Data-Syndrome codes project

Weilei Zeng, Alexey Ashikhmin, and Leonid Pryadko
(Dated: March 23, 2018)

I. ERROR MODELS

1. Physical level error model (including coherent errors). Coherent errors of various kind can also be used in the following.

2. Gate error mode (with back propagation, etc) with additional penalty for non-local entangling gates

3. Gate error model (with back propagation, etc) but no penalty for non-local gates

4. more detailed phenomenological model: qubit errors and measurement error scaling with syndrome weight, each layer of syndrome measurements gets to increment the qubit errors.

5. less detailed phenomenological model: same as standard, but each layer of syndrome measurements gets to increment qubit errors.

6. standard phenomenological error model (depolarizing noise or independent X/Z errors combined with syndrome measurement errors, separate layers for repeated measurement of the entire code)

7. simplified phenomenological error model (channel model with syndrome measurement errors which have no effect on the qubits; syndrome measurement errors remain the same no matter how complicated or how non-local a generator is measured)

II. FAULT TOLERANT SYNDROME MEASUREMENT

One should think about flagged error correction and how it helps (apparent nutshell is that the use of these gates kills back propagation and makes the effective model closer to channel/phenomenological)

III. HYPERCUBIC ANSATZ

See thesis of Breuckmann arXiv:1802.01520 “PhD thesis: Homological Quantum Codes Beyond the Toric Code”

Also our paper where somewhat similar codes are discussed[1].

The inspiration comes from cubic and hypercubic toric codes. For the cubic codes: these are CSS codes. G_X: qubits on bonds of the cubic lattice, vertex operators are products of qubits on the six bonds of each vertex. G_Z: three types of plaquette operators: XY, XZ, and YZ. Statement: each vertex commutes with any plaquette.

Algebraic description is given by the following block matrix:

$$G_X = \left(\begin{array}{c|c|c} aee & eae & eea \end{array} \right)$$

$$G_Z = \left(\begin{array}{c|c|c} ebe & bee & \\ & eeb & ebe \\ eeb & & bee \end{array} \right)$$

where each block is a Kronecker product of three matrices (an identity matrix and a circulant matrix). If we take $b = a^T$, this gives an orthogonal ansatz which includes the usual cubic codes if a is the circulant check matrix of the repetition code.

For hypercubic codes the construction is similar. For both G_x and G_z the qubits live on plaquettes, there are total of six types of qubits: tx, ty, tz, xy, xz, and yz. The operators are cubes and three-vertices, there are total of four types of each; each row should contain contribution from three types of the planes. The structure of the matrices is shown in Tab. I.

	tx	ty	tz	xy	xz	yz
txy	y=$eeae$	x=$eaee$		t=$aeee$		
xyz				z=$eeea$	y=$eeae$	x=$eaee$
yzt		z=$eeea$	y=$eeae$			t=$aeee$
ztx	z=$eeea$		x=$eaee$		t=$aeee$	
t	X=$ebee$	Y=$eebe$	Z=$eeeb$			
x	T			Y	Z	
y		T		X		Z
z			T		X	Y

TABLE I. Algeberaic structure of G_X and G_Z matrices for the hypercubic circulant ansatz.

With much luck, we should be able to generalize this to non-square matrices, and prove properties similar to those of the hypergraph product codes, as well as various rotated constructions as in our old paper[1].

With a bit less luck, we should be able to prove some nice properties just for circulant matrices, e.g., that the distance scales as the square of the distance, whereas k is the fourth power of k for circulant codes, which would give a new family of finite-rate LDPC codes, with square-root scaling of the distance.

Even if we are not able to prove anything, there is numerics, we should play with circulant matrices a and see what kind of codes we get.

Notice that if we arrange qubits in one big line, these would be tail-biting codes; by making one dimension very large we shold be able to get convolutional codes.

Finally, we should also investigate the binary syndrome code, numerically and analytically; by analogy with hypercubic codes this should be able to correct syndrome in one round. I believe this code (or combined d+s code) would be similar to a quantum code, in the sense that there are going to be some small-weight operators commuting with the code but acting trivially… I do not think Terhal and Breuckmann did this right — although empirically they saw that min-weight decoding works even though the distance they claimed was puny.

[1] A. A. Kovalev and L. P. Pryadko, "Quantum Kronecker sum-product low-density parity-check codes with finite rate," Phys. Rev. A **88**, 012311 (2013).