PROPOSITIONAL LOGIC

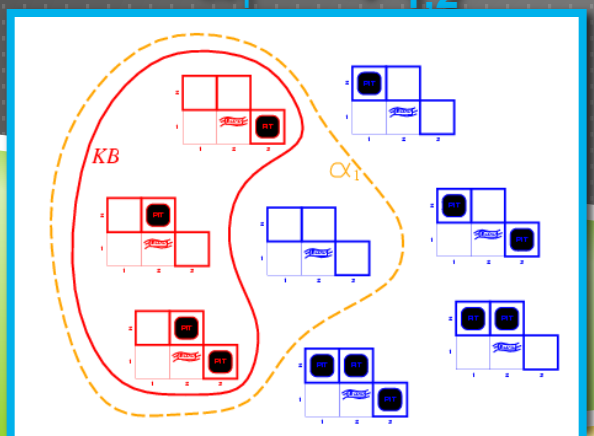
Prof Tejada stejada@usc.edu

USING TRUTH TABLES IN INFERENCE

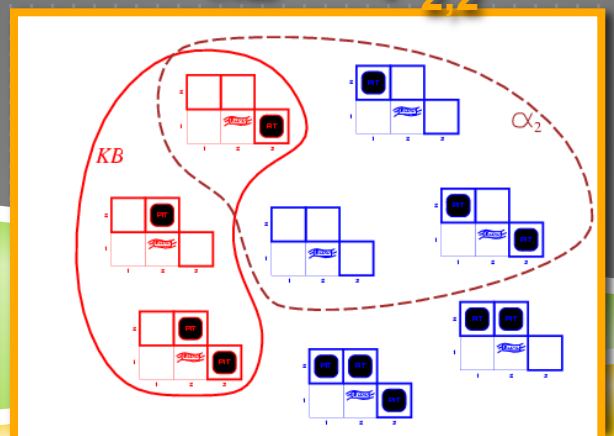
$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

R_5 R_3 R_3 R_2 R_1

$\alpha_1 = \neg P_{1,2}$



$\alpha 2 = \neg P_{2,2}$



KB

R_1: $\neg P_{1,1}$
R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
R_4: $\neg B_{1,1}$
R_5: $B_{2,1}$

EXERCISE 7.4

1. False |= True.
2. True |= False.
3. $(A \wedge B) \models (A \Leftrightarrow B)$.
4. $A \Leftrightarrow B \models A \vee B$.
5. $A \Leftrightarrow B \models \neg A \vee B$.
6. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.
7. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
9. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable.
10. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ Is satisfiable.
11. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

EXERCISE 7.4

False $\vDash$ True.

EXERCISE 7.4

False $\models$ True. TRUE

- False has no models and hence entails every sentence
- True is true in all models and hence is entailed by every sentence.

EXERCISE 7.4

- True $\nvDash$ False.

EXERCISE 7.4

True $\models$ False. FALSE

- False is not true in any models

EXERCISE 7.4

- $(A \wedge B) \models (A \Leftrightarrow B)$.

EXERCISE 7.4

- $(A \wedge B) \models (A \Leftrightarrow B)$. TRUE

- The left-hand side $(A \wedge B)$ has exactly one model: A=True and B=True then $(A \wedge B)$=True
- That model is one of the two models of the right-hand side $(A \Leftrightarrow B)$. Two models:
  - A=True and B=True then $(A \Leftrightarrow B)$ =True
  - A=False and B=False then $(A \Leftrightarrow B)$ =True

EXERCISE 7.4

- $A \Leftrightarrow B \models A \vee B$.

EXERCISE 7.4

- $A \Leftrightarrow B \models A \vee B$. FALSE

- $(A \Leftrightarrow B)$ has two models:
  - A=True and B=True then $(A \Leftrightarrow B)$ =True
  - A=False and B=False then $(A \Leftrightarrow B)$ =True
- $(A \vee B)$ is not True in this model
  - A=False and B=False then $(A \vee B)$ = False

EXERCISE 7.4

- $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

EXERCISE 7.4

- $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable. TRUE

- This sentence $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is True, when A=True and B=True

EXERCISE 7.4

1. False |= True.
2. True |= False.
3. $(A \wedge B) \models (A \Leftrightarrow B)$.
4. $A \Leftrightarrow B \models A \vee B$.
5. $A \Leftrightarrow B \models \neg A \vee B$.
6. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.
7. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
9. $(A \vee B) \wedge \neg (A \Rightarrow B)$ is satisfiable.
10. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ Is satisfiable.
11. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.

1. Smoke ⇒ Smoke
2. Smoke ⇒ Fire
3. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
4. Smoke ∨ Fire ∨ ¬Fire
5. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
6. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
7. Big ∨ Dumb ∨ (Big ⇒ Dumb)

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- Smoke ⇒ Smoke

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- Smoke ⇒ Smoke Valid

1. Convert to ¬Smoke V Smoke
2. When Smoke=True then ¬Smoke V Smoke = True
3. When Smoke=False then ¬Smoke V Smoke = True
4. Always True = Valid.

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- Smoke ⇒ Fire

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- Smoke ⇒ Fire Neither

1. Convert to ¬Smoke V Fire
2. When Smoke=True and Fire=True then ¬Smoke V Fire = True
3. When Smoke=True and Fire=False then ¬Smoke V Fire =False
4. Not always True and not always False

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.
- (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire) Valid

1. Convert to ¬(Smoke ⇒ Fire) ∨ ((Smoke ∧ Heat) ⇒ Fire)
2. Convert to (Smoke ∧ ¬Fire) ∨ ((¬Smoke ∨ ¬Heat) ∨ Fire)
3. (Smoke ∧ ¬Fire) ∨ Fire ∨ (¬Smoke ∨ ¬Heat)
4. ((Smoke ∨ Fire) ∧ (¬Fire ∨ Fire)) ∨ (¬Smoke ∨ ¬Heat)
5. ((Smoke ∨ Fire) ∧ True) ∨ (¬Smoke ∨ ¬Heat)
6. (Smoke ∨ Fire) ∨ ((¬Smoke ∨ ¬Heat)

EXERCISE 7.10

- Decide whether each of the following sentences is valid, unsatisfiable, or neither.

1. Smoke ⇒ Smoke
2. Smoke ⇒ Fire
3. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
4. Smoke ∨ Fire ∨ ¬Fire
5. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
6. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
7. Big ∨ Dumb ∨ (Big ⇒ Dumb)

REVIEW OF KEY TERMS

- *Syntax*: formal structure of sentences
- *Semantics*: truth of sentences with respect to models
  - *Model:* A possible world that defines truth values for all sentences
- *Entailment*: necessary truth of one sentence given another
- *Inference*: determine whether sentence entailed by KB
  - Or equivalently, derive new sentences from old
- *Soundness*: produce only entailed sentences
- *Completeness*: can produce all entailed sentences
- *Equivalence*: sentences are true in the same models
- *Validity*: sentence is true in all models
- *Satisfiability*: sentence is true in some model