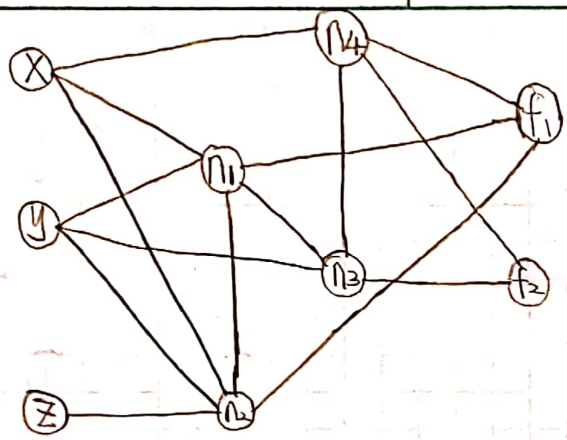
1.(a)



$n_1 = x \cdot y$

$n_2 = n_1 \cdot z = x \cdot y \cdot z$

$n_3 = \text{sigmoid}(n_1 + y)$

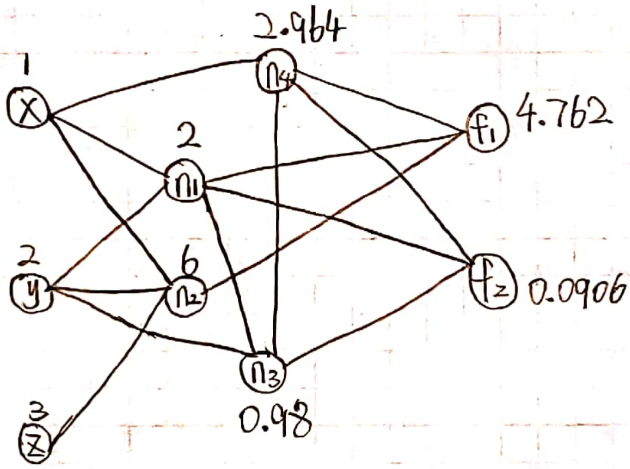
$= \frac{1}{1+e^{-xy-y}}$

$n_4 = 2 \cdot n_3 + x = 2 \cdot \frac{1}{1+e^{-xy-y}} + x$

$f_1 = \log(1 + e^{n_1+n_2+n_4})$

$f_2 = \frac{e^{n_3}}{e^{n_1} + e^{n_3} + e^{n_4}}$

(b) $x = 1$; $y = 2$; $z = 3$



$n_1 = x \cdot y = 2$

$n_2 = n \cdot z = 2 \cdot 3 = 6$

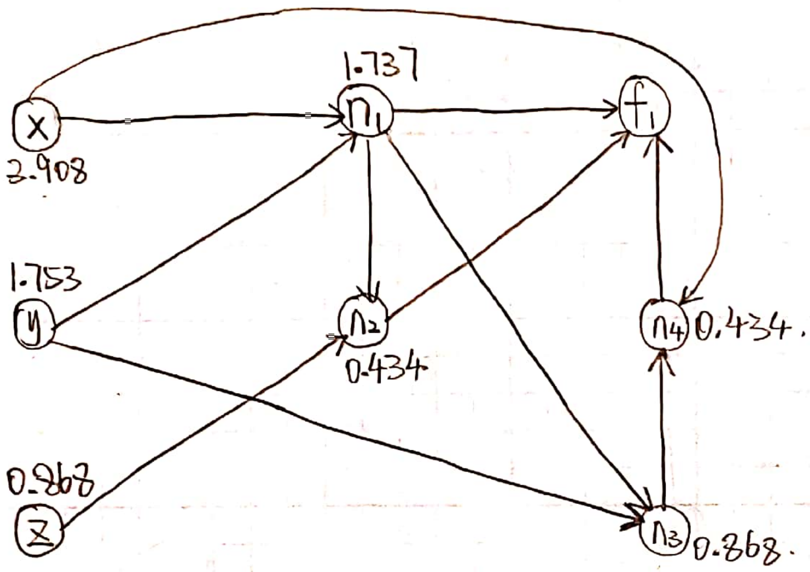
$n_3 = \text{sigmoid}(n_1 + y) = \frac{1}{1+e^{-n_1-y}} = 0.98$

$n_4 = 2 \cdot n_3 + x = 1.96$

$f_1 = 4.762$

$f_2 = 0.0906$

(c). Compute the gradient $\nabla f_1(x,y,z)$ at $x=1,\ y=2,\ z=3$



$$\frac{\partial f_1}{\partial n_4} = \frac{e^{n_1+n_2+n_4}}{(1+e^{n_1+n_2+n_4})\cdot \ln 10} = 0.434$$

$$\frac{\partial f_1}{\partial n_2} = \frac{\partial f_1}{\partial n_4} = 0.434$$

$$\frac{\partial f_1}{\partial n_1} = \frac{\partial f_1}{\partial n_1} + \frac{\partial f_1}{\partial n_2}\cdot\frac{\partial n_2}{\partial n_1} + \frac{\partial f_1}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_1}$$

$$= \frac{e^{n_1+n_2+n_4}}{(1+e^{n_1+n_2+n_4})\ln 10}\cdot(1+z+0) = 1.737$$

$$\frac{\partial f_1}{\partial n_3} = \frac{\partial f_1}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_3} = 0.434\times 2 = 0.868$$

$$\frac{\partial n_3}{\partial y} = \frac{-e^{-(n_1+y)}}{(1+e^{-(n_1+y)})^2} = -0.018$$

$$\frac{\partial f_1}{\partial x} = \frac{\partial f_1}{\partial n_1}\cdot\frac{\partial n_1}{\partial x} + \frac{\partial f_1}{\partial n_4}\cdot\frac{\partial n_4}{\partial x} = 1.737\times 2 + 0.434\times 1 = 3.908$$

$$\frac{\partial f_1}{\partial y} = \frac{\partial f_1}{\partial n_1}\cdot\frac{\partial n_1}{\partial y} + \frac{\partial f_1}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_3}\cdot\frac{\partial n_3}{\partial y} = 1.737\times 1 + 0.434\times 2\times(-0.018) = 1.753$$

$$\frac{\partial f_1}{\partial z} = \frac{\partial f_1}{\partial n_2}\cdot\frac{\partial n_2}{\partial z} = 0.434\times 2 = 0.868$$

(d). $x=y=z=1$

$n_1 = x\cdot y = 1 \quad n_2 = n_1 \cdot z = 1$

$n_3 = \frac{1}{1+e^{-xy-y}} = \frac{1}{1+e^{-2}} = 0.881 \qquad n_4 = 2\times n_3 + x = 2\times 0.88+1 = 2.762$

$f_1 = \log(1+e^{n_1+n_2+n_4}) = \log(1+e^{1+1+2.762}) = 2.072$

$f_2 = \frac{e^{n_3}}{e^{n_1}+e^{n_3}+e^{n_4}} = \frac{e^{0.881}}{e^1+e^{0.881}+e^{2.762}} = 0.115$

$\frac{\partial n_1}{\partial y} = x = 1$

$\frac{\partial n_2}{\partial y} = \frac{\partial n_2}{\partial n_1}\cdot\frac{\partial n_1}{\partial y} = z\cdot x = 1$

$\frac{\partial n_3}{\partial y} = \frac{\partial n_3}{\partial n_1}\cdot\frac{\partial n_1}{\partial y} + \frac{\partial n_3}{\partial y} = \frac{-e^{-(n_1+y)}}{(1+e^{-(n_1+y)})^2}\cdot x + \frac{-e^{-(n_1+y)}}{(1+e^{-(n_1+y)})^2} = \frac{-2e^{-2}}{(1+e^{-2})^2} = -0.21$

$\frac{\partial n_4}{\partial y} = \frac{\partial n_4}{\partial n_3}\cdot\frac{\partial n_3}{\partial y} = 2\cdot(-0.21) = 0.42$

$\frac{\partial f_1}{\partial y} = \frac{\partial f_1}{\partial n_1}\cdot\frac{\partial n_1}{\partial y} + \frac{\partial f_1}{\partial n_2}\cdot\frac{\partial n_2}{\partial y} + \frac{\partial f_1}{\partial n_4}\cdot\frac{\partial n_4}{\partial y}$

$= \frac{e^{n_1+n_2+n_4}}{(1+e^{n_1+n_2+n_4})\ln 10}\cdot(x+z\cdot x+(-0.42))$

$= \frac{e^{4.762}}{(1+e^{4.762})\ln 10}\cdot(1+1-0.42)$

$= 0.68$

$\frac{\partial f_2}{\partial y} = \frac{\partial f_2}{\partial n_1}\cdot\frac{\partial n_1}{\partial y} + \frac{\partial f_2}{\partial n_3}\cdot\frac{\partial n_3}{\partial y} + \frac{\partial f_2}{\partial n_4}\cdot\frac{\partial n_4}{\partial y}$

$= \frac{e^{n_3}}{(e^{n_1}+e^{n_3}+e^{n_4})}\cdot(1-0.42) + \frac{e^{n_3}\cdot(e^{n_1}+e^{n_3}+e^{n_4}) - e^{n_3}\cdot e^{n_3}}{(e^{n_1}+e^{n_3}+e^{n_4})^2}$

$= -0.018$