

## Verification of hypergraph states

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Hypergraph states are generalizations of graph states where controlled-Z gates on edges are replaced with generalized controlled-Z gates on hyperedges. Hypergraph states have several advantages over graph states. For example, certain hypergraph states, such as the Union Jack states, are universal resource states for measurement-based quantum computing with only Pauli measurements, while graph state measurement-based quantum computing needs non-Clifford basis measurements. Furthermore, it is impossible to classically efficiently sample measurement results on hypergraph states unless the polynomial hierarchy collapses to the third level. Although several protocols have been proposed to verify graph states with only sequential single-qubit Pauli measurements, there was no verification method for hypergraph states. In this paper, we propose a method for verifying a certain class of hypergraph states with only sequential single-qubit Pauli measurements. Importantly, no i.i.d. property of samples is assumed in our protocol: any artificial entanglement among samples cannot fool the verifier. As applications of our protocol, we consider verified blind quantum computing with hypergraph states, and quantum computational supremacy demonstrations with hypergraph states.

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### I. INTRODUCTION

Many-point correlations in quantum many-body systems are one of the most essential ingredients in condensed-matter physics and statistical physics. Correlations of sequential single-qubit measurements on quantum states are also important drive forces for quantum information processing. For example, measurement-based quantum computing [1], which is nowadays one of the standard quantum computing models, enables universal quantum computing with only adaptive single-qubit measurements on certain quantum states, such as graph states [1] and other condensed-matter-physically motivated states including the AKLT state [2–17]. Furthermore, not only adaptive but also nonadaptive single-qubit measurements on graph states can demonstrate a quantumness which cannot be classically efficiently simulated: it is known that if probability distributions of nonadaptive sequential single-qubit measurements on graph states are classically efficiently sampled, then the polynomial hierarchy collapses to the third level [18–20] or the second level [21]. The polynomial hierarchy is a hierarchy of complexity classes generalizing P and NP, and it is not believed to collapse in computer science. It is an example of recently well studied “quantum computational supremacy” of subuniversal quantum computing models, which are expected to be easier to experimentally implement, but can outperform classical computing. (For details, see Refs. [18–24] and their supplementary materials.)

For practical implementations of measurement-based quantum computing and experimental demonstrations of quantum computational supremacy, verifying graph states is essential,

since in reality a generated state cannot be the ideal graph state due to some experimental noises. The problem becomes more serious if we consider delegated secure quantum computing, so-called blind quantum computing [25,26]. It is known that the ability of sequentially measuring single qubits is enough to secretly delegate quantum computing to a remote server [27,28]. The honest server sends each qubit of a graph state one by one to the user, and user can realize any quantum computing with only sequential single-qubit measurements. If the server is malicious, however, a completely wrong state might be sent to the user. The user therefore needs to test the state sent from the server. In such a quantum cryptographic scenario, the situation is worse than the single-party laboratory experiments, since the noises on the given state are caused by malicious servers and therefore not necessarily physically natural ones. Several methods of verifying graph states with only sequential single-qubit Pauli measurements have been proposed [28,29]. (If more than two noncommunicating servers are available, a completely classical user can verify stabilizer states [30–32].) In the protocol of Ref. [28], the user does a test so-called the stabilizer test on some parts of the state sent from the server. The stabilizer test can be done with only sequential single-qubit Pauli measurements. If the user passes the test, the remaining state is guaranteed to be close to the ideal graph state.

Since the protocol of Ref. [28] makes no assumption (such as the i.i.d. sample or physically natural noises) on the given state, the verification method can be used in quantum cryptographic contexts. In particular, verified blind quantum computing and verified quantum computational supremacy demonstrations can be realized with graph states verified through the protocol. There are, however, two problems. First, in the verified blind protocol of Ref. [28], the user needs non-Clifford basis measurements for computing (the verification itself can be done with only Pauli measurements). It would

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be better if both the verification and the computation can be done with only Pauli measurements [33]. Second, the quantum computational supremacy demonstration with graph states [18], which needs only nonadaptive measurements, requires somehow a strict approximation, namely a multiplicative-error approximation.

Recently, two breakthroughs that solve these drawbacks of graph states have been done. These results use hypergraph states [34–38] instead of graph states. (For the definition of hypergraph states and their properties, see below.) First, certain hypergraph states, such as the Union Jack states, are universal resource states for measurement-based quantum computing with only Pauli measurements [39]. This result solves the first problem, namely, the requirement of non-Clifford basis measurements for the user. Therefore, by using the hypergraph states, the one-way secure delegated quantum computing is possible for the user who can do only Pauli measurements. Reference [39] also pointed out that hypergraph states are important in the study of symmetry-protected topological orders. Second, it was shown in Ref. [19] that if hypergraph states are considered, the multiplicative error requirement can be replaced with an  $L1$ -norm one, which is more relaxed. This result solves the second problem.

In short, hypergraph states are promising novel resource states for many quantum information processing tasks. However, how can we verify hypergraph states? Without any verification, the above advantages of hypergraph states cannot be enjoyed. The verification protocol of Ref. [28] can be applied to only bipartite graph states, and therefore useless for general hypergraph states. Recently, a protocol of verifying Union-Jack states was proposed [40]. They also mention that their protocol can be generalized to other hypergraph states. Their protocol, however, assumes i.i.d. property of samples.

In this paper, to solve the problem, we invent a test for general hypergraph states and, by using it, introduce a protocol for verifying certain class of hypergraph states. Our protocol needs only sequential single-qubit Pauli measurements. Neither quantum memory nor entangling gate operation is necessary. Furthermore, our protocol makes no assumption on the i.i.d. property of samples: any malicious and artificial entanglement among samples cannot fool the verifier. As applications of our protocol, we consider verified blind quantum computing with hypergraph states, and verified quantum computational supremacy demonstrations of IQP with hypergraph states.

Note that our protocol works only for the class of hypergraph states such that at most constant number of generalized  $CZ$  gates are applied on every qubit. As we will see later, however, the class contains several useful hypergraph states such as the Union Jack states for measurement-based quantum computing and output states of IQP circuits. We leave the generalization for a future study.

The idea of decomposing  $CZ$  gates into Pauli operators, which we use in our protocol, was also considered in Ref. [36] to study nonlocality of hypergraph states.

## II. HYPERGRAPH STATES

We first define hypergraph states and explain their properties. A hypergraph  $G \equiv (V, E)$  is a pair of a set  $V$  of vertices

and a set  $E$  of hyperedges, where  $n \equiv |V|$ . A hyperedge may link more than two vertices. For simplicity, in this paper, we assume that  $2 \leq |e| \leq 3$  for all  $e \in E$ , where  $|e|$  is the number of vertices linked to the hyperedge  $e$ . (Generalizations to other cases would be possible.) Let

$$|G\rangle \equiv \left( \prod_{e \in E} \widetilde{CZ}_e \right) |+\rangle^{\otimes n}$$

be the hypergraph state corresponding to the hypergraph  $G$ , where

$$\widetilde{CZ}_e \equiv \bigotimes_{i \in e} I_i - 2 \bigotimes_{i \in e} |1\rangle\langle 1|_i$$

is the generalized  $CZ$  gate acting on vertices in the hyperedge  $e$ . Here,  $I$  is the two-dimensional identity operator. For example, if  $|e| = 2$ , it is nothing but the standard  $CZ$  gate. If  $|e| = 3$ , it is the  $CCZ$  gate,

$$CCZ \equiv (I^{\otimes 2} - |11\rangle\langle 11|) \otimes I + |11\rangle\langle 11| \otimes Z.$$

The **stabilizer**  $g_i$  of  $|G\rangle$  associated with the vertex  $i$  is defined by

$$\begin{aligned} g_i &\equiv \left( \prod_{e \in E} \widetilde{CZ}_e \right) X_i \left( \prod_{e \in E} \widetilde{CZ}_e \right) \\ &= X_i \left( \prod_{j \in W_i^Z} Z_j \right) \left( \prod_{(j,k) \in W_i^{CZ}} CZ_{j,k} \right), \end{aligned}$$

where

$$W_i^Z \equiv \{j \in V | (i, j) \in E\},$$

$$W_i^{CZ} \equiv \{(j, k) \in V \times V | (i, j, k) \in E\}.$$

It is easy to check that the following properties are satisfied:

$$[g_i, g_j] = 0,$$

$$g_i |G\rangle = |G\rangle,$$

$$g_i^2 = I^{\otimes n},$$

$$\prod_{i=1}^n \frac{I^{\otimes n} + g_i}{2} = |G\rangle\langle G|.$$

## III. STABILIZER TEST FOR $g_i$

Before introducing our verification protocol, we define the stabilizer test for each  $g_i$ , which is an essential ingredient of the protocol. Note that

$$CZ_{j,k} = \frac{1}{2}(I_j \otimes I_k + I_j \otimes Z_k + Z_j \otimes I_k - Z_j \otimes Z_k).$$

Therefore,

$$\begin{aligned} g_i &= X_i \left( \prod_{j \in W_i^Z} Z_j \right) \left( \frac{1}{2^r} \sum_{t \in \{1,2,3,4\}^r} \prod_{(j,k) \in W_i^{CZ}} \sigma_{j,k}(t_{j,k}) \right) \\ &= \frac{1}{2^r} \sum_{t \in \{1,2,3,4\}^r} s_t, \end{aligned}$$

where

$$\begin{aligned} r &\equiv |W_i^{CZ}|, \\ t &\equiv \{t_{j,k}\}_{(j,k) \in W_i^{CZ}}, \\ \sigma_{j,k}(1) &\equiv I_j \otimes I_k, \\ \sigma_{j,k}(2) &\equiv I_j \otimes Z_k, \\ \sigma_{j,k}(3) &\equiv Z_j \otimes I_k, \\ \sigma_{j,k}(4) &\equiv -Z_j \otimes Z_k, \\ s_t &\equiv X_i \left( \prod_{j \in W_i^Z} Z_j \right) \left( \prod_{(j,k) \in W_i^{CZ}} \sigma_{j,k}(t_{j,k}) \right). \end{aligned}$$

Let us define a bit  $\alpha_t \in \{0, 1\}$  and a subset  $D_t \subseteq V$  such that

$$s_t = (-1)^{\alpha_t} X_i \left( \prod_{j \in D_t} Z_j \right).$$

Note that  $\alpha_t$  and  $D_t$  can be calculated in polynomial time (see Appendix A). ( $\alpha_t$  and  $D_t$  actually depend on  $i$ , but for simplicity, we omit it.)

Let  $\rho$  be an  $n$ -qubit state. We define the “stabilizer test for  $g_i$  on  $\rho$ ” as the following Alice’s action: (1) Alice randomly generates  $t \in \{1, 2, 3, 4\}^r$ . (2) She measures  $i$ th vertex of  $\rho$  in  $X$  and  $j$ th vertex of  $\rho$  in  $Z$  for all  $j \in D_t$ .

Let  $x \in \{+1, -1\}$  be the measurement result of the  $X$  measurement and  $z_j \in \{+1, -1\}$  be that of the  $Z$  measurement on vertex  $j \in D_t$ . We say that Alice passes the stabilizer test for  $g_i$  on  $\rho$  if

$$x \prod_{j \in D_t} z_j = (-1)^{\alpha_t}.$$

The probability  $p_{\text{test},i}$  that Alice passes the stabilizer test for  $g_i$  on  $\rho$  is

$$p_{\text{test},i} \equiv \frac{1}{4^r} \sum_{t \in \{1, 2, 3, 4\}^r} \text{Tr} \left( \rho \frac{I^{\otimes n} + s_t}{2} \right) = \frac{1}{2} + \frac{\text{Tr}(\rho g_i)}{2^{r+1}}.$$

Here we can see that if  $r = \text{poly}$ , then  $p_{\text{test},i} = \frac{1}{2} + O(2^{-\text{poly}})$ , which means that exponentially many measurements are required to gain useful information about  $\text{Tr}(\rho g_i)$ . It suggests that our verification method does not work if  $r = \text{poly}$ .

#### IV. VERIFICATION PROTOCOL

We now explain our verification protocol. Bob sends Alice an  $n(nk + 1 + m)$ -qubit state  $\Psi$ , where  $k = 2^{2r+3}n^7$  and  $m \geq 2n^7k^2 \ln 2$ . The state  $\Psi$  consists of  $nk + 1 + m$  registers (Fig. 1). Each register stores  $n$  qubits. (If Bob is honest, every

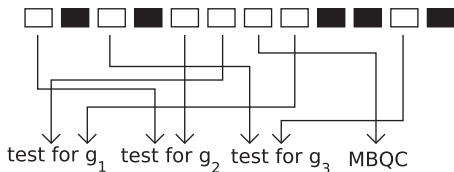


FIG. 1. Example for  $n = 3$ ,  $k = 2$ , and  $m = 5$ . Each square represents a register that stores  $n$  qubits. Registers represented by black squares are discarded.

register is in the state  $|G\rangle$ . If Bob is malicious, on the other hand,  $\Psi$  can be any  $n(nk + 1 + m)$ -qubit entangled state.) Alice randomly permutes registers and discards  $m$  registers. (As we will see later, this random permutation and discarding of some registers are necessary to guarantee that the remaining state is close to an i.i.d. sample by using the quantum de Finetti theorem [41].) Let  $\Psi'$  be the remaining state. The state  $\Psi'$  consists of  $nk + 1$  registers. She chooses one register from  $\Psi'$ , which is used for the measurement-based quantum computing. We call the register computing register. The remaining  $nk$  registers of  $\Psi'$  are divided into  $n$  groups. Each group consists of  $k$  registers. The stabilizer test for  $g_i$  is performed on every register in the  $i$ th group for  $i = 1, 2, \dots, n$ . (Note that Alice does not need to do the permutation “physically,” which requires a quantum memory. Bob just sends each qubit of  $\Psi$  one by one to Alice, and Alice randomly chooses her action from the test, discarding, or computation.)

Let  $K_i$  be the number of times that Alice passes the stabilizer test for  $g_i$ , i.e., the random variable to describe the number of Alice’s observation of the event

$$\frac{1}{4^r} \sum_t \frac{I^{\otimes n} + s_t}{2}.$$

If

$$\frac{K_i}{k} \geq \frac{1}{2} + \frac{1 - \epsilon}{2^{r+1}},$$

we say that the  $i$ th group passes the test. Here,  $\epsilon = \frac{1}{2n^3}$ . If all groups pass the test, we say that Alice accepts Bob.

The main results of the present paper are the following two items.

(1) Completeness: if every register of  $\Psi$  is in the state  $|G\rangle$ , then the probability that Alice accepts Bob is larger than  $1 - ne^{-n}$ .

(2) Soundness: if Alice accepts Bob, the state  $\rho_{\text{comp}}$  of the computing register satisfies

$$\langle G | \rho_{\text{comp}} | G \rangle \geq 1 - \frac{1}{n}$$

with a probability larger than  $1 - \frac{1}{n}$ .

Proofs are given in Appendixes B and C.

#### V. APPLICATIONS

To conclude this paper, we discuss two applications of our results. First, our verification protocol can be used in verified blind quantum computing. Blind quantum computing [25] is a secure quantum computing protocol where Alice, who does not have enough quantum technology, can delegate her quantum computing to Bob, who has a full-fledged quantum computer, without leaking any her privacy. Several verification protocols have been proposed that enable Alice to check the correctness of Bob’s quantum computing [26, 28]. In particular, in the protocol of Ref. [28], Bob sends each qubit of the graph state to Alice one by one, and Alice checks the correctness of the graph state by measuring stabilizer operators. However, in the protocol, Alice needs non-Clifford basis measurements to implement quantum computing (note that the verification itself can be done with only Pauli measurements). If Bob sends Alice the Union Jack state [39] instead of the graph

state, for example, Alice needs only Pauli measurements for both the verification and the computation, which is a great advantage over the previous protocols. The verification protocol introduced in this paper can be used to verify the Union Jack state.

The second application of our verification protocol is the verified quantum computational supremacy demonstration of subuniversal quantum computing. It was shown in Ref. [19] that, for several hypergraph states, if there exists a classical sampler that outputs  $z$  with probability  $q_z$  such that

$$\sum_{z \in \{0,1\}^n} |p_z - q_z| \leq \frac{1}{192},$$

then the polynomial hierarchy collapses to the third level. Here,  $p_z$  is the probability of obtaining the result  $z \in \{0,1\}^n$  when certain single-qubit measurements are done on an  $n$ -qubit hypergraph state. Since the collapse of the polynomial hierarchy is not believed to happen, the result suggests the “quantumness” of hypergraph states that cannot be classically simulated. However, in reality, not the ideal hypergraph states but some noisy ones are available in laboratories. Here, we show that the verified state  $\rho_{\text{comp}}$  via our protocol is enough to demonstrate the same quantum advantage. In fact, let us assume that there exists a classical sampler such that

$$\sum_z |p'_z - q_z| \leq \frac{1}{192},$$

where  $p'_z$  is the output probability distribution of the single-qubit measurements on  $\rho_{\text{comp}}$ . Then, from the triangle inequality,

$$\begin{aligned} \sum_z |p_z - q_z| &\leq \sum_z |p_z - p'_z| + \sum_z |p'_z - q_z| \\ &\leq o(1) + \frac{1}{192}, \end{aligned}$$

which means that the classical sampler can also sample  $p_z$  with the  $\sim 1/192$   $L_1$ -norm error, and therefore the polynomial hierarchy collapses.

For example, the hypergraph states that are outputs of IQP circuits corresponding to the nonadaptive Union Jack state measurement-based quantum computing [39] can be used for that purpose. Since the nonadaptive Union Jack state measurement-based quantum computing is universal with postselections, **a multiplicative error calculation of its output probability distribution is #P-hard** [20]. If we make the “average case vs worst case” conjecture (as in Refs. [19,22]) that the worst case hardness can be lifted to the average case one, we obtain the hardness of the classical constant  $L_1$ -norm error sampling of the Union Jack states with a similar proof as that of Ref. [19].

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#### APPENDIX A: CALCULATION OF $\alpha_t$ AND $D_t$

Here we show that  $\alpha_t$  and  $D_t$  can be calculated in polynomial time.

First,  $\alpha_t$  can be calculated in the following algorithm.

- (1) First set  $\alpha_t = 0$ .
  - (2) Choose  $(j,k)$ . Calculate  $t_{j,k}$ .
  - (3) If  $t_{j,k} = 4$ , flip  $\alpha_t$ .
  - (4) Repeat 2. and 3. for all  $(j,k) \in W_i^{CZ}$ .
- Since

$$|W_i^{CZ}| \leq \binom{n-1}{2} = O(n^2),$$

the above algorithm takes at most polynomial time.

Next,  $D_t$  can be calculated in the following algorithm.


- (1) First set  $D_t = W_i^Z$ .
- (2) Choose  $(j,k)$ . Calculate  $t_{j,k}$ .
- (3) Update  $D_t$  according to  $t_{j,k}$ .
- (4) Repeat 2. and 3. for all  $(j,k) \in W_i^{CZ}$ .

Again,  $|W_i^{CZ}| \leq O(n^2)$  means that the algorithm takes at most polynomial time.

#### APPENDIX B: PROOF OF THE COMPLETENESS

If every register of  $\Psi$  is in the state  $|G\rangle$ , then

$$p_{\text{test},i} = \frac{1}{2} + \frac{1}{2^{r+1}}$$

for all  $i = 1, 2, \dots, n$ . From the union bound and the **Hoeffding inequality**, 

$$\begin{aligned} \Pr[\text{Alice accepts Bob}] &= \Pr\left[\bigwedge_{i=1}^n \left(\frac{K_i}{k} \geq \frac{1}{2} + \frac{1-\epsilon}{2^{r+1}}\right)\right] \\ &\geq 1 - \sum_{i=1}^n \Pr\left[\frac{K_i}{k} < \frac{1}{2} + \frac{1-\epsilon}{2^{r+1}}\right] \\ &= 1 - \sum_{i=1}^n \Pr\left[\frac{K_i}{k} < p_{\text{test},i} - \frac{\epsilon}{2^{r+1}}\right] \\ &\geq 1 - n e^{-2\frac{\epsilon^2}{2^{2r+2}}k}. \end{aligned}$$

#### APPENDIX C: PROOF OF THE SOUNDNESS

We next show the soundness. We define the  $n$ -qubit projection operator

$$\Pi_G^\perp \equiv I^{\otimes n} - |G\rangle\langle G|.$$

Let  $T$  be the POVM element corresponding to the event that Alice accepts Bob. We can show that for any  $n$ -qubit state  $\rho$ ,

$$\text{Tr}[(T \otimes \Pi_G^\perp) \rho^{\otimes nk+1}] \leq \frac{1}{2n^2}. \quad (\text{C1})$$

Its proof is given later. Due to the quantum de Finetti theorem (for the one-way LOCC norm version) [41],

$$\begin{aligned} \text{Tr}[(T \otimes \Pi_G^\perp) \Psi'] &\leq \text{Tr}\left[(T \otimes \Pi_G^\perp) \int d\mu(\rho) \rho^{\otimes nk+1}\right] \\ &\quad + \frac{1}{2} \sqrt{\frac{2n^2 k^2 n \ln 2}{m}} \\ &\leq \frac{1}{2n^2} + \frac{1}{2} \sqrt{\frac{2n^3 k^2 \ln 2}{2n^7 k^2 \ln 2}} = \frac{1}{n^2}. \end{aligned}$$

(Note that the reason why we use the version of Ref. [41] is that other versions require exponentially many subsystems to discard. The version of Ref. [41] needs only polynomially many, but restricted to only one-way LOCC. Fortunately, the one-way LOCC is enough for our purpose, and therefore we can use this version.)

We have

$$\text{Tr}[(T \otimes \Pi_G^\perp) \Psi'] = \text{Tr}(\Pi_G^\perp \rho_{\text{comp}}) \text{Tr}[(T \otimes I) \Psi'].$$

Therefore, if

$$\text{Tr}(\Pi_G^\perp \rho_{\text{comp}}) > \frac{1}{n},$$

then

$$\text{Tr}[(T \otimes I) \Psi'] < \frac{1}{n},$$

which means that if Alice accepts Bob,

$$\langle G | \rho_{\text{comp}} | G \rangle \geq 1 - \frac{1}{n}$$

with a probability larger than  $1 - \frac{1}{n}$ .

*Proof of Eq. (C1).* First, let us assume that  $\text{Tr}(\rho g_i) \geq 1 - \delta$  for all  $i = 1, 2, \dots, n$ , where  $\delta = \frac{1}{n^3}$ . Due to the union

bound,

$$\begin{aligned} 1 - \langle G | \rho | G \rangle &= 1 - \text{Tr}\left(\prod_{i=1}^n \frac{I^{\otimes n} + g_i}{2} \rho\right) \\ &\leq \sum_{i=1}^n \left[1 - \text{Tr}\left(\rho \frac{I^{\otimes n} + g_i}{2}\right)\right] \leq \frac{n\delta}{2}. \end{aligned}$$

Therefore,

$$\begin{aligned} \text{Tr}[(T \otimes \Pi_G^\perp) \rho^{\otimes nk+1}] &= \text{Tr}(T \rho^{\otimes nk}) \text{Tr}(\Pi_G^\perp \rho) \\ &\leq 1 \times \frac{n\delta}{2} = \frac{1}{2n^2}. \end{aligned} \quad (\text{C2})$$

Next let us assume that  $\text{Tr}(\rho g_i) < 1 - \delta$  for at least one  $i$ . In this case,

$$p_{\text{test},i} = \frac{1}{2} + \frac{\text{Tr}(\rho g_i)}{2^{r+1}} < \frac{1}{2} + \frac{1-\delta}{2^{r+1}}$$

for the  $i$ . Then, due to the Hoeffding inequality,

$$\begin{aligned} \text{Tr}[(T \otimes I) \rho^{\otimes nk+1}] &\leq \Pr[\text{group } i \text{ passes the test}] \\ &= \Pr\left[\frac{K_i}{k} \geq \frac{1}{2} + \frac{1-\epsilon}{2^{r+1}}\right] \\ &= \Pr\left[\frac{K_i}{k} \geq \frac{1}{2} + \frac{1-\delta}{2^{r+1}} + \frac{\delta-\epsilon}{2^{r+1}}\right] \\ &\leq \Pr\left[\frac{K_i}{k} > p_{\text{test},i} + \frac{\delta-\epsilon}{2^{r+1}}\right] \\ &\leq e^{-2\frac{(\delta-\epsilon)^2}{2^{2r+2}}k} = e^{-n}. \end{aligned}$$

Hence

$$\begin{aligned} \text{Tr}[(T \otimes \Pi_G^\perp) \rho^{\otimes nk+1}] &= \text{Tr}(T \rho^{\otimes nk}) \text{Tr}(\Pi_G^\perp \rho) \\ &\leq e^{-n} \times 1. \end{aligned} \quad (\text{C3})$$

From Eqs. (C2) and (C3), for any state  $\rho$ ,

$$\text{Tr}[(T \otimes \Pi_G^\perp) \rho^{\otimes nk+1}] \leq \max\left(\frac{1}{2n^2}, e^{-n}\right) = \frac{1}{2n^2}.$$

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