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4.24

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}, x=0,1,2,3 \quad y=0,1,2 \quad 1 \leq x+y \leq 4$$

x	0	1	2	3	y	0	1	2
g(x)	5/70	30/70	30/70	5/70	h(y)	15/70	40/70	15/70

$$(a) E(X^2Y - 2XY) = \sum_{x=0}^3 \sum_{y=0}^2 (x^2y - 2xy)f(x,y) = -\frac{3}{7}$$

$$(b) \mu_x - \mu_y = E(X) - E(Y) = 0 \cdot 5/70 + 1 \cdot 30/70 + 2 \cdot 30/70 + 3 \cdot 5/70 - (0 \cdot 15/70 + 1 \cdot 40/70 + 2 \cdot 15/70) = 1/2$$

4.44

$$\begin{aligned} \sigma_{XY} &= E(XY) - \mu_x \mu_y = \sum_{x=0}^3 \sum_{y=0}^2 (xy)f(x,y) - 105/70 \\ &= 1 \cdot 1 \cdot 18/70 + 2 \cdot 1 \cdot 18/70 + 3 \cdot 1 \cdot 2/70 + 1 \cdot 2 \cdot 9/70 + 2 \cdot 2 \cdot 3/70 - 3/2 = 9/7 - 3/2 \\ &= -3/14 \end{aligned}$$

4.60

$$E(X) = 2(0.15+0.25+0.15) + 4(0.1+0.25+0.1) = 1.1 + 1.8 = 2.9$$

$$E(Y) = 1(0.15+0.1) + 3(0.25+0.25) + 5(0.15+0.1) = 0.25 + 1.5 + 1.25 = 3$$

$$(a) E(2X-3Y) = 2E(X) - 3E(Y) = 2 \cdot 2.9 - 3 \cdot 3 = -3.2$$

$$(b) E(XY) = E(X)E(Y) = 2.9 \cdot 3 = 8.7 \quad (\text{independent})$$

4.78

$$\mu = E(X) = \int_0^1 x(30x^2(1-x)^2) dx = 30\left(\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right)\bigg|_0^1 = 0.5$$

$$E(X^2) = \int_0^1 x^2(30x^2(1-x)^2) dx = 30\left(\frac{x^5}{5} - \frac{2x^6}{6} + \frac{x^7}{7}\right)\bigg|_0^1 = \frac{2}{7}$$

$$\sigma^2 = E(X^2) - \mu^2 = 2/7 - 1/4 = 1/28, \quad \sigma \approx 0.19$$

$$\text{用 Chebyshev's theorem} \quad P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 1 - 1/4 = 0.75$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(0.12 < X < 0.88) = \int_{0.12}^{0.88} (30x^2(1-x)^2) dx$$

$$= 30\left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5}\right)\bigg|_{0.12}^{0.88} \approx 30(0.227 - 0.3 + 0.1055) - 30(0.000576 - 0.0001 + 0)$$

$$= 0.96072 \geq 0.75$$

4.98

(a)

x	0	1	2
g(x)	0.2	0.32	0.48

y	0	1	2
h(y)	0.26	0.35	0.39

x	0	1	2
$f(x 2) = \frac{f(x,2)}{h(2)}$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{10}{13}$

(b) $E(X) = 0 \cdot 0.2 + 1 \cdot 0.32 + 2 \cdot 0.48 = 1.28$

$$\text{Var}(x) = E(X^2) - \mu^2 = 0 \cdot 0.2 + 1 \cdot 0.32 + 4 \cdot 0.48 - 1.28 \cdot 1.28 = 0.6016$$

(c) $E(X|Y=2) = 0 \cdot \frac{4}{39} + 1 \cdot \frac{5}{39} + 2 \cdot \frac{10}{13} = \frac{65}{39}$

$$\text{Var}(X|Y=2) = E(X^2|Y=2) - E(X|Y=2)^2 = 0 \cdot \frac{4}{39} + 1 \cdot \frac{5}{39} + 4 \cdot \frac{10}{13} - \frac{65}{39} \cdot \frac{65}{39}$$

$$= \frac{125}{39} - \frac{4225}{1521} = \frac{650}{1521} = \frac{50}{117}$$