Uncapacitated Plant Location

A rural county would like to promote sustainable and environment friendly living conditions and also to decrease the carbon footprint. One component is to build disposal plants where people can get rid of hazardous chemicals, paints etc.

- We know all town locations, the roads and distances between them.
- We know the number of people living in each towns, and we can estimate from past data the average number of trips a person will do yearly to a disposal plant.
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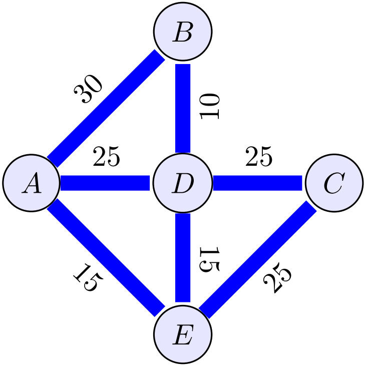
Map of County



Town	# People	Plant Cost
A	150,000	$2,000,000
B	200,000	$1,600,000
C	160,000	$1,200,000
D	80,000	$4,000,000
E	45,000	$3,500,000

- Average number of trips a person makes to a plant in a year: $n = 2$
- Average cost of driving a mile: $c = 0.2$

Main Entities of the Problem

SETS:

- Set of locations (towns): L

PARAMETERS:

- Population: $p\{L\}$ (person)
- Plant building cost: $b\{L\}$ ($)
- Trips: n (trips/person*year)
- Unit cost of travel: c ($/mile)
- Time horizon: T (years)
- Total personal cost of NOT RECYCLING at all: M ($)

DERIVED PARAMETERS:

- Distances: $d\{L, L\}$ (mile)

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DECISION VARIABLES:

- To build or not to build: $x\{L\}$, binary

$$x_\ell = \begin{cases} 1 & \text{if we build a plant} \\ 0 & \text{if we do not build a plant} \end{cases} \quad \text{at location } \ell \in L$$

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Model Components

- Expected number of total trips to a recycling plant from town $\ell \in L$:

$$n * p[\ell] * T$$

- Expected total mileage cost for citizens of town $\ell \in L$:

$$n * p[\ell] * T * c$$

- Total miles of trips to a recycling plant from town $\ell \in L$:

$$\sum_{k \in L} d[\ell, k] * \left(x_k * \prod_{\substack{j \in L \\ d[\ell, j] < d[\ell, k]}} (1 - x_j) \right)$$

- Total personal cost of not recycling at all:

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Model Objective

INVESTMENT COST:

$$\sum_{\ell \in L} b[\ell] * x_\ell$$

LONG TERM SOCIAL COST for $\ell \in L$:

$$(n * p[\ell] * T) * c * \sum_{k \in L} d[\ell, k] * \left(x_k * \prod_{\substack{j \in L \\ d[\ell,j] < d[\ell,k]}} (1 - x_j) \right)$$

No distance ties here!

LONG TERM SOCIAL COST of NOT RECYCLING:

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Unconstrained Nonlinear Binary Model

OBJECTIVE FUNCTION:

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\min \quad & \sum_{\ell \in L} b[\ell] * x_\ell \\
& + \sum_{\ell \in L} (n * p[\ell] * T) * c * \sum_{k \in L} d[\ell, k] * \left(x_k * \prod_{\substack{j \in L \\ d[\ell,j] < d[\ell,k]}} (1 - x_j) \right) \\
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CONSTRAINTS:

$$x \in \{0, 1\}^L$$

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Unconstrained Nonlinear Binary Model Simpl'r

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Linearization

We introduce new binary variables in place of the nonlinear products of variables and their complements. Namely, we introduce

$$\begin{aligned} z_{k,\ell} &= x_k * \prod_{\substack{j \in L \\ d[\ell,j] < d[\ell,k]}} (1 - x_j) \quad \forall k, \ell \in L, \text{ and} \\ z_L &= \prod_{\ell \in L} (1 - x_\ell) \end{aligned}$$

Standard Linearization

- For $k, \ell \in L$ we have $z_{k,\ell} = x_k * \prod_{\substack{j \in L \\ d[\ell,j] < d[\ell,k]}} (1 - x_j)$ if and only if $x \in \{0,1\}^L$ and the following inequalities hold

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\begin{aligned}
z_{k,\ell} &\leq x_k, \\
z_{k,\ell} &\leq 1 - x_j \qquad \forall j \in L \ s.t. \ d[\ell,j] < d[\ell,k], \\
z_{k,\ell} &\geq 0, \\
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General Principle of Standard Linearization

- Both of the previous equivalences are special cases of a more general rule.
- Assume $X_1, ..., X_m$ are binary variables, $\bar{X}_j = 1 - X_j$ for $j = 1, ..., m$, and that $S \subseteq \{X_1, ..., X_m, \bar{X}_1, ..., \bar{X}_m\}$.
- Then we have

$$Z = \prod_{u \in S} u$$

if and only if $X \in \{0,1\}^m$, and the following inequalities hold:

$$\begin{aligned} Z &\leq u && \forall u \in S, \\ Z &\geq 0, \\ Z &\geq 1 - |S| + \sum_{u \in S} u. \end{aligned}$$

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A MILP model of Uncapacitated Plant Location

$$\min \quad \sum_{\ell \in L} b[\ell] * x_\ell + n * T * c * \sum_{\ell \in L} p[\ell] * \sum_{k \in L} d[\ell, k] * z_{k,\ell} + M * \left(\sum_{\ell \in L} p[\ell] \right) * z_L$$

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z_L &\le 1 - x_j && \forall j \in L, \\
z_L &\ge 0, \\
z_L &\ge 1 - \sum_{j \in L} x_j, \\
x_\ell &\in \{0, 1\} && \forall \ell \in L.
\end{aligned}$$

SIRS Survey

Perhaps I forgot to mention

PLEASE fill out the SIRS survey!

I really appreciate your feedback ...

Lifting: Another Model in Higher Dimension

- Our main (primary) decision variables are x_ℓ, $\ell \in L$: "To build or not to build?"
- Implied (hidden) variables describe which plant is serving which community (remember: citizens are rational and travel to the nearest built plant).
- Introduce these hidden "decisions" $y\{L, L\} \in \{0,1\}^{L \times L}$:

$$y_{k,\ell} = \begin{cases} 1 & \text{if citizens of township } \ell \text{ served by plant built at } k \\ 0 & \text{otherwise.} \end{cases}$$

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- **Everybody is served** (NO "DO NOTHING" OPTION!):

$$\sum_{k \in L} y_{k,\ell} \;=\; 1 \quad \forall\, \ell \in L.$$

- **Do not go if no plant is built there:**

$$x_k \;\geq\; y_{k,\ell} \quad \forall\, k, \ell \in L.$$

- How to model rationality of citizens in terms of these variables?
- **RATIONALITY** of service:

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x_\ell & \in \{0, 1\} & \forall \ell \in L, \\
y_{k,\ell} & \in \{0, 1\} & \forall k, \ell \in L.
\end{array}$$

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