Project1 Report

Weigeng Li G44406413

Water Pitcher Problem Without Steps Counting

For Water Pitcher Problem **without steps counting**, we got several pitchers with different volumes and a **target pitcher with infinite volume**. By pouring in or out water into pitchers, the goal is to get a "target" amount of water in the target pitcher.
My mathematical way of expressing the problem is.

$$\text{Given Target } T \text{ and a set of pitchers } S = \{x_i |\ x_i > 0\ , i \in [1,n]\}$$

$$\text{Find Set } M = \{k_i |\ i \in [1,n]\}\ that\ T = \sum_{i=1}^{n} k_i x_i \qquad Formula\ 1$$

This problem may have none or several solutions. For a two-pitcher problem (e.g. pitcher A and B, target C), a formulaic problem-solving ideas (not optimal) is to find

$$mA\ mod\ B = nC \qquad Formula\ 2$$

By Extended Euclidean Algorithm, we knows that

$$if\ \exists\ integer\ p, C == p * \gcd(A,B)\ ,$$
$$then\ Formula\ 2\ have\ one\ \ or\ more\ solution$$

note: gcd () Greatest Common Divisor Function

Therefore, if $C\ mod \gcd(A,B) \neq 0$ Water Pitcher Problem must have a solution.
For more pitchers, The Water Pitcher Problem has a solution if:

$$T\ mod \gcd(S) \neq 0 \qquad Formula\ 3$$

Minimum Step of Water Pitcher Problem

Step Counting

Moving water from A pitcher to B to get A-B amount of water, then adding A-B to the target pitcher is 4 steps. Moving water from A pitcher to the infinity pitcher and then removing B from the target pitcher is also 4 steps. Therefore, we only consider adding water from the pitcher to the target pitcher or removing water from the target pitcher to other pitchers. This helps simplify the problem. The target pitcher can have any positive amount of water, while the other pitcher has 2 states, empty or full.

Adding water to the target pitcher is 2 steps. Fill a pitcher, then pour it to the pitcher.

Removing water from the target pitcher is 1 or 2 steps. If that pitcher is empty then it is 1 move, else need to empty that pitcher first, taking 2 steps.

State

Because Formula 1 is a summarization problem, the order of adding or removing which pitcher does not influence the result. My State is the amount of water in the target pitcher. Because the A* algorithm guarantee reaching each state in the optimal steps (proved in class).

Lower Bound

$$f(n) = g(n) + h(n)$$

$g(n)$ is number of Step that has taken. Therefore $g(0) = 0$. From step counting section. $g(n)$ can be calculated as

$$g(n) = \begin{cases} 0, & if\ n = 0 \\ g(n-1) + 1, & remove\ to\ an\ empty\ pitcher \\ g(n-1) + 2, & otherwise \end{cases}$$

$h(n)$ is Heuristics function that estimates how close a state is to a goal.

$$Let\ Target\ T, State\ n, distance\ D = T - n;\ S\ is\ the\ set\ of\ pitchers;$$

$$h(n) \begin{cases} \infty, & if\ D\ mod \gcd(S) \neq 0 \\ ceil\left(\dfrac{|D|}{\max(S)}\right), & otherwise \end{cases}$$

note: gcd () Greatest Common Divisor Function

Prove of Admissible and Consistent

The first condition of $h(n)$ means that there is no path to the target from the current state. Proved in First Section if the target is not reachable $D\ mod \gcd(S) \neq 0$. Therefore, $h^*(n) = h(n) = \infty$. The first condition is admissible.

The second of $h(n)$ is the estimate of the cost/distance. We assume that:

$$\exists n, h^*(n) < h(n)$$

$$\frac{|D|}{h^*(n)} > \frac{|D|}{h(n)} = \max(S)$$

This means there is a bigger pitcher than the biggest pitcher in the set ($\max(S)$). Which is not possible. Therefore, the assumption does not hold.

$$0 \leq h(n) \leq h^*(n)$$

The Heuristics function is admissible.

Assumption 2:

$$\exists\ A, B;\ h(A) > cost(A\ to\ B) + h(B)$$

$$h^*(A) \geq h(A) > cost(A\ to\ B) + h(B)$$

$h^*(A)$ is the minimum distance to A. Therefore, the assumption does not hold.

$$\forall\ A, B;\ h(A) \leq cost(A\ to\ B) + h(B)$$

In conclusion this **Heuristics function is admissible and consistent.**

Code:

Code explaining part is written as the in-code comment.

https://github.com/WeigengLi/6511_Project1

Result:

```
-----------Testing test_data/input.txt File-----------
Pitchers: [1, 4, 10, 15, 22]
Target: 181
Start with a 0 volume Pitcher
Operation: Fill a 15 Pitcher then Add the water to infinite cup.(2 move) Total: 15
Operation: Move 10 water from infinite cup to the 10 pitcher.(1 move) Total: 5
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 27
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 49
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 71
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 93
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 115
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 137
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 159
Operation: Fill a 22 Pitcher then Add the water to infinite cup.(2 move) Total: 181
Total: 19 move
Result:  19
-----------End of test_data/input.txt File Test-----------
```

```
----------Testing test_data/input4.txt File----------
Pitchers: [2, 3, 5, 19, 121, 852]
Target: 11443
Start with a 0 volume Pitcher
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 852
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 1704
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 2556
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 3408
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 4260
Operation: Fill a 121 Pitcher then Add the water to infinite cup.(2 move) Total: 4381
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 5233
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 6085
Operation: Fill a 2 Pitcher then Add the water to infinite cup.(2 move) Total: 6087
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 6939
Operation: Fill a 2 Pitcher then Add the water to infinite cup.(2 move) Total: 6941
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 7793
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 8645
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 9497
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 10349
Operation: Fill a 852 Pitcher then Add the water to infinite cup.(2 move) Total: 11201
Operation: Fill a 121 Pitcher then Add the water to infinite cup.(2 move) Total: 11322
Operation: Fill a 121 Pitcher then Add the water to infinite cup.(2 move) Total: 11443
Total: 36 move
Result:  36
----------End of test_data/input4.txt File Test----------
Time cost  3.8121845722198486 s
```

```
----------Testing test_data/input1.txt File----------
Pitchers: [2, 5, 6, 72]
Target: 143
Start with a 0 volume Pitcher
Operation: Fill a 72 Pitcher then Add the water to infinite cup.(2 move) Total: 72
Operation: Move 6 water from infinite cup to the 6 pitcher.(1 move) Total: 66
Operation: Fill a 5 Pitcher then Add the water to infinite cup.(2 move) Total: 71
Operation: Fill a 72 Pitcher then Add the water to infinite cup.(2 move) Total: 143
Total: 7 move
Result:  7
----------End of test_data/input1.txt File Test----------
----------Testing test_data/input2.txt File----------
Pitchers: [3, 6]
Target: 2
Result:  -1  No results found
----------End of test_data/input2.txt File Test----------
----------Testing test_data/input3.txt File----------
Pitchers: [2]
Target: 143
Result:  -1  No results found
----------End of test_data/input3.txt File Test----------
All Tests Successfully
```

All Sample Files from the project introduction has been calculated correctly!