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Some families of asymmetric quantum codes and quantum convolutional codes from constacyclic codes

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A B S T R A C T

Quantum maximal-distance-separable (MDS) codes are an important class of quantum codes. Recently, many scholars utilize constacyclic codes to construct some quantum MDS codes. In this paper, several new families of optimal asymmetric quantum codes and optimal quantum convolutional codes are constructed by using constacyclic codes. Moreover, these quantum codes constructed in this paper are different from the ones in the literature.

1. Introduction

The constructions of quantum error-correcting codes have become an important subject in quantum information and quantum computing. Some families of good quantum

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codes have been constructed in [4,9,24,26,27,40,45]. Nowadays, some scholars studied some families of constacyclic codes in [5–8,10,20,25,39]. As we know, constacyclic codes contain cyclic codes and negacyclic codes. In [21], the authors used negacyclic codes to construct two families of quantum MDS codes. In [22], the authors constructed some families of good quantum codes by using negacyclic codes. The authors utilized some classes of constacyclic codes to construct some families of quantum MDS codes in [23]. In [11], the authors studied some families of constacyclic codes that are different from the ones in [23] and constructed quantum MDS codes. Now, constacyclic codes have become a good source to search for quantum codes.

Quantum codes defined over quantum channels where qubit-flip errors and phase-shift errors may have different probabilities are called asymmetric quantum codes. In many quantum mechanical systems, the occurrence of qubit-flip and phase-shift errors is quite different [12]. For the past two decades, the constructions of good asymmetric quantum codes have been investigated by some researchers [28–30,32,33,38]. Qian and Zhang utilized q^2-ary cyclotomic cosets to construct a family of optimal asymmetric quantum codes in [43]. In [12], we studied the optimal asymmetric quantum codes by using negacyclic codes. For more constructions of asymmetric quantum codes, the readers can consult [13–15,37] for more detail.

Now, the constructions of good quantum convolutional codes have been studied by many authors [1–3,16–19,41,42]. In [31], the author utilized some classes of cyclic codes to construct some good quantum convolutional codes compared with the ones in [2]. In [34], the author studied a family of optimal quantum convolutional codes by using BCH cyclic codes. In [35], G.G. La Guardia also utilized negacyclic codes to construct two families of optimal quantum convolutional codes. In [36], G.G. La Guardia constructed some families of optimal convolutional codes and asymmetric quantum codes by using constacyclic codes. In this work, we use some constacyclic codes in [11] to construct some families of optimal asymmetric quantum codes and optimal quantum convolutional codes.

In this paper, we obtain some families of optimal asymmetric quantum codes and optimal quantum convolutional codes as follows:

(1) $[[(q^2-1)/3, (q^2-1)/3-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $3|(q+1)$, δ_1 and δ_2 are positive integers, and $1 \leq \delta_2 \leq \delta_1 \leq \frac{2(q-2)}{3}$;

(2) $[[(q^2-1)/5, (q^2-1)/5-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $5|(q+1)$, δ_1 and δ_2 are positive integers and $1 \leq \delta_2 \leq \delta_1 \leq \frac{3(q+1)}{5}-2$;

(3) $[[(q^2-1)/7, (q^2-1)/7-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $7|(q+1)$, δ_1 and δ_2 are positive integers and $1 \leq \delta_2 \leq \delta_1 \leq \frac{4(q+1)}{7}-2$;

(4) $[[(q^2+1)/10, (q^2+1)/10-2(\delta_1+\delta_2), 2\delta_1+1/2\delta_2+1]]_{q^2}$, where q is an odd prime power with the form $10m+3$ or $10m+7$, δ_1 and δ_2 are positive integers, $1 \leq \delta_2 \leq \delta_1 \leq 2m$ and m is a positive integer;

(5) $[((q^2-1)/3, (q^2-1)/3-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $3|(q+1)$, δ' is a positive integer and $2 \leq \delta' \leq \frac{2(q-2)}{3}$;

(6) $[((q^2-1)/5, (q^2-1)/5-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $5|(q+1)$, δ' is a positive integer and $2 \leq \delta' \leq \frac{3(q+1)}{5}-2$;

(7) $[((q^2-1)/7, (q^2-1)/7-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $7|(q+1)$, δ' is a positive integer and $2 \le \delta' \le \frac{4(q+1)}{7} - 2$;

(8) $[((q^2+1)/10, (q^2+1)/10-4\delta'+4, 1; 2, 2\delta'+1)]_q$, where q is an odd prime power with the form $10m+3$ or $10m+7$, δ' is a positive integer, $2 \le \delta' \le 2m$ and m is a positive integer.

The organization of this paper is as follows: In Section 2, we present some definitions and basic results of constacyclic codes. In Section 3, we recall some definitions and basic results of asymmetric quantum codes, then we construct some families of optimal asymmetric quantum codes. In Section 4, we state some basic concepts and results of classical and quantum convolutional codes, then we construct some families of optimal quantum convolutional codes.

2. Review of constacyclic codes

In this section, we recall some basic results about constacyclic codes in [11,23].

Throughout this paper, let F_{q^2} be the finite field with q^2 elements, where p is an odd prime number and q is a power of p. We assume that n is a positive integer relatively prime to q, i.e. $gcd(n,q)=1$. If $\mathcal{C}$ is a k-dimensional subspace of $F_{q^2}^n$, then $\mathcal{C}$ is said to be an $[n,k]$-linear code. The number of nonzero components of $c \in \mathcal{C}$ is said to be the weight $wt(c)$ of the codeword c. The minimum nonzero weight d of all codewords in $\mathcal{C}$ is said to be the minimum weight of $\mathcal{C}$. For $\lambda \in F_{q^2}^*$, a linear code $\mathcal{C}$ of length n over F_{q^2} is said to be λ-constacyclic if $(\lambda c_{n-1}, c_0, c_1, \cdots, c_{n-2}) \in \mathcal{C}$ for every $(c_0, c_1, \cdots, c_{n-1}) \in \mathcal{C}$. When $\lambda = -1$, $\mathcal{C}$ is a negacyclic code. When $\lambda = 1$, $\mathcal{C}$ is a cyclic code. We know that a q^2-ary λ-constacyclic code $\mathcal{C}$ of length n is an ideal of $F_{q^2}[x]/\langle x^n - \lambda\rangle$ and $\mathcal{C}$ can be generated by a monic polynomial $g(x)$ which divides $x^n - \lambda$. From [23,11], we can see that the Hermitian dual $\mathcal{C}^{\perp_h}$ of a λ-constacyclic code over F_{q^2} is a λ^{-q}-constacyclic code. We assume that $\lambda \in F_{q^2}^*$ is a primitive r-th root of unity, then there exists a primitive rn-th root of unity over some extension field of F_{q^2} such that $\eta^n = \lambda$. Hence, the elements η^{1+ri} are the roots of $x^n - \lambda$ for $0 \le i \le n-1$. Let $\mathcal{O}_{rn} = \{1+jr | 0 \le j \le n-1\}$. For each $i \in \mathcal{O}_{rn}$, let C_i be the q^2-cyclotomic coset modulo rn containing i. The defining set of a constacyclic code $\mathcal{C} = \langle g(x)\rangle$ of length n is the set $Z = \{i \in \mathcal{O}_{rn} \mid \eta^i \text{ is a root of } g(x)\}$. Let $\mathcal{C}$ be an $[n,k]$ constacyclic code over F_{q^2} with defining set Z, then the Hermitian dual $\mathcal{C}^{\perp_h}$ has defining set $Z^{\perp_h} = \{z \in \mathcal{O}_{rn} | -qz (mod\ rn) \notin Z\}$.

Let $a^q = (a_0^q, a_1^q, \cdots, a_{n-1}^q)$ denote the conjugation of the vector $a = (a_0, a_1, \cdots, a_{n-1})$. For $u = (u_0, u_1, \cdots, u_{n-1})$, $v = (v_0, v_1, \cdots, v_{n-1}) \in F_{q^2}^n$, the Hermitian inner product is defined as

$$\langle u, v\rangle_h = u_0 v_0^q + u_1 v_1^q + \cdots + u_{n-1} v_{n-1}^q.$$

The Hermitian dual code of $\mathcal{C}$ can be defined as

$$\mathcal{C}^{\perp_h} = \{u \in F_{q^2}^n \mid \langle u, v\rangle_h = 0 \text{ for all } v \in \mathcal{C}\}.$$

If $\mathcal{C} \subseteq \mathcal{C}^{\perp_h}$, then $\mathcal{C}$ is called a Hermitian self-orthogonal code. If $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$, then $\mathcal{C}$ is a Hermitian dual-containing code.

The following proposition in [11,23,25] plays an important role in constructing quantum codes.

Proposition 1 *(The BCH bound for constacyclic codes). Assume that $gcd(n, q) = 1$. Let $\mathcal{C}$ be a q^2-ary λ-constacyclic code of length n. If the generator polynomial $g(x)$ of $\mathcal{C}$ has the elements $\{\eta^{1+ri} \mid 0 \leq i \leq d-2\}$ as the roots where η is a primitive rn-th root of unity, then the minimum distance of $\mathcal{C}$ is at least d.*

3. Constructions of optimal asymmetric quantum codes

In this section, we state some definitions and some basic results in [28–30,32,33] firstly, then we will use constacyclic codes from [11] to construct some families of optimal asymmetric quantum codes.

Let H be the Hilbert space $H = \mathbf{C}^{q^n} = \mathbf{C}^q \bigotimes \cdots \bigotimes \mathbf{C}^q$. Let $|x\rangle$ be the vectors of an orthonormal basis of $\mathbf{C}^q$, where the notions x are elements of F_q. Consider $a, b \in F_q$, the unitary operators $X(a)$ and $Z(b)$ in $\mathbf{C}^q$ are defined by $X(a) \mid x\rangle = \mid x+a\rangle$ and $Z(b) \mid x\rangle = \omega^{tr(bx)} \mid x\rangle$ respectively, where $\omega = exp(2\pi i/p)$ is a p-th root of unity and tr is the trace map from F_q to F_p. Consider $a = (a_1, a_2, \cdots, a_n) \in F_q^n$ and $b = (b_1, b_2, \cdots, b_n) \in F_q^n$. Let $X(a) = X(a_1) \otimes X(a_2) \otimes \cdots \otimes X(a_n)$ and $Z(a) = Z(b_1) \otimes Z(b_2) \otimes \cdots \otimes Z(b_n)$ be the tensor products of n error operators. The set $E_n = \{X(a)Z(b) \mid a, b \in F_q^n\}$ is an error basis on the complex vector space $\mathbf{C}^{q^n}$ and the set $G_n = \{\omega^c X(a)Z(b) \mid a, b \in F_q^n, c \in F_p\}$ is the error group associated with E_n. For a quantum error $e = \omega^c X(a)Z(b) \in G_n$ the quantum weight $\omega_Q(e)$, the X-weight $\omega_X(e)$ and the Z-weight $\omega_Z(e)$ of e, are defined respectively by $\omega_Q(e) = \sharp\{i : 1 \leq i \leq n, (a_i, b_i) \neq (0, 0)\}$, $\omega_X(e) = \sharp\{i : 1 \leq i \leq n, a_i \neq 0\}$, $\omega_Z(e) = \sharp\{i : 1 \leq i \leq n, b_i \neq 0\}$.

Definition 1. (See [28].) A q-ary asymmetric quantum code Q, denoted by $[[n, k, d_z/d_x]]$, is a q^k-dimensional subspace of the Hilbert space $\mathbf{C}^{q^n}$ and can control all qubit-flip errors up to $\lfloor (d_x - 1)/2 \rfloor$ and all phase-shift errors up to $\lfloor (d_z - 1)/2 \rfloor$.

The following basic result in [38,43] will be applied to construct asymmetric quantum codes. This result holds for Euclidean and Hermitian case.

Theorem 1 *(CSS Construction). (See [38,43].) Let $\mathcal{C}_i$ be a classical code with parameters $[n, k_i, d_i]$ for $i = 1, 2$, with $\mathcal{C}_1^\perp \subseteq \mathcal{C}_2$. Then there exists an asymmetric quantum code Q with parameters $[[n, k_1 + k_2 - n, d_z/d_x]]$, where $d_x = wt(\mathcal{C}_1 \backslash \mathcal{C}_2^\perp)$ and $d_z = wt(\mathcal{C}_2 \backslash \mathcal{C}_1^\perp)$.*

Proposition 2 *(Singleton bound). (See [39].) If an $[n, k, d]$ linear code $\mathcal{C}$ exists, then*

$$k \leq n - d + 1.$$

Proposition 3. *(See [43].) If $\mathcal{C}$ is an asymmetric quantum code with parameters $[[n, k, d_z/d_x]]$, then $\mathcal{C}$ satisfies the quantum Singleton bound $k \leq n - d_z - d_x + 2$. If $\mathcal{C}$ satisfies the equality $k = n - d_z - d_x + 2$, then it is called an optimal code.*

Lemma 1. *(See [11].) Assume that q is an odd prime power with $3|(q+1)$. Let $n = (q^2-1)/3$ and $r = 3$. Suppose that $\mathcal{C}$ is a λ-constacyclic code with defining set $Z = \cup_{j=1}^{\delta} C_{1+3(\frac{q-2}{3}+j)}$, where $C_{1+3(\frac{q-2}{3}+j)} = \{1 + 3(\frac{q-2}{3} + j)\}$ and $1 \leq \delta \leq \frac{2(q-2)}{3}$, then $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$.*

Theorem 2. *Assume that q is an odd prime power with $3|(q+1)$. Let $n = (q^2-1)/3$ and $r = 3$. Then there exist asymmetric quantum codes with parameters $[[(q^2-1)/3, (q^2-1)/3 - (\delta_1 + \delta_2), \delta_1 + 1/\delta_2 + 1]]_{q^2}$, where δ_1 and δ_2 are positive integers and $1 \leq \delta_2 \leq \delta_1 \leq \frac{2(q-2)}{3}$.*

Proof. Let $\mathcal{C}_1$ be a λ-constacyclic code with defining set $Z_1 = \cup_{j=1}^{\delta_1} C_{1+3(\frac{q-2}{3}+j)}$, where $1 \leq \delta_1 \leq \frac{2(q-2)}{3}$, then $\mathcal{C}_1$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta_1, d_1 \geq \delta_1 + 1]_{q^2}$ from Proposition 1 and Lemma 1. From Proposition 2, $\mathcal{C}_1$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta_1, \delta_1 + 1]_{q^2}$. Let $\mathcal{C}_2$ be a λ-constacyclic code with defining set $Z_2 = \cup_{j=1}^{\delta_2} C_{1+3(\frac{q-2}{3}+j)}$, where $1 \leq \delta_2 \leq \delta_1 \leq \frac{2(q-2)}{3}$, then $\mathcal{C}_2$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta_2, d_2 \geq \delta_2 + 1]_{q^2}$ from Proposition 1 and Lemma 1. From Proposition 2, $\mathcal{C}_2$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta_2, \delta_2 + 1]_{q^2}$. From the definition of λ-constacyclic codes and Lemma 1, we know that $\mathcal{C}_2^{\perp_h} \subseteq \mathcal{C}_1$. Therefore, from Theorem 1, we have asymmetric quantum codes with parameters $[[(q^2-1)/3, (q^2-1)/3 - (\delta_1 + \delta_2), \delta_1 + 1/\delta_2 + 1]]_{q^2}$, where δ_1 and δ_2 are positive integers, and $1 \leq \delta_2 \leq \delta_1 \leq \frac{2(q-2)}{3}$. From Proposition 3, we know that these codes constructed here are optimal. □

Lemma 2. *(See [11].) Assume that q is an odd prime power with the form $10m + 3$ or $10m+7$, $n = (q^2+1)/10$ and $r = q+1$. Let $\mathcal{C}$ be a λ-constacyclic code with defining set $Z = \cup_{j=0}^{\delta} C_{s,(q+1)(\frac{n-1}{2}-j)}$, where $C_{s,(q+1)(\frac{n-1}{2}-j)} = \{s - (q+1)(\frac{n-1}{2} - j), s + (q+1)(\frac{n-1}{2} - j)\}$, $0 \leq \delta \leq 2m - 1$ and m is a positive integer, then $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$.*

Theorem 3. *Assume that q is an odd prime power with the form $10m+3$ or $10m+7$, $n = (q^2+1)/10$ and $r = q+1$. Then there exist asymmetric quantum codes with parameters $[[(q^2+1)/10, (q^2+1)/10 - 2(\delta_1 + \delta_2), 2\delta_1 + 1/2\delta_2 + 1]]_{q^2}$, where δ_1 and δ_2 are positive integers, $1 \leq \delta_2 \leq \delta_1 \leq 2m$ and m is a positive integer.*

Proof. Here, we use the same method as in Theorem 2 and present its proof here for completeness. Let $\mathcal{C}_1$ be a λ-constacyclic code with defining set $Z_1 = \cup_{j=0}^{\delta_1-1} C_{s,(q+1)(\frac{n-1}{2}-j)}$, where $1 \leq \delta_1 \leq 2m$, then $\mathcal{C}_1$ is a λ-constacyclic code with the parameters $[(q^2+1)/10, (q^2+1)/10 - 2\delta_1, d_1 \geq 2\delta_1 + 1]_{q^2}$ from Proposition 1 and Lemma 2. From Proposition 2, $\mathcal{C}_1$ is a λ-constacyclic code with the parameters $[(q^2+1)/10, (q^2+1)/10 - 2\delta_1, 2\delta_1 + 1]_{q^2}$.

We can also define the defining set of λ-constacyclic code $\mathcal{C}_2$ that is given by $Z_2 = \cup_{j=0}^{\delta_2-1} C_{s,(q+1)(\frac{n-1}{2}-j)}$, where $1 \le \delta_2 \le \delta_1 \le 2m$, then we can see that $\mathcal{C}_2$ is a λ-constacyclic code with the parameters $[(q^2+1)/10, (q^2+1)/10 - 2\delta_2, d_2 \ge 2\delta_2 + 1]_{q^2}$ from Proposition 1 and Lemma 2. From Proposition 2, $\mathcal{C}_2$ is a λ-constacyclic code with the parameters $[(q^2+1)/10, (q^2+1)/10-2\delta_2, 2\delta_2+1]_{q^2}$. From the definition of constacyclic codes and Lemma 2, we know that $\mathcal{C}_2^{\perp_h} \subseteq \mathcal{C}_1$. Hence, from Theorem 1, we have asymmetric quantum codes with parameters $[[(q^2+1)/10, (q^2+1)/10-2(\delta_1+\delta_2), 2\delta_1+1/2\delta_2+1]]_{q^2}$, where δ_1 and δ_2 are positive integers, and $1 \le \delta_2 \le \delta_1 \le 2m$. From Proposition 3, we know that these codes constructed here are optimal. $\square$

Lemma 3. *(See [11].) Assume that q is an odd prime power with $5|(q+1)$, $n = (q^2-1)/5$ and $r = 5$. Let $\mathcal{C}$ be a λ-constacyclic code with defining set $Z = \cup_{j=1}^{\delta} C_{1+5(\frac{2(q+1)}{5}-1+j)}$, where $C_{1+5(\frac{2(q+1)}{5}-1+j)} = \{1+5(\frac{2(q+1)}{5}-1+j)\}$ and $1 \le \delta \le \frac{3(q+1)}{5} - 2$, then $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$.*

Theorem 4. *Assume that q is an odd prime power with $5|(q+1)$,$n = (q^2-1)/5$ and $r = 5$. Then there exist asymmetric quantum codes with parameters $[[(q^2-1)/5, (q^2-1)/5-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where δ_1 and δ_2 are positive integers and $1 \le \delta_2 \le \delta_1 \le \frac{3(q+1)}{5} - 2$.*

Proof. From Lemma 3, we can assume that $\mathcal{C}_1$ is a λ-constacyclic code with defining set $Z_1 = \cup_{j=1}^{\delta_1} C_{1+5(\frac{2(q+1)}{5}-1+j)}$, where $1 \le \delta_1 \le \frac{3(q+1)}{5} - 2$. Let $\mathcal{C}_2$ be the λ-constacyclic code with defining set $Z_2 = \cup_{j=1}^{\delta_2} C_{1+5(\frac{2(q+1)}{5}-1+j)}$, where $1 \le \delta_2 \le \delta_1 \le \frac{3(q+1)}{5} - 2$. Then, the result follows by using a similar process to Theorem 2. $\square$

Lemma 4. *(See [11].) Assume that q is an odd prime power with $7|(q+1)$, $n = (q^2-1)/7$ and $r = 7$. Let $\mathcal{C}$ be a λ-constacyclic code with defining set $Z = \cup_{j=1}^{\delta} C_{1+7(\frac{3(q+1)}{7}-1+j)}$, where $C_{1+7(\frac{3(q+1)}{7}-1+j)} = \{1+7(\frac{3(q+1)}{7}-1+j)\}$ and $1 \le \delta \le \frac{4(q+1)}{7} - 2$, then $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$.*

Theorem 5. *Assume that q is an odd prime power with $7|(q+1)$, $n = (q^2-1)/7$ and $r = 7$. Then there exist asymmetric quantum codes with parameters $[[(q^2-1)/7, (q^2-1)/7 - (\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where δ_1 and δ_2 are positive integers and where $1 \le \delta_2 \le \delta_1 \le \frac{4(q+1)}{7} - 2$.*

Proof. From Lemma 4, we can assume that $\mathcal{C}_1$ is a λ-constacyclic code with defining set $Z_1 = \cup_{j=1}^{\delta_1} C_{1+7(\frac{3(q+1)}{7}-1+j)}$, where $1 \le \delta_1 \le \frac{4(q+1)}{7} - 2$. Let $\mathcal{C}_2$ be the λ-constacyclic code with defining set $Z_2 = \cup_{j=1}^{\delta_2} C_{1+7(\frac{3(q+1)}{7}-1+j)}$, where $1 \le \delta_2 \le \delta_1 \le \frac{4(q+1)}{7} - 2$. Then, the results follow by using a similar process to Theorem 2. $\square$

4. Constructions of optimal quantum convolutional codes

In this section, we recall some definitions and some basic results about classical convolutional codes and quantum convolutional codes from [18,19,41,42,31,34–36] firstly, then we utilize constacyclic codes from [11] to construct some families of optimal quantum convolutional codes. These codes constructed are different from the ones in the literature.

A polynomial encoder matrix $G(D) \in F_q[D]^{k\times n}$ is called basic if $G(D)$ has a polynomial right inverse. If the overall constraint length $\gamma = \sum_{i=1}^{k} \gamma_i$ has the smallest value among all basic generator matrices of convolutional code $\mathcal{C}$, then the basic generator matrix of the convolutional code $\mathcal{C}$ is said to be reduced. For this case, the overall constraint length γ is called the degree of the convolutional code $\mathcal{C}$. The weight of an element $v(D) \in F_q[D]^n$ is defined as $wt(v(D)) = \sum_{i=1}^{n} wt(v_i(D))$, where $wt(v_i(D))$ is the number of nonzero coefficients of $v_i(D)$.

Definition 2. (See [18,19].) A rate k/n convolutional code $\mathcal{C}$ with parameters $(n, k, \gamma; \mu, d_f)$ is a submodule of $F_q[D]^n$ generated by a reduced basic matrix $G(D) = (g_{ij}) \in F_q[D]^{k\times n}$, that is, $\mathcal{C} = \{u(D)G(D)|u(D) \in F_q[D]^k\}$, where n is the length, k is the dimension, $\gamma = \sum_{i=1}^{k} \gamma_i$ is the degree, where $\gamma_i = max_{1\le j\le n}\{deg\ g_{ij}\}$, $\mu = max_{1\le i\le k}\{\gamma_i\}$ is the memory and $d_f = wt(\mathcal{C}) = min\{wt(v(D))|v(D) \in \mathcal{C}, v(D) \ne 0\}$ is the free distance of the code.

For $u(D) = \sum_i u_i D^i$ and $v(D) = \sum_j v_j D^j$ in $F_q[D]^n$, the Euclidean inner product is defined as $\langle u(D)|v(D)\rangle = \sum_i u_i v_i$, where $u_i, v_i \in F_q^n$ and $v_i = (v_{1i}, v_{2i}, \cdots, v_{ni})$. The Euclidean dual of a convolutional code $\mathcal{C}$ is defined as $\mathcal{C}^\perp = \{u(D) \in F_q[D]^n|\langle u(D)|v(D)\rangle = 0$ for all $v(D) \in \mathcal{C}\}$.

Now, we recall some results about classical convolutional codes available in [18,19, 34–36].

Let $[n, k, d]_q$ be a block code with parity check matrix H, which can be partitioned into $\mu + 1$ disjoint submatrices H_i such that $H = [H_0, H_1, \cdots H_\mu]^T$, where each H_i has n columns. Therefore, we have the polynomial matrix as follows:

$$G(D) = \widetilde{H}_0 + \widetilde{H}_1 D + \widetilde{H}_2 D^2 + \cdots + \widetilde{H}_\mu D^\mu. \tag{1}$$

The matrix $G(D)$ has κ rows and it can generate a convolutional code V, where κ is the maximum number of rows among the matrices H_i. The matrices $\widetilde{H}_i$ can be derived from the matrices H_i by adding zero-rows at the bottom such that the matrix $\widetilde{H}_i$ has κ rows in total.

Now, we recall the concept of Hermitian inner product. For two n-tuples $u(D) = \sum_i u_i D^i$ and $v(D) = \sum_j v_j D^j$ in $F_{q^2}[D]^n$, the Hermitian inner product is defined as $\langle u(D)|v(D)\rangle_h = \sum_i u_i v_i^q$, where $u_i, v_i \in F_{q^2}^n$ and $v_i^q = (v_{1i}^q, v_{2i}^q, \cdots, v_{ni}^q)$. The Hermitian dual of a convolutional code $\mathcal{C}$ is defined as $\mathcal{C}^{\perp_h} = \{u(D) \in F_{q^2}[D]^n|\langle u(D)|v(D)\rangle_h = 0$ for all $v(D) \in \mathcal{C}\}$. The following result holds for Euclidean and Hermitian case, the readers can consult [35] for more detail.

Theorem 6. *(See [18,19,34–36].) Let $\mathcal{C} \subseteq F_q^n$ be an $[n, k, d]_q$ code with parity check matrix $H \in F_q^{(n-k)\times n}$. Assume that H is partitioned into submatrices $H_0, H_1, \cdots, H_\mu$ as above* *such that $\kappa = rkH_0$ and $rkH_i \leq \kappa$ for $1 \leq i \leq \mu$. Consider the matrix $G(D)$ in (1). Then we have:*

(a) The matrix $G(D)$ is a reduced basic generator matrix.

(b) If $\mathcal{C}^{\perp} \subseteq \mathcal{C}$ (resp. $\mathcal{C}^{\perp_h} \subseteq \mathcal{C}$), then the convolutional code $V = \{v(D) = u(D)G(D)|u(G) \in F_q^{n-k}[D]\}$ satisfies $V \subset V^{\perp}$ (resp. $V \subset V^{\perp_h}$).

(c) If d_f and $d_f^{\perp}$ denote the free distance of V and $V^{\perp}$, respectively, d_i denotes the minimum distance of the code $\mathcal{C}_i = \{v \in F_q^n | v\widetilde{H}_i^t = 0\}$ and $d^{\perp}$ is the minimum distance of $\mathcal{C}^{\perp}$, then one has $\min\{d_0 + d_{\mu}, d\} \leq d_f^{\perp} \leq d$ and $d_f \geq d^{\perp}$.

Here, we recall some basic results of quantum convolutional code in [16,18,19,34,35].

The stabilizer can be given by a matrix of the form

$$S(D) = \big(X(D)|Z(D)\big) \in F_q[D]^{(n-k)\times 2n}$$

which satisfies $X(D)Z(1/D)^t - Z(D)X(1/D)^t = 0$. Now, we can consider a quantum convolutional code $\mathcal{C}$ defined by the full-rank stabilizer matrix $S(D)$ given above. Then $\mathcal{C}$ is a rate k/n quantum convolutional codes with parameters with $[(n, k, \mu; \gamma, d_f)]_q$, where n is called the frame size, k is the number of logical qudits per frame. The memory of the quantum convolutional codes is $\mu = max_{1\leq i\leq n-k, 1\leq j\leq n}\{max\{degX_{ij}(D), degZ_{ij}(D)\}\}$, d_f is the free distance and γ is the degree of the code. We also can define the constraint lengths of quantum convolutional codes as $\gamma_i = max_{1\leq j\leq n}\{max\{degX_{ij}(D), degZ_{ij}(D)\}\}$. Then, the overall constraint length is defined as $\gamma = \sum_{i=1}^{n-k} \gamma_i$. For more details about quantum convolutional codes, the readers can consult [18,19,34–36].

For quantum convolutional stabilizer codes, the following theorem shows how to construct quantum convolutional stabilizer codes by using classical convolutional codes:

Theorem 7. *(See [18,19].) Let $\mathcal{C}$ be an $(n, (n-k)/2, \gamma; \mu, d_f^*)_{q^2}$ convolutional code such that $\mathcal{C} \subseteq \mathcal{C}^{\perp_h}$. Then there exists an $[(n, k, \mu; \gamma, d_f)]_q$ convolutional stabilizer code, where $d_f = wt(\mathcal{C}^{\perp_h} \setminus \mathcal{C})$.*

Proposition 4 *(Quantum Singleton bound). (See [18,19].) The free distance of an $[(n, k, \mu; \gamma, d_f)]_q$ F_{q^2}-linear pure convolutional stabilizer code is bounded by*

$$d_f \leq \frac{n-k}{2}\left(\left\lfloor \frac{2\gamma}{n+k} \right\rfloor + 1\right) + 1 + \gamma.$$

Theorem 8. *Assume that q is an odd prime power with $3|(q+1)$, $n = (q^2-1)/3$ and $r = 3$. Then there exist quantum convolutional codes with parameters $[((q^2-1)/3, (q^2-1)/3 - 2\delta' + 2, 1; 1, \delta' + 1)]_q$, where $2 \leq \delta' \leq \frac{2(q-2)}{3}$.*

Proof. Let $\mathcal{C}$ be a λ-constacyclic code with defining set $Z = C_{q+2} \cup C_{q+5} \cup \cdots \cup C_{q-1+3\delta'}$, where $2 \le \delta' \le \frac{2(q-2)}{3}$. Since $1 = ord_{3n}(q^2)$, from Lemma 4 in [25] (the readers also can see Theorem 4.2 in [36]), the parity check matrix H of $\mathcal{C}$ can be denoted as

$$H_{\delta'+1,q-1+3\delta'} = \begin{bmatrix} 1 & \eta^{q+2} & \eta^{2(q+2)} & \dots & \eta^{(n-1)(q+2)} \\ 1 & \eta^{q+5} & \eta^{2(q+5)} & \dots & \eta^{(n-1)(q+5)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \eta^{q-4+3\delta'} & \eta^{2(q-4+3\delta')} & \dots & \eta^{(n-1)(q-4+3\delta')} \\ 1 & \eta^{q-1+3\delta'} & \eta^{2(q-1+3\delta')} & \dots & \eta^{(n-1)(q-1+3\delta')} \end{bmatrix}.$$

We can obtain that $\mathcal{C}$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta', \delta'+1]_{q^2}$, where $2 \le \delta' \le \frac{2(q-2)}{3}$.

Similarly, consider $\mathcal{C}_0$ a λ-constacyclic code over F_{q^2} with defining set $Z_0 = C_{q+2} \cup C_{q+5} \cup \cdots \cup C_{q-4+3\delta'}$. From Lemma 4 in [25], the parity check matrix H_0 of $\mathcal{C}_0$ can be denoted as

$$H_{\delta',q-4+3\delta'} = \begin{bmatrix} 1 & \eta^{q+2} & \eta^{2(q+2)} & \dots & \eta^{(n-1)(q+2)} \\ 1 & \eta^{q+5} & \eta^{2(q+5)} & \dots & \eta^{(n-1)(q+5)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \eta^{q-4+3\delta'} & \eta^{2(q-4+3\delta')} & \dots & \eta^{(n-1)(q-4+3\delta')} \end{bmatrix}.$$

We can see that $\mathcal{C}_0$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - \delta' + 1, \delta']_{q^2}$.

Now, we can assume that $\mathcal{C}_1$ is a λ-constacyclic code over F_{q^2} with defining set $Z_1 = C_{q-1+3\delta'}$. From Lemma 4 in [25], the parity check matrix H_1 of $\mathcal{C}_1$ can be denoted as

$$H_{2,q-1+3\delta'} = \begin{bmatrix} 1 & \eta^{q-1+3\delta'} & \eta^{2(q-1+3\delta')} & \dots & \eta^{(n-1)(q-1+3\delta')} \end{bmatrix}.$$

We know that $\mathcal{C}_1$ is a λ-constacyclic code with parameters $[(q^2-1)/3, (q^2-1)/3 - 1, d \ge 2]_{q^2}$.

From the above discussion, we can see that $rkH_0 \ge rkH_1$. Therefore, the convolutional code V generated by the matrix $G(D) = \widetilde{H}_0 + \widetilde{H}_1 D$ has parameters $((q^2-1)/3, \delta'-1, 1; 1, d_f^*)_{q^2}$, where $\widetilde{H}_0 = H_0$, $\widetilde{H}_1$ can be obtained from H_1 by adding zero-rows at the bottom such that $\widetilde{H}_1$ has the number rows of H_0. Since $wt(V^{\perp}) = wt(V^{\perp_h})$, then we can see that $d_f^{\perp_h} = \delta'+1$ from Theorem 6. From Theorem 6 and Lemma 1, one has $V \subset V^{\perp_h}$. Then, there exist convolutional codes with parameters $[((q^2-1)/3, (q^2-1)/3 - 2\delta' + 2, 1; 1, \delta'+1)]_q$ from Theorem 7, where $2 \le \delta' \le \frac{2(q-2)}{3}$. From Proposition 4, we can see that the codes constructed here are optimal. $\square$

Theorem 9. *Assume that q is an odd prime power with the form $10m+3$ or $10m+7$, $n = (q^2+1)/10$ and $r = q+1$. Then there exist quantum convolutional codes with* *parameters* $[((q^2+1)/10, (q^2+1)/10-4\delta'+4, 1; 2, 2\delta'+1)]_q$, *where* $2 \le \delta' \le 2m$ *and* m *is a positive integer.*

Proof. Since the proof presented here uses the same method as Theorem 8, we just give a sketch of the proof. Let $\mathcal{C}$ be a λ-constacyclic code with defining set $Z = C_{s,(q+1)(\frac{n-1}{2})} \cup C_{s,(q+1)(\frac{n-1}{2}-1)} \cup \cdots \cup C_{s,(q+1)(\frac{n-1}{2}-\delta'+1)}$, where $2 \le \delta' \le 2m$. Since $2 = ord_{(q+1)n}(q^2)$, we can see that $\mathcal{C}$ is a λ-constacyclic code with parameters $[(q^2+1)/10, (q^2+1)/10 - 2\delta', 2\delta'+1]_{q^2}$ and its parity check matrix can be denoted as H, where $2 \le \delta' \le 2m$.

Similarly, consider $\mathcal{C}_0$ a λ-constacyclic code over F_{q^2} with defining set $Z_0 = C_{s,(q+1)(\frac{n-1}{2})} \cup C_{s,(q+1)(\frac{n-1}{2}-1)} \cup \cdots \cup C_{s,(q+1)(\frac{n-1}{2}-\delta'+2)}$. We can see that $\mathcal{C}_0$ is a λ-constacyclic code with parameters $[(q^2+1)/10, (q^2+1)/10-2\delta'+2, 2\delta'-1]_{q^2}$ and its parity check matrix can be denoted as H_0.

Now, we can assume that $\mathcal{C}_1$ is a λ-constacyclic code over F_{q^2} with defining set $Z_1 = C_{s,(q+1)(\frac{n-1}{2}-\delta'+1)}$. We know that $\mathcal{C}_1$ is a λ-constacyclic code with parameters $[(q^2+1)/10, (q^2+1)/10-2, d \ge 2]_{q^2}$ and its parity check matrix can be denoted as H_1.

Then, we can see that $rkH_0 \ge rkH_1$ and the convolutional codes V generated by the matrix $G(D) = \widetilde{H}_0 + \widetilde{H}_1 D$ with parameters $((q^2+1)/10, 2\delta'-2, 2; 1, d_f^*))_{q^2}$. Proceeding similarly as in Theorem 8, there exist quantum convolutional codes with parameters $[((q^2+1)/10, (q^2+1)/10-4\delta'+4, 1; 2, 2\delta'+1)]_q$ from Theorem 7. From Proposition 4, we can see that these quantum convolutional codes constructed here are optimal. □

Theorem 10. *Let* q *be an odd prime power with* $5|(q+1)$. *Let* $n = (q^2-1)/5$ *and* $r = 5$. *Then there exist quantum convolutional codes with parameters* $[((q^2-1)/5, (q^2-1)/5 - 2\delta'+2, 1; 1, \delta'+1)]_q$, *where* $2 \le \delta' \le \frac{3(q+1)}{5} - 2$.

Proof. Consider the defining sets of $\mathcal{C}$, $\mathcal{C}_0$ and $\mathcal{C}_1$ of length $\lambda(q+1)$ given by

$$Z = C_{1+5(\frac{2(q+1)}{5})} \cup C_{1+5(\frac{2(q+1)}{5}+1)} \cup \cdots \cup C_{1+5(\frac{2(q+1)}{5}-1+\delta')},$$
$$Z_0 = C_{1+5(\frac{2(q+1)}{5})} \cup C_{1+5(\frac{2(q+1)}{5}+1)} \cup \cdots \cup C_{1+5(\frac{2(q+1)}{5}-2+\delta')},$$

and

$$Z_1 = C_{1+5(\frac{2(q+1)}{5}-1+\delta')},$$

where $2 \le \delta' \le \frac{3(q+1)}{5} - 2$. Since $1 = ord_{5n}(q^2)$, then we can use the same method as in Theorem 8 to proceed and the result follows. □

Theorem 11. *Let* q *be an odd prime power with* $7|(q+1)$. *Let* $n = (q^2-1)/7$ *and* $r = 7$. *Then there exist quantum convolutional codes with parameters* $[((q^2-1)/7, (q^2-1)/7 - 2\delta'+2, 1; 1, \delta'+1]]_q$, *where* $2 \le \delta' \le \frac{4(q+1)}{7} - 2$.

Proof. Consider the defining sets of $\mathcal{C}$, $\mathcal{C}_0$ and $\mathcal{C}_1$ of length $\lambda(q+1)$ given by

Table 1
Some asymmetric quantum codes from Theorems 2, 4 and 5.

$[[(q^2-1)/3, (q^2-1)/3-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $3\mid(q+1)$, δ_1 and δ_2 are positive integers and $1 \le \delta_2 \le \delta_1 \le \frac{2(q-2)}{3}$
$[[40, 33, 7/2]]_{121}$ $[[40, 32, 7/3]]_{121}$ $[[40, 31, 7/4]]_{121}$ $[[40, 34, 6/2]]_{121}$ $[[40, 33, 6/3]]_{121}$ $[[40, 32, 6/4]]_{121}$ $[[40, 35, 5/2]]_{121}$ $[[40, 34, 5/3]]_{121}$ $[[40, 33, 5/4]]_{121}$ $[[40, 36, 4/2]]_{121}$ $[[40, 35, 4/3]]_{121}$ $[[40, 34, 4/4]]_{121}$
$[[(q^2-1)/5, (q^2-1)/5-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $5\mid(q+1)$, δ_1 and δ_2 are positive integers and $1 \le \delta_2 \le \delta_1 \le \frac{3(q+1)}{5} - 2$
$[[16, 11, 5/2]]_{81}$ $[[16, 10, 5/3]]_{81}$ $[[16, 9, 5/4]]_{81}$ $[[16, 8, 5/5]]_{81}$ $[[16, 12, 4/2]]_{81}$ $[[16, 11, 4/3]]_{81}$ $[[16, 10, 4/4]]_{81}$ $[[16, 13, 3/2]]_{81}$ $[[16, 12, 3/3]]_{81}$
$[[(q^2-1)/7, (q^2-1)/7-(\delta_1+\delta_2), \delta_1+1/\delta_2+1]]_{q^2}$, where q is an odd prime power with $7\mid(q+1)$, δ_1 and δ_2 are positive integers and $1 \le \delta_2 \le \delta_1 \le \frac{4(q+1)}{7} - 2$
$[[24, 17, 7/2]]_{169}$ $[[24, 16, 7/3]]_{169}$ $[[24, 15, 7/4]]_{169}$ $[[24, 18, 6/2]]_{169}$ $[[24, 17, 6/3]]_{169}$ $[[24, 16, 6/4]]_{169}$ $[[24, 19, 5/2]]_{169}$ $[[24, 18, 5/3]]_{169}$ $[[24, 17, 5/4]]_{169}$ $[[24, 20, 4/2]]_{169}$ $[[24, 19, 4/3]]_{169}$ $[[24, 18, 4/4]]_{169}$

$$Z = C_{1+7(\frac{3(q+1)}{7})} \cup C_{1+7(\frac{3(q+1)}{7}+1)} \cup \cdots \cup C_{1+7(\frac{3(q+1)}{7}-1+\delta')},$$

$$Z_0 = C_{1+7(\frac{3(q+1)}{7})} \cup C_{1+7(\frac{3(q+1)}{7}+1)} \cup \cdots \cup C_{1+7(\frac{3(q+1)}{7}-2+\delta')},$$

and

$$Z_1 = C_{1+7(\frac{3(q+1)}{7}-1+\delta')},$$

Table 2
Some quantum convolutional codes from Theorems 8, 10 and 11.

$[((q^2-1)/3, (q^2-1)/3-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $3\|(q+1)$ and $2 \leq \delta' \leq \frac{2(q-2)}{3}$
$[(96, 94, 1; 1, 3)]_{17}$
$[(96, 92, 1; 1, 4)]_{17}$
$[(96, 90, 1; 1, 5)]_{17}$
$[(96, 88, 1; 1, 6)]_{17}$
$[(96, 86, 1; 1, 7)]_{17}$
$[(96, 84, 1; 1, 8)]_{17}$
$[(96, 82, 1; 1, 9)]_{17}$
$[(96, 80, 1; 1, 10)]_{17}$
$[(96, 78, 1; 1, 11)]_{17}$
$[((q^2-1)/5, (q^2-1)/5-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $5\|(q+1)$ and $2 \leq \delta' \leq \frac{3(q+1)}{5}-2$
$[(72, 70, 1; 1, 3)]_{19}$
$[(72, 68, 1; 1, 4)]_{19}$
$[(72, 66, 1; 1, 5)]_{19}$
$[(72, 64, 1; 1, 6)]_{19}$
$[(72, 62, 1; 1, 7)]_{19}$
$[(72, 60, 1; 1, 8)]_{19}$
$[(72, 58, 1; 1, 9)]_{19}$
$[(72, 56, 1; 1, 10)]_{19}$
$[(72, 54, 1; 1, 11)]_{19}$
$[((q^2-1)/7, (q^2-1)/7-2\delta'+2, 1; 1, \delta'+1)]_q$, where q is an odd prime power with $7\|(q+1)$ and $2 \leq \delta' \leq \frac{4(q+1)}{7}-2$
$[(104, 102, 1; 1, 3)]_{27}$
$[(104, 100, 1; 1, 4)]_{27}$
$[(104, 98, 1; 1, 5)]_{27}$
$[(104, 96, 1; 1, 6)]_{27}$
$[(104, 94, 1; 1, 7)]_{27}$
$[(104, 92, 1; 1, 8)]_{27}$
$[(104, 90, 1; 1, 9)]_{27}$
$[(104, 88, 1; 1, 10)]_{27}$
$[(104, 86, 1; 1, 11)]_{27}$
$[(104, 84, 1; 1, 12)]_{27}$
$[(104, 82, 1; 1, 13)]_{27}$
$[(104, 80, 1; 1, 14)]_{27}$
$[(104, 78, 1; 1, 15)]_{27}$

where $2 \leq \delta' \leq \frac{4(q+1)}{7} - 2$. Since $1 = ord_{7n}(q^2)$, then we can use the same method as in Theorem 8 to proceed and the result follows. □

5. Summary

In this work, we construct some optimal asymmetric quantum codes and quantum convolutional codes by using constacyclic codes in [11]. G.G. La Guardia utlized Reed–Solomon codes to construct optimal asymmetric quantum codes with parameters $[[p-1, p-2d+2, d/d-1]]_p$ in [29], where p is a prime number. We compare the optimal asymmetric quantum codes constructed in this paper with the ones in [29]. We can find that the upper bound d_z of the new asymmetric quantum codes is greater than the lower bound for d_x. These asymmetric quantum codes are able to correct quantum errors with great asymmetry. To illustrate the results of optimal asymmetric quantum codes constructed in this paper, three families of optimal asymmetric quantum codes are listed in Table 1. In [35], G.G. La Guardia utilized negacyclic codes to construct two families of optimal quantum convolutional codes. The optimal quantum convolutional codes constructed in this paper are different from those optimal quantum convolutional codes constructed in [35]. To illustrate the results of optimal quantum convolutional codes constructed in this paper, three families of optimal quantum convolutional codes are listed in Table 2. Moreover, we can see that constacyclic codes are a good source in the search for optimal quantum codes. It is also an interesting topic to utilize the constacyclic codes in this paper to construct quantum subsystem codes by using the method in [44].

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References

[1] A.C.A. deAlmeida, R. Palazzo Jr., A concatenated $[(4,1,3)]$ quantum convolutional code, in: Proc. IEEE Inform. Theory Workshop (ITW), 2004, pp. 28–33.
[2] S.A. Aly, M. Grassl, A. Klappenecker, M. Rötteler, P.K. Sarvepalli, Quantum convolutional BCH codes, in: Proc. 10th Canadian Workshop on Information Theory, 2007, pp. 180–183.
[3] S.A. Aly, A. Klappenecker, P.K. Sarvepalli, Quantum convolutional codes derived from Reed–Solomon and Reed–Muller codes, arXiv:quant-ph/0701037.
[4] A. Ashikhmin, E. Knill, Non-binary quantum stabilizer codes, IEEE Trans. Inform. Theory 47 (7) (2001) 3065–3072.
[5] N. Aydin, I. Siap, D.K. Ray-Chaudhuri, The structure of 1-generator quasi-twisted codes and new linear codes, Des. Codes Cryptogr. 24 (2001) 313–326.
[6] G.K. Bakshi, M. Raka, A class of constacyclic codes over a finite field, Finite Fields Appl. 18 (2012) 362–377.
[7] E.R. Berlekamp, Negacyclic codes for the Lee metric, in: Proc. Symp. Combinatorial Math. Appl., 1967, pp. 1–27.
[8] T. Blackford, Negacyclic duadic codes, Finite Fields Appl. 14 (2008) 930–943.
[9] A.R. Calderbank, E.M. Rains, P.W. Shor, N.J.A. Sloane, Quantum error correction via codes over GF(4), IEEE Trans. Inform. Theory 44 (4) (1998) 1369–1387.
[10] B. Chen, Y. Fan, L. Lin, H. Liu, Constacyclic codes over finite fields, Finite Fields Appl. 18 (2012) 1217–1231.
[11] B. Chen, S. Ling, G. Zhang, Application of constacyclic codes to quantum MDS codes, arXiv: 1403.2499.
[12] J.Z. Chen, J.P. Li, J. Lin, New optimal asymmetric quantum codes derived from negacyclic codes, Internat. J. Theoret. Phys. 53 (1) (2014) 72–79.

[13] M.F. Ezerman, S. Jitman, S. Ling, D.V. Pasechnik, CSS-like constructions of asymmetric quantum codes, IEEE Trans. Inform. Theory 59 (10) (2013) 6732–6754.
[14] M.F. Ezerman, S. Jitman, H.M. Kiah, S. Ling, Pure asymmetric quantum MDS codes from CSS construction: a complete characterization, Int. J. Quantum Inf. 11 (03) (2013) 1350027.
[15] M.F. Ezerman, S. Jitman, P. Solé, Xing–Ling codes, duals of their subcodes, and good asymmetric quantum codes, Des. Codes Cryptogr. (2013), http://dx.doi.org/10.1007/s10623-013-9885-5.
[16] G.D. Forney Jr., M. Grassl, S. Guha, Convolutional and tail-biting quantum error-correcting codes, IEEE Trans. Inform. Theory 53 (3) (2007) 865–880.
[17] M. Grassl, M. Rötteler, Quantum block and convolutional codes from self-orthogonal product codes, in: Proc. Int. Symp. Inform. Theory (ISIT), 2005, pp. 1018–1022.
[18] M. Grassl, M. Rötteler, Non-catastrophic encoders and encoder inverses for quantum convolutional codes, in: Proc. Int. Symp. Inform. Theory (ISIT), 2006, pp. 1109–1113.
[19] M. Grassl, M. Rötteler, Constructions of quantum convolutional codes, in: Proc. IEEE Int. Symp. on Inform. Theory, 2007, pp. 816–820.
[20] W.C. Huffman, V. Pless, Fundamentals of Error-Correcting Codes, University Press, Cambridge, 2003.
[21] X. Kai, S. Zhu, New quantum MDS codes from negacyclic codes, IEEE Trans. Inform. Theory 59 (2) (2013) 1193–1197.
[22] X. Kai, S. Zhu, Y. Tang, Quantum negacyclic codes, Phys. Rev. A 88 (1) (2013) 012326.
[23] X. Kai, S. Zhu, P. Li, Constacyclic codes and some new quantum MDS codes, IEEE Trans. Inform. Theory 60 (4) (2014) 2080–2086.
[24] A. Ketkar, A. Klappenecker, S. Kumar, P.K. Sarvepalli, Nonbinary stabilizer codes over finite fields, IEEE Trans. Inform. Theory 52 (11) (2006) 4892–4914.
[25] A. Krishna, D.V. Sarwate, Pseudocyclic maximum-distance-separable codes, IEEE Trans. Inform. Theory 36 (4) (1990) 880–884.
[26] G.G. La Guardia, Constructions of new families of nonbinary quantum codes, Phys. Rev. A 80 (4) (2009) 042331.
[27] G.G. La Guardia, New quantum MDS codes, IEEE Trans. Inform. Theory 57 (8) (2011) 5551–5554.
[28] G.G. La Guardia, New families of asymmetric quantum BCH codes, Quantum Inf. Comput. 11 (3) (2011) 239–252.
[29] G.G. La Guardia, Asymmetric quantum Reed-Solomon and generalized Reed–Solomon codes, Quantum Inf. Process. 11 (2) (2012) 591–604.
[30] G.G. La Guardia, Asymmetric quantum product codes, Int. J. Quantum Inf. 10 (01) (2012).
[31] G.G. La Guardia, On nonbinary quantum convolutional BCH codes, Quantum Inf. Comput. 12 (9–10) (2012) 820–842.
[32] G.G. La Guardia, Asymmetric quantum codes: new codes from old, Quantum Inf. Process. 12 (8) (2013) 2771–2790.
[33] G.G. La Guardia, On the construction of asymmetric quantum codes, Internat. J. Theoret. Phys. 53 (7) (2014) 2312–2322.
[34] G.G. La Guardia, On classical and quantum MDS-convolutional BCH codes, IEEE Trans. Inform. Theory 60 (1) (2014) 304–312.
[35] G.G. La Guardia, On negacyclic MDS-convolutional codes, Linear Algebra Appl. 448 (2014) 85–96.
[36] G.G. La Guardia, On optimal constacyclic codes, arXiv:1311.2505v1.
[37] R. Li, G. Xu, L. Guo, On two problems of asymmetric quantum codes, Internat. J. Modern Phys. B 28 (6) (2013) 1450017.
[38] R.G. Leng, Z. Ma, Constructions of new families of nonbinary asymmetric quantum BCH codes and subsystem BCH codes, Sci. China Phys. Mech. 55 (3) (2012) 465–469.
[39] F.J. MacWilliams, N.J.A. Sloane, The Theory of Error-Correcting Codes, North-Holland, 1977.
[40] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, 2000.
[41] H. Ollivier, J.-P. Tillich, Quantum convolutional codes: fundamentals, arXiv:quant-ph/0401134.
[42] H. Ollivier, J.-P. Tillich, Description of a quantum convolutional code, Phys. Rev. Lett. 91 (17) (2003) 1779021.
[43] J. Qian, L. Zhang, New optimal asymmetric quantum codes, Modern Phys. Lett. B 27 (02) (2013) 1350010.
[44] J. Qian, L. Zhang, New optimal subsystem codes, Discrete Math. 313 (21) (2013) 2451–2455.
[45] A. Steane, Enlargement of Calderbank–Shor–Steane quantum codes, IEEE Trans. Inform. Theory 45 (7) (1999) 2492–2495.