2.

a) $x_1, x_2, \dots x_m$ are drawn uniformly from the interval $[0, a]$.

Propapility of picking any of these number from the interval will be given $1/a$

Likelihood function can be written as

$$\text{Likelihood}\left(\frac{x_1 x_2 \dots x_m}{a}\right)$$

$$= \frac{1}{a} \dots \frac{1}{a} \times \frac{1}{a} \quad (m\text{ times})$$

$$= \prod_{i=1}^{m} \frac{1}{a}$$

Log- likelihood function

$$\log \prod_{i=1}^{m} 1/a = \log(1/a)^m = \cancel{\log a}$$

$$= -m\log a$$

finding the values of a which will that maximize the log likelihood can be found by taking derivative w.r.t. a.

$$\frac{d}{da}(-\log a^m) = \frac{-m\,d(\log a)}{da} = \frac{-m}{a}$$

derivative w.r.t. a is monotonically decreasing.

Thus, the mle for $\hat{a}$ would be the smallest $\hat{a}$ possible which is $\max(x_1, x_2 \dots x_m)$

b) for uniform distripution, Likelihood function can be written as $\prod_{i=1}^{m} f(x_i / a, b) = \prod_{i=1}^{n} \frac{1}{b-a} = \frac{1}{(b-a)^n}$

log likelihood function will be

$$\log\left(\prod_{i=1}^{n} \frac{1}{b-a}\right) = \log \frac{1}{(b-a)^n} = -n\log(b-a)$$

$$\frac{d}{da}(-n\log(b-a)) = \frac{n}{b-a}$$

$$\frac{d}{db}(-n\log(b-a)) = \frac{-n}{b-a}$$

as derivative w.r.t. a is monotonically increasing

a will be the ~~min~~ larges a possible

i.e. $\min(x_1, x_2 \cdots x_n)$

as derivative w.r.t. b is monotonically decreasing

b will be the smalles possible value.

i.e $\max(x_1, x_2 \cdots x_n)$

2.c)

Volume of Sphere is $= \frac{4}{3}\pi r^3$

Vector representor of C in d dimension will be $(c_1, c_2 \cdots c_d)$

Vector represetaion of any point x_i in d dimension will $(x_{i1}, x_{i2} \cdots x_{id})$

Euclidean distance will be

$$= \|x_1 - c\| = \sqrt{(x_1 - c_1)^2 \cdots (x_{id} - c_d)^2}$$

Propability of drawing any number from Sphere will be

$$= \frac{\|x_1 - c_1\|}{4/3\,\pi r^3}$$

Likelihood function will be $= \frac{\|x_1 - c_1\|}{4/3\,\pi r^3} \cdots \frac{\|x_n - c\|}{4/3\,\pi r^3}$

$$= \prod_{i=1}^{n} \frac{x_i - c}{4/3\,\pi r^3} = \frac{\prod_i^n (x_i - c)}{(4/3\,\pi r^3)^n}$$

Log Likelihood function will be

$$= \log \prod_1^n (x_i - c) - n \log \frac{4}{3}\pi r^3$$