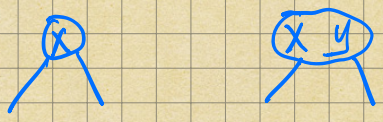
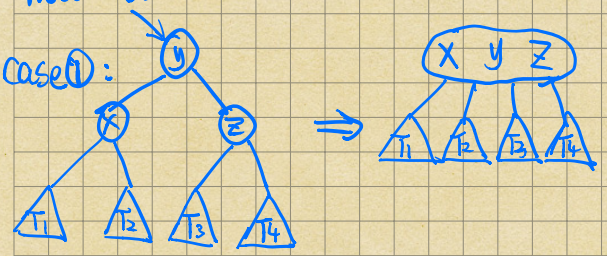
- 2-3 trees:



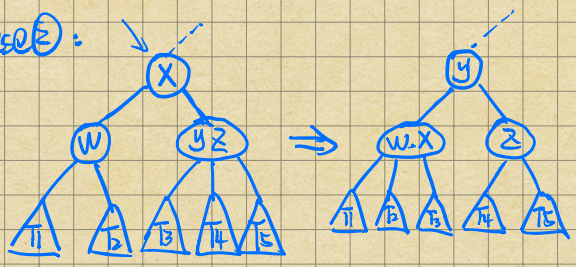
- Delete min:

- Rule: If the left child of the current node is a 2-node, somehow turn it into 3-node (or a 4-node)

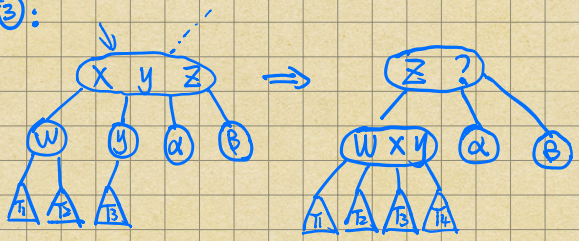
Case ①:



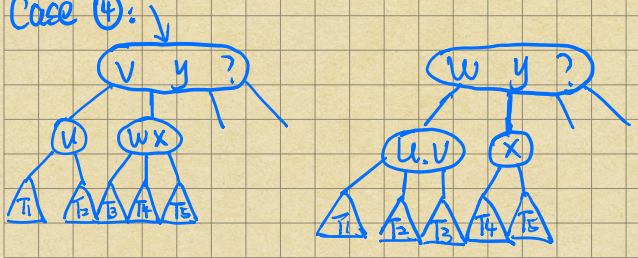
Case ②:



Case ③:



Case ④:



- Red Black Tree: (figure out the problems of 2-3 trees)

1. A Binary Search Tree

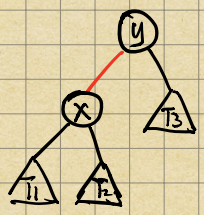
2. Value stored as Red or Black.

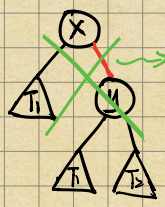
2. Links are colored Red or Black.

3. "Black-depth": the # of black links from the root to any leaf are the same.

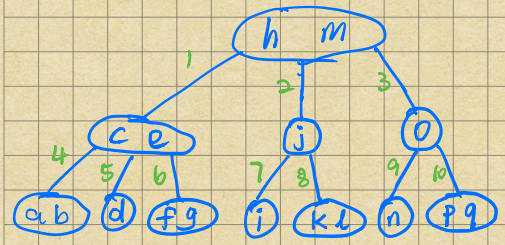
4. There are no more than one red link attached to a node.

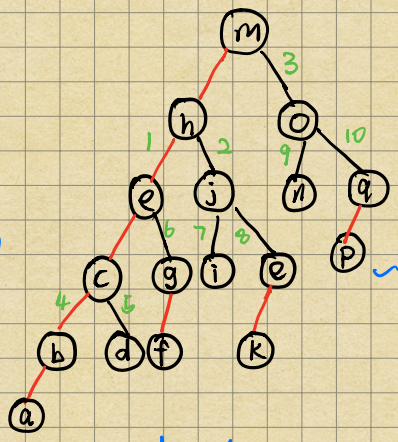
5. All red links are left-learning





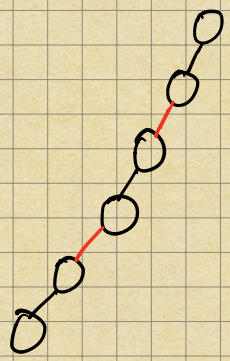
→ not left-learning!!



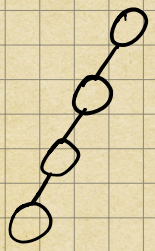


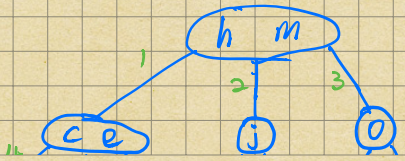
→ depth = O(lg N)

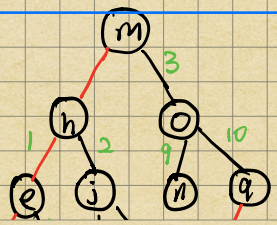
- worst case senario:

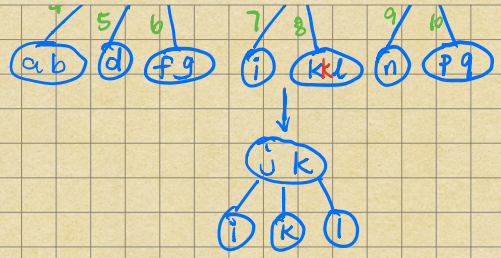


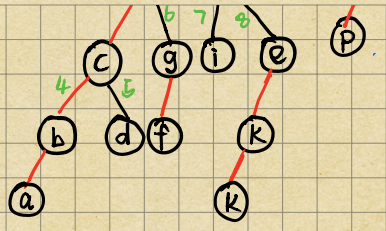
- best case senario:

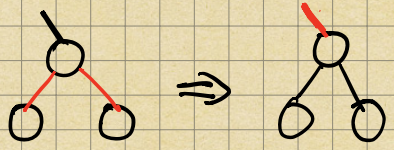


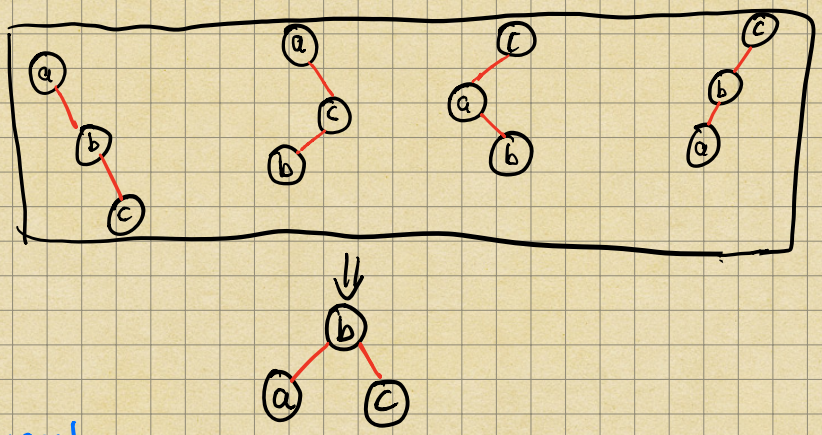




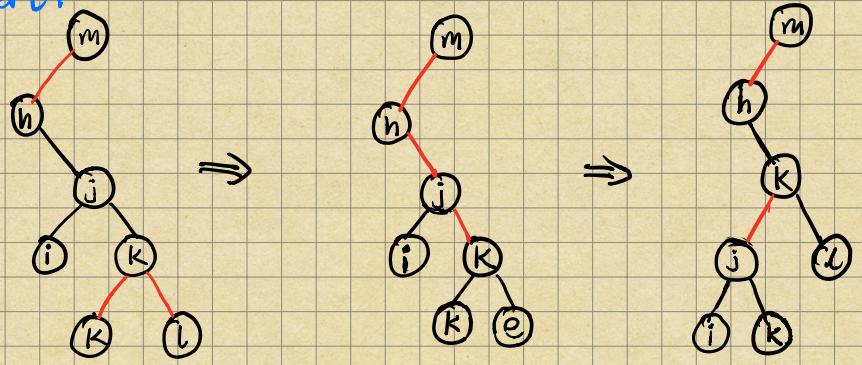




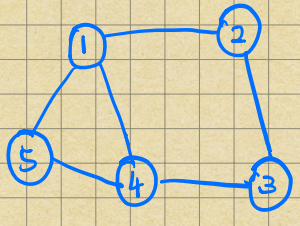




- insert:



- Graph and networds



undirected-graph

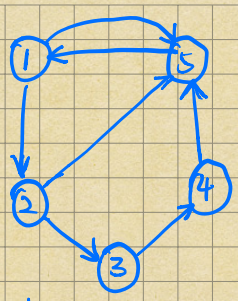
$G=(V, E)$

vertex/node

Edge/arc/link

$\{2,3\}$

	1	2	3	4	5
1	0	1	0	1	1
2	1	0	1	0	0
3	0	1	0	1	0
4	1	0	0	0	1
5	1	0	0	1	0



$E=(4,5)$

	1	2	3	4	5
1	0	1	0	0	1
2	0	0	1	0	1
3	0	0	0	1	0
4	0	0	0	0	1
5	1	0	0	0	0

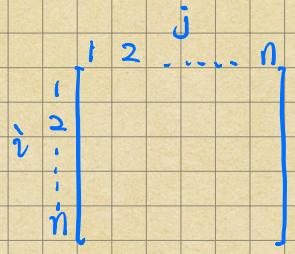
directed-graph

$G=(V,E)$

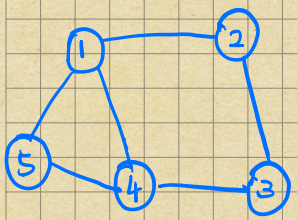
$|E|=O(n^2)$

- Adjacency Matrix:

$G=(V, E)$ $|V|=n$

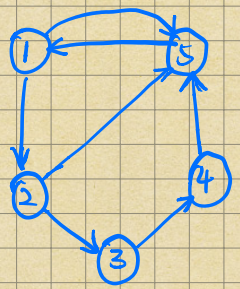


- Incidence Matrix:



	1	2	3	4	5
12	1	1	0	0	0
14	1	0	0	1	0
15	1	0	0	0	1
23	0	1	1	0	0
24	0	0	1	1	0
45	0	0	0	1	1

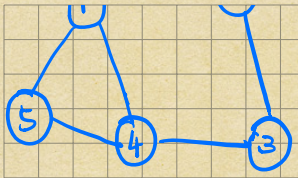
$|E| \times |V|$

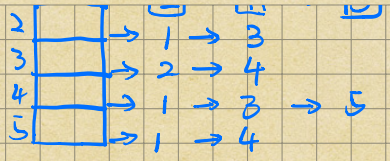


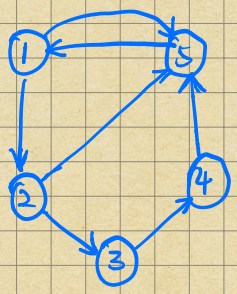
	1	2	3	4	5
12	-1	1	0	0	0
15	-1	0	0	0	1
23	0	-1	1	0	0
25	0	-1	0	0	1
24	0	0	-1	1	0
45	0	0	0	-1	1
51	1	0	0	0	-1

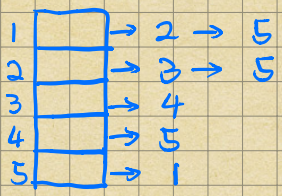
$|E| \times |V|$

- Adjacency list:







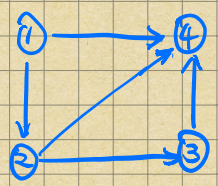


- Undirected : degree(V)

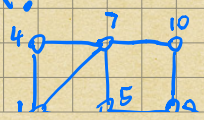
  directed : { indegree(V) = row sum (adj matrix)
  outdegree(V) = colsum (adj matrix)

- { Path / Walks — lengths
  Cycles / Circuits — length

- Undirected : connected / unconnected / connected components
  - Connected : If there is a path from any node to any other node
- Directed :
  - Strongly connected : for each pair of vertices (i, j), there is a directed path from i to j.
  - Weakly connected : Connected if direction of edges are disregarded



ex.



① depth-first search → stack

② breadth-first search → queue

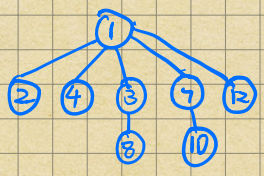
• nodes: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

• breadth-first search:

1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	1	1	0	1	1	0	1	0	1

→ ~~10~~ ~~8~~ ~~11~~ ~~7~~ 5 ~~4~~ ~~12~~ →

1-2-4-5-7-12-8-10



↪ Breadth-first Tree

1	2	3	4	5	6	7	8	9	10	11	12
1	1	~~0~~	1	1	~~0~~	1	1	~~0~~	1	~~0~~	1

→ 11 9 ~~6~~ ~~3~~ →

3-6-9-11