Maximum Flows in Networks

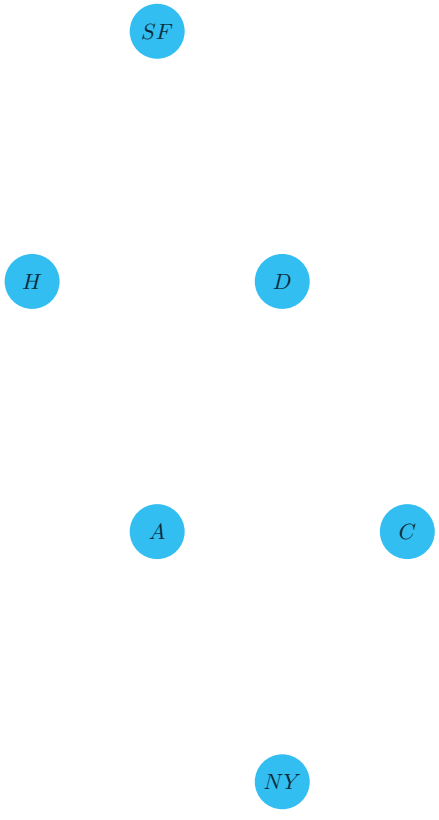
Augmenting Path Method – Ford and Fulkerson (1957)

Endre Boros

26:711:653: Discrete Optimization

March 5, 2018

A travel agency example (Chvatal, 1983)



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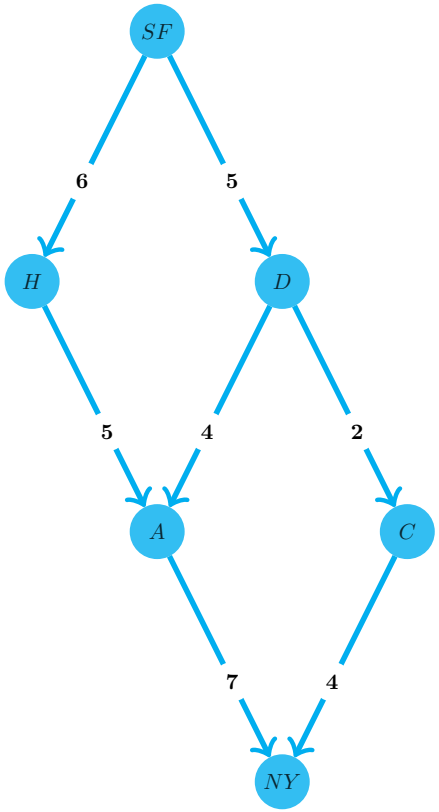

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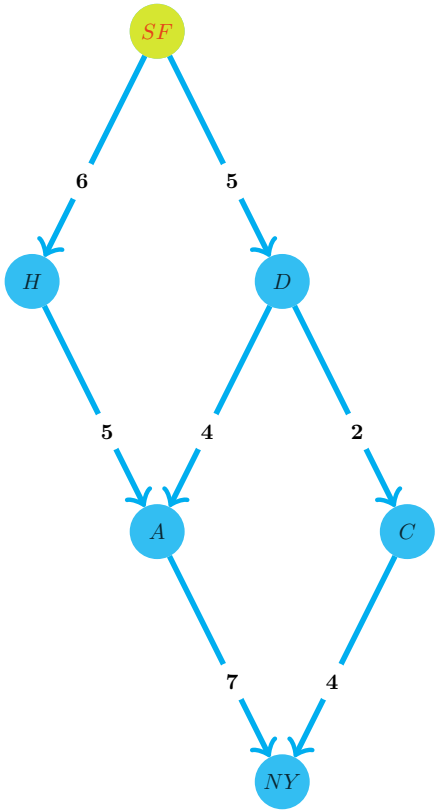

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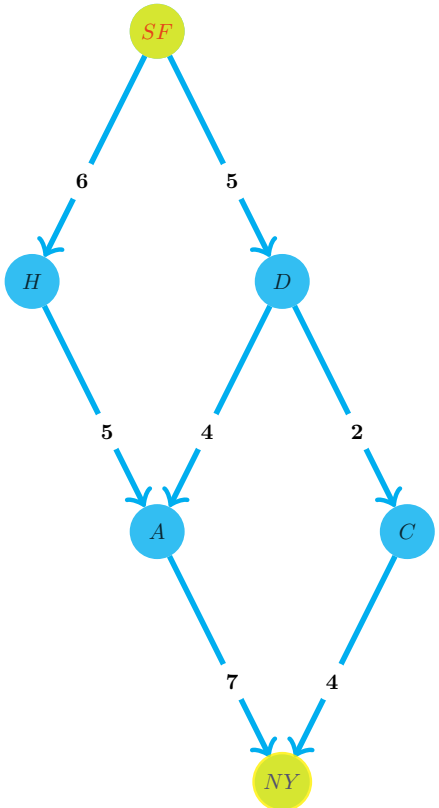

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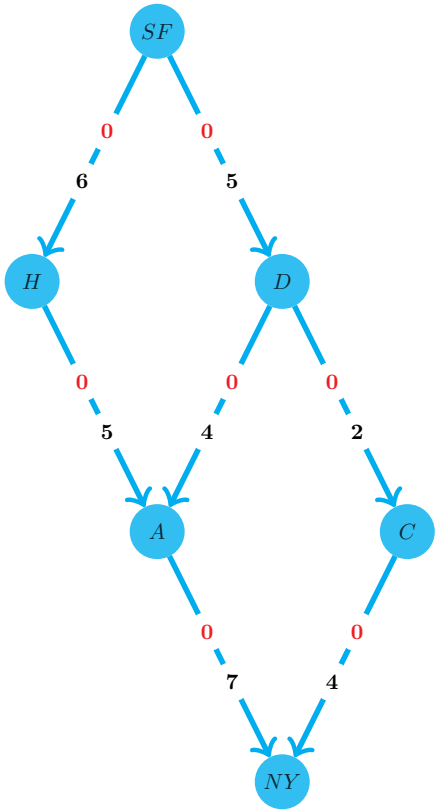

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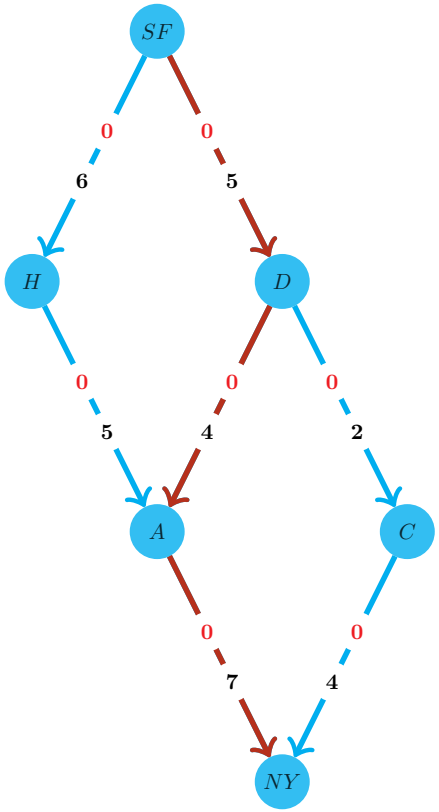

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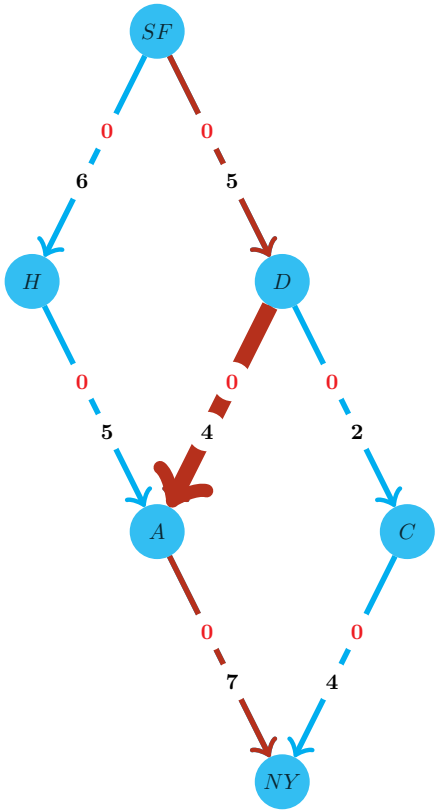

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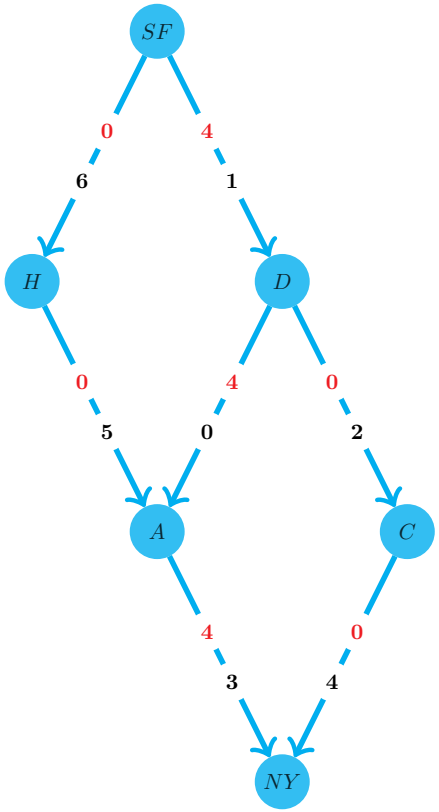

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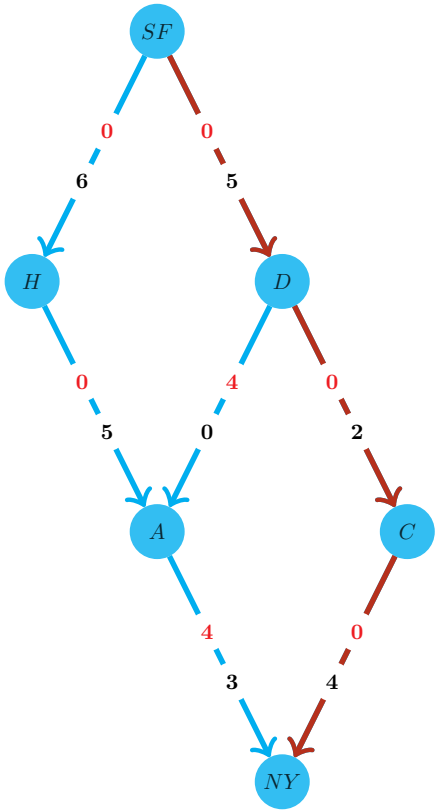

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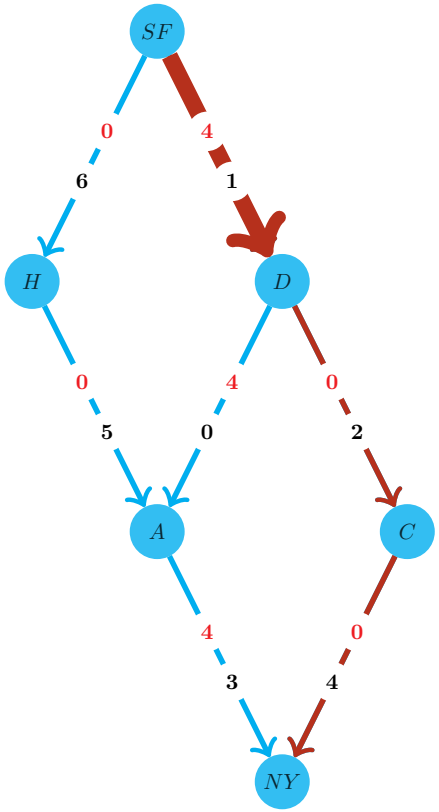

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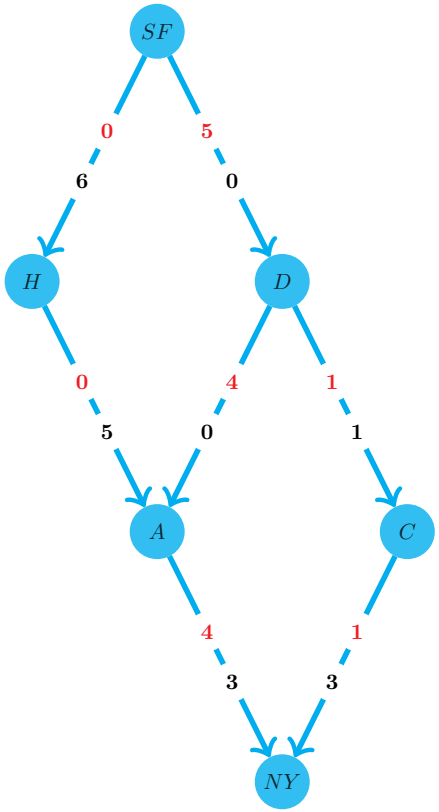

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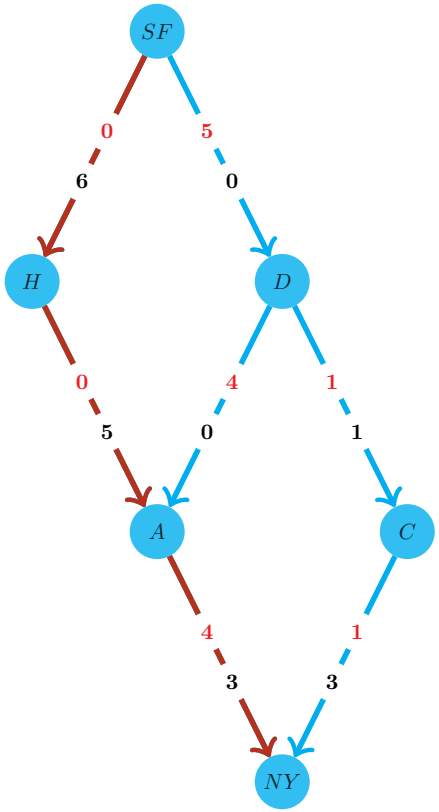

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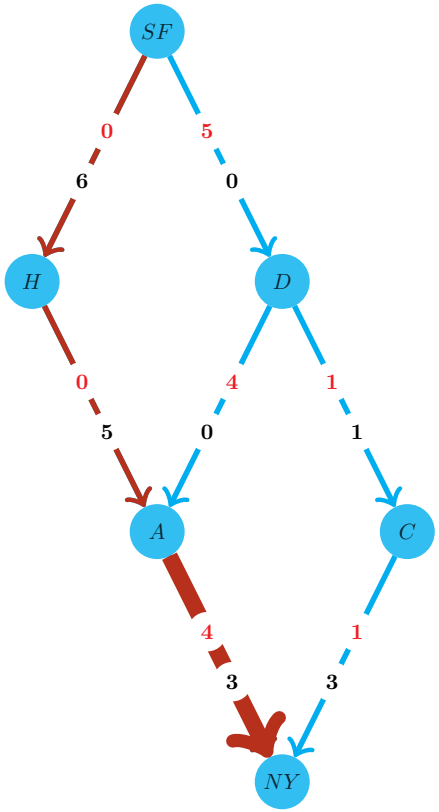

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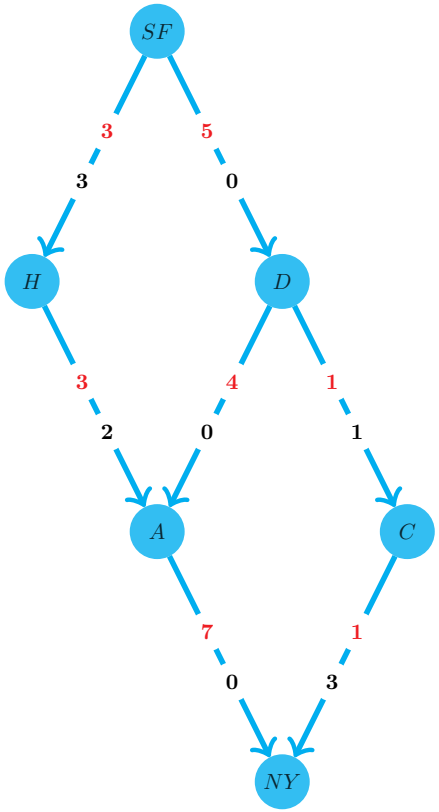

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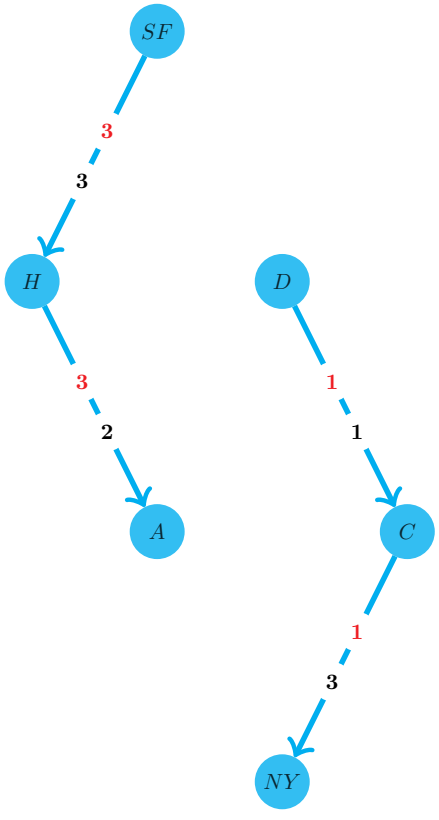

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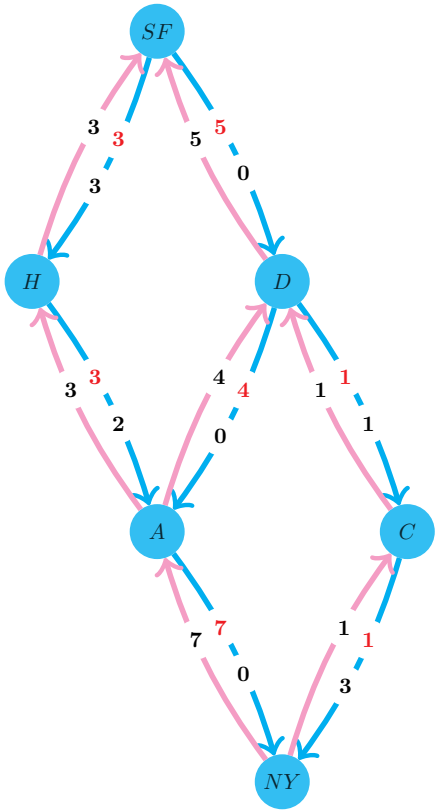

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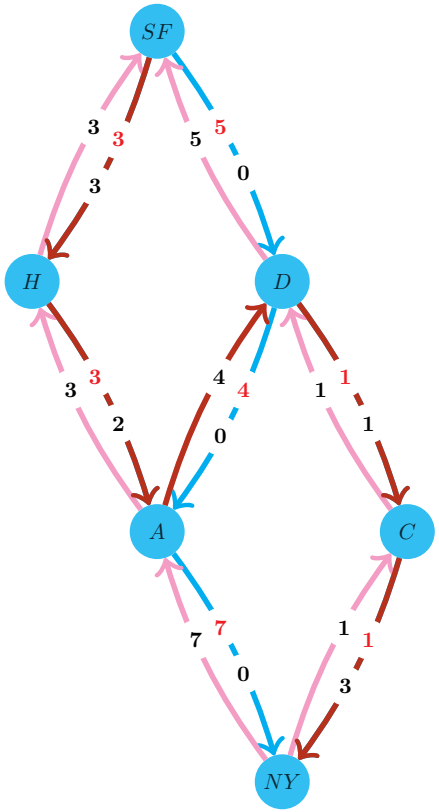

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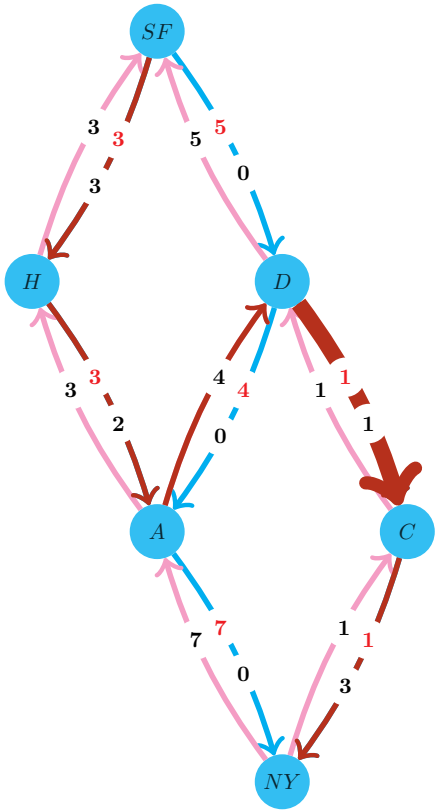

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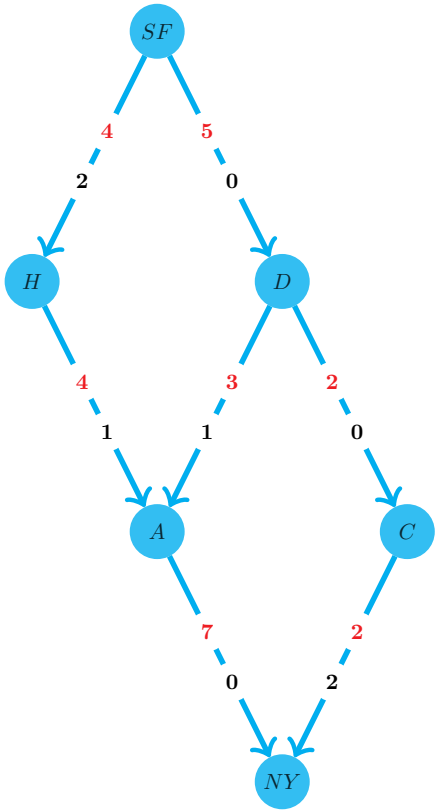

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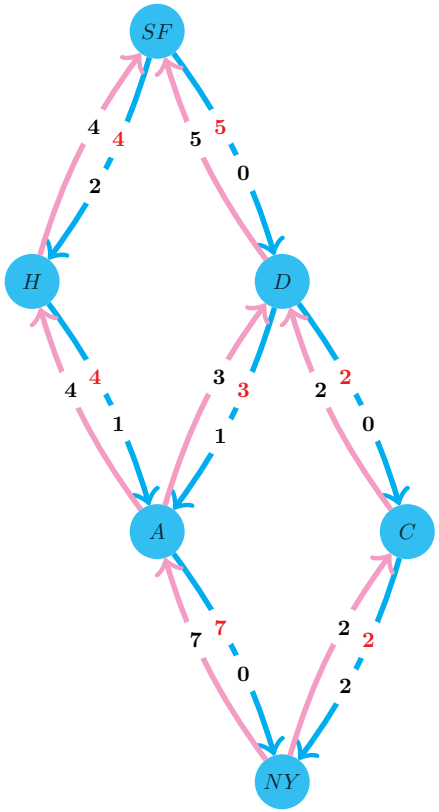

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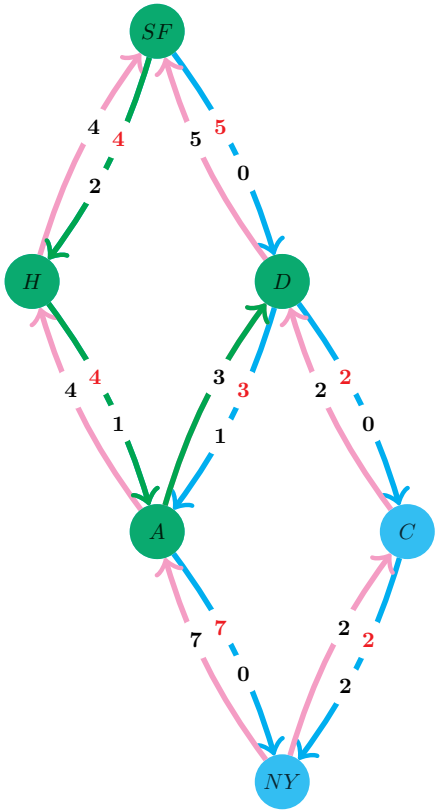

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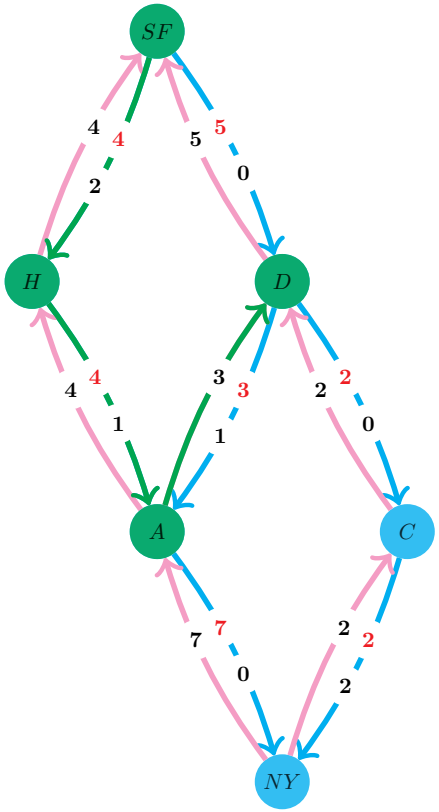

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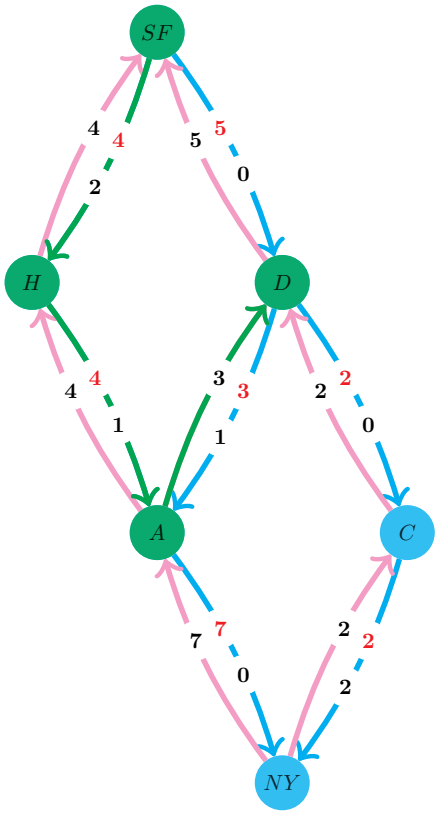

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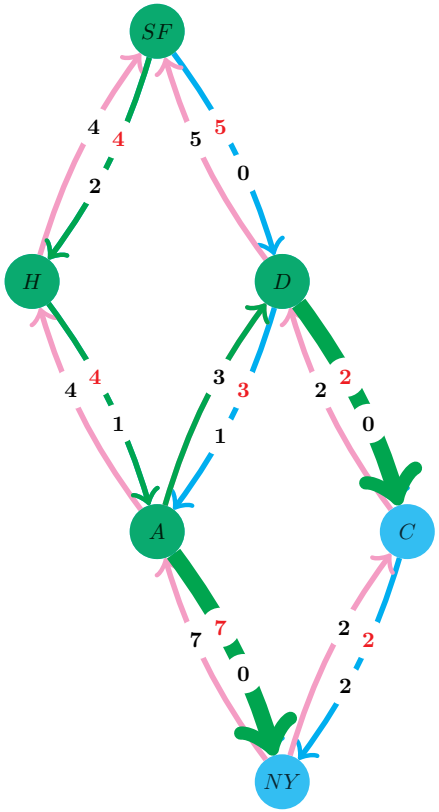

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