Deep learning

Homework # 1

Weijun Zhu 191004254

2(a). The PDF of each observation has following form:

$$f(x|a)=\begin{cases} 1/a ; & 0\le x\le a \\ 0 ; & \text{otherwise} \end{cases}$$

The likelihood function is:

$$L(a)=\prod_{i=1}^{m} f(x_i|a)=\prod_{i=1}^{m}\frac{1}{a}=a^{-m}$$

The log-likelihood is:

$$\ln L(a)=-n\ln(a).$$

Setting its derivative with respect to a to zero, we get

$$\frac{d}{da}\ln L(a)=-\frac{m}{a}<0 \quad \text{for } a>0$$

Hence, $L(a)$ is decreasing function and its maximized at

$$a=\max(X_1, X_2, \ldots, X_m)$$

(b). The PDF of each observation has following form:

$$f(x|a,b)=\begin{cases} \frac{1}{b-a} , & a\le x\le b \\ 0 , & \text{otherwise} \end{cases}$$

The likelihood function is:

$$L(b,a)=\prod_{i=1}^{m} f(x_i|a,b)=\prod_{i=1}^{m}\frac{1}{b-a}=(b-a)^{-m}$$

The log-likelihood is:

$$L\ln(b,a)=-m\ln(b-a)$$

Setting its derivative with respect to a and b to zero, we get

$$\frac{d}{da}\ln L(b,a)=-1\cdot\left(-\frac{m}{b-a}\right)$$
$$=\frac{m}{b-a}>0$$

Hence, this is increasing function and its maximized at

$$a=\min(X_1, X_2, \ldots, X_m)$$

The derivative of b, we get

$$\frac{d}{db}\ln L(b,a)=1\cdot\left(-\frac{m}{b-a}\right)$$
$$=-\frac{m}{b-a}<0$$

Hence, this is decreasing function and its maximized at

$$b=\max(X_1, X_2, \ldots, X_m).$$

(c). Assume that $X=(X_1, X_2, \dots, X_n)$ is a point on n-dimensions space and a circle with center $C=(C_1, C_2)$ and radius $= r$

The PDF is: d is the distance between center of the cirle and point

$$f(X|C_1,C_2,r)=\begin{cases}\frac{1}{\pi r^2}[(X_1-C_1)^2+(X_2-C_2)^2] & ; d\le r\\ 0 & ; d>r\end{cases}$$

The likelihood function of two dimension:

$$\text{like } f(X|C_1,C_2,r)=\arg\max_{r,C}\prod_{i=1}^{n}\frac{1}{\pi r^2}[(X_1-C_1)^2+(X_2-C_2)^2]$$

The likelihood function of three dimension

$$\text{like } f(X|C_1,C_2,r)=\arg\max_{r,C}\prod_{i=1}^{n}\frac{3}{4\pi r^3}[(X_1-C_1)^2+(X_2-C_2)^2+(X_3-C_3)^2]$$

Then we do log-likelihood function for both 2 and 3 dimensions.
For more higher-dimensions, we can do in the similar scheme above.