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A contractor has a few employees, and his wife is taking orders on the phone. The contractor calls back the customers in the evening setting up appointments to do the job next day. All jobs are in a small neighborhood, and the employees can quickly move from one task to another, if needed. For each job the contractor can estimate the time needed (in hours) and the profit expected (in $).

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and

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- Claim: Inequalities (1) and (2) are equivalent.

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