

# Firehouse Location

A previously unpopulated large area is developing fast, due to rezoning. The area is divided into several districts  $d \in D$ . The township plans to build new firehouses in some of these districts (one or more). **How to locate these so that we minimize the total cost?**

We know for each district how much it would cost there to build a firehouse and operate it for several years,  $B_d$ ,  $d \in D$ . We also know the travel time  $t_{d,e}$  between any pair of these districts  $d, e \in D$ , and based on these distances we estimated the cost  $c_{d,e}$  of a fire event in district  $e$  if trucks were sent from  $d$ , for all  $d, e \in D$ , on average. (Clearly, arriving later cost more, since fire can do more damage; these are estimates based on many years of experience in other similar areas.) We also know the expected number  $m_d$  of fires (on average) in district  $d \in D$ . **We can assume that if there is a fire in a district we send fire trucks from the nearest location. This must be wired in for 911 calls!**

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## SETS:

- Districts  $D$ .

## PARAMETERS:

- Fixed costs of building  $B_d$ ,  $d \in D$ .
- Expected number of fires  $m_d$ ,  $d \in D$ .
- Travel times  $t_{d,e}$ ,  $d, e \in D$ .
- Estimated extra damage in district  $e \in D$  if fire truck is sent from district  $d \in D$ :  $c_{d,e}$ .

## VARIABLES:

- Binary  $x_d$  indicating whether we build or not a firehouse in district  $d \in D$ .
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# Firehouse Location

## CONSTRAINTS:

- Cannot send fire truck if no firehouse is built:

$$y_{d,e} \leq x_d \quad \text{for all } d, e \in D.$$

- All fires are extinguished:

$$\sum_{d \in D} y_{d,e} = 1 \quad \text{for all } e \in D.$$

- The closest district sends fire truck:

$$x_f + y_{d,e} \leq 1 \quad \text{for all } d, e, f \in D \text{ s.t. } t_{f,e} < t_{d,e}.$$

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**OBJECTIVE:**

$$\min \sum_{d \in D} B_d * x_d + \sum_{d,e \in D} c_{d,e} * m_e * y_{d,e}$$

**VARIABLE TYPES:**

$$\begin{aligned} x_d &\in \{0, 1\} && \text{for all } d \in D, \\ y_{d,e} &\in \{0, 1\} && \text{for all } d, e \in D. \end{aligned}$$



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