Corner Polyhedra

Endre Boros
26:711:653: Discrete Optimization

Spring 2019

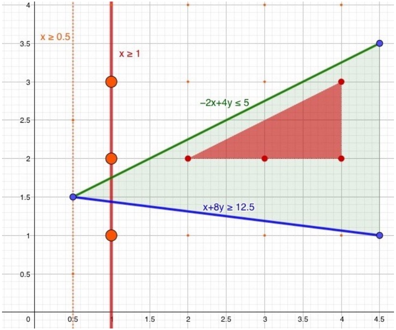
Chvátal-Gomory Cuts

- Consider the following inequalities over integer variables x and y:

$$
\begin{array}{llll}
\min & x & & \\
 & -2x & +4y & \le 5 \\
 & -x & -8y & \le -12.5
\end{array}
$$

- If we minimize x over these inequalities we get that the LP-optimum is at $\mathbf{A} = (0.5, 1.5)$
- $2-1$ CG-cut is

$$-\mathbf{x} \le \lfloor -\mathbf{0.5} \rfloor = -\mathbf{1}$$



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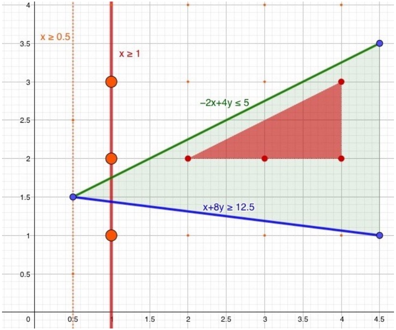

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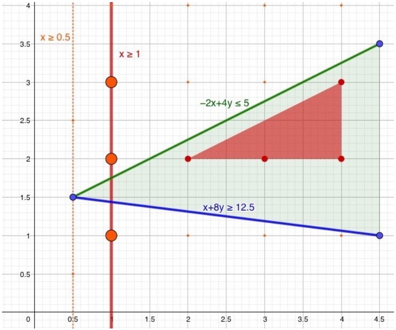

Corner Polyhedron

- IP problem over polyhedron **P** is defined by

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- LP-optimum is at **A** $= (0.5, 2.5)$
- $\frac{1}{3}$-$\frac{2}{3}$ CG-cut is

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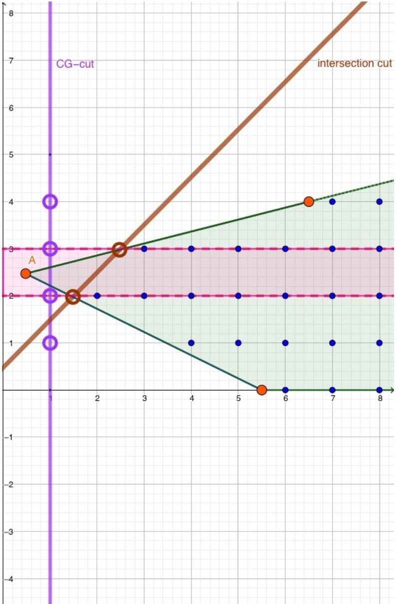
- Intersection cut for the stripe $2 \leq y \leq 3$ is

$$-\mathbf{x} + \mathbf{y} \leq \frac{1}{2}$$



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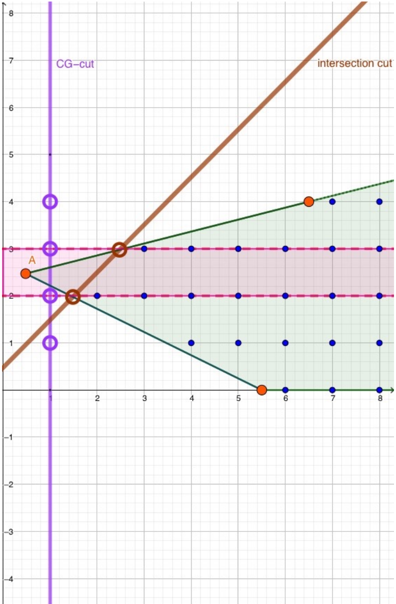

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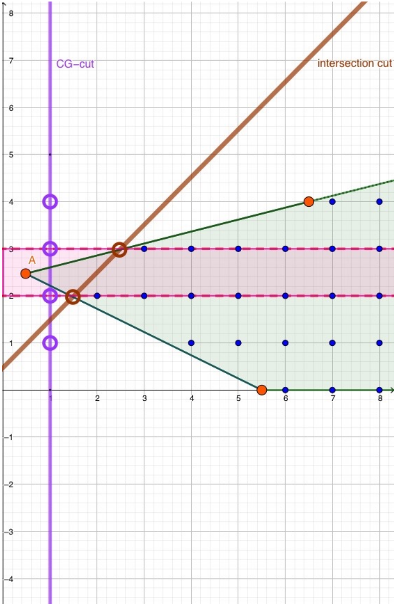

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