations in factors within and between North America and Europe.

6.2 Summary statistics for portfolio returns

Table 2 shows means and standard deviations of returns on 75 portfolios studied in the paper. None of the portfolio returns is statistically different from zero although most of them are economically large. Table 3 shows summary statistics for the differences in portfolio returns between North America and Europe. We see most of differences are economically significant although none is statistically different from zero.

One might doubt the normality of the portfolio returns then using t-statistic to test significances is not a good idea. We use Kolmogorov–Smirnov tests to check the normality of portfolio returns.[16] For every portfolio, the null hypothesis that the distribution of differences is normal is rejected at the confidence level of 95%. Thus, we use Wilcoxon tests to test significances of the differences and find that the differences are not significantly different from zero.[17] Suppose the differences are not economically significant, it is still interesting to decompose the differences and examine the decomposition components. The non-normality of the distribution of the differences makes it necessary to decompose the differences in quantile returns.

[16]See Massey Jr (1951), Lilliefors (1967), Lilliefors (1969), Friedman and Rafsky (1979), and Stephens (1974) for Kolmogorov–Smirnov test.

[17]The results of Kolmogorov–Smirnov tests and Wilcoxon tests are not shown and are offered upon request.

Table 2: Summary statistics for portfolio returns

(a) Mean

	North America					Europe				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.43	0.40	0.79	0.82	0.64	-0.08	0.29	0.34	0.49	0.34
2	0.60	0.62	0.68	0.70	0.66	0.37	0.49	0.56	0.53	0.53
3	0.91	0.79	0.78	0.81	0.63	0.45	0.55	0.54	0.59	0.57
4	0.88	0.80	0.78	0.77	0.65	0.59	0.71	0.54	0.55	0.64
High B/M	1.16	0.86	0.89	0.86	0.57	0.75	0.76	0.74	0.65	0.54
Low Inv	1.21	0.88	0.91	0.90	0.74	0.55	0.59	0.64	0.64	0.57
2	1.12	0.90	0.90	0.94	0.65	0.71	0.77	0.67	0.63	0.58
3	0.99	0.88	0.90	0.85	0.65	0.71	0.76	0.70	0.64	0.48
4	0.96	0.87	0.79	0.88	0.62	0.65	0.64	0.48	0.63	0.45
High Inv	0.54	0.36	0.54	0.51	0.55	0.16	0.37	0.30	0.39	0.45
Low OP	0.84	0.44	0.59	0.57	0.25	0.15	0.25	0.26	0.20	0.16
2	1.06	0.78	0.78	0.79	0.57	0.67	0.57	0.56	0.55	0.55
3	1.05	0.95	0.81	0.93	0.63	0.75	0.70	0.73	0.71	0.56
4	1.07	1.01	0.90	0.78	0.71	0.90	0.75	0.63	0.69	0.47
High OP	1.09	1.14	1.06	0.93	0.72	0.76	0.96	0.81	0.70	0.60

(b) Standard deviation

	North America					Europe				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	7.96	7.31	6.88	6.44	4.53	5.49	5.63	5.71	5.39	4.84
2	6.82	6.49	5.70	5.06	4.10	5.27	5.25	5.23	4.99	4.77
3	6.13	5.47	5.02	4.57	4.21	4.95	5.02	5.14	5.01	5.21
4	5.39	4.92	4.72	4.58	4.12	4.84	5.06	5.18	5.35	5.42
High B/M	5.28	5.16	4.88	4.76	5.23	4.83	5.30	5.59	5.80	6.34
Low Inv	6.42	5.68	5.09	4.76	4.05	4.98	5.22	5.46	5.20	4.85
2	4.97	4.76	4.36	4.21	3.69	4.46	4.82	5.04	5.00	4.81
3	4.92	4.78	4.65	4.38	4.05	4.56	4.80	4.89	4.85	5.17
4	5.29	5.42	5.23	4.87	4.77	4.76	5.14	5.17	5.23	5.45
High Inv	6.80	6.92	7.26	6.64	5.82	5.69	5.74	5.94	6.07	5.35
Low OP	6.68	6.65	6.77	6.26	5.60	5.18	5.43	5.56	5.52	6.09
2	4.91	4.94	4.87	4.69	4.89	4.67	5.00	5.09	5.14	5.53
3	4.98	4.99	4.60	4.40	4.31	4.81	5.02	5.07	5.25	5.08
4	5.34	5.27	4.93	4.45	4.07	4.79	5.09	5.17	5.08	4.99
High OP	5.66	5.44	5.40	4.83	3.96	4.94	5.32	5.33	5.28	4.76

Note: The table shows summary statistics for 75 portfolios in North America and Europe, respectively. The top panel shows means of monthly portfolio returns in excess of the one-month Treasury bill rate. The bottom panel shows standard deviations.

We calculate the average portfolio returns by averaging out the portfolio returns for all portfolios in North America and Europe, respectively. Figure 1 plots time series of the average portfolio returns in North America against Europe. It clearly shows that the portfolio returns between North America and Europe are positively correlated. The correlation is 0.747. That simply means that the portfolio returns in Europe are high when the portfolio returns in North America are high. Positive perfect correlation, that is all portfolio returns lining on the 45-degree line, means that the rank of the portfolio return for month t in the distribution of portfolio returns in North

America is the same as the rank of the portfolio return for month t in the distribution of portfolio returns in Europe.

This is related to rank invariance in the quantile treatment effect literature. Rank invariance requires that the rank of a counterfactual portfolio return in the counterfactual distribution remains the same as the rank of the portfolio return in the observed distribution.[18] Rank invariance is important because it is required for the decomposition for differences in quantile portfolio returns. It is also important for analyses of the decomposition results in the quantile differences from the perspective of asset pricing at the end of the paper. Thus, we implicitly assume that the rank invariance assumption holds without statistically testing it.

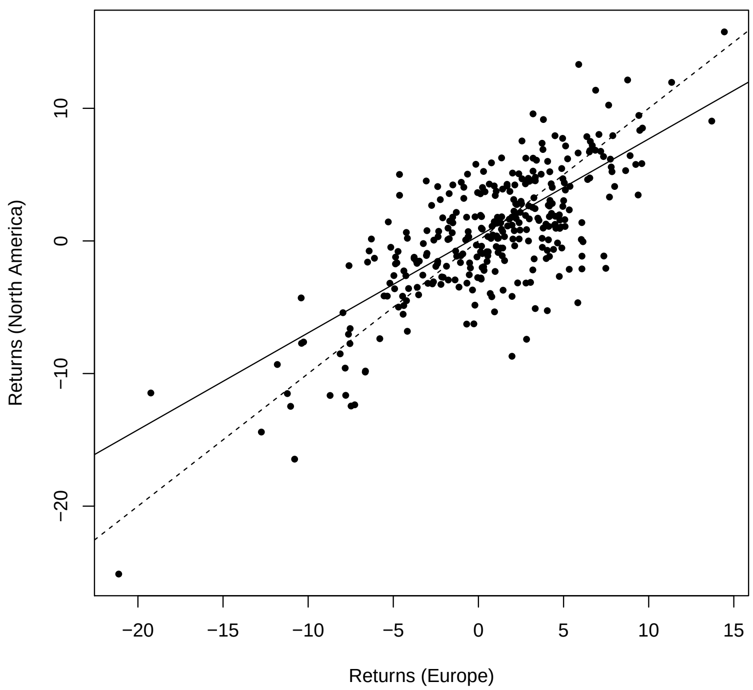
Table 3: Summary statistics for differences in portfolio returns

	Mean					Standard deviation				
	Small	2	3	4	Big	Small	2	3	4	Big
Low B/M	0.52	0.11	0.45	0.33	0.30	5.83	5.60	4.91	4.41	3.27
2	0.23	0.14	0.12	0.17	0.13	5.28	5.22	4.44	3.67	3.30
3	0.45	0.24	0.24	0.22	0.06	4.68	4.41	4.06	3.45	3.18
4	0.29	0.09	0.23	0.21	0.00	4.48	4.13	3.85	3.70	3.62
High B/M	0.41	0.10	0.15	0.21	0.04	4.35	4.29	4.22	4.10	4.37
Low Inv	0.66	0.29	0.26	0.26	0.17	5.11	4.35	4.16	3.54	3.25
2	0.41	0.13	0.22	0.31	0.06	4.26	4.13	3.83	3.48	3.14
3	0.28	0.12	0.20	0.21	0.17	4.26	4.28	3.78	3.58	3.26
4	0.31	0.23	0.32	0.24	0.17	4.35	4.38	4.08	3.75	3.76
High Inv	0.38	0.00	0.24	0.12	0.10	5.01	5.12	5.08	4.40	4.55
Low OP	0.69	0.19	0.33	0.36	0.09	5.10	5.02	5.14	4.70	4.36
2	0.39	0.21	0.23	0.23	0.02	4.40	4.31	3.99	3.78	3.74
3	0.30	0.25	0.08	0.22	0.06	4.36	4.20	3.79	3.47	3.17
4	0.16	0.26	0.27	0.09	0.25	4.43	4.47	4.16	3.56	3.36
High OP	0.32	0.18	0.25	0.23	0.11	4.49	4.61	4.27	3.72	3.30

Note: The table shows summary statistics for means of differences in monthly excess portfolio returns between North America and Europe.

[18] In the quantile treatment effect literature, rank invariance is as known as rank preservation. See, e.g., Heckman, Smith, and Clements (1997), Chernozhukov and Hansen (2005), Chernozhukov and Hansen (2006), and Chernozhukov and Hansen (2008), Horowitz and Lee (2007), Chernozhukov, Imbens, and Newey (2007), Abadie, Angrist, and Imbens (2002), Frölich and Melly (2013), Firpo (2007), Firpo and Pinto (2016), Imbens and Newey (2009), and Dong and Shen (2018).

Figure 1: Average portfolio returns in North America against Europe



Note: The figure plots time series of the average portfolio returns in North America against Europe. The true line is the regression fitted line and the dashed line is the 45 degree line.

6.3 Decomposition results

Before showing the main results, we first note that we choose the structures in North America as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors. In Section 7, we show robustness checks by changing the reference or the decomposition order. We also note that the decomposition results are averaged out over the decomposition results for all portfolios. That is, we do decomposition for each portfolio and average out the results. The decomposition results for each portfolio are offered upon request.

We first show the true and observed overall differences in portfolio returns between North America and Europe. We note that we decompose the observed overall differences in the paper. We then compare the decomposition results in the observed overall mean difference using Blinder-Oaxaca decomposition and distribution regressions. Lastly, we present the decomposition results in the observed quantile differences across a sequence of quantiles.

6.3.1 True and observed overall differences

Figure 2 shows the observed and true overall quantile and mean differences in portfolio returns between North America and Europe.[19] The observed overall mean (OLS) difference and the true overall mean difference coincide with each other, both of which are 0.23%. The standard deviation of the observed mean (OLS) difference is 0.045%. The observed overall mean (DR) difference is much lower, which is 0.11% (see Table 4). The observed and true overall quantile differences in portfolio returns are quite close in the lower quantiles. In the higher quantiles, however, the observed ones is lower than the true ones.[20]

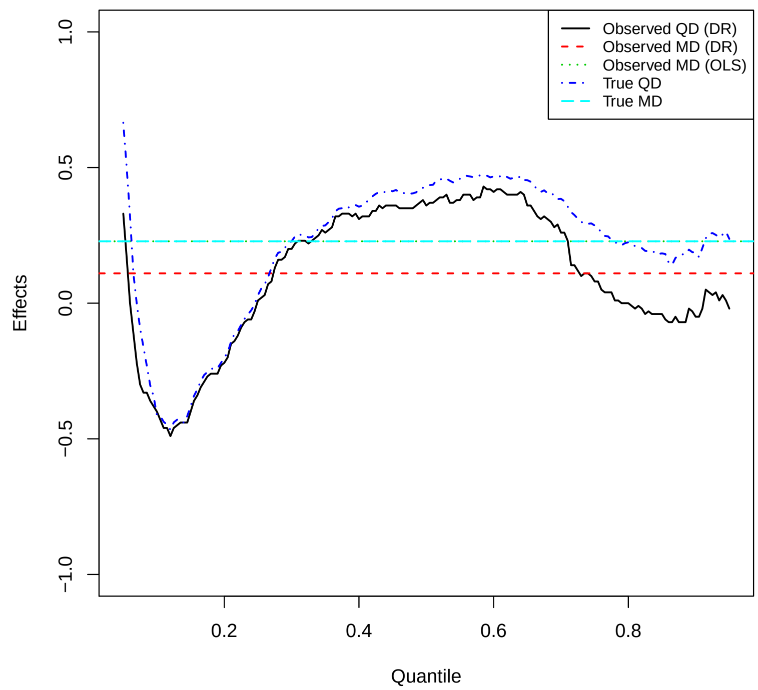
The overall quantile differences jump down in the quantiles below 10%. The differences then start to climb up and go down across quantiles. We observe that the differences in the lower quantiles are mostly negative and the differences in the middle quantiles are positive. We recall that the overall mean difference in portfolio returns between North America and Europe is positively 0.23% per month. This positiveness in mean difference is mostly contributed by the positive quantile differences in the middle quantiles. The negative quantile differences in the low quantiles mean that the portfolios in North America have higher down-side risk. This higher down-side risk may partially explain the higher mean returns on portfolios in North America. Higher risks are compensated by higher returns.

In the paper, we decompose the observed overall differences.

[19] The observed overall quantile and mean (DR) differences are obtained by following steps: Firstly, estimate the quantile and mean returns by distribution regressions for each portfolio. Secondly, compute the differences between two regions for each portfolio. Thirdly, average out the differences for all portfolios. The observed overall mean difference (OLS) is obtained by the same way as the observed mean difference (DR) except it is computed by ordinary least squares. The true overall quantile and mean differences are computed by following steps: Firstly, sort the portfolio returns for each portfolio and directly compute the quantiles and mean. Secondly, compute the differences in mean and quantile returns between two regions for each portfolio. Lastly, average out the differences in mean and quantile returns for all portfolios.

[20] For the sake of clarity, we do not show the confidence interval for the observed overall quantile differences. The confidence interval contains zero across all quantiles. The confidence intervals for aggregate decomposition components shown below are also not plotted for the same reason.

Figure 2: Observed and True overall quantile and mean differences



Note: The figure plots the overall differences in quantiles and mean of portfolio returns between North America and Europe. QD and MD denote quantile and mean differences, repectively. DR and OLS represent methods used to compueted Observed QD and MD, which are distribution regressions and ordinary least squares, respectively.

6.3.2 Decomposing mean difference

Table 4 shows the decomposition results for the mean difference using Blinder-Oaxaca decomposition and distribution regressions. We observe that using Blinder-Oaxaca decomposition 89.5% of the overall difference, 0.228%, comes from the aggregate composition component, 0.204%. Using distribution regressions, the composition component, 0.16%, over explains the overall difference, 0.11%. The aggregate composition component from Blinder-Oaxaca decomposition is both statistically and economically significant. The aggregate structure component is weakly statistically different from zero but economically trivial. The composition component from distribution regressions are economically significant but statistically insignificant. The structure component, however, is not only statistically insignificant but also economically trivial and negative.

For the detailed composition components from Blinder-Oaxaca decomposition, the composition component linked to market factor is 0.163% per month, which explains 71.9% of the overall

Table 4: Decomposition results for mean difference (in percentage)
North America: MSHRC

	Mean (BO)	SD	Mean (DR)	SD
Overall	0.228	0.045	0.11	0.37
Agg_c	0.204	0.043	0.16	0.36
Agg_s	0.024	0.012	-0.05	0.06
Mkt_c	0.163	0.039	0.11	0.34
SMB_c	0.052	0.015	0.06	0.11
HML_c	-0.010	0.007	-0.03	0.03
RMW_c	0.001	0.006	0.01	0.01
CMA_c	-0.002	0.006	0.00	0.01
Constant_s	0.010	0.012	0.10	0.15
Mkt_s	0.005	0.002	-0.11	0.11
SMB_s	0.000	0.002	-0.02	0.04
HML_s	0.009	0.004	-0.01	0.02
RMW_s	0.007	0.004	0.00	0.03
CMA_s	-0.006	0.003	0.00	0.06

Note: The table shows the deocomposition results for the mean difference in portfolio returns between North America and Europe. "BO" and "DR" denote the approches used, which are Blinder-Oaxaca decomposition and distribution regressions, respectively. "Overall" represents the observed overall mean differences. "Agg" denotes aggregate component. "c" and "s" represent composition and structure components, respectively. For instance, "Agg_c" denotes aggregate composition component and "Mkt_c" represents the detailed composition component associated to market factor. "Constant_s" denotes the structure component linked to constant term. "North America: MSHRC" represents that the structure in North America are used as reference and the order of decomposition is the constant, market, size, value, profitability, investment factors.

difference and 79.9% of the aggregate composition component. It is economically and statically significant. The second largest is the composition component linked to size factor which is 0.052% per month and explaining 22.8% of the overall difference and 25.5% of the aggregate composition component. Although it is statistically significant, it is economically smaller relative to market factor. The other components are all economically insignificant although the structure components linked to market, value and investment factors are statistically significant.

For the detailed decomposition results from distribution regressions, we observe that the composition component linked to market factor is 0.11% per month, explaining the entire overall difference and 68.8% of the aggregate composition component. The composition component linked to size factor is 0.06%. The structure component linked to market factor is -0.11%. The other components are economically trivial. Notice that none of the components is statistically significant.

In sum, the results in the mean difference from two approaches seem to be different in terms of the sizes of the decomposition components.[21] However, the results from both approaches show

[21]The sizes are different mainly because we are decomposing the observed overall mean difference instead of the true overall mean difference. Also, the results from distribution regressions are path dependent and the ones from Blinder-Oaxaca decomposition are path independent.

that the mean difference in portfolio returns is mainly contributed by the differences in factor risk premia. More specifically, the differences in the market and size factor risk premia play a significant role in explaining the mean difference in portfolio returns and the differences in the factor risks play an insignificant role in aggregate.

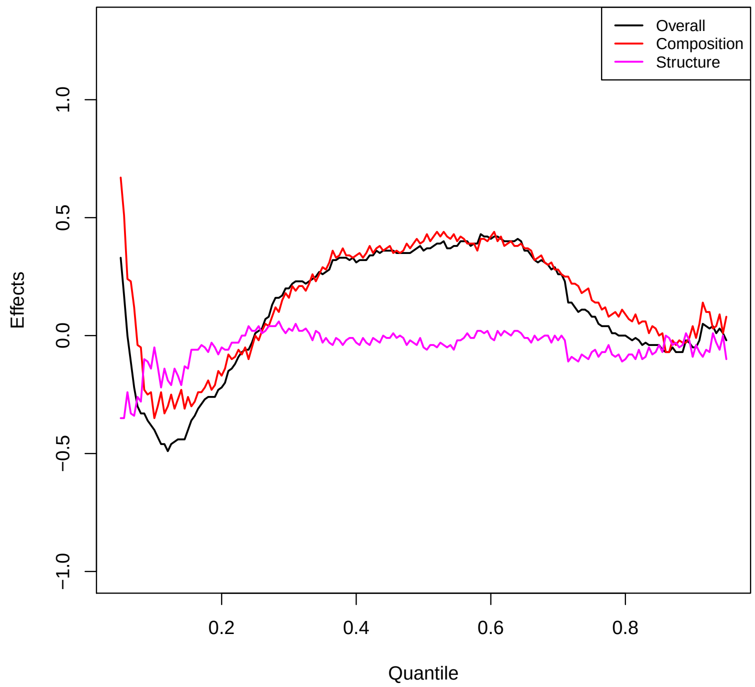
6.3.3 Decomposing quantile differences

Aggregate components

Figure 3 shows the aggregate decomposition results for a series of quantile differences using distribution regressions. It is obvious to observe that the aggregate composition components play a significant role in explaining the observed overall quantile differences in portfolio returns. The composition components are various across quantiles and follow tightly the pattern of the overall quantile differences, first jumping down in the low quantiles and climbing up then going down across quantiles. However, the structure components seem to move around zero and are trivial relative to the composition components.

In sum, we find that the differences in quantiles of portfolio returns between North America and Europe are mostly contributed by the differences in the distributions of factors, i.e., factor risk premia. The differences in the structures, i.e., factor risks, seem to play a trivial role in aggregate. Next we will look at the detailed decomposition to further examine the role of each factor.

Figure 3: Aggregate decomposition across quantiles
North America: MSHRC



Note: The figure shows the aggregate decomposition decomponents across a series of quantiles. "Overall", "Composition" and "Structure" denotes the observed overall quantile differences, aggregate composition and structre components, respectively. "North America: MSHRC" represents that the structures in North America are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

Detailed composition components

The top panel of Figure 4 shows the detailed composition components for quantile differences across quantiles and Figure 5 shows each component with its 95% confidence interval separately. We observe that the composition components across quantiles are mostly contributed by the components associated to market and size factors. The composition components linked to value, profitability and investment factors are economically trivial and statistically insignificant.

In the extreme low quantiles, the positive quantile differences are mainly from the composition component linked to market factor. It is interesting to further examine the "anomaly", i.e., the big jump, in the extreme low quantiles. Except in the extreme low quantiles, the composition component linked to market factor seems to go down smoothly across quantiles and the composition component associated to size factor goes up monotonically across quantiles. However, the speeds are different, which leads to the aggregate composition component first going up in the low quantiles and going down in the high quantiles.

The positive composition component linked to market factor implies higher returns on market factor in North America than the ones in Europe in the lower quantiles and the negative component in the higher quantiles means lower returns on market factor. We recall the mean difference in market factor between North America and Europe is 0.16% per month (see Table 1a). This positive mean difference in market factor is mainly contributed by the positive differences in the lower quantiles.

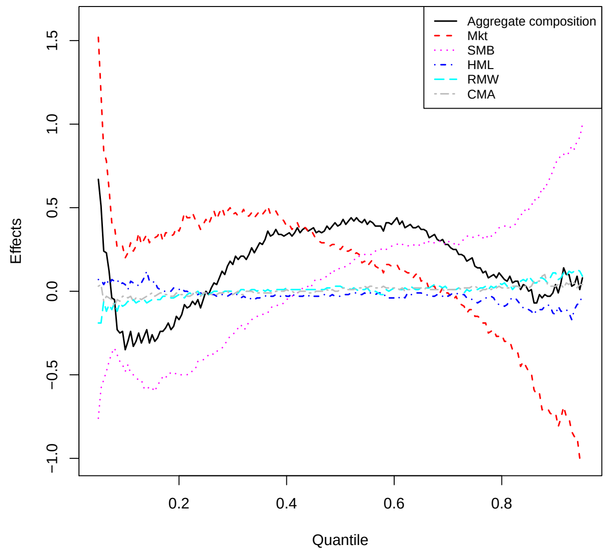
The composition component linked to size factor is negative in the low quantiles but positive in the high quantiles. The component seems to be statistically significant. This implies that the size factor returns in North America are lower than in Europe in the low quantiles but higher in the high quantiles. More specifically, when the portfolio has low returns (in the low quantile returns) the difference in size factor returns between North America and Europe is negative and when the portfolio has high returns (in the high quantile returns) the difference in size factor is positive.

In sum, the aggregate composition component, i.e., the contribution of the difference in factor risk premia, is mainly contributed by the differences in market and size factor premia. When the portfolio returns are low, the market factor risk premium positively contributes to the quantile differences and the size factor risk premium negatively contributes to the difference. When the portfolio returns are high, the market factor risk premium negatively contributes to the quantile differences and the size factor risk premium positively contributes to the difference. The variations in the differences in factor risk premia across quantiles imply variations in factor risk premia themselves. That is, factor risk premia are various at different levels of portfolio returns, adding to the literature on variations in risk premia.

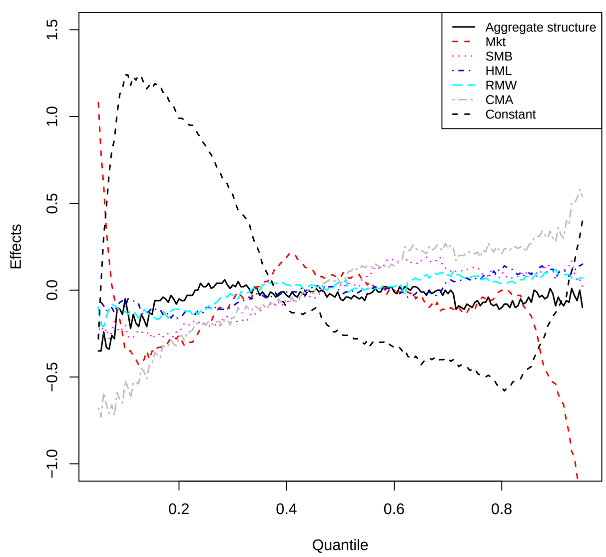
Detailed structure components

Figure 4 (Panel b) and 6 show the detailed structure component across quantiles. We recall that the aggregate structure component is trivial relative to the aggregate composition component. The components associated to size, value and profitability factors are all insignificant. The component associated to market factor is flat across quantiles except in the extreme quantiles. The large

Figure 4: Detailed decomposition
North America: MSHRC



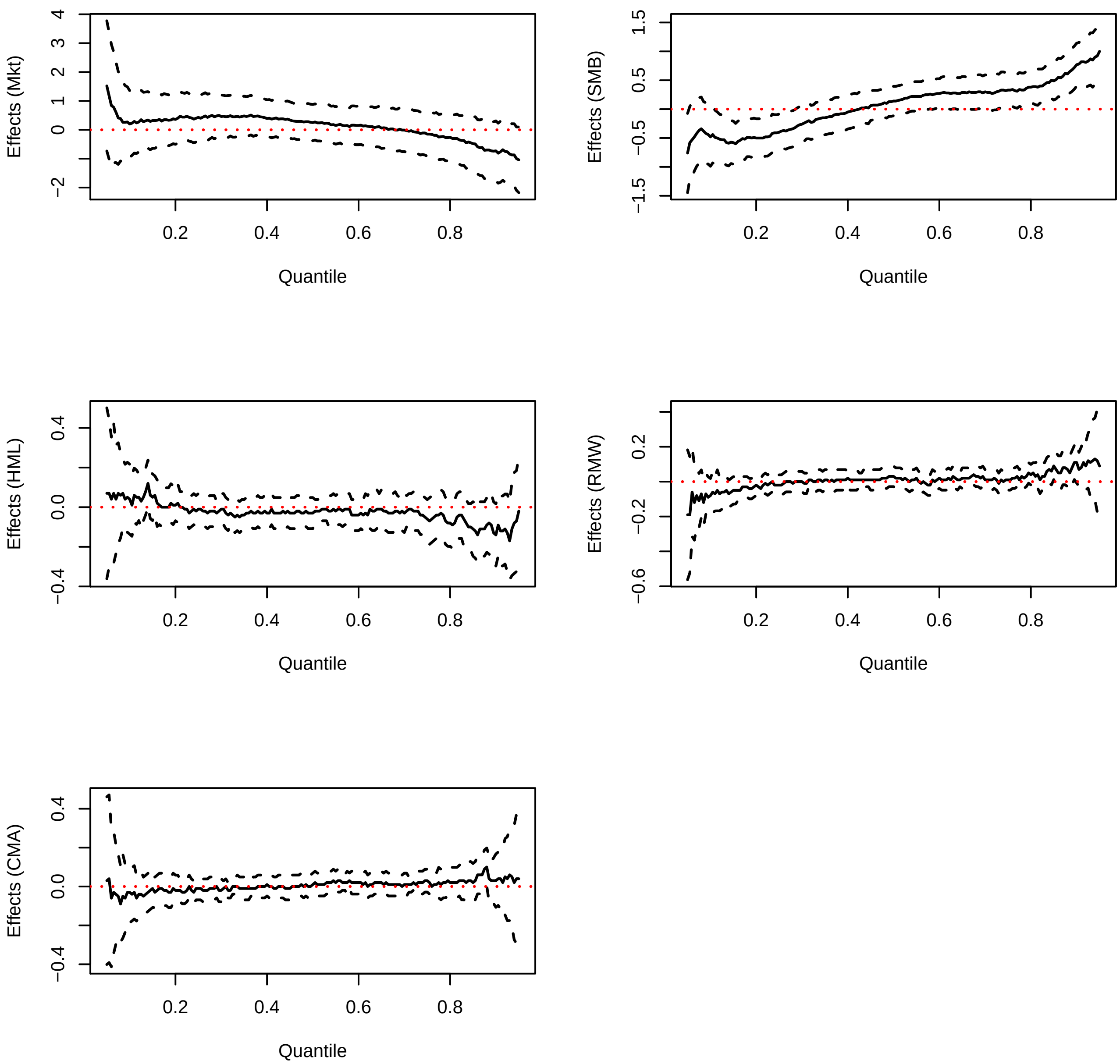
(a) Detailed composition components across quantiles (1)



(b) Detailed structure components across quantiles (1)

Note: The figure shows the detailed decomposition results. The top panel shows the detailed composition components across a series of quantiles. The bottom panel shows the detailed structure components across a series of quantiles. "North America: MSHRC" represents that the structures in North America are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

Figure 5: Detailed composition components across quantiles (2)
North America: MSHRC



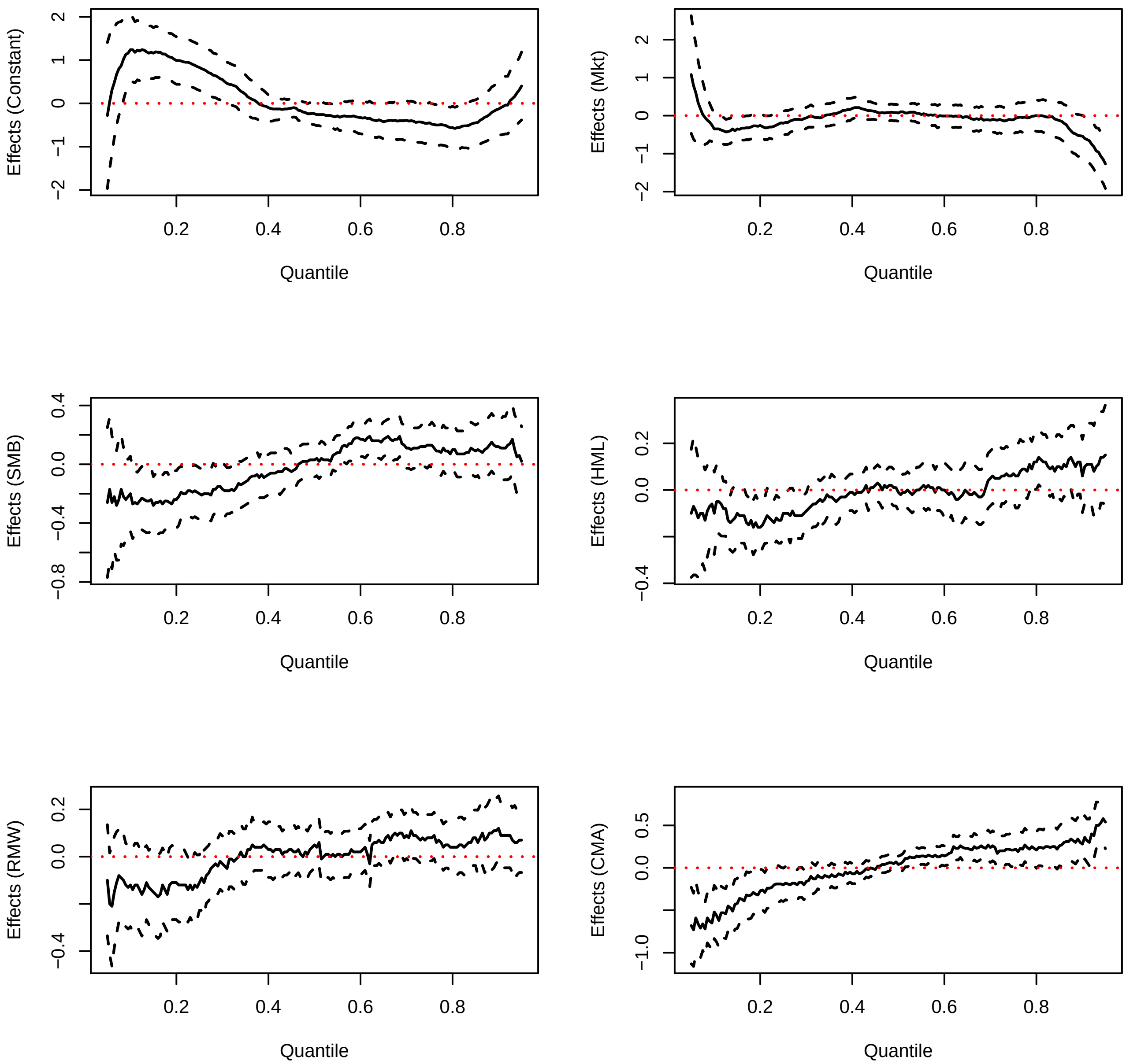
Note: The figure shows separately across quantiles the detailed composition component associated to each factor with 95% confidence interval. "North America: MSHRC" represents that the structures in North America are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

and positive market component in the extreme low quantiles implies that the market factor risk in North America is much higher than in Europe when the portfolio returns are low. And the large and negative market component in the extreme high quantiles implies that the market factor risk in North America is much lower than in Europe when the portfolio returns are high. It is interesting to further explore about the large difference in market factor risk between two regions in the extreme quantile portfolio returns.

The component linked to investment factor seems to play a statistically and economically significant role, which starts negative and monotonically goes up across quantiles. This means that the investment factor risk in North America is lower than in Europe when the portfolio returns are low and the investment factor risk in North America is higher than in Europe when the portfolio returns are high. This variation in the difference in investment factor risk across quantiles implies the variation in investment factor risk itself in North America and Europe. That is, the investment factor risk is various at different levels of portfolio returns, adding to the literature on variations in risks.

In sum, the market and investment factor risks seem to play a significant role in explaining the overall quantile difference although in aggregate the difference in factor risks do not seem to play an important role. Also, the roles of factor risks are various at different levels of portfolio returns, implying variations in the factor risks themselves.

Figure 6: Detailed structure components across quantiles (2)
North America: MSHRC



Note: The figure shows separately across quantiles the detailed structure component associated to each factor with 95% confidence interval. “North America: MSHRC” represents that the structures in North America are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

7 Robustness

In this section, we show some results for robustness checks. We need robustness checks for the following reasons. Firstly, the aggregate decomposition results from two approaches depend on the structures used as reference. Secondly, although the detailed decomposition using Blinder-Oaxaca decomposition is path independent, the detailed decomposition results from distribution regressions are path dependent. That is, the order of computing detailed components could affect the decomposition results.

We refer to the main results shown above as the reference results. We summarize the main differences between the robustness checks and the reference results. We offer the results in detail upon request.

7.1 North America: size, market, value, profitability and investment

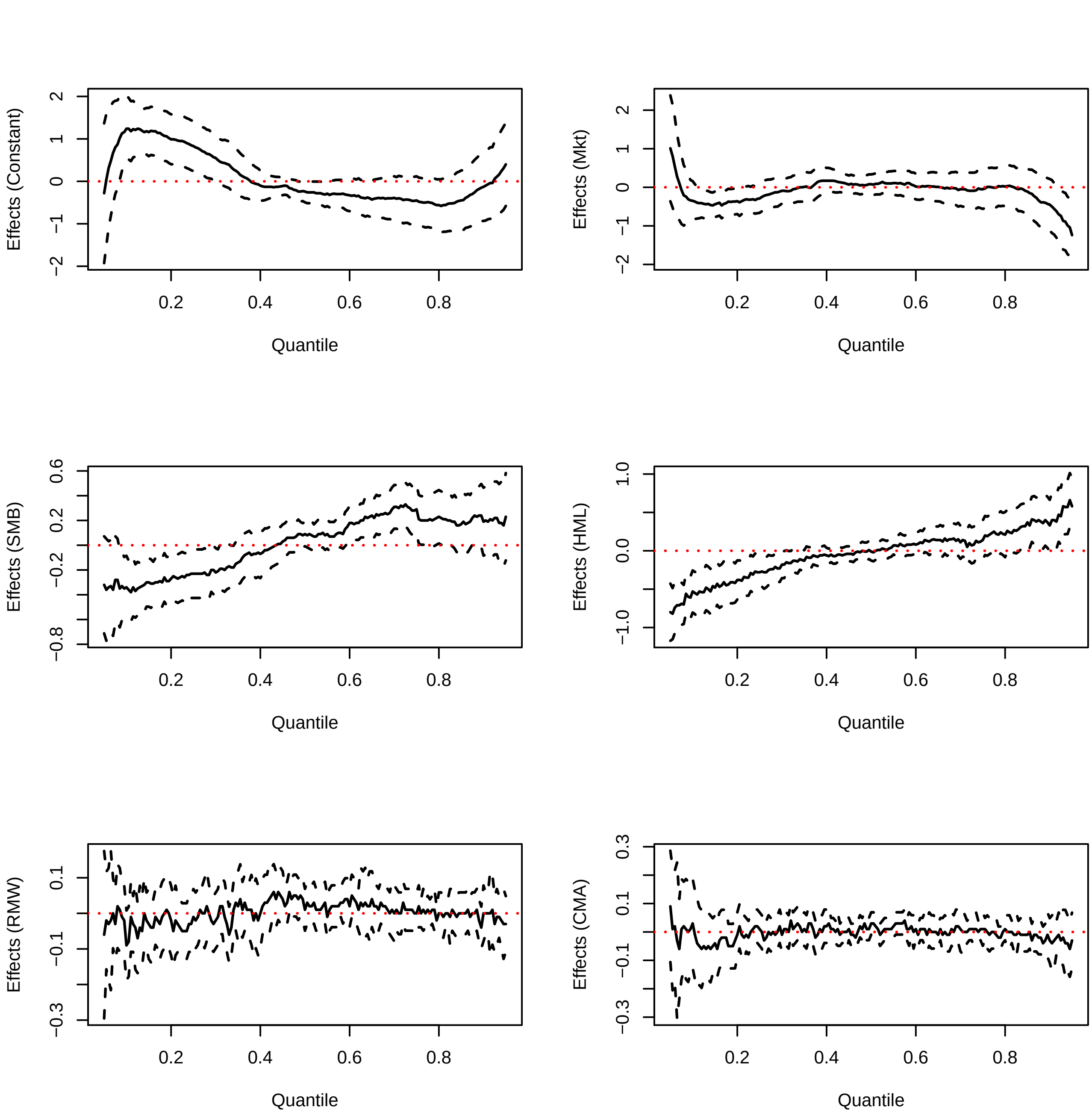
This section shows the results, in which the structures in North America are used as reference and the order of decomposition is the constant, size, market, value, profitability and investment factors. In this case, we do not change the reference structures. Thus, the results for mean difference using Blinder-Oaxaca decomposition will be the same as the ones in the reference results because the results from Blinder-Oaxaca decomposition are path independent. The results for the mean difference using distribution regressions in this case are quite similar to the reference results.

For the quantile differences, by construction if the results are path dependent, the detailed components linked to market and size factors will be significantly different from the ones in the reference results. However, in this case we do not see significant differences in the detailed components associated to market and size factors. That is, the reference results are robust to this change in the decomposition order.

7.2 North America: profitability, investment, market, size and value

The results, in which the structures in North America are used as reference and the order of decomposition is the constant, profitability, investment, market, size and value factors, are also quite similar to the reference results. The exception is that the structure component associated to value factor seems to be significant and the structure component linked to investment factor is not significant any more (Figure 7).

Figure 7: Detailed structure components across quantiles
North America: RCMSH



Note: The figure shows separately across quantiles the detailed structure component associated to each factor with 95% confidence interval. "North America: RCMSH" represents that the structures in North America are used as reference and the order of decomposition is the constant, profitability, investment, market, size and value factors.

7.3 Europe: market, size, value, profitability and investment

This section shows the results in which the structures in Europe are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors. Table 5 shows the decomposition results for the mean difference using both Blinder-Oaxaca decomposition and distribution regressions. Comparing to the reference results shown in Table 4, we observe that the results from Blinder-Oaxaca decomposition are quite close to the reference results. Comparing the results from distribution regressions to the reference results, we can come to the same mian conclusions as the ones drawn from the reference results although the sizes of the components are different.

Table 5: Decomposition results for mean difference (in percentage)
Europe: MSHRC

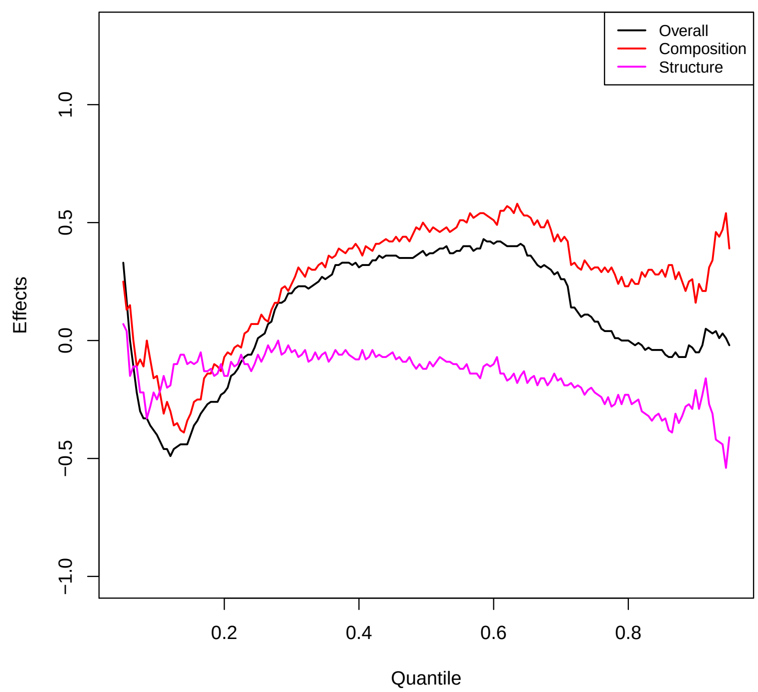
	Mean (BO)	SD	Mean (DR)	SD
Overall	0.228	0.045	0.11	0.43
Agg_c	0.209	0.043	0.26	0.42
Agg_s	0.019	0.013	-0.15	0.06
Mkt_c	0.161	0.039	0.14	0.39
SMB_c	0.053	0.014	0.11	0.12
HML_c	-0.006	0.007	0.00	0.02
RMW_c	0.002	0.004	0.03	0.02
CMA_c	0.000	0.005	-0.01	0.02
Constant_s	0.010	0.013	-0.04	0.17
Mkt_s	0.007	0.003	-0.07	0.44
SMB_s	-0.001	0.002	0.01	0.09
HML_s	0.005	0.004	-0.09	0.05
RMW_s	0.006	0.004	-0.03	0.03
CMA_s	-0.008	0.004	-0.02	0.03

Note: The table shows the deocomposition results for the mean difference in portfolio returns between North America and Europe. “BO” and “DR” denote the approches used, which are Blinder-Oaxaca decomposition and distribution regressions, respectively. “Overall” represents the observed overall mean differences. ”Agg” denotes aggregate component. “c” and “s” represent composition and structure components, respectively. For instance, “Agg_c” denotes aggregate composition component and “Mkt_c” represents the detailed composition component associated to market factor. “Constant_s” denotes the structure component linked to constant term. “Europe: MSHRC” represents that the structures in Europe are used as reference and the order of decomposition is the constant, market, size and value, profitability and investment factors.

For the quantile differences, we observe the detailed composition components are quite similar, both of which show that the market and size factors play a significant role (see Panel a of Figure 9 and 10). Comparing the aggregate components for the quantile differences shown in Figure 3 and 8, however, we observe that the aggregate composition component in this case does not follow the pattern of the overall quantile differences as tightly as in reference results, especially in the high quantiles.

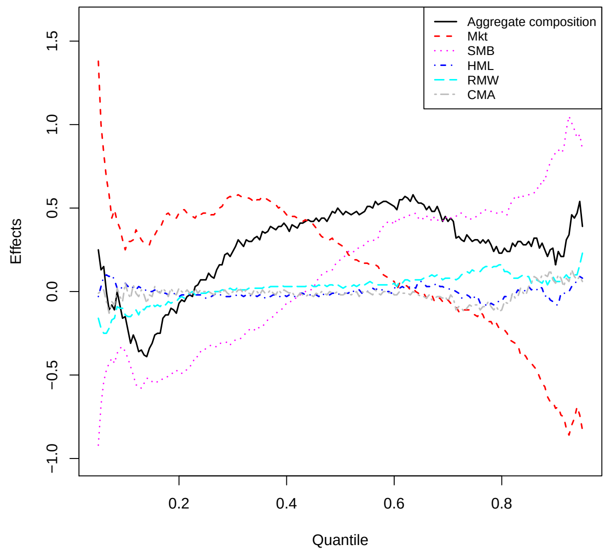
Also, the detailed structure components in this case are quite different from the ones in the reference results (Figure 9 and 11). In this case, the structure component associated to the constant is negative in the low quantiles but positive in the reference results. The components linked to market and size factors are also different from the reference results. The component linked to investment factor is not significant any more.

Figure 8: Aggregate decomposition across quantiles
Europe: MSHRC

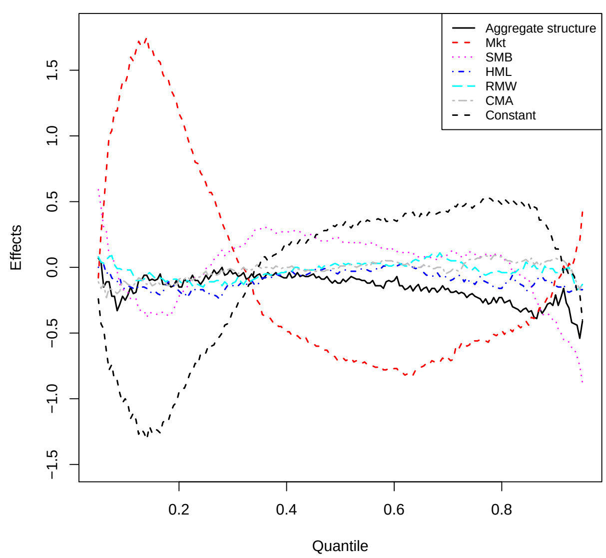


Note: The figure shows the aggregate decomposition decomponents across a series of quantiles. “Overall”, “Composition” and “Structure” denotes the observed overall quantile differences, aggregate composition and structre components, respectively. “Europe: MSHRC” represents that the structures in Europe are used as reference and the order of decomposition is the constant, market, size and value, profitability and investment factors.

Figure 9: Detailed decomposition
Europe: MSHRC



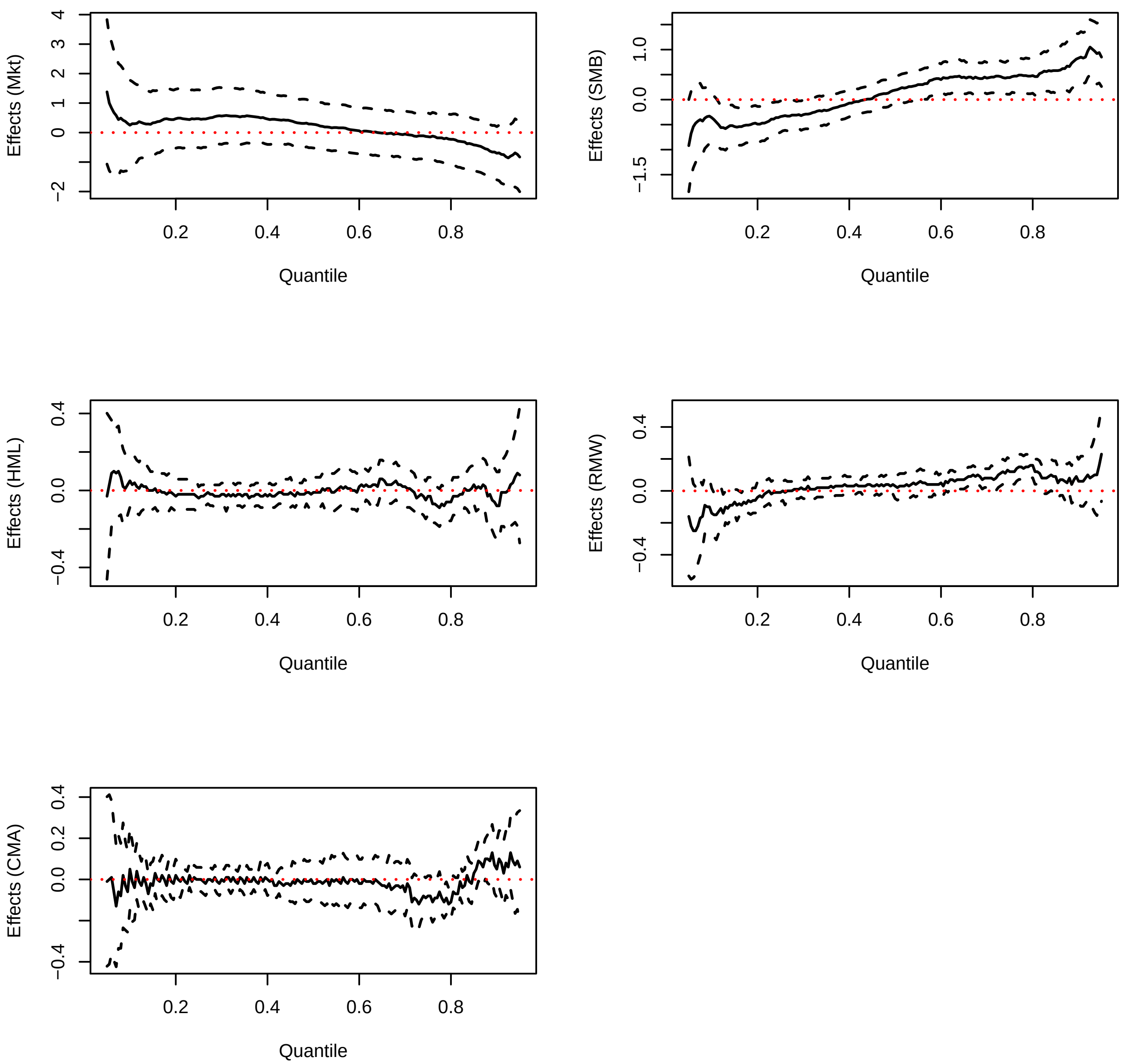
(a) Detailed composition components across quantiles (1)



(b) Detailed structure components across quantiles (1)

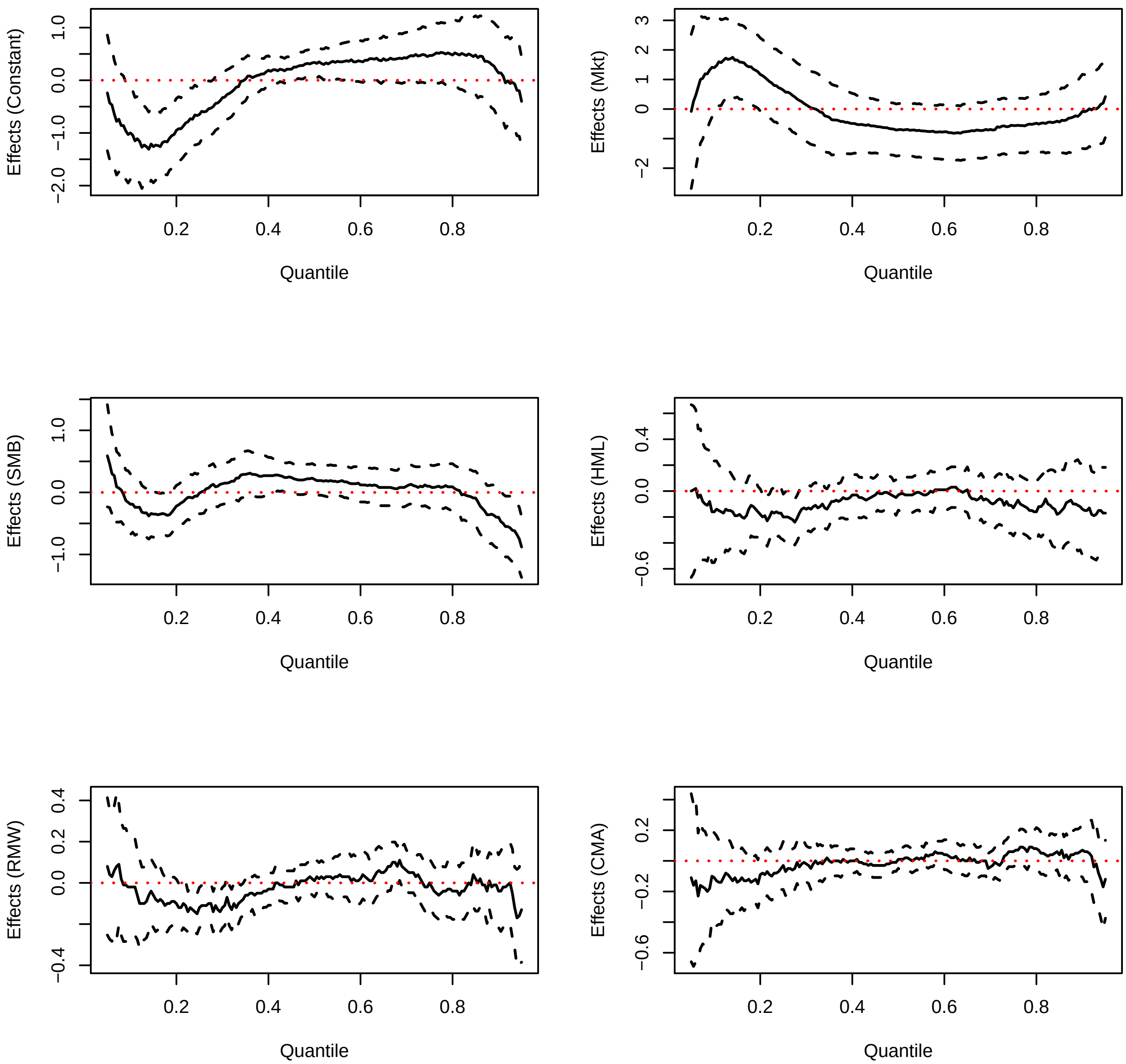
Note: The figure shows the detailed decomposition results. The top panel shows the detailed composition components across a series of quantiles. The bottom panel shows the detailed structure components across a series of quantiles. “Europe: MSHRC” represents that the structures in Europe are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

Figure 10: Detailed composition components across quantiles (2)
Europe: MSHRC



Note: The figure shows separately across quantiles the detailed composition component associated to each factor with 95% confidence interval. “Europe: MSHRC” represents that the structures in Europe are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

Figure 11: Detailed structure components across quantiles (2)
Europe: MSHRC



Note: The figure shows separately across quantiles the detailed structure component associated to each factor with 95% confidence interval. "Europe: MSHRC" represents that the structures in Europe are used as reference and the order of decomposition is the constant, market, size, value, profitability and investment factors.

In sum, the main conclusions drawn from the reference results are mostly robust to changing the reference structures and the composition order in terms of that the differences in portfolio returns between North America and Europe are mostly explained by the differences in factor risk premia and the differences in factor risks play a relatively trivial role. Further, the differences are mostly explained by the differences in market and size factor risk premia between two regions. Also, the factor risk premia are various at different levels of portfolio returns.

Huang (2018) applies the recentered influence function regressions to decompose the differences in quantiles of the same portfolio returns. His results are path independent and he comes to conclusions which are consistent with ours.

We note that the significance of the detailed structure components associated to some factors are not that robust to the changes in the reference structures and the decomposition order. However, we are inclined to conclude that the factor risks are likely various at different levels of portfolio returns.

8 Conclusion

In the paper, we decompose the large differences in mean portfolio returns between North America and Europe into Fama and French's five factors. We show that the mean differences are mainly contributed by the differences in risk premia on factors, especially on market and size factors. The differences in factor risks do not seem to play an important role in explaining the mean differences. We also decompose a series of differences in quantiles of portfolio returns. Again, we find that the quantile differences are also mainly contributed by the differences in risk premia on factors, especially on market and size factors and the differences in factor risks seem to play an insignificant role in aggregate. The detailed decomposition for the aggregate composition component further shows that the roles that the market and size factors play in explaining the quantile differences are various at different levels of portfolio returns. These findings imply the market and size factor risk premia are various at different levels of portfolio returns. The detailed decomposition for the aggregate structure component helps us find that the differences in risks on some factors possibly play a role in explaining the quantile differences too although they are not that robust to changing the reference structures and the decomposition order. Also, the roles that the factor risks play seems to be various at different levels of portfolio returns too. These findings imply that the factor risks could be various at different levels of portfolio returns.

These findings shed further light on empirical asset pricing in terms of that one needs to think about differences in factor risk premia and risks in different markets and the variation of factor risk premia and risks at different levels of portfolio returns. In future studies, we will decompose the differences in portfolio returns between normal time periods and financial crises or recessions to further explore the roles of factor risks and risk premia.

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