Algorithm HW#02

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1.(a). $F(n)=3\cdot F(\frac{n}{2})+n^2$ with $F(1)=1$

$a=3$ $b=3$ $g(n)=n^2$ $\log_a b=\log_3 3=1$

$g(n)=n^2=\Omega(n^{\log_3 3+\varepsilon})$, and $3\cdot f(\frac{n}{3})<c\cdot n^2$ where $\varepsilon=1$, $\Rightarrow$ Case ③

$\Rightarrow f(n)=\Theta(g(n))=\Theta(n^2)$

(b). $G(n)=2G(\sqrt{n})+\log_2(n)$, with $G(1)=1$

Assume that $k=\log_2 n \Rightarrow G(2^k)=2G(2^{\frac{k}{2}})+k$

$\Rightarrow H(k)=2H(\frac{k}{2})+k$

We have $a=2$, $b=2$, $g(k)=k$, and $\log_2 2=1$

$g(k)=k=\Theta(k^{\log_2 2-0})=\Theta(k^{\log_2 2}\cdot\log_2 k)=\Theta(k\log_2 k)$ ⇝ Case ②

Thus, $\Theta(\log_2 n\cdot\log_2(\log_2 n))$

(c). $\begin{cases} nH(n)=H(n-1)+H(n-2)+\cdots+H(1)+2n & ① \\ nH(n-1)=H(n-2)+H(n-3)+\cdots+H(1)+2(n-1) & ② \end{cases}$

①-②: $nH(n)-nH(n-1)=H(n-1)+2n-2n+2$

$=H(n-1)+2$

$\Rightarrow H(n)-H(n-1)=2/n$

$H(n-1)-H(n-2)=2/(n-1)$

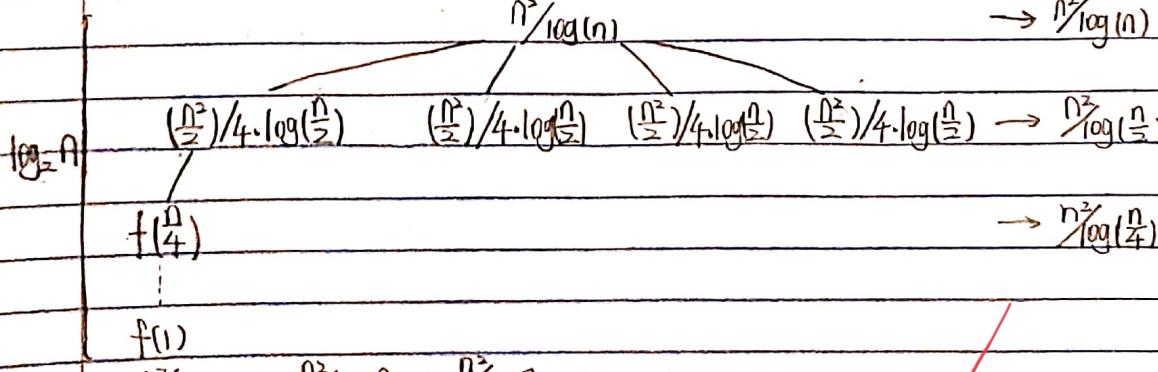
$\vdots$

$H(2)-H(1)=2/2$

$\Rightarrow H(n)=2(1+1/2+1/3+\cdots+1/n)+2$, and $1+1/2+1/3+\cdots+1/n=\log n$

$\Rightarrow \Theta(\log n)$

(d). $f(n)=4\cdot f(n/2)+n^2/\log(n)$. use recurison tree because master theorem does not work



$\Rightarrow n^2/\log(n)+n^2/\log(\frac{n}{2})+n^2/\log(\frac{n}{4})+\cdots$

$=n^2/\log(n)+n^2/\log(n)-1+n^2/\log(n)-2+\cdots$ Assume that $n=2^k$ & $k=\log n$

$=n^2(\frac{1}{k}+\frac{1}{k-1}+\frac{1}{k-2}+\cdots+1)=n^2\cdot\log(k)$ ⇝ $\Theta(n^2\log(\log n))$

2.(a). The boolean operation has "+" and "·", does not have "−", but in Strassen's algorithm, there are "−" operations. We don't know how to define the "−" operation in Strassen's algorithm.

(-2)

(b). For ordinary multiplication:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$ → For $2 = 1\times1 + 1\times1$ if we change to boolean,

and we have $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

⇒ We can find if some elements in Ordinary is greater than 1, it will be changed to 1 in boolean.

For Strassen's Algorithm, We have:

$$\begin{cases} C_{11} = A_{11}\cdot B_{11} + A_{12}\cdot B_{21} & \& \quad C_{12} = A_{11}\cdot B_{12} + A_{12}\cdot B_{22} \\ C_{21} = A_{21}\cdot B_{11} + A_{22}\cdot B_{21} & \& \quad C_{22} = A_{22}\cdot B_{22} + A_{21}\cdot B_{12} \end{cases}$$

We can find in Strassen's Algorithm, it's same as the Ordinary multiplication. The elements which bigger than 1 will change to 1.

what are boolean operation?

(-2)

(c).

$$f(n) = \begin{cases} \Theta(1) & ; n = 1 \\ 7f(\frac{n}{2}) + \Theta(n^{2+w}) & ; n > 1 \text{ and } 0 < w < 1 \end{cases}$$

$a = 7,\ b = 2 \quad \log_2 7 = 2.81$

$g(n) = n^{2+w} = n^{2.81}$ ⇝ Case 3

$\therefore O(n^{\log 7} \cdot \log n)$.

(-2)

3.

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Define function unimodal(L, left, right)

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Unimodal(L, left, right)
    if right == left+1:
        return left
    mid=floor((right-left+1)/2)
    if mid > L[mid-1]:
        return unimodal(L, mid, right).
    elif mid < L[mid-1]
        return unimodal(L, left, mid).
    else:
        i=unimodal(L, left, mid)
        j=unimodal(L, mid, right)
        if a[i] > a[i]:
                return i
        if a[j] > a[i]:
                return j
        if a[i] == a[j]
                return min(i,j)
```

$\begin{cases} f(1)=C \\ f(n)=f(\frac{n}{2})+C \end{cases}$

Assume that $n=2^k$ & $k=\log n$

$\Rightarrow f(2^k)=f(2^{k-1})+C_1$, and define $g(k)=f(2^k)$, so $\begin{cases} g(k)=g(k-1)+C_1 \\ g(0)=1 \end{cases}$

$\Rightarrow g(k)=g(k-2)+C_1+C_1$

$=g(k-3)+C_1+C_1+C_1$

$\vdots$

$=g(0)+kC_1$

$=1+kC_1$

$\Rightarrow f(2^k)=kC_1+C_1 \Rightarrow f(n)=\log n \cdot C_1+1$

$\Rightarrow \Theta(\log n)$ ✓ → worst case $\Theta(n)$ (-1)

4.(a). Use Euclid's Algorithm, $\gcd(726,693) = \gcd(693,33)$
$= \gcd(33,0)$
$= 33$

By Bezoutly Theorem: $\gcd(726,693) = 33 = ax+by$, x & y are integers
And $\gcd(a,b) = \gcd(b, b \bmod a)$.
$\Rightarrow a = qb + r$ where $r = b \bmod a$; $0 \le r \le a$
$\gcd(a,b) = \gcd(b,r) = g \Rightarrow g = bx + ry = bx + (a - bq)\cdot y = ay + b(x - qy)$.
Thus, $33 = \gcd(33,0) = 33 \times 1 - 0 \times 1$
$= \gcd(693,33) = 693 \times 1 - 33 \times 20$
$= \gcd(726,693) = 726 \times 1 - 693 \times 1$
So $\gcd(726,693) = 33 = 726 \times 1 - 693 \times 1$

(b). $77x = 4 \bmod 20$
Let $\begin{cases} a = 77 \\ b = 4 \end{cases}$ and $n = 20$
Using Ecalid on (77, 20), then $\gcd(77,20) = 1$
$\Rightarrow 77x_k + 20y_k = d = 1$
$\Rightarrow \begin{cases} x_k = 13 \\ y_k = -50 \end{cases}$
By theorem: $x_0 = x_k'(b/d) \pmod n$
$= 13(4/1) \pmod{20} = 52$
We set the valuse of x_0, then
$x_1 = 52 + i(20/1) = 52 + 20i$; $i = 0$
$x_k = \{52\}$