Ride Sharing[1]

A large company has parking problems and also would like to promote its carbon saving image. They decide to encourage employees to form carpools, and drive together to and from the workplace.

- How to organize carpools such that nobody drives "much more" than he/she would driving alone from his/her home?
- How to organize a carpool system that minimizes the total mileage driven?
- Can we do both?

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Ride Sharing - Data

- The company has N (≈ 500) employees, all living in a 50 mile radius of the workplace.
- We know the location of each employee's home, and can compute the driving distance from any one to any other one.
- We assume that in a carpool some of the employees drive to another one's home, park there, and then travel together in one car; maybe they pick up additional employees on their way to the workplace. For sake of simplicity, we assume that all cars can accommodate 4 people - the driver and 3 passengers.
- For "*fairness*" we require that in a carpool nobody travels "much more" than his/her own travel distance to the workplace. (Say, no more than M extra miles.)

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Ride Sharing: Main Entities

SETS:

- Employees E – each with its home location.

PARAMETERS:

- Workplace w – with its location.
- Driving distances $d[i, j]$, $i \in E$, $j \in E \cup \{w\}$ (in miles).
- Extra mileage allowed to be traveled: M (in miles).
- Car capacities: $s[i]$ is the number of seats in the car of employee $i \in E$. *We assume that those who arrive to the home of i will travel together in the car of employee i; for simplicity, we assume $s[i] = 4$ for all $i \in E$.*

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Ride Sharing: Decision Variables

VARIABLES:

- $f[i, j]$ denotes the number of people traveling together from i to j, $i \in E$, $j \in E \cup \{w\}$. (Sounds like a **FLOW** looks like a **FLOW**, smells like a **FLOW** — it is a **FLOW**!)
- **Looks like a transshipment problem**: employee homes $i \in E$ have a supply $= 1$, and workplace w has a demand $= N$.

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\begin{aligned}
1 + \sum_{i \in E} f[i, j] &= \sum_{k \in E} f[j, k] && \forall j \in E, \\
\sum_{i \in E} f[i, w] &= N, \\
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Ride Sharing: How to Formulate the Objective?

OBJECTIVE FUNCTION:

- Total mileage driven:

$$\min \sum_{\substack{i \in E \\ j \in E \cup \{w\}}} d[i,j] * f[i,j]$$

- **Is this correct? What does it measure?**

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Ride Sharing: ADDITIONAL Decision Variables

VARIABLES:

- $y \in \{0,1\}^{E \times E \cup \{w\}}$

$$y[i,j] = \begin{cases} 1 & \text{if } f[i,j] > 0, \\ 0 & \text{otherwise.} \end{cases}$$

- Enforced by

$$\begin{aligned} y[i,j] &\leq f[i,j] \quad \forall i \in E,\ j \in E \cup \{w\}, \\ s[i] * y[i,j] &\geq f[i,j] \quad \forall i \in E,\ j \in E \cup \{w\}. \end{aligned}$$

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$$\ell[i] \;=\; \min_{j \in E} y[i,j] * (d[i,j] + \ell[j]) \quad \forall i \in E,$$

- **Does NOT work if $\sum_{j \in E} y[i,j] > 1$ for some $i \in E$!**
- Assume (restricting solutions)

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- **STILL NONLINEAR!**

Linearizing the product of a binary variable and a linear function[2]

- Assume that L is a linear expression of our variables and X is a binary variable.
- We would like to enforce $Z = X * L$ for all feasible solutions.
- Assume that $0 \leq L \leq U$ for all feasible solutions, where U is a constant.
- We have $Z = X * L$ if and only if

$$
\begin{aligned}
Z &\leq L, \\
Z &\leq U * X, \\
Z &\geq 0, \\
Z &\geq L - U * (1 - X).
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[2] Glover (1975), Adams and Forester (2005)

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$$\begin{array}{lcll} z[i,j] & \leq & d[i,j] + \ell[j] & \forall\, i \in E,\ j \in E \cup \{w\} \\ z[i,j] & \leq & 100 * y[i,j] & \forall\, i \in E,\ j \in E \cup \{w\} \\ z[i,j] & \geq & 0 & \forall\, i \in E,\ j \in E \cup \{w\} \\ z[i,j] & \geq & d[i,j] + \ell[j] - 100 * (1 - y[i,j]) & \forall\, i \in E,\ j \in E \cup \{w\} \end{array}$$

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$$\ell[i] = \sum_{j \in E \cup \{w\}} z[i,j] \quad \forall\, i \in E.$$

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$$\ell[i] \leq d[i,w] + M \quad \forall\, i \in E.$$

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z[i,j] & \geq & d[i,j] + \ell[j] - 100 * (1 - y[i,j]) & \forall\, i \in E,\ j \in E \cup \{w\}
\end{array}
$$

- Then we can write

$$\ell[i] = \sum_{j \in E \cup \{w\}} z[i,j] \quad \forall\, i \in E.$$

- With these **fairness** can be ensured by

$$\ell[i] \leq d[i,w] + M \quad \forall\, i \in E.$$

VARIABLES:

- Introduce $\ell[i]$ to denote the distance traveled by employee $i \in E$, $\ell[w] = 0$.
- Introduce $z[i,j] = y[i,j] * (d[i,j] + \ell[j])$, enforced by

$$\begin{array}{lcll} z[i,j] & \leq & d[i,j] + \ell[j] & \forall\, i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \leq & 100 * y[i,j] & \forall\, i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \geq & 0 & \forall\, i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \geq & d[i,j] + \ell[j] - 100 * (1 - y[i,j]) & \forall\, i \in E,\ \ j \in E \cup \{w\} \end{array}$$

- Then we can write

$$\ell[i] \;=\; \sum_{j \in E \cup \{w\}} z[i,j] \quad \forall\, i \in E.$$

- With these **fairness** can be ensured by

$$\ell[i] \;\leq\; d[i,w] + M \quad \forall\, i \in E.$$

Ride Sharing: ADDITIONAL Decision Variables

VARIABLES:

- Introduce $\ell[i]$ to denote the distance traveled by employee $i \in E$, $\ell[w] = 0$.
- Introduce $z[i,j] = y[i,j] * (d[i,j] + \ell[j])$, enforced by

$$\begin{array}{lcll} z[i,j] & \leq & d[i,j] + \ell[j] & \forall\ i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \leq & 100 * y[i,j] & \forall\ i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \geq & 0 & \forall\ i \in E,\ \ j \in E \cup \{w\} \\ z[i,j] & \geq & d[i,j] + \ell[j] - 100 * (1 - y[i,j]) & \forall\ i \in E,\ \ j \in E \cup \{w\} \end{array}$$

- Then we can write

$$\ell[i] \ = \sum_{j \in E \cup \{w\}} z[i,j] \quad \forall\ i \in E.$$

- With these **fairness** can be ensured by

$$\ell[i] \ \leq \ d[i,w] + M \quad \forall\ i \in E.$$