Lecture 3: Finite Automata

Administrivia

- Please get a Unix instructional account (cs164-xxx) using Webacct.
- I'd like to have teams formed by next week.

An Alternative Style for Describing Languages

- Rather than *giving* a single pattern, we can *give* a set of rules of the form:

$$A:\ \alpha_1\alpha_2\cdots\alpha_n,\ \ n \geq 0,$$

  where

  - A is a symbol that is intended to stand for a language (set of strings)—a *metavariable* or *nonterminal symbol*.
  - Each α_i is either a literal character (like `"a"`) or a nonterminal symbol.

- The interpretation of this rule is

  One way to form a string in $L(A)$ (the language denoted by A) is to concatenate one string each from $L(\alpha_1),\ L(\alpha_2), \ldots$.

  (where $L(\texttt{"}c\texttt{"})$ is just the language $\{\texttt{"}c\texttt{"}\}$).

- This is *Backus-Naur Form (BNF)*. A set of rules is a *grammar*. One of the nonterminals is designated as its *start symbol* denoting the language described by the grammar.

- **Aside:** You'll see that ':' written many different ways, such as '::=', '$\longrightarrow$', etc.

Some Abbreviations

- The basic form from the last slide is good for formal analysis, but not for writing.
- So, we can allow some abbreviations that are obviously exandable into the basic forms:

Abbreviation	Meaning
$A: \ \mathcal{R}_1 \,\vert \cdots \vert\, \mathcal{R}_n$	$A: \ \mathcal{R}_1$ $\vdots$ $A: \ \mathcal{R}_n$
$A: \ \cdots (\mathcal{R}) \cdots$	$B: \ \mathcal{R}$ $A: \cdots B \cdots$
$A:$ "c_1" | $\cdots$ | "c_n" (likewise other character classes)	$[c_1 \cdots c_n]$

Some Technicalities

- From the definition, each nonterminal in a grammar defines a language. Often, we are interested in just one of them (the *start symbol*), and the others are auxiliary definitions.
- The definition of what a rule means ("One way to form a string in $L(A)$ is...") leaves open the possibility that there are other ways to form items in $L(A)$ than covered in the rule.
- We need that freedom in order to allow multiple rules for A, but we don't really want to include strings that aren't covered by some rule.
- So precise mathematical definitions throw in sentences like:

  A grammar defines the *minimal* languages that contain all strings that satisfy the rules.

A Big Restriction (for now)

- For the time being, we'll also add a restriction. In each rule:

$$A: \ \alpha_1\alpha_2\cdots\alpha_n, \ \ n \geq 0,$$

  we'll require that if α_i is a nonterminal symbol, then either

  - All the rules for that symbol have to occured before all the rules for A, or
  - $i = n$ (i.e., is the last item) and α_n is A.

- We call such a restricted grammar a *Type 3* or *regular* grammar. The languages definable by regular grammars are called *regular languages.*

Claim: Regular languages are exactly the ones that can be defined by regular expressions.

Proof of Claim (I)

- Start with a regular expression, $\mathcal{R}$, and make a (possibly not yet valid) rule,

  R: $\mathcal{R}$

- Create a new (preceding) rule for each parenthesized expression.
- This will leave just the constructs '$X*$', '$X+$', and '$X?$'. What do we do with them?

Proof of Claim (II)

Replace construct. . .	. . . with Q, where
$R*$	

Proof of Claim (II)

Replace construct...	...with Q, where
$R*$	`Q :` `Q : R Q`
$R+$	

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R*	`Q :` `Q : R Q`
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$R+$	`Q : R` `Q : R Q`
$R?$	`Q :` `Q : R`

Example

- Consider the regular expression `("+"|"-")?("0"|"1")+`

1. R: `("+"|"-")?("0"|"1")+` *replace with ...*

2. Q_1: `"+" | "-"`
   Q_2: `"0" | "1"`
   R: Q_1? Q_2+ *replace with ...*

3. Q_3: ϵ | Q_1
   Q_4: Q_2 | Q_2 Q_4
   R: Q_3 Q_4

Side Note: The Empty Language

- The grammar for the empty language is a bit non-intuitive:

```
Q: Q
```

- *Any* set of strings, `Q`, satisfies this rule.
- Hence, by the implicit rule that we choose the *smallest* solution that satisfies all rules, `Q` represents the empty set.

Classical Pattern-Matching Implementation

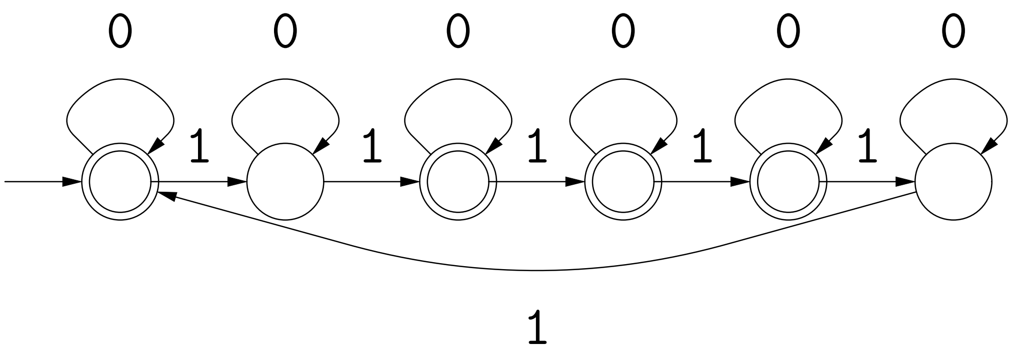
- For compilers, can generally make do with "classical" regular expressions.
- Implementable using *finite(-state) automata* or *FAs.* ("Finite state" = "finite memory").
- Classical construction:

  regular expression
  $\Rightarrow$ nondeterministic FA (NFA)
  $\Rightarrow$ deterministic FA (DFA)
  $\Rightarrow$ table-driven program.

Review: FA operation

- A FA is a graph whose nodes are states (of memory) and whose edges are state transitions. There are a finite number of nodes.
- One state is the designated start state.
- Some subset of the nodes are final states.
- Each transition is labeled with a set of symbols (characters, etc.) or ϵ.
- A FA recognizes a string $c_1c_2\cdots c_n$ if there is a path (sequence of edges) from the start state to a final state such that the labels of the edges in sequence, aside from ϵ edges, respectively contain $c_1, c_2, \ldots, c_n$.
- If the edges leaving any node have disjoint sets of characters and if there are no ϵ nodes, FA is a DFA, else an NFA.

Example: What does this DFA recognize?



What is the simplest equivalent NFA you can think of?

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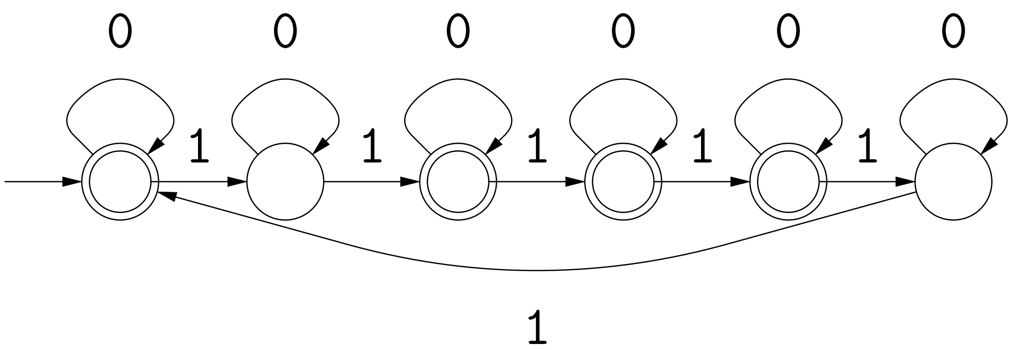

Bit strings with # of 1's divisible by 2 or 3.

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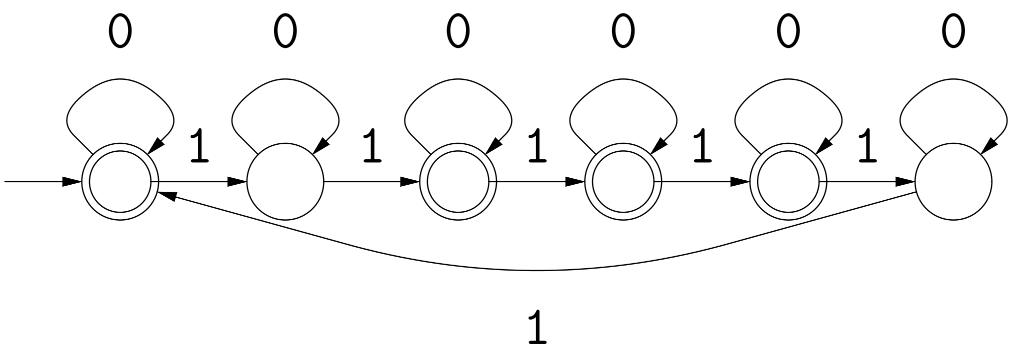

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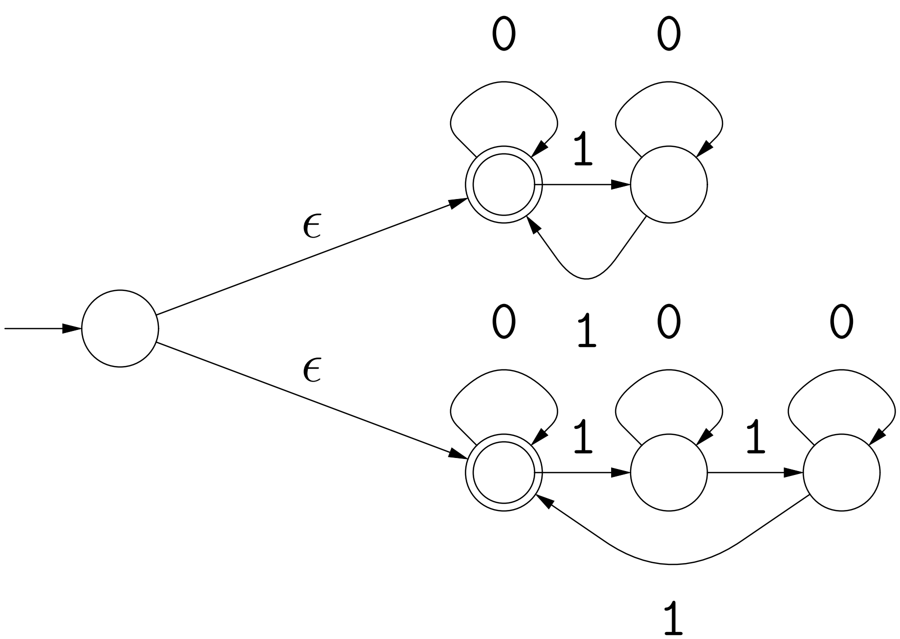

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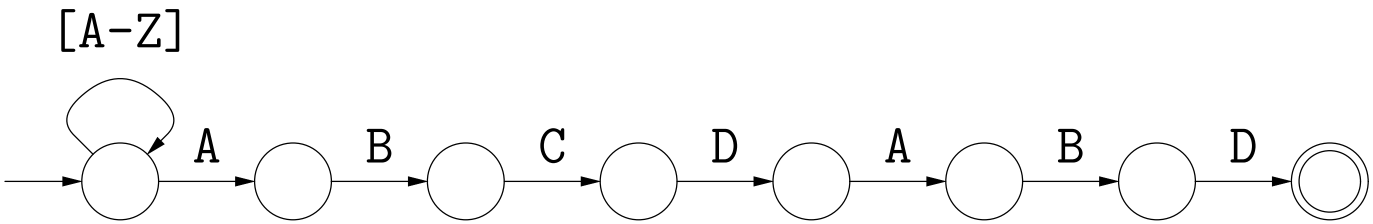

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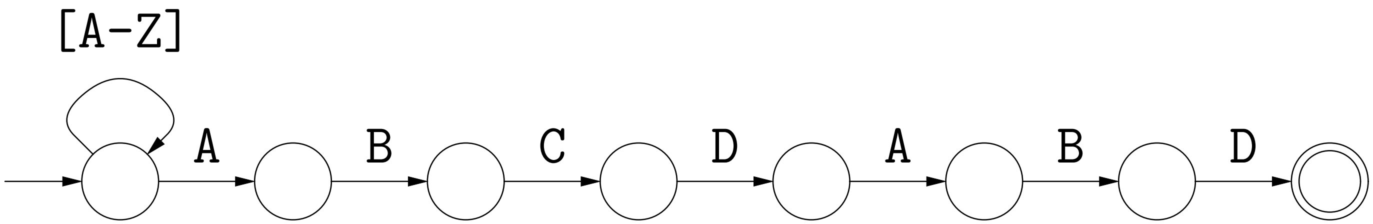

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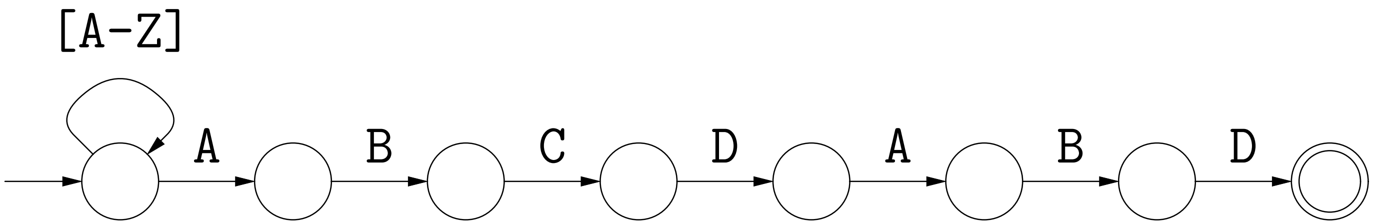

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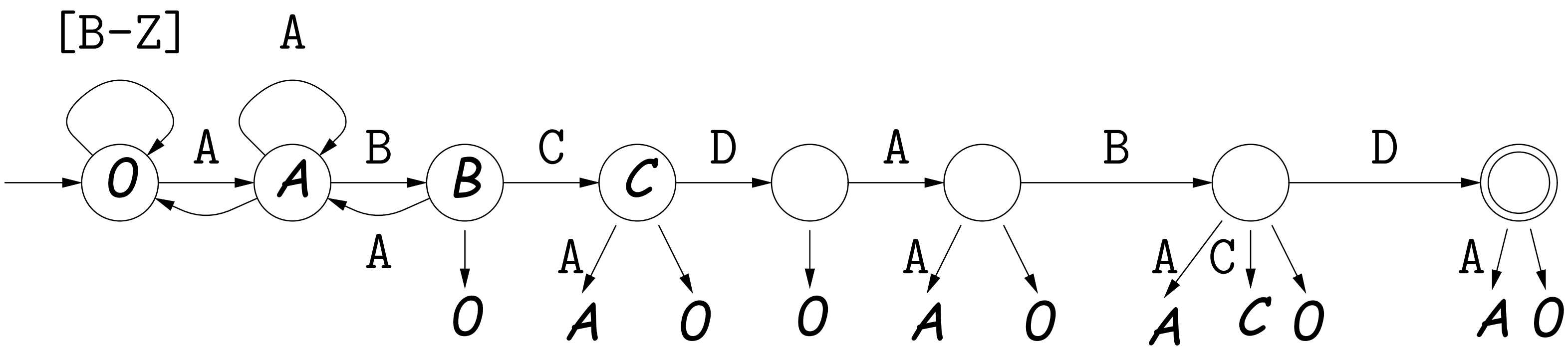

(Edges without labels mean "any character not covered by another edge.")

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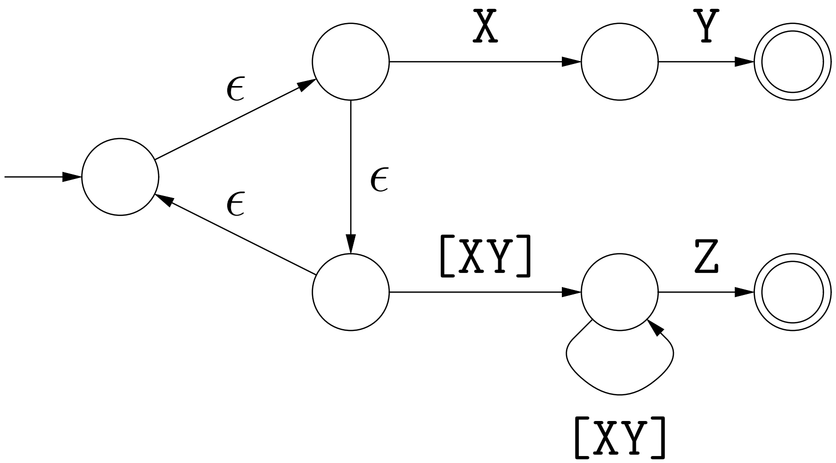

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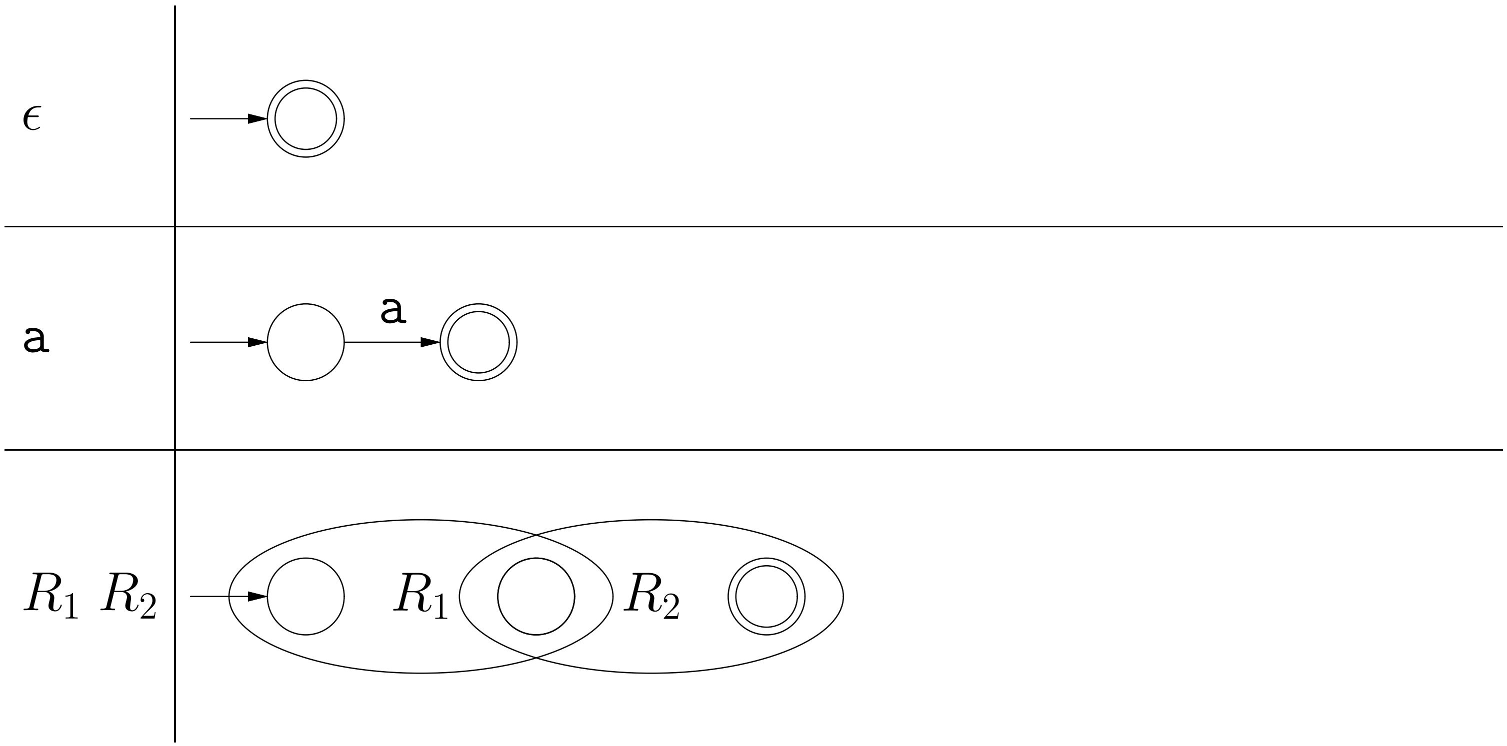
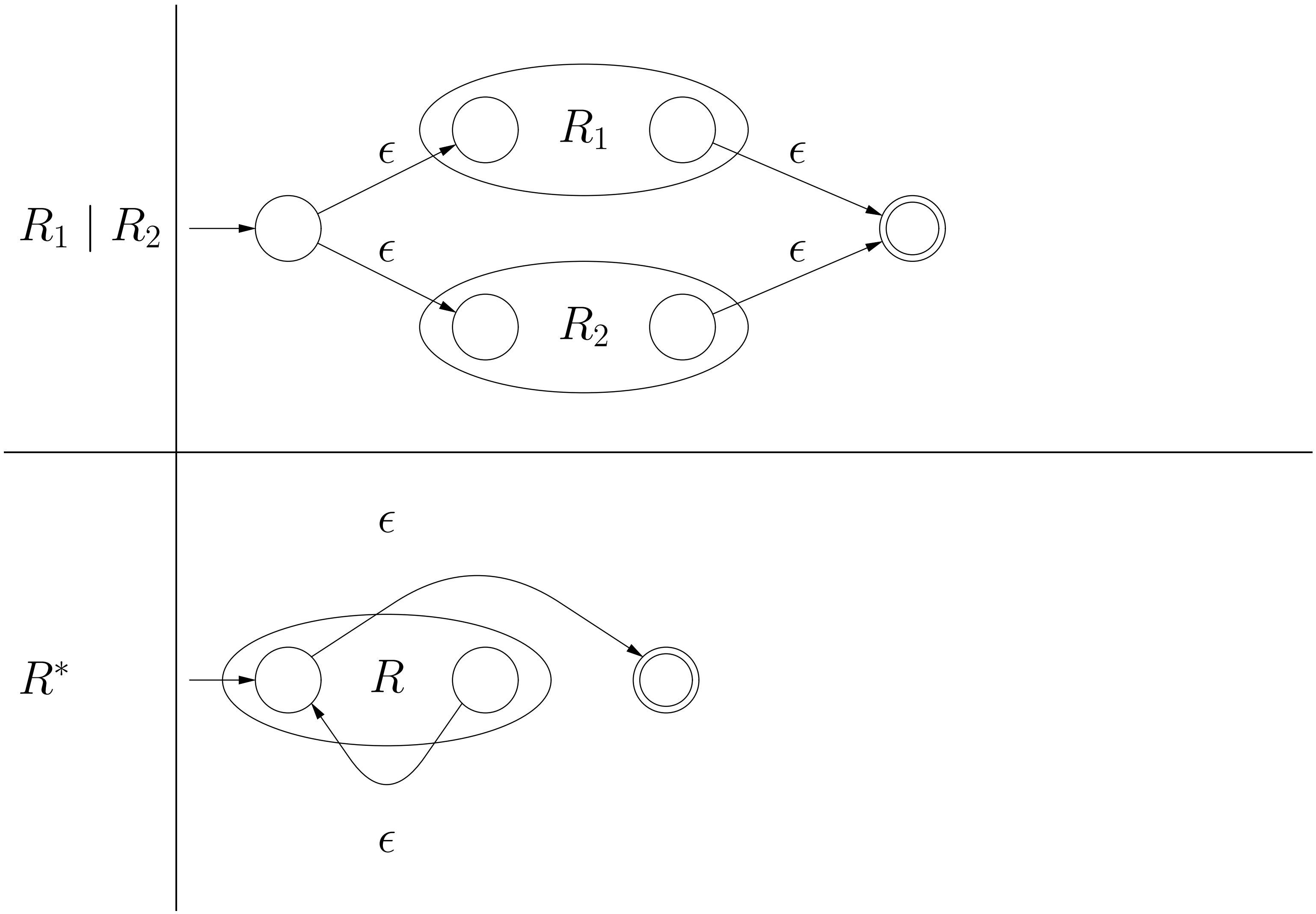
Review: Classical Regular Expressions to NFAs (I)



Review: Classical Regular Expressions to NFAs (II)



Extensions?

- How would you translate ϕ (the empty language, containing no strings) into an FA?
- How could you translate 'R?' into an NFA?
- How could you translate 'R+' into an NFA?
- How could you translate '$R_1|R_2|\cdots|R_n$' into an NFA?

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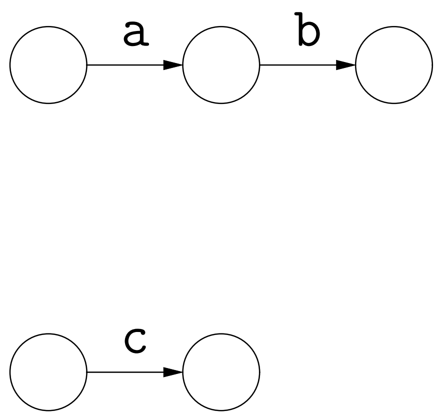

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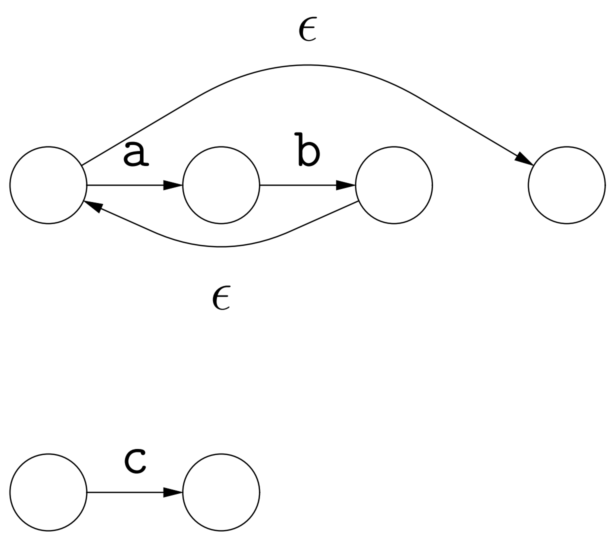

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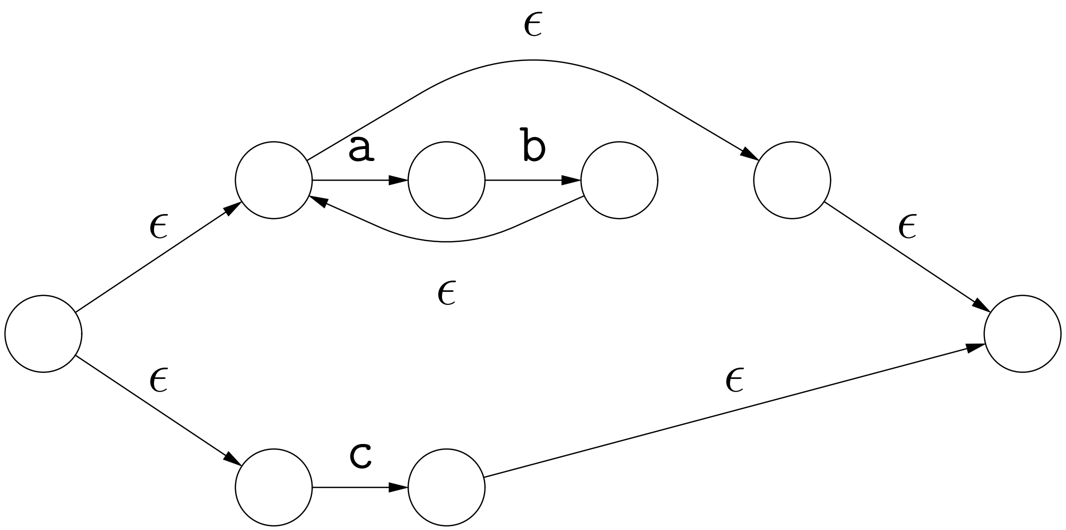

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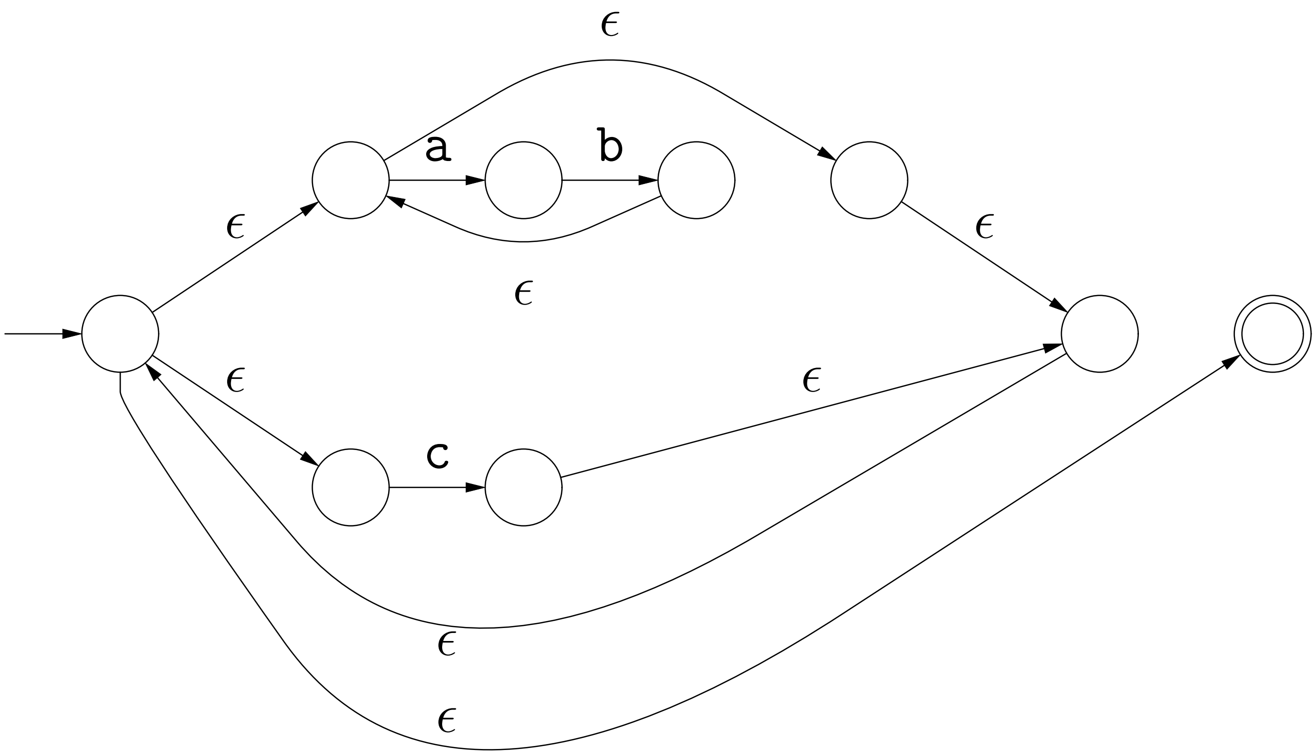
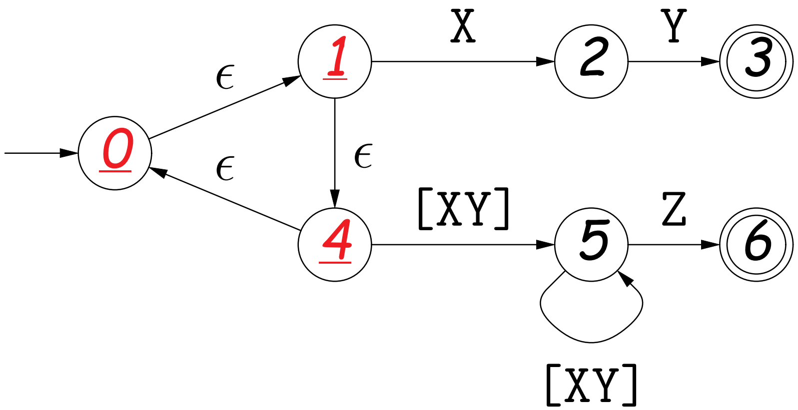
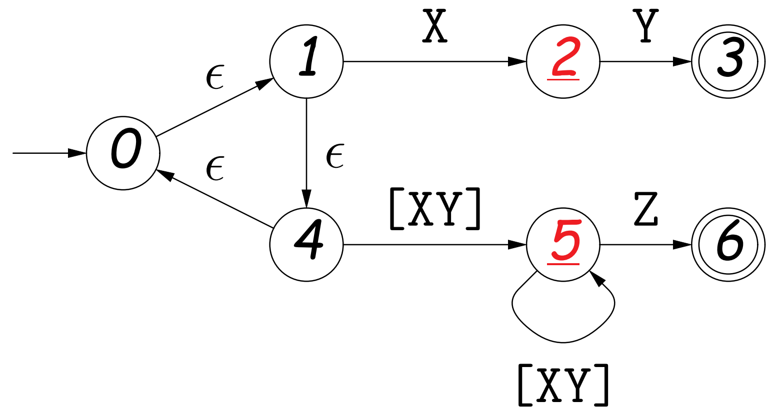
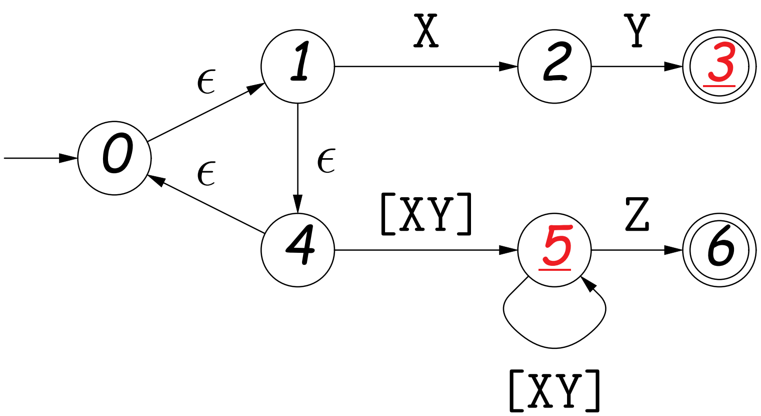
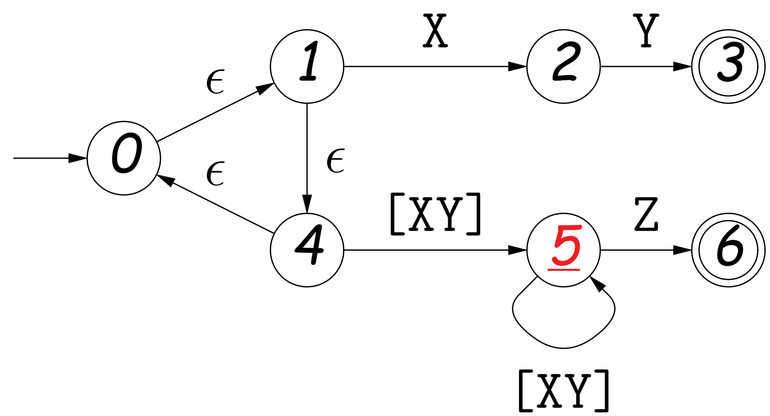


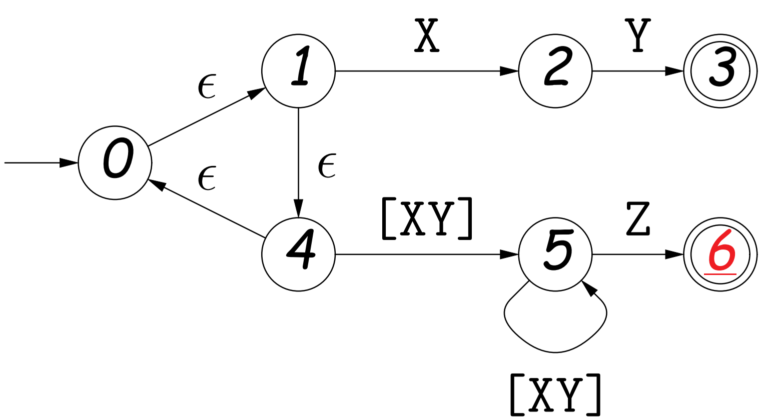
Abstract Implementation of NFAs







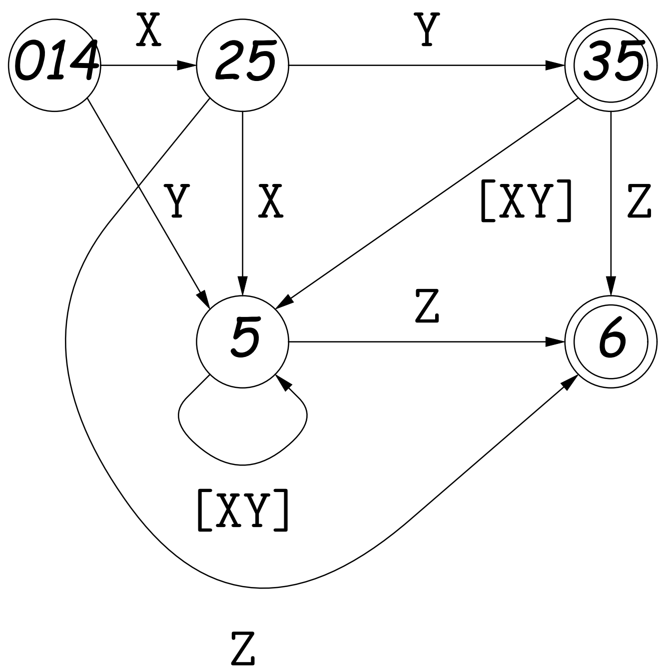




String: XYYZ

Review: Converting to DFAs

- **OBSERVATION:** The set of states that are marked (colored red) changes with each character in a way that depends only on the set and the character.
- In other words, machine on previous slide acted like this DFA:



DFAs as Programs

- Can realize DFA in program with control structure:

```
state = INITIAL;
for (s = input; *s != '\0'; s += 1) {
   switch (state):
   case INITIAL:
      if (*s == 'a') state = A_STATE; break;
   case A_STATE:
      if (*s == 'b') state = B_STATE; else state = INITIAL; break;
   ...
   }
}
return state == FINAL1 || state == FINAL2;
```

- Or with data structure (table driven):

```
state = INITIAL;
for (s = input; *s != '\0'; s += 1)
   state = transition[state][s];
return isfinal[state];
```

What JLex and Flex Do

- A JLex or Flex program specification is a giant regular expression of the form $R_1|R_2|\cdots|R_n$, where none of the R_i match ϵ.
- Each final state labeled with some action.
- Converted, by previous methods, into a table-driven DFA.
- But, this particular DFA is used to recognize *prefixes* of the (remaining) input: initial portions that put machine in a final state.
- Which final state(s) we end up in determine action. To deal with multiple actions:
  - Match *longest* prefix ("maximum munch").
  - If there are multiple matches, apply *first* rule in order.

How Do They Do It (I)?

Q: How can we use a DFA to recognize longest match?

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Answer:

- Use the DFA to scan the input until there is no transition on the current symbol.
- Every time the DFA enters a final state, record the input position and the state.
- When the scan stops, reset the input position to the last one saved, and use the last-saved final state as the result.

How Do They Do It (II)?

Q: How can we use DFA to act on the first of equal-length matches? Example:

```
while|[a-zA-Z]+
```

That is, we want our DFA to distinguish the keyword "while" from non-keyword identifiers.

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Answer:

- 1. The NFA for patterns of the form $R_1|R_2|\cdots|R_n$ may be formed from the NFAs for each of the R_is.
- 2. In those NFAs, label the final state for R_i the integer i.
- 3. Take the labels of the DFA to be sets of states from the NFA.
- 4. When we determine the final state of the DFA, look at its label and find the smallest of the integer labels from step 2 among the NFA states that label it.

How Do They Do It (III)?

Q: How can we use a DFA to handle the R_1/R_2 pattern (matches just R_1 but only if followed by R_2, like R_1`(?=`R_2`)` in Python)?

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Answer:

- Construct the NFAs for R_1 and R_2 and glue them together to get an NFA for R_1R_2.
- When scanning the string, record the state and position whenever you pass through a final state of the original R_1.
- When you get to a final state of the combined pattern for R_1R_2, use the last recorded final state and position for R_1.