

# Classification

## Logistic Regression

Response in the logistic regression formula is the log odds of a binary outcome of 1. We only observe the binary outcome, not the log odds, so special statistical methods are needed to fit the equation. Logistic regression is a special instance of generalized linear model (GLM) developed to extend linear regression to other settings.

In R, to fit a logistic regression, glm function is used with family set to "binomial". The following code fits a logistic regression to the personalloan data.

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```
loan_data = read.csv("~/Dropbox/Priya-PhD- Documents/Courses/Data Analysis and Visualization-Spring 2019/Datasets/loan_data.csv", header=TRUE)
head(loan_data)
```

```

X      status loan_amnt      term annual_inc  dti payment_inc_ratio
1 1 Charged Off    2500 60 months    30000  1.00      2.39320
2 2 Charged Off    5600 60 months    40000  5.55      4.57170
3 3 Charged Off    5375 60 months    15000 18.08      9.71600
4 4 Charged Off    9000 36 months    30000 10.08     12.21520
5 5 Charged Off   10000 36 months   100000  7.06      3.90888
6 6 Charged Off   21000 36 months   105000 13.22      8.01977
  revol_bal revol_util      purpose home_ownership
1    1687      9.4          car          RENT
2    5210     32.6  small_business          OWN
3    9279     36.5          other          RENT
4   10452     91.7 debt_consolidation          RENT
5    11997     55.5          other          RENT
6   32135     90.3 debt_consolidation          RENT
 delinq_2yrs_zero pub_rec_zero open_acc grade outcome emp_length
1              1              1         3   4.8 default         1
2              1              1        11   1.4 default         5
3              1              1         2   6.0 default         1
4              1              1         4   4.2 default         1
5              1              1        14   5.4 default         4
6              1              1         7   5.8 default        11
      purpose_ home_ emp_len_ borrower_score
1  major_purchase RENT > 1 Year         0.65
2  small_business  OWN > 1 Year         0.80
3         other RENT > 1 Year         0.60
4 debt_consolidation RENT > 1 Year         0.50
5         other RENT > 1 Year         0.55
6 debt_consolidation RENT > 1 Year         0.40
```

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```
logistic_model = glm(outcome ~ payment_inc_ratio + purpose_ + home_ + emp_len_ + borrower_score,
data = loan_data, family = 'binomial' )
logistic_model
```

```
Call: glm(formula = outcome ~ payment_inc_ratio + purpose_ + home_ +
emp_len_ + borrower_score, family = "binomial", data = loan_data)
```

Coefficients:

(Intercept)	payment_inc_ratio
-1.63809	-0.07974
purpose_debt_consolidation	purpose_home_improvement
-0.24937	-0.40774
purpose_major_purchase	purpose_medical
-0.22963	-0.51048
purpose_other	purpose_small_business
-0.62066	-1.21526
home_OWN	home_RENT
-0.04833	-0.15732
emp_len_ > 1 Year	borrower_score
0.35673	4.61264

Degrees of Freedom: 45341 Total (i.e. Null); 45330 Residual

Null Deviance: 62860

Residual Deviance: 57510 AIC: 57540

The response is outcome, which takes a 0 if the loan is paid off and 1 if the loan defaults. purpose\_ and home\_ are factor variables representing the purpose and the home ownership status. As in regression, a factor variable with P levels is represented using P-1 columns. By default, reference coding is used and the levels are all compared to the reference level. The reference level for these factors are credit\_card and MORTGAGE respectively. The variable borrower\_score is a score from 0 to 1 (poor to excellent) representing the creditworthiness of the borrower.

## Generalized Linear Models

GLMs are characterized by two main components: - A probability distribution or family (binomial in case of logistic regression) - A link function mapping the response to the predictors (logit in case of logistic regression)

Logistic regression is the most common form of GLM. Sometimes log link function is used instead of logit. Poisson distribution is used to model count data (number of times user visit a web page in certain amount of time). Other families include negative binomial and gamma (often used to model elapsed time).

## Predicted Values from Logistic Regression

The predicted value from logistic regression is in terms of log odds.  $\hat{Y} = \log(\text{Odds}(Y = 1))$ .

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```
pred = predict(logistic_model)
summary(pred)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-3.509606	-0.505061	0.008539	-0.002564	0.518825	2.704774

Converting these values to probabilities

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```
prob = 1/(1 + exp(-pred))
summary(prob)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.02904	0.37635	0.50213	0.50000	0.62687	0.93731

These are on a scale from 0 to 1 and don't indicate whether the predicted value is default or paid off. We could declare any value greater than 0.5 as default. A lower cutoff is often appropriate if the goal is to identify members of a rare class.

## Interpreting the Coefficients and Odds Ratios

Odds ratio is given by – \$odds ratio =

This is interpreted as the odds that  $Y=1$  when  $X=1$  versus the odds that  $Y=1$  when  $X=0$ . If the odds ratio is 2, then the odds that  $Y=1$  are two times higher when  $X=1$  versus  $X=0$ .

We work with odds because the coefficient  $\beta_j$  in the logistic regression is the log of the odds ratio for  $X_j$ .

## Assessing the Model

Logistic regression is assessed by how accurately the model classifies new data.

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```
summary(logistic_model)
```

Call:

```
glm(formula = outcome ~ payment_inc_ratio + purpose_ + home_ +
     emp_len_ + borrower_score, family = "binomial", data = loan_data)
```

Deviance Residuals:

Min	1Q	Median	3Q	Max
-2.15528	-1.07421	0.05853	1.06908	2.51951

Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	-1.638092	0.073708	-22.224	< 2e-16 ***
payment_inc_ratio	-0.079737	0.002487	-32.058	< 2e-16 ***
purpose_debt_consolidation	-0.249373	0.027615	-9.030	< 2e-16 ***
purpose_home_improvement	-0.407743	0.046615	-8.747	< 2e-16 ***
purpose_major_purchase	-0.229628	0.053683	-4.277	1.89e-05 ***
purpose_medical	-0.510479	0.086780	-5.882	4.04e-09 ***
purpose_other	-0.620663	0.039436	-15.738	< 2e-16 ***
purpose_small_business	-1.215261	0.063320	-19.192	< 2e-16 ***
home_OWN	-0.048330	0.038036	-1.271	0.204
home_RENT	-0.157320	0.021203	-7.420	1.17e-13 ***
emp_len_ > 1 Year	0.356731	0.052622	6.779	1.21e-11 ***
borrower_score	4.612638	0.083558	55.203	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 62857 on 45341 degrees of freedom  
 Residual deviance: 57515 on 45330 degrees of freedom  
 AIC: 57539

Number of Fisher Scoring iterations: 4

Interpretation of p-value comes with the same caveat as in regression, and should be viewed more as a relative indicator of variable importance than a formal measure of statistical significance. Logistic regression model, which has a binary response, does not have an associated RMSE or R-squared. Logistic regression model is typically evaluated using more general metrics for classification.

Fit generalized additive model using "mgcv" package.

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```
library(mgcv)
```

Loading required package: nlme

Attaching package: 'nlme'

The following object is masked from 'package:dplyr':

collapse

This is mgcv 1.8-28. For overview type 'help("mgcv-package")'.

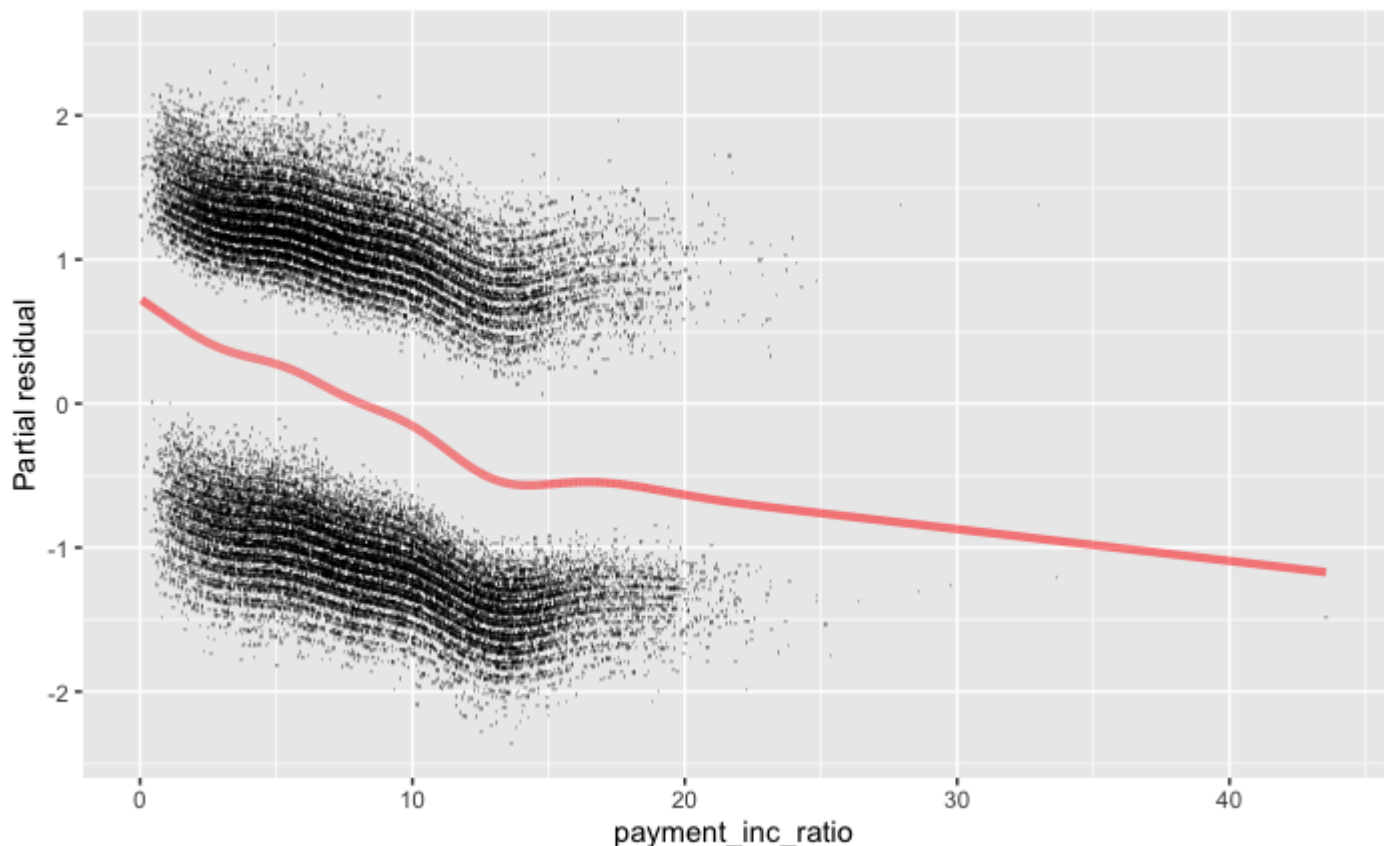
Hide

```
logistic_gam = gam(outcome~ s(payment_inc_ratio)+ purpose_ + home_ + emp_len_ + s(borrower_score), data = loan_data, family = "binomial")
```

One area where logistic regression differs from linear regression is in the analysis of residuals.

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```
terms = predict(logistic_gam, type = "terms")
partial_resid = resid(logistic_model) + terms
df = data.frame(payment_inc_ratio = loan_data[, 'payment_inc_ratio'],
                terms = terms[, 's(payment_inc_ratio)'],
                partial_resid = partial_resid[, 's(payment_inc_ratio)'])
ggplot(df, aes(x = payment_inc_ratio, y = partial_resid, solid = FALSE)) +
  geom_point(shape = 46, alpha = .4) +
  geom_line(aes(x = payment_inc_ratio, y = terms), color = 'red', alpha = 0.5, size = 1.5) +
  labs(y = "Partial residual")
```



The estimated fit, shown by the line goes between two sets of point clouds. The top cloud corresponds to a response of 1 (defaulted loans), and bottom cloud corresponds to a response of 0 (loans paid off). This is typical of residuals from a logistic regression since the output is binary. Partial residuals in logistic regression are useful to confirm non linear behavior and identify highly influential records.

## Evaluating Classification Models

Applying holdout set approach

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```
# Random sample indexes
train_index = sample(1:nrow(loan_data), 0.75 * nrow(loan_data))
test_index = setdiff(1:nrow(loan_data), train_index)
# Build train and test sets
train_set = loan_data[train_index, ]
test_set = loan_data[test_index, ]
fit_glm = glm(outcome ~ payment_inc_ratio + purpose_ + home_ + emp_len_ + borrower_score, data =
loan_data, family=binomial(link="logit"))
glm_link_scores = predict(fit_glm, test_set, type="link")
glm_response_scores = predict(fit_glm, test_set, type="response")
```

## Confusion Matrix

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```
pred = predict(fit_glm, newdata = test_set)
pred_y = as.numeric(pred > 0)
true_y = as.numeric(test_set$outcome == "default")
true_pos = (true_y == 1) & (pred_y == 1)
true_neg = (true_y == 0) & (pred_y == 0)
false_pos = (true_y == 0) & (pred_y == 1)
false_neg = (true_y == 1) & (pred_y == 0)
conf_mat = matrix(c(sum(true_pos),sum(false_pos),
                     sum(false_neg),sum(true_neg)),2,2)
colnames(conf_mat) = c('Yhat = 1', 'yhat = 0')
rownames(conf_mat) = c('Y = 1', 'Y = 0')
conf_mat
```

	Yhat = 1	yhat = 0
Y = 1	2148	3635
Y = 0	3595	1958

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```
#precision
precision = conf_mat[1,1]/sum(conf_mat[,1])
precision
```

```
[1] 0.3740205
```

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```
#recall  
recall = conf_mat[1,1]/sum(conf_mat[1,])  
recall
```

```
[1] 0.3714335
```

[Hide](#)

```
#specificity  
specificity = conf_mat[2,2]/sum(conf_mat[2,])  
specificity
```

```
[1] 0.3526022
```

[Hide](#)

```
#FPR = 1 - specificity
```

## Plot ROC Curve

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```
library(pROC)  
plot(roc(true_y, pred_y),  
      col="yellow", lwd=3, main="The turtle finds its way")
```

```
Setting levels: control = 0, case = 1  
Setting direction: controls > cases
```

