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7.12

$$\begin{cases} Y_1 = X_1 + X_2 \\ Y_2 = \frac{X_1}{X_1+X_2} \end{cases} \Rightarrow \begin{cases} X_1 = Y_1 Y_2 \\ X_2 = Y_1(1-Y_2) \end{cases} \qquad J = \begin{vmatrix} y_2 & y_1 \\ 1-y_2 & -y_1 \end{vmatrix} = y_1$$

Joint density function : $g(y_1,y_2) = e^{-y_1y_2}e^{-y_1(1-y_2)}y_1 = \begin{cases} y_1 e^{-y_1}, & y_1 > 0 \ \ 0 < y_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$

Marginal distribution : $h_1(y_1) = \int_0^1 y_1 e^{-y_1}\, dy_2 = y_1 e^{-y_1}$

$h_2(y_2) = \int_0^\infty y_1 e^{-y_1}\, dy_1 = 1$

$g(y_1,y_2) = h_1(y_1)\, h_2(y_2) \Longrightarrow$ independent

7.14

$y = x^2 \Longrightarrow x_1=\sqrt{y} \quad x_2=-\sqrt{y} \qquad J_1=-\frac{1}{2\sqrt{y}}\ , J_2=\frac{1}{2\sqrt{y}}$

$g(y) = \frac{1+\sqrt{y}}{2}\cdot\frac{1}{2\sqrt{y}} + \frac{1-\sqrt{y}}{2}\cdot\frac{1}{2\sqrt{y}} = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$

7.18

$M_x(t) = \sum_{x=1}^{\infty} e^{tx}\, pq^{x-1} = pe^t\sum_{x=1}^{\infty} e^{t(x-1)}\, q^{x-1} = pe^t\sum_{k=0}^{\infty}(e^t\, q)^k = pe^t\frac{1}{1-qe^t}$,

$|qe^t|<1 \Longrightarrow t<-\ln(q)$

$\frac{dM_X(t)}{dt}\Big|_{t=0} = pe^t\frac{1}{1-qe^t} + pe^t\frac{-qe^t}{(1-qe^t)^2}\Big|_{t=0} = \frac{pe^t}{(1-qe^t)^2}\Big|_{t=0} = \frac{1}{p}$

$\frac{d^2M_X(t)}{dt^2}\Big|_{t=0} = \frac{pe^t-pq^2e^t}{(1-qe^t)^4}\Big|_{t=0} = \frac{1+q}{p^2}$

$E(X) = \frac{1}{p}$

$Var(X) = \frac{1+q}{p^2} - \frac{1}{p^2} = \frac{q}{p^2}$

7.22

$\frac{dM_X(t)}{dt}\Big|_{t=0} = \frac{-\mathrm{v}}{2}(1-2\mathrm{t})^{-\frac{v}{2}-1}(-2)\Big|_{t=0} \quad = \mathrm{v}(1-2\mathrm{t})^{-\frac{v}{2}-1}\Big|_{t=0} = \mathrm{v}$

$\frac{d^2M_X(t)}{dt^2}\Big|_{t=0} = \mathrm{v}\left(-\frac{v}{2}-1\right)(1-2\mathrm{t})^{-\frac{v}{2}-2}(-2)\Big|_{t=0} = \mathrm{v}^2+2\mathrm{v}$

E(X) = v

Var(X) = $\mathrm{v}^2+2\mathrm{v}-\mathrm{v}^2 = 2\mathrm{v}$