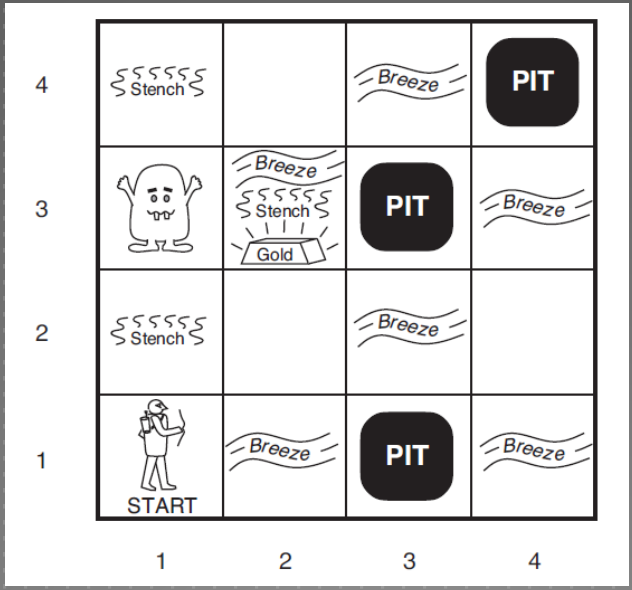
CSCI561 FALL 2015

Prof Tejada stejada@usc.edu

EXERCISE 8.17

- Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:
- $\forall$ x, y Adjacent([x, y], [x + 1, y]) $\wedge$ Adjacent([x, y], [x, y + 1])



- How about this one?
- Adjacency of cells
- $\forall x,y,a,b\ Adjacent([x, y], [a, b]) \Leftrightarrow$
- $[a, b] \in \{[x+1, y], [x-1, y], [x, y+1], [x, y-1]\}$

EXERCISE 8.21

- In Chapter 6, we wrote WA=red to mean that Western Australia is colored red.

- Representing this in first-order logic, we must write more verbosely ColorOf(WA)=red.
- What incorrect inference could be drawn if we wrote sentences such as WA = red directly as logical assertions?

WA = red
Q = red
WA = Q (!)

EXERCISE 8.24

- Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

a. Some students took French in spring 2001.
b. Every student who takes French passes it.
c. Only one student took Greek in spring 2001.
d. The best score in Greek is always higher than the best score in French.
e. Every person who buys a policy is smart.
f. No person buys an expensive policy.
g. There is an agent who sells policies only to people who are not insured.
h. There is a barber who shaves all men in town who do not shave themselves
i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.
j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.
k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.
l. All Greeks speak the same language. (Use Speaks(x,l) to mean that person x speaks language l.)

- Student(x): x is a student

- Takes(x, c, s): student x takes course c in semester s;
- Passes(x, c, s): student x passes course c in semester s;
- Score(x, c, s): the score obtained by student x in course c in semester s;

- Policy(x): x is a policy
- Expensive(x): x is expensive
- Buys(x, y, z): x buys y from z (using a binary predicate with unspecified seller is OK but less expressive);

EXERCISE 8.24

a. Some students took French in spring 2001.

b. Every student who takes French passes it.

c. Only one student took Greek in spring 2001.

f. No person buys an expensive policy.

EXERCISE 8.24

a. Some students took French in spring 2001.

- $\exists x\ \text{Student}(x) \wedge \text{Takes}(x,F,\text{Spring2001})$.

- b. Every student who takes French passes it.
- $\forall x,s\ \text{Student}(x) \wedge \text{Takes}(x,F,s) \Rightarrow \text{Passes}(x,F,s)$.

- c. Only one student took Greek in spring 2001.
- $\exists x\ \text{Student}(x) \wedge \text{Takes}(x,G,\text{Spring2001}) \wedge$
- $\forall y\ y \neq x \Rightarrow \neg\text{Takes}(y,G,\text{Spring2001})$.

- f. No person buys an expensive policy.

$\forall x,y,z\ \text{Person}(x) \wedge \text{Policy}(y) \wedge \text{Expensive}(y) \Rightarrow \neg\text{Buys}(x,y,z)$.

T akes(x, c, s): student x takes course c in semester s;
P asses(x, c, s): student x passes course c in semester s;
Score(x, c, s): the score obtained by student x in course c in semester s;
x > y: x is greater than y;
F and G: specific French and Greek courses (one could also interpret these sentences as referring to *any* such course, in which case one could use a predicate Subject(c,f) meaning that the subject of course c is field f ;
Buys(x, y, z): x buys y from z (using a binary predicate with unspecified seller is OK but less felicitous);
Sells(x, y, z): x sells y to z;
Shaves(x, y): person x shaves person y
Born(x, c): person x is born in country c;
P arent(x, y): x is a parent of y;
Citizen(x, c, r): x is a citizen of country c for reason r;
Resident(x, c): x is a resident of country c;
F ools(x, y, t): person x fools person y at time t;
Student(x), Person(x), Man(x), Barber(x), Expensive(x), Agent(x), Insured(x), Smart(x), Politician(x): predicates satisfied by members of the corresponding categories.

- **Some students took French in spring 2001.**
  $\exists x$ Student(x) $\wedge$ Takes(x,F,Spring2001).
- **Every student who takes French passes it.**
  $\forall x,s$ Student(x) $\wedge$ Takes(x,F,s) $\Rightarrow$ Passes(x,F,s).
- **Only one student took Greek in spring 2001.**
  $\exists x$ Student(x) $\wedge$ Takes(x,G,Spring2001) $\wedge$ $\forall y$ y$\square$=x$\Rightarrow\neg$Takes(y,G,Spring2001).
- **The best score in Greek is always higher than the best score in French. $\forall s$ $\exists x$ $\forall y$ Score(x,G,s) > Score(y,F,s).**
- **Every person who buys a policy is smart.**
  $\forall x$ Person(x) $\wedge$ ($\exists y,z$ Policy(y) $\wedge$ Buys(x,y,z)) $\Rightarrow$ Smart(x).
- **No person buys an expensive policy.**
  $\forall x,y,z$ Person(x) $\wedge$ Policy(y) $\wedge$ Expensive(y) $\Rightarrow \neg$Buys(x,y,z).
- **There is an agent who sells policies only to people who are not insured.**
  $\exists x$ Agent(x) $\wedge$ $\forall y,z$ Policy(y) $\wedge$ Sells(x,y,z) $\Rightarrow$ (Person(z) $\wedge \neg$Insured(z)).
- **There is a barber who shaves all men in town who do not shave themselves. $\exists x$ Barber(x) $\wedge$ $\forall y$**
  Man(y) $\wedge \neg$Shaves(y,y) $\Rightarrow$ Shaves(x,y).
- **A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.**
  $\forall x$ Person(x) $\wedge$ Born(x,UK) $\wedge$ ($\forall y$ Parent(y,x) $\Rightarrow$ (($\exists r$ Citizen(y,UK,r)) $\vee$ Resident(y,UK))) $\Rightarrow$ Citizen(x,UK,Birth).
- **A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.**
- **$\forall x$ Person(x) $\wedge \neg$Born(x,UK) $\wedge$ ($\exists y$ Parent(y,x) $\wedge$ Citizen(y,UK,Birth)) $\Rightarrow$ Citizen(x,UK,Descent).**
- **Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.**
- **$\forall x$ Politician(x) $\Rightarrow$**
  ($\exists y$ $\forall t$ Person(y) $\wedge$ Fools(x,y,t)) $\wedge$ ($\exists t$ $\forall y$ Person(y) $\Rightarrow$ Fools(x,y,t)) $\wedge \neg$($\forall t$ $\forall y$ Person(y) $\Rightarrow$ Fools(x,y,t))
- **I.** All Greeks speak the same language.
  $\forall x,y,l$ Person(x) $\wedge$ [$\exists r$ Citizen(x,Greece,r)] $\wedge$ Person(y) $\wedge$ [$\exists r$ Citizen(y,Greece,r)]
- $\wedge$ Speaks(x,l) $\Rightarrow$ Speaks(y,l)

EXERCISE 8.28

- Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs).
- The vocabulary contains the following symbols:
- CopyOf (d, a): Predicate. Disk d is a copy of album a.

Owns(p, d): Predicate. Person p owns disk d.
Sings(p, s, a): Album a includes a recording of song s sung by person p.
Wrote(p, s): Person p wrote song s.
McCartney , Gershwin , BHoliday , Joe , EleanorRigby , TheManILove , Revolver : Constants with the obvious meanings.

EXERCISE 8.28

- Express the following statements in first-order logic:

a. Gershwin wrote "The Man I Love."
b. Gershwin did not write "Eleanor Rigby."
c. Either Gershwin or McCartney wrote "The Man I Love."
d. Joe has written at least one song.
e. Joe owns a copy of *Revolver*.
f. Every song that McCartney sings on *Revolver* was written by McCartney.
g. Gershwin did not write any of the songs on *Revolver*.
h. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
i. There is a single album that contains every song that Joe has written.
j. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."
k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each
l. different album is instantiated in a different physical CD.)
m. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.

EXERCISE 8.28

Gershwin wrote "The Man I Love."

b. Gershwin did not write "Eleanor Rigby."

c. Either Gershwin or McCartney wrote "The Man I Love."

d. Joe has written at least one song.

EXERCISE 8.28

a. Gershwin wrote "The Man I Love."

- Wrote(Gershwin, The_Man_I_Love).

- b. Gershwin did not write "Eleanor Rigby."
- ¬Wrote (Gershwin, Eleanor_Rigby).

- c. Either Gershwin or McCartney wrote "The Man I Love."

- Wrote(Gershwin, The_Man_I_Love) ∨ Wrote (McCartney, The_Man_I_Love).

- d. Joe has written at least one song.
- ∃ s Wrote(Joe,s)

- $W(G,T)$.
- $\neg W(G,E)$.
- $W(G,T) \vee W(M,T)$.
- $\exists s\, W(J,s)$.
- $\exists x\, C(x,R) \wedge O(J,x)$.
- $\forall s\, S(M,s,R) \Rightarrow W(M,s)$.
- $\neg[\exists s\, W(G,s) \wedge \exists p\, S(p,s,R)]$.
- $\forall s\, W(G,s) \Rightarrow \exists p,a\, S(p,s,a)$.
- $\exists a\ \forall s\, W(J,s) \Rightarrow \exists p\, S(p,s,a)$.
- $\exists d,a,s\, C(d,a) \wedge O(J,d) \wedge S(B,T,a)$.
- $\forall a\, [\exists s\, S(M,s,a)] \Rightarrow \exists d\, C(d,a) \wedge O(J,d)$.
- $\forall a\, [\forall s,p\, S(p,s,a) \Rightarrow S(B,s,a)] \Rightarrow \exists d\, C(d,a) \wedge O(J,d)$.