Algorithmic Machine Learning

HW #3

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3.(a). likelihood function $= f_e(t|\lambda_1)\cdots f_e(t|\lambda_n)$

$= \lambda e^{-t_i\lambda}\cdots\lambda e^{-t_n\lambda}$

$= \lambda^n e^{-\lambda\Sigma t_i}$ $\Rightarrow$ known constant : $t_1 \ldots t_n$ unknown : λ

(b). $\log(\text{likelihood function}) = \log(\lambda^n e^{-\lambda\Sigma t_i})$

$= n\log\lambda - \lambda\Sigma t_i$

(c). $\frac{d}{d\lambda}(n\log\lambda - \lambda\Sigma t_i) = \frac{n}{\lambda} + \Sigma t_i = 0$

$\Rightarrow \frac{n}{\lambda} = \Sigma t_i$

$\Rightarrow \lambda_{ML} = \frac{n}{\Sigma t_i}$

(d). Posterior distribution $= f_e(t_1|\lambda)\cdots f_e(t_n|\lambda)\cdot$ Prior distribution

$= \lambda e^{-t_i\lambda}\cdots\lambda e^{-t_n\lambda}\cdot\frac{\lambda^{3.5}}{\Gamma(3.5)}\lambda^{2.5}e^{-\lambda\lambda_p}$

$= \lambda^n e^{-\lambda(\Sigma t_i+\lambda_p)}\cdot\frac{\lambda^{3.5}}{\Gamma(3.5)}\cdot\lambda^{n+2.5}$

(e). $\log(\text{Posterior}) = \log(e^{-\lambda(\Sigma t_i+\lambda_p)}\cdot\frac{\lambda^{3.5}}{\Gamma(3.5)}\cdot\lambda^{n+2.5})$

$= -\lambda(\Sigma t_i+\lambda_p) + \log(\frac{\lambda^{3.5}}{\Gamma(3.5)}) + (n+2.5)\log\lambda$

(f). $\frac{d}{d\lambda}\log(\text{posterior}) = \frac{d}{d\lambda}[-\lambda(\Sigma t_i+\lambda_p) + \log(\frac{\lambda^{3.5}}{\Gamma(3.5)}) + (n+2.5)\log\lambda)$

$\Rightarrow -(\Sigma t_i+\lambda_p) + \frac{n+2.5}{\lambda} = 0$

$\Rightarrow \lambda = \frac{n+2.5}{\Sigma t_i+\lambda_p}$

The data $t_1,\ldots,t_N$ exerts influence in the outcome.