Hilbert Bases
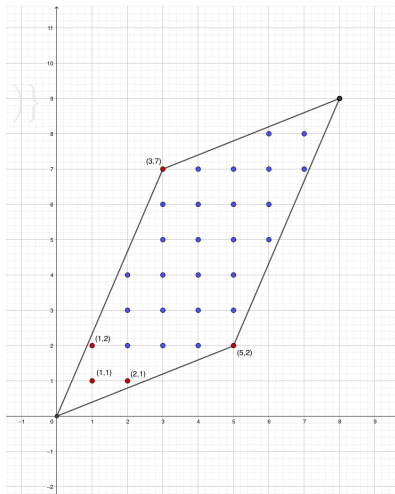
Endre Boros
26:711:653: Discrete Optimization

Spring 2019
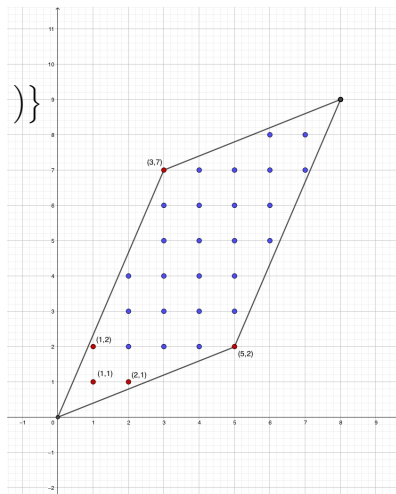
Hilbert Bases of Pointed Cones

▶ $\mathbf{C} = cone\left(\left(\begin{array}{c} 3 \\ 7 \end{array} \right), \left(\begin{array}{c} 5 \\ 2 \end{array} \right) \right)$

▶ $H^*(\mathbf{C}) = \left\{ \left(\begin{array}{c} 3 \\ 7 \end{array} \right), \left(\begin{array}{c} 1 \\ 2 \end{array} \right), \left(\begin{array}{c} 1 \\ 1 \end{array} \right), \left(\begin{array}{c} 2 \\ 1 \end{array} \right), \left(\begin{array}{c} 5 \\ 2 \end{array} \right) \right\}$

▶ Note the determinant values of the 2×2 matrices formed by the consecutive vectors.

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