# CS520 Assignment 3 Probabilistic Search

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November 2018

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## 1 A Stationary Target

1)We want to compute $P(Target\ in\ Cell_i \mid Observations_t \wedge Failure\ in\ Cell_j)$.
Consider the 50*50 map. At first, our probability for each cell containing the target is 1/2500. Every time we will determine a cell and search it. If the search result is failure, we will update the belief state for all cells. For the cell containing the target, update the probability using the following formula.
$P(Target\ in\ Cell_i) * P(Target\ not\ found\ in\ Cell_i | Target\ is\ in\ Cell_i)$
For other cells, compute the following statement.

$$P(Target\ in\ current\ cell) * [1 + P(Target\ in\ searched\ cell) * \frac{P(Target\ found\ in\ cell \mid Target\ in\ Cell)}{[1\ -\ P(Initial\ target\ in\ Searched\ Cell)]}]$$

2) Given the observations up to time t, the belief state captures the current probability the target is in a given cell. What is the probability that the target will be found in Cell i if it is searched:

Compute $P(Target\ found\ in\ Cell_i | Observations_t)$.
$P(Target\ found\ in\ Cell_i | Target\ in\ Cell_i)$ has four values according to four different terrain types. The values is 0.9 when the terrain type is flat. And it is 0.7, 0.3, 0.1 when the terrain type is hilly, forest and cave. When time $t = 0$, all the cells have the probability of $\frac{1}{2500}$.
And when time comes to t, we use the following formula for computing:

$$P(Target\ found\ in\ Cell_i | Observations_t) =$$
$$P(Target\ found\ in\ Cell_i | Target\ in\ Cell_i) * P(Target\ in\ Cell_i | Observations_{t-1})$$

3) For Rule 1, every time we choose the cell which has the highest probability of containing the target and search it. For Rule 2, every time we choose the cell which has the highest probability of finding the target and search it.

For a fixed map, we run our program for target in all kinds of terrains 100 times each, and we compute the weighted average cost of Rule1 and Rule2.
Following data shows the average cost for each rule according to different terrain type:

Rule 1

| Terrain Type | Average Cost |
|---|---|
| Flat | 4131.6 |
| Hill | 3949.45 |
| Forest | 8015.35 |
| Cave | 9553.95 |

Rule 2

| Terrain Type | Average Cost |
|---|---|
| Flat | 1168.55 |
| Hill | 2232.5 |
| Forest | 7956.9 |
| Cave | 14027.95 |

And we use the weighted average cost to compare Rule 1 and Rule 2. From the problem implementation, we know that for each terrain type, the probability of flat is 0.2, hilly with probability 0.3, forest with probability 0.3 and caves with probability 0.2.

| Rule Type | Weighted Average Cost |
|---|---|
| Rule 1 | 6327.43 |
| Rule 2 | 6096.11 |

For Rule 1, it will search the cell with the highest probability of containing the target. For Rule 2, it will search the cell with the highest probability of finding the target. Every time when we search, Rule 1 searches impartially. And in terms of the false negative rates, Rule 1 will search the cave area more times than other terrain types. But if we use Rule 2 to search, it means that we will always choose to use the easiest way to find the target, even if the target is in the cave or forest.

Consider the following two situations, the target is in flat land or hill and forest or cave. If the target is in flat land or hill, using Rule 2 to search means that the cost of searching flat land and hill will be smaller. As we can see from the table, the average cost for cave using Rule 2 is 14027.95 and it is much more greater than that of Rule 1, that is 9553.95. But the average cost of searching in flat land or hill using Rule 2 is smaller than that of Rule 1. And if the target is in the cave or forest, using Rule 2 to search does not mean it will search forest or cave first. It still searches flat and hill before searching other terrain types. Consequently, we can see from the table that the average cost of using Rule 2 to search is very large if the target is in cave. In this situation, using Rule 1 will cost less.

Although the weighted average cost of Rule 1 is bigger than Rule 2, it does not mean Rule 2 performs better than Rule 1. If we increase $P(Target\ found\ in\ Cell_i|Target\ is\ in\ Cell_i)$ for the flat and hilly terrain types, search using Rule 2 will improve and cost less time and on average Rule 2 is better.

4) Our decision rule is that we will search the cell which is the neighboring cell or other cell that need to be travel according to Rule 1 or Rule 2. For our rule, we will use the following formula for Rule 1:

$$\frac{P(Target\ in\ Cell)}{\sqrt{distance - total\ number\ of\ actions\ required}}$$

And we will the following formula for Rule 2:

$$\frac{P(Target\ in\ Cell) * P(Target\ found\ in\ Cell|Target\ is\ in\ Cell)}{\sqrt{distance - total\ number\ of\ actions\ required}}$$

The distance we calculate is bigger than the probability so we let the square root of distance to be the denominator. And the following table are results of average cost according to Rule 1 or Rule 2.

Rule 1

| Terrain Type | Average Cost |
|---|---|
| Flat | 7768.05 |
| Hill | 9975.55 |
| Forest | 11200.5 |
| Cave | 17066.4 |

Rule 2

| Terrain Type | Average Cost |
|---|---|
| Flat | 2456.5 |
| Hill | 4254.25 |
| Forest | 9490.9 |
| Cave | 25994.85 |

And same as question 3, consider the weighted average cost of Rule 1 and Rule 2.

| Rule Type | Weighted Average Cost |
|---|---|
| Rule 1 | 11319.705 |
| Rule 2 | 9813.8 |

In fact, the hunter will not travel too far to search next cell according to our decision rule, that is because difference of possibilities between two cells is not big, especially at the beginning of the search.
Also, because of the difference between rule1 and rule2, we need to take different steps to find the target with different terrain types. For rule1, we will always search the cell who has the largest possibility that contains the target. However, we will search the cell that with the largest possibility that we can find the target. When target is in flat and hilly, rule2 has advantages to find the target because rule2 tells us that we need to search flat and hilly first because we may find the target with larger possibility, so we can find the target with less steps than decision made by rule1. But when target is in cave, rule2 tell us to search flat first, even for many times and then hilly and forest, so we will waste many steps to search flat, hilly and forest before we search cave. In this case, Rule1 is smarter.

5) This old joke shows that the man wanted to find his key under the streetlight according to the highest probability of finding the target. Therefore, the man used Rule 2 in question 3 and his key is the target. Although the man knew that he lost keys in the park, he didn't use Rule 1 to find because the park is quite dark. It is difficult for him to find keys in the park even if the park is the place which has the highest probability of containing the target.
Furthermore, from the result of question 3, we can see that the man will find the key faster if the probability of the key under the streetlight is greater than that of in the park.

## 2 A Moving Target

In this problem, we can use Type1 x Type2 to get a lot of information about the target. For example, if we know that the target is moving between flat and hilly, we can immediately know that the target cannot be in cave or forest, so the possibility of the target in cave or forest is 0. Moreover, some flats who has no hilly in its neighbors are impossibility to contain targets, because the target just moves between neighbors. So we can update possibilities of the whole map.

| | | | |
|---|---|---|---|
| Forest | Forest | Cave | Cave |
| Forest | Flat | Forest | Forest |
| Cave | Hill | Flat | Forest |
| Cave | Cave | Forest | Cave |

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | $\frac{1}{20}$ | 0 | 0 |
| 0 | $\frac{1}{20}$ | $\frac{1}{20}$ | 0 |
| 0 | 0 | 0 | 0 |

As we can see from the map bellow, the possibility of the target in hilly is 0.1 because the target can move from both flats. And each flat has possibility of 0.025 because the target can move to each one of the flats from the hilly with half possibility.

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | $\frac{1}{40}$ | 0 | 0 |
| 0 | $\frac{1}{10}$ | $\frac{1}{40}$ | 0 |
| 0 | 0 | 0 | 0 |

Based on the possibilities we get, we can use rule1, rule2, rule1(location based) and rule2(location based) to make decision and search.

Moving target

| Rule Type | Average Cost |
|---|---|
| Rule 1 | 33 |
| Rule 2 | 26 |
| Rule 1(Location based) | 83 |
| Rule 2(Location based) | 135 |

As we can see from the result, the number of steps needed is much less than the stationary target. That is because we can get more information about the target when it moves. So we can narrow down possible locations significantly, which makes our search efficiently.