Fault-tolerant quantum computation with asymmetric Bacon-Shor codes

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Biased noise

- Many useful systems expect dephasing to be dominant source of noise
- i.e. superconducting flux qubits
- Can we design error-correcting codes to take advantage of dephasing bias?

Bacon-Shor codes

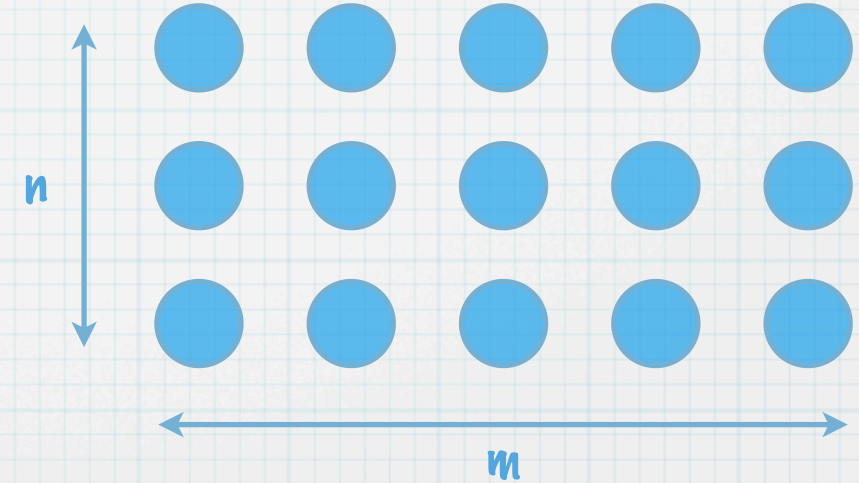
- Family of quantum error-correcting subsystem codes
- Encode a single qubit in n×m block of physical qubits
- Independently tunable levels of Z and X error correction

Logical qubits

$$\mathcal{H} = (\mathcal{C} \otimes \mathcal{D}) \oplus \mathcal{E}$$

Gauge qubits

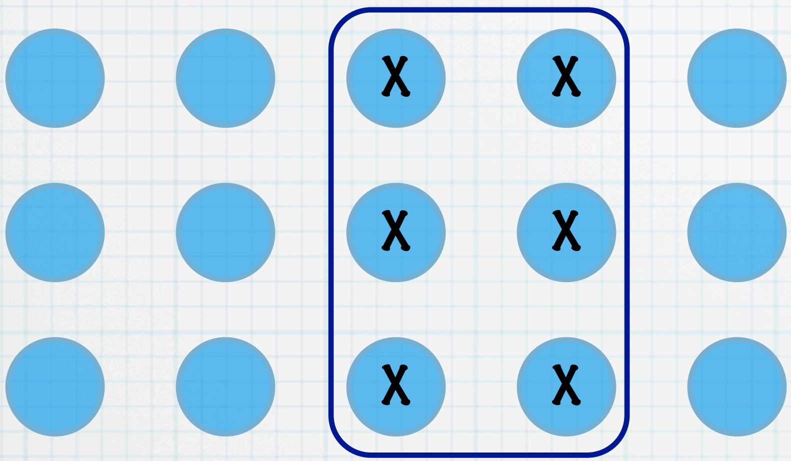
Error subspace

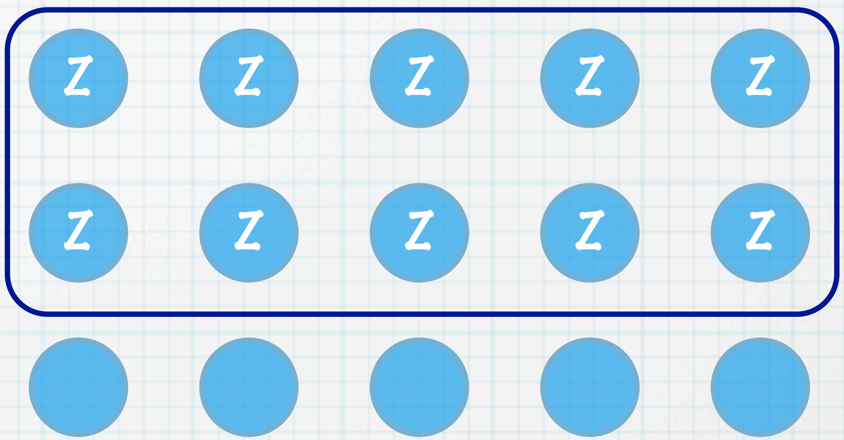


[1] Peter Shor, Phys. Rev. A **52**, 2493 (1995)
[2] Dave Bacon, Phys. Rev. A **73**, 012340 (2006)

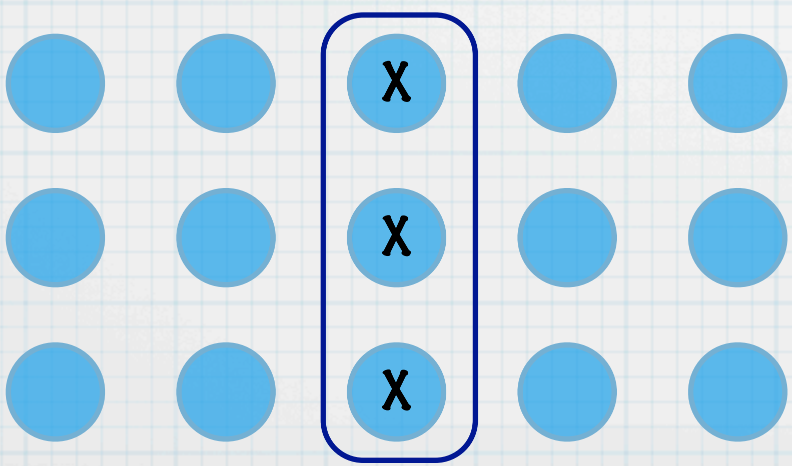
Bacon-Shor codes

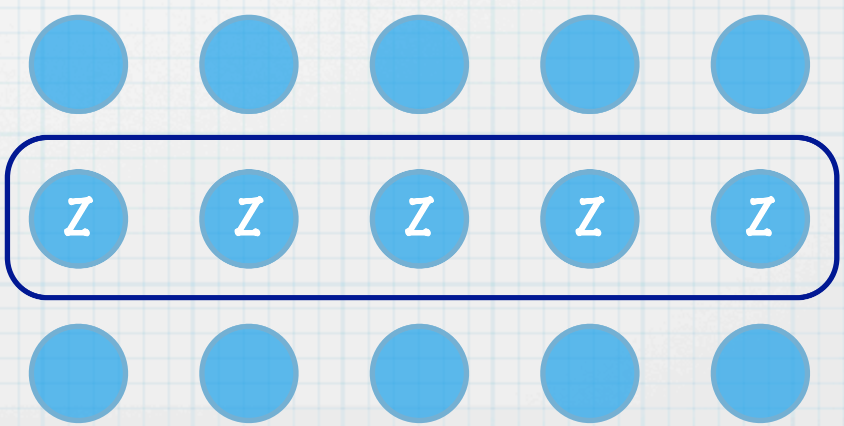
Stabilizers





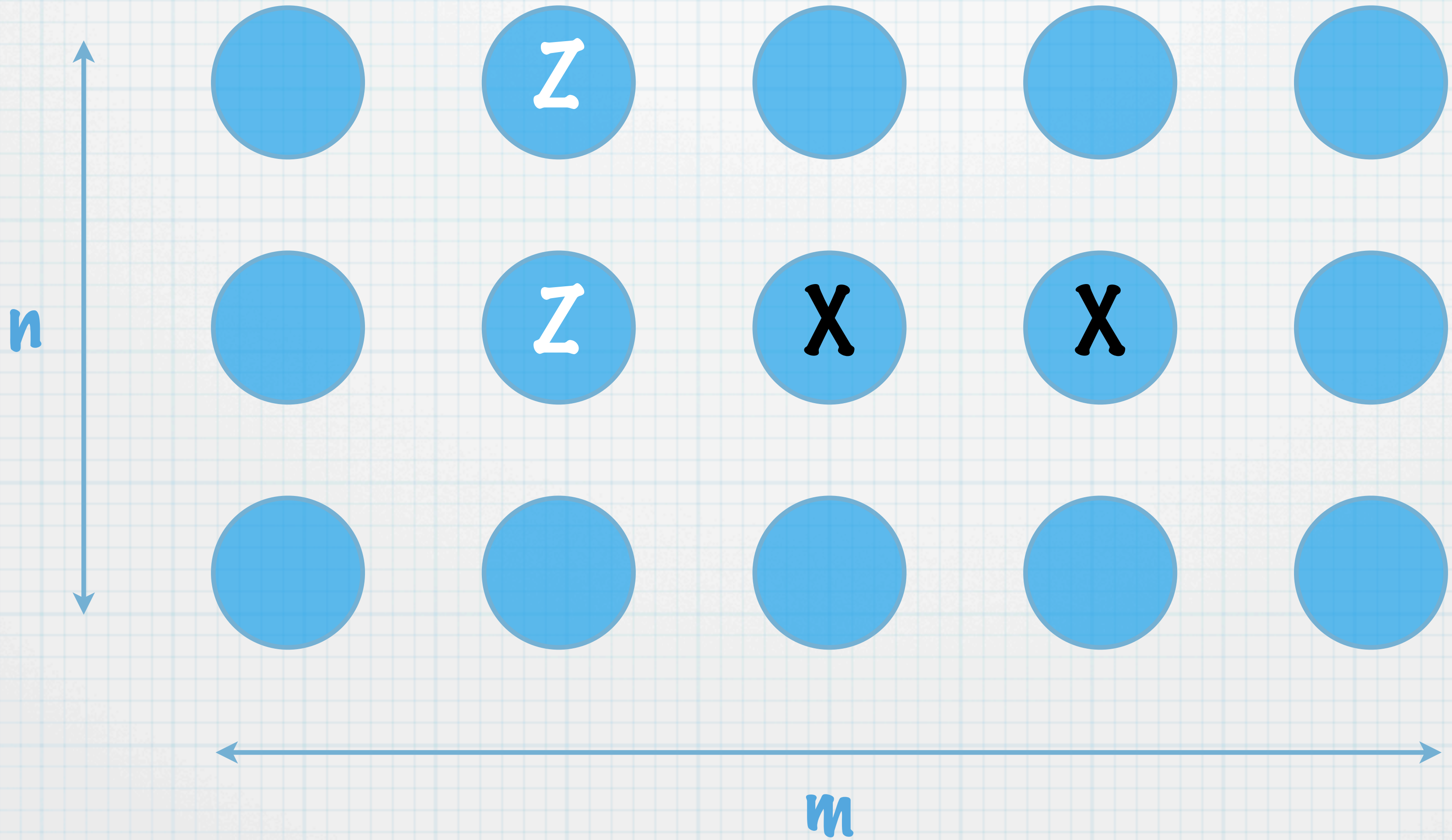
Logical operators





Bacon-Shor codes

Gauge algebra



Bacon-Shor codes

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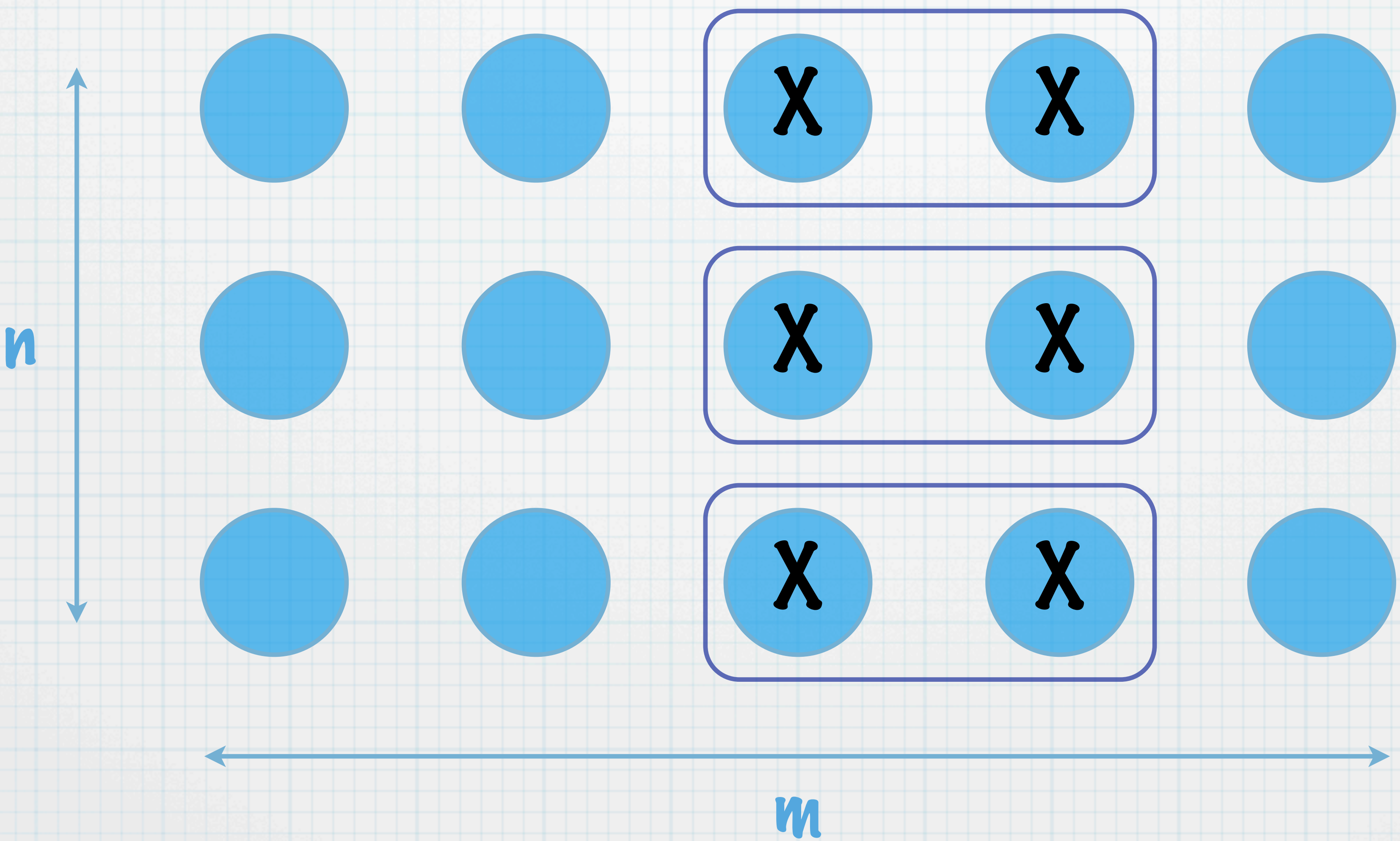

Bacon-Shor codes

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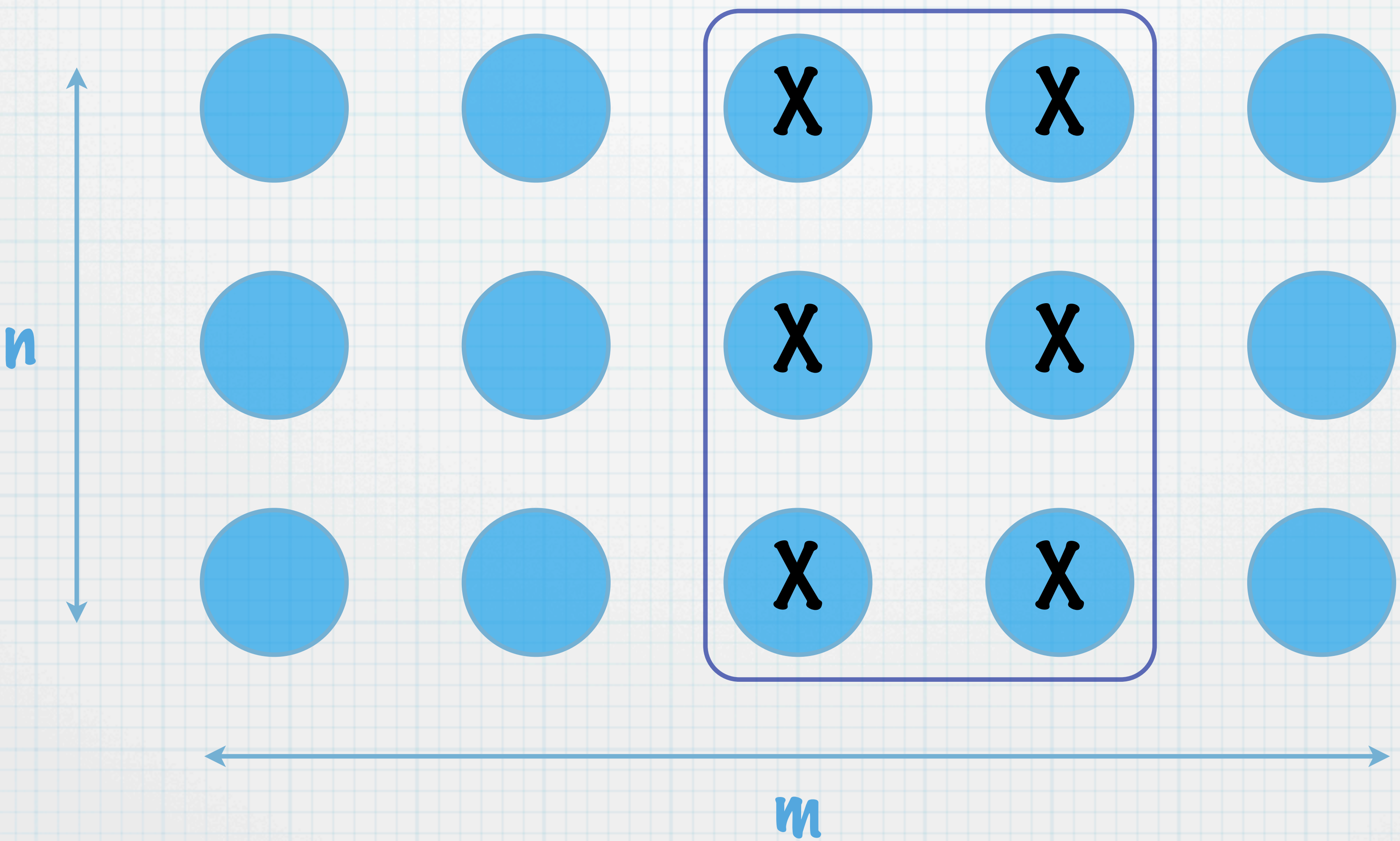

Bacon-Shor codes

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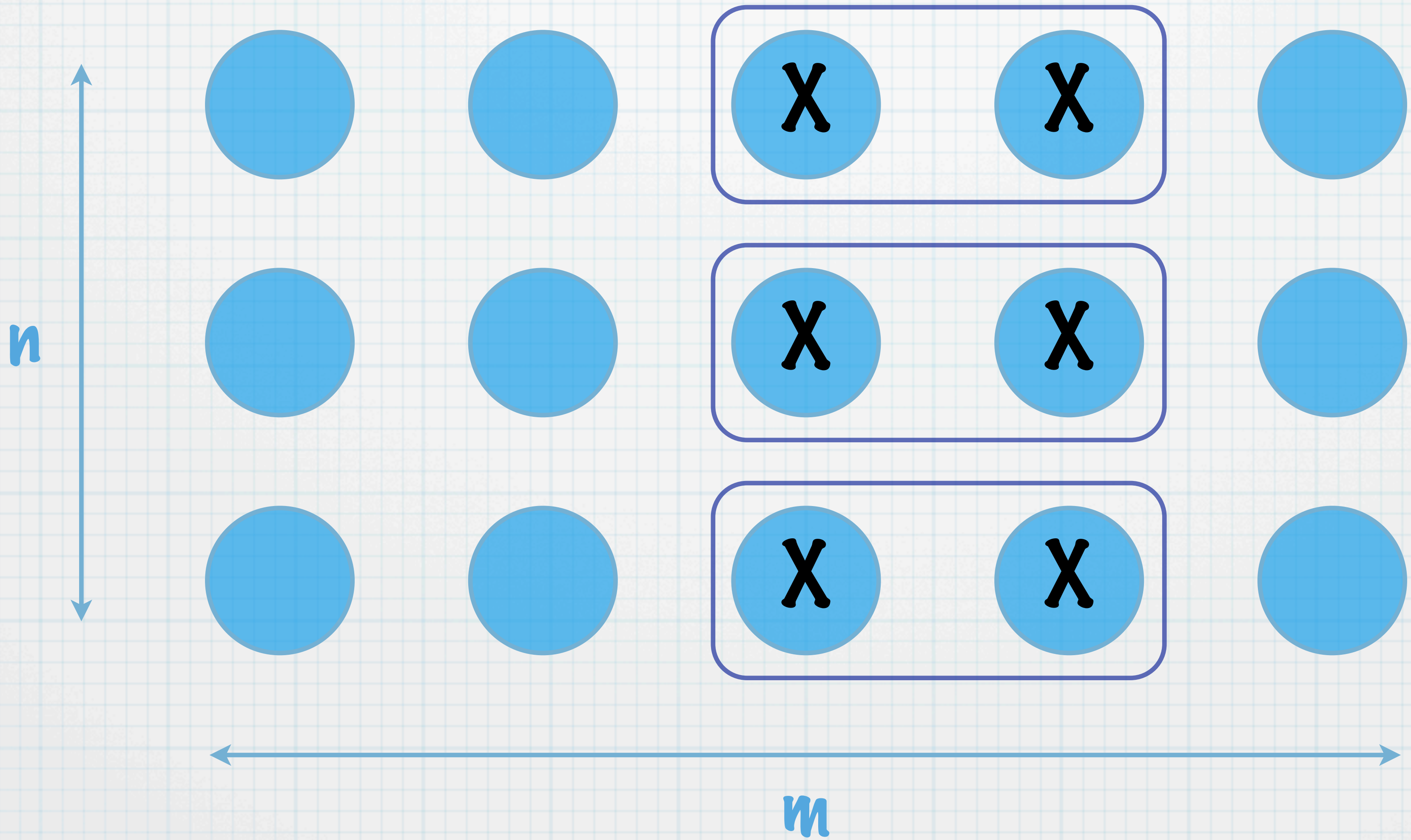
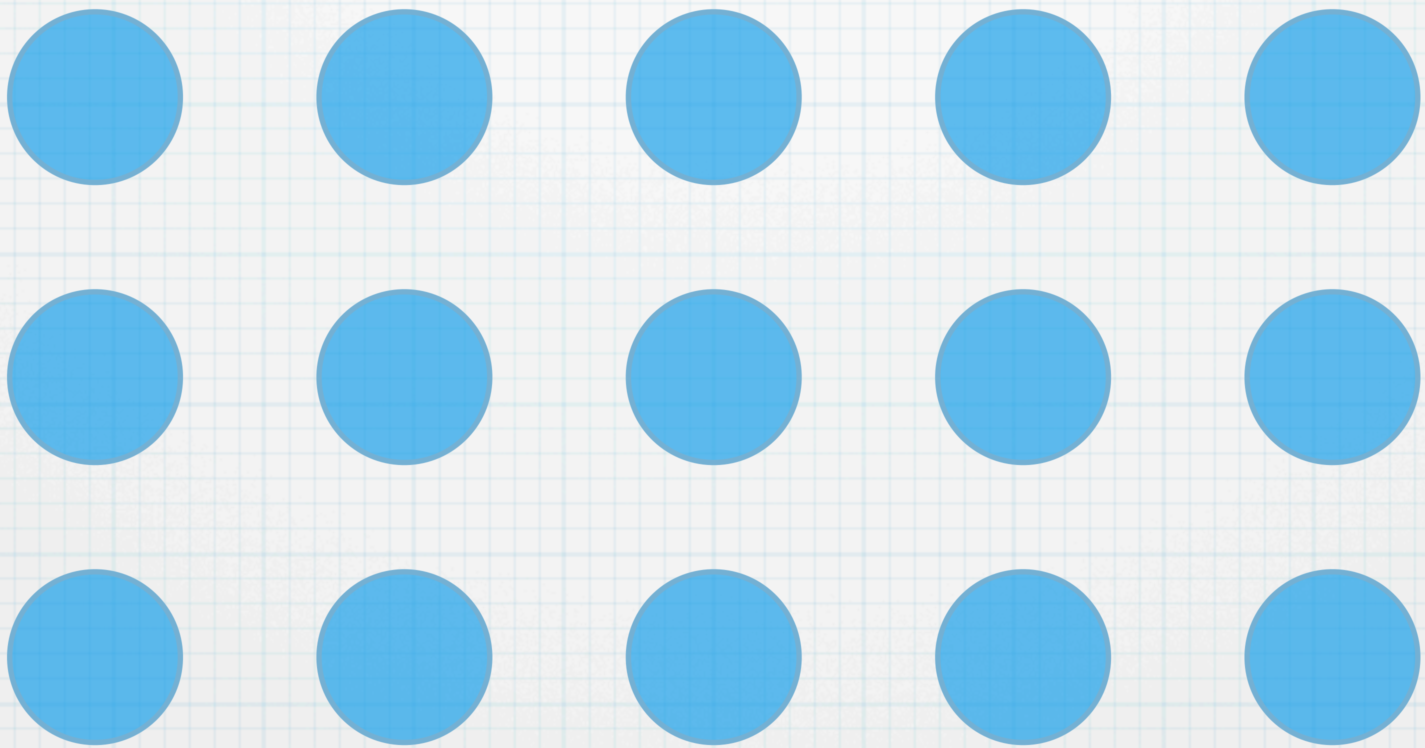


Error correction



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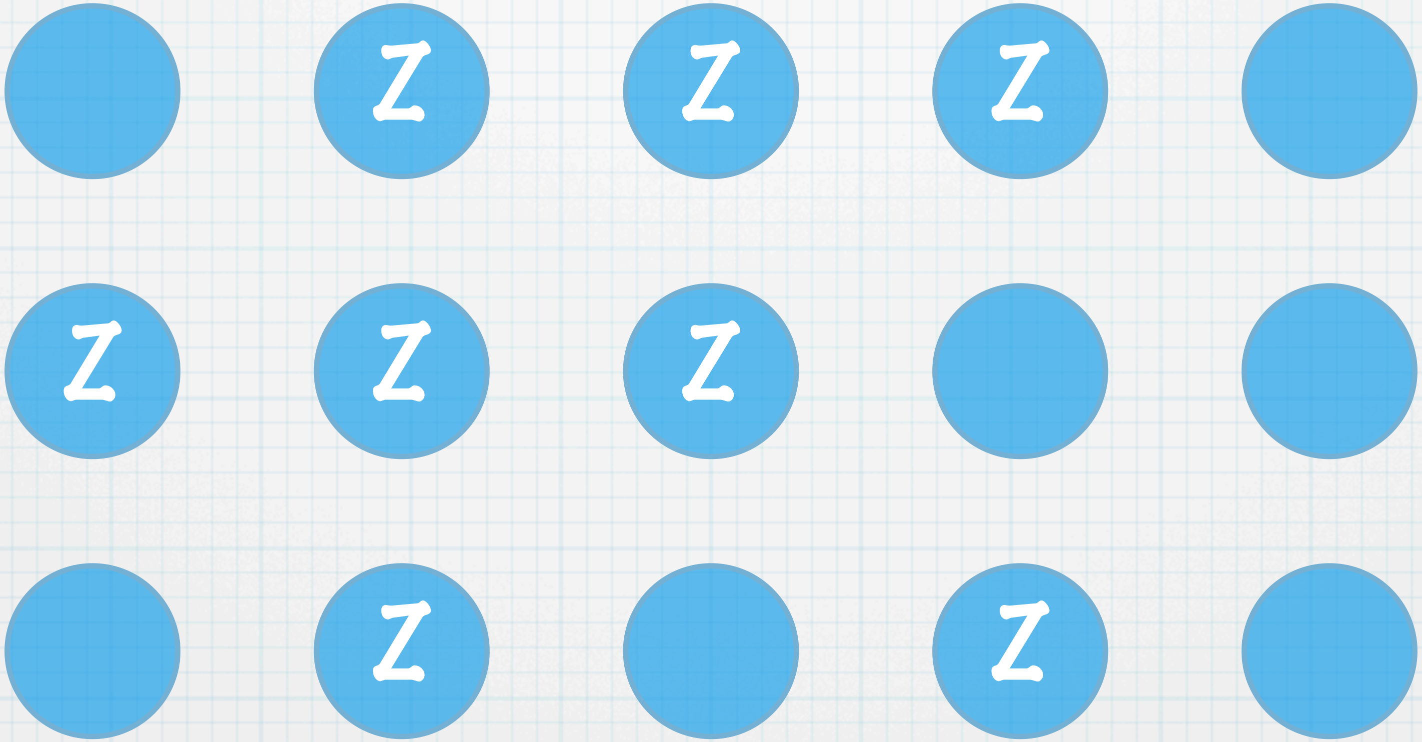

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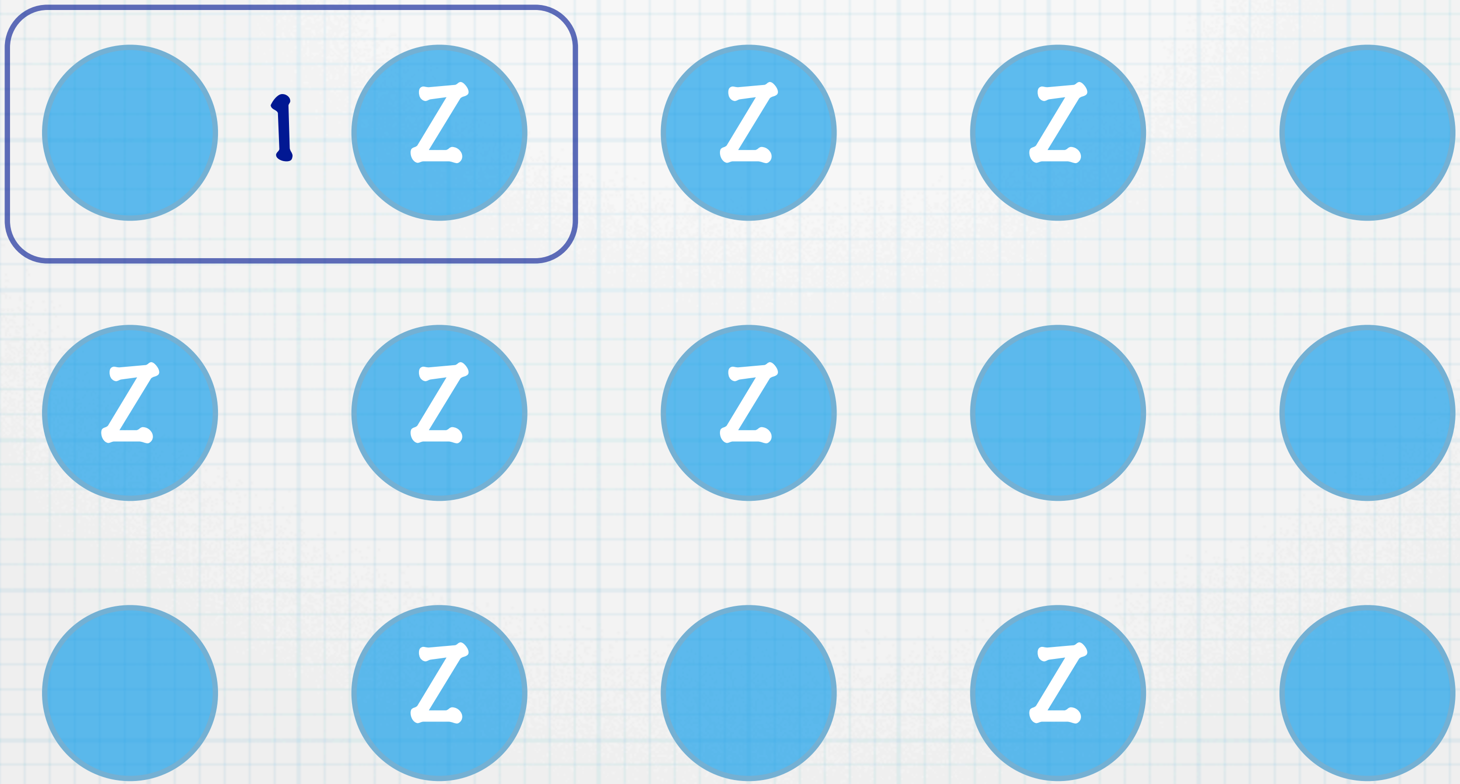

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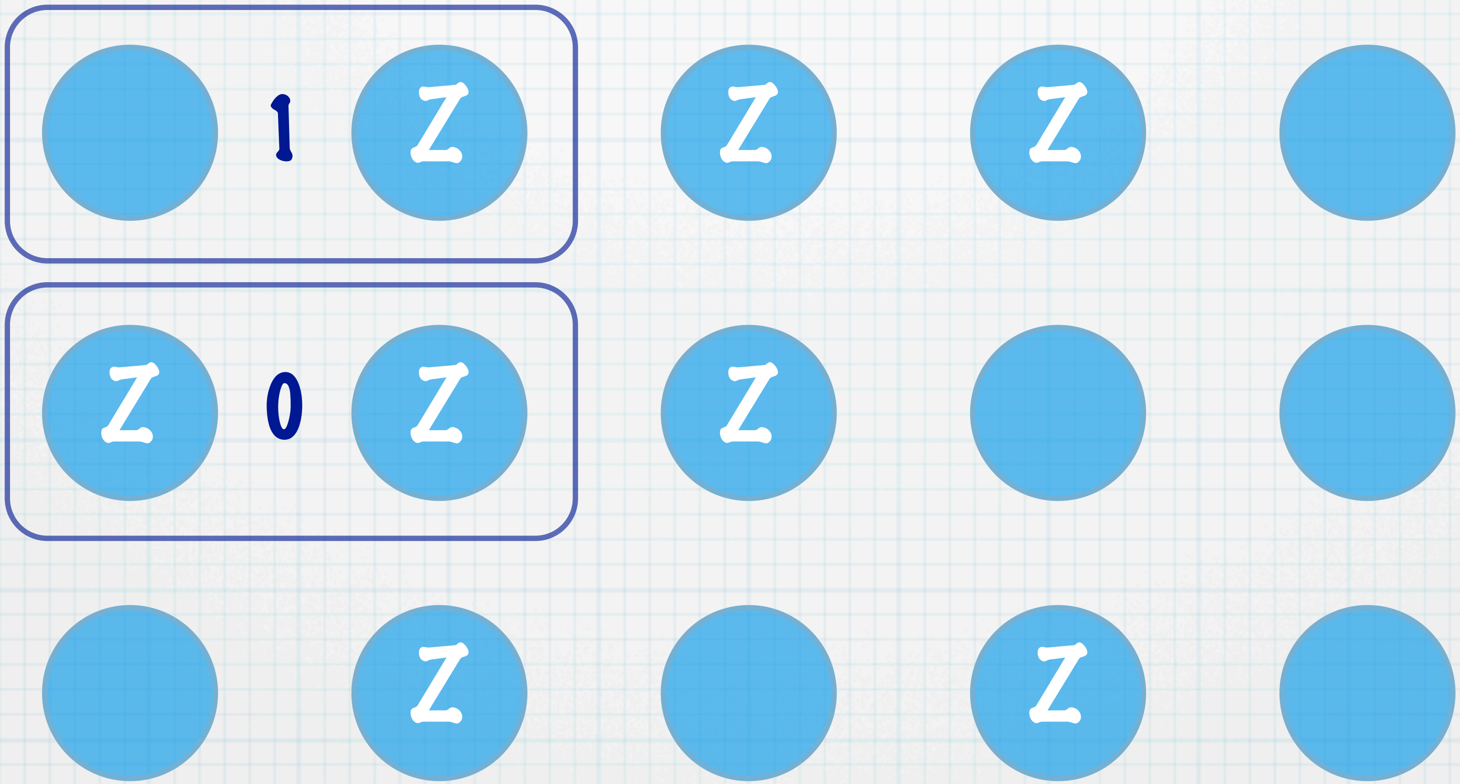

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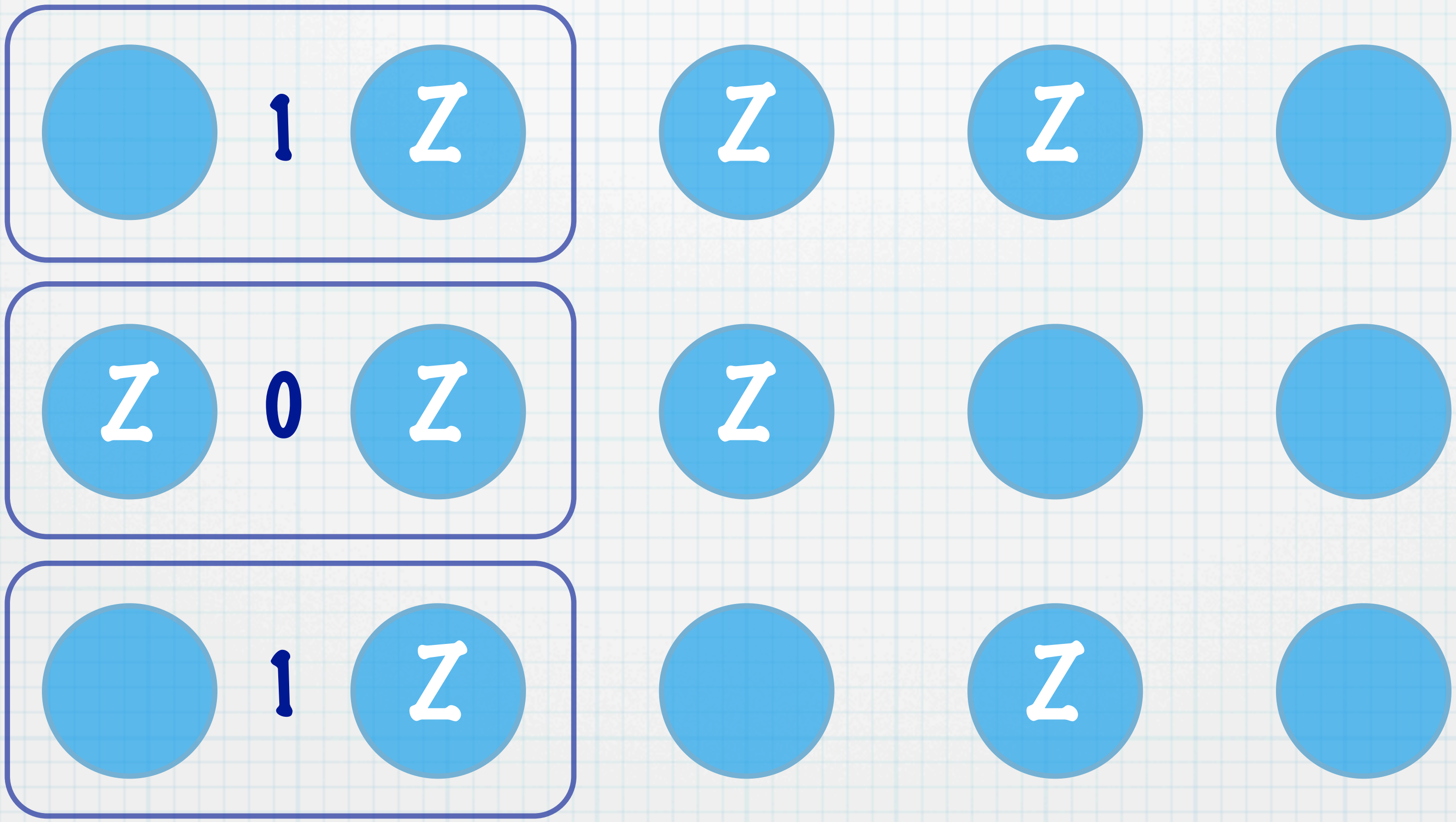

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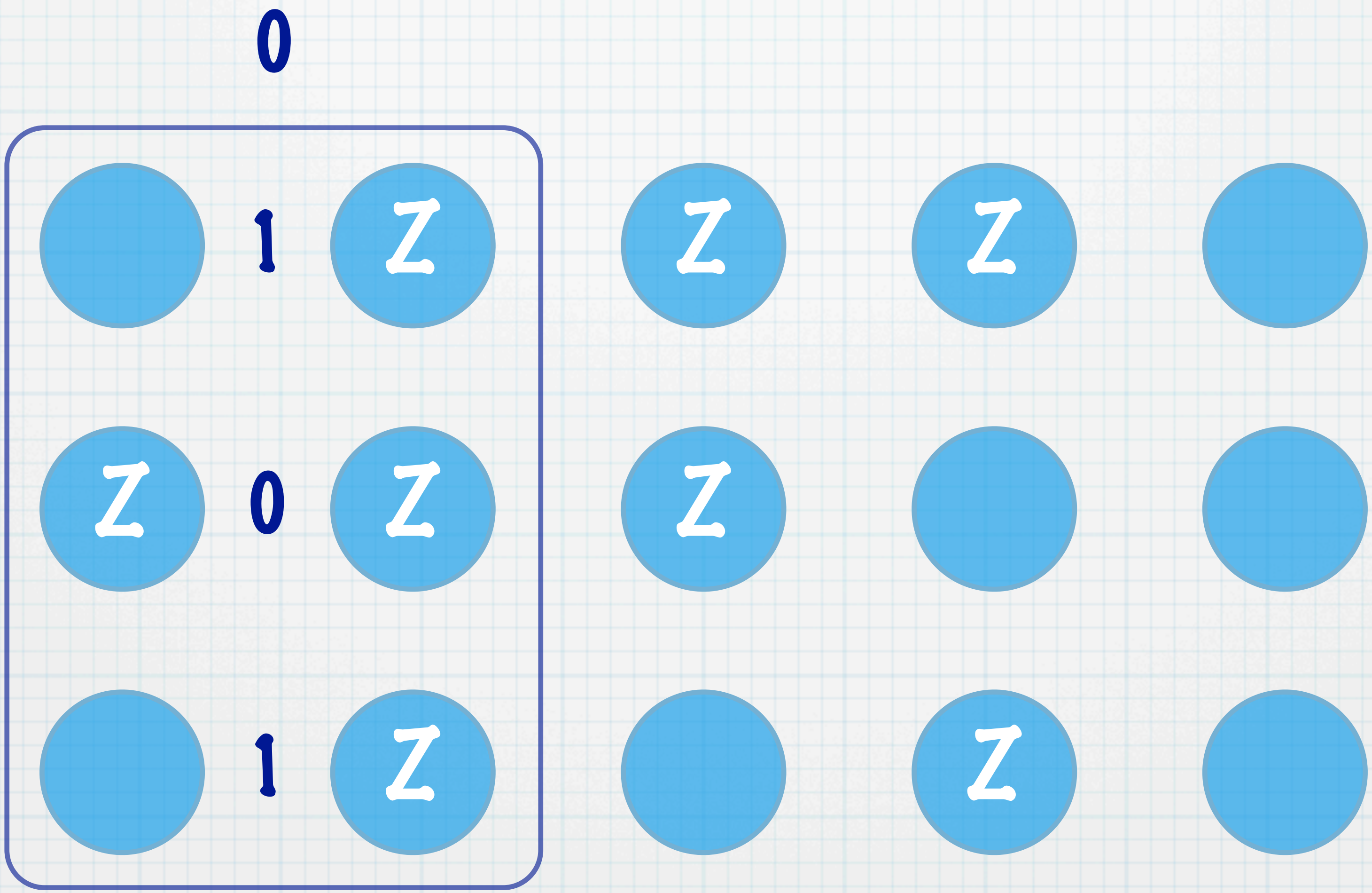

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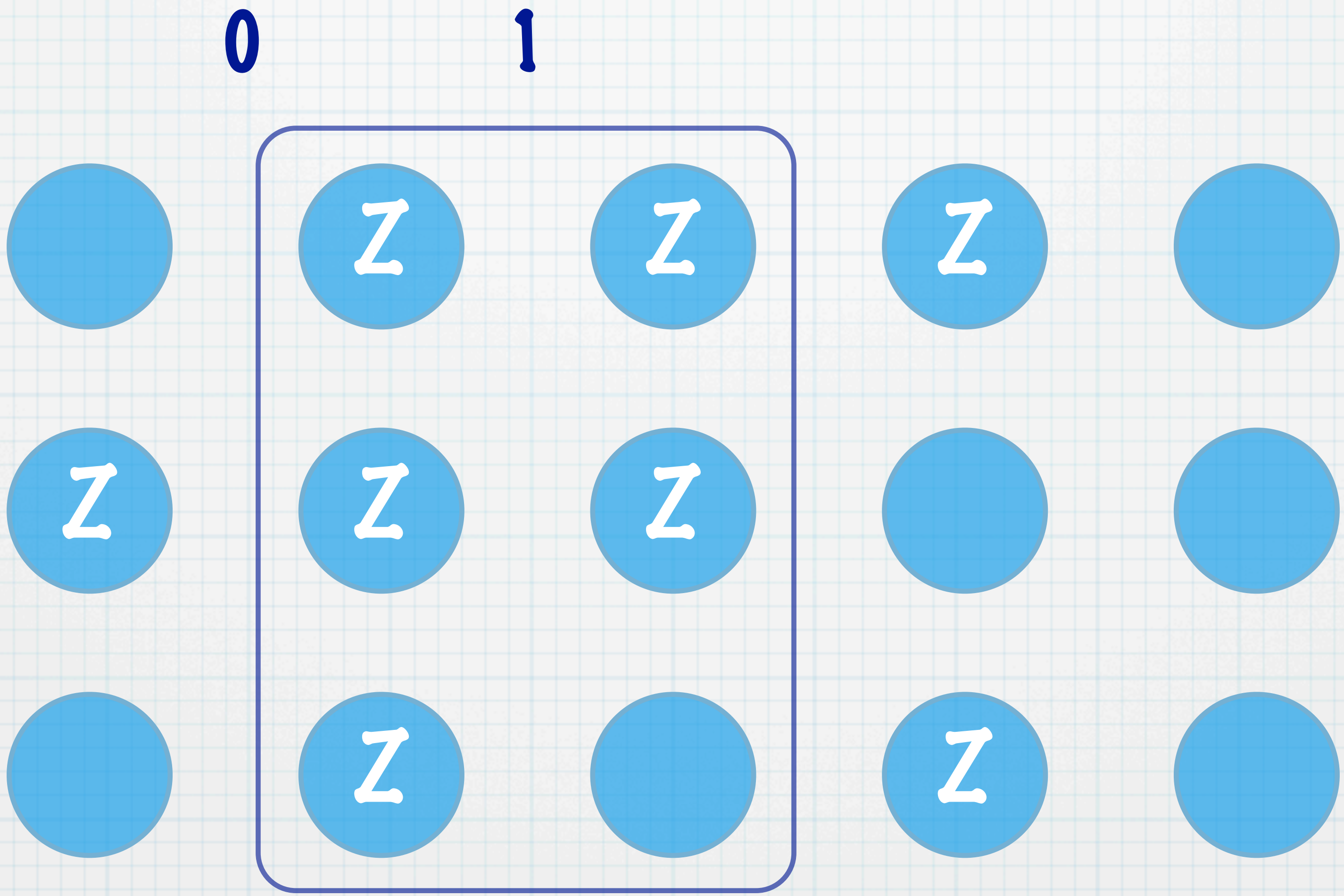

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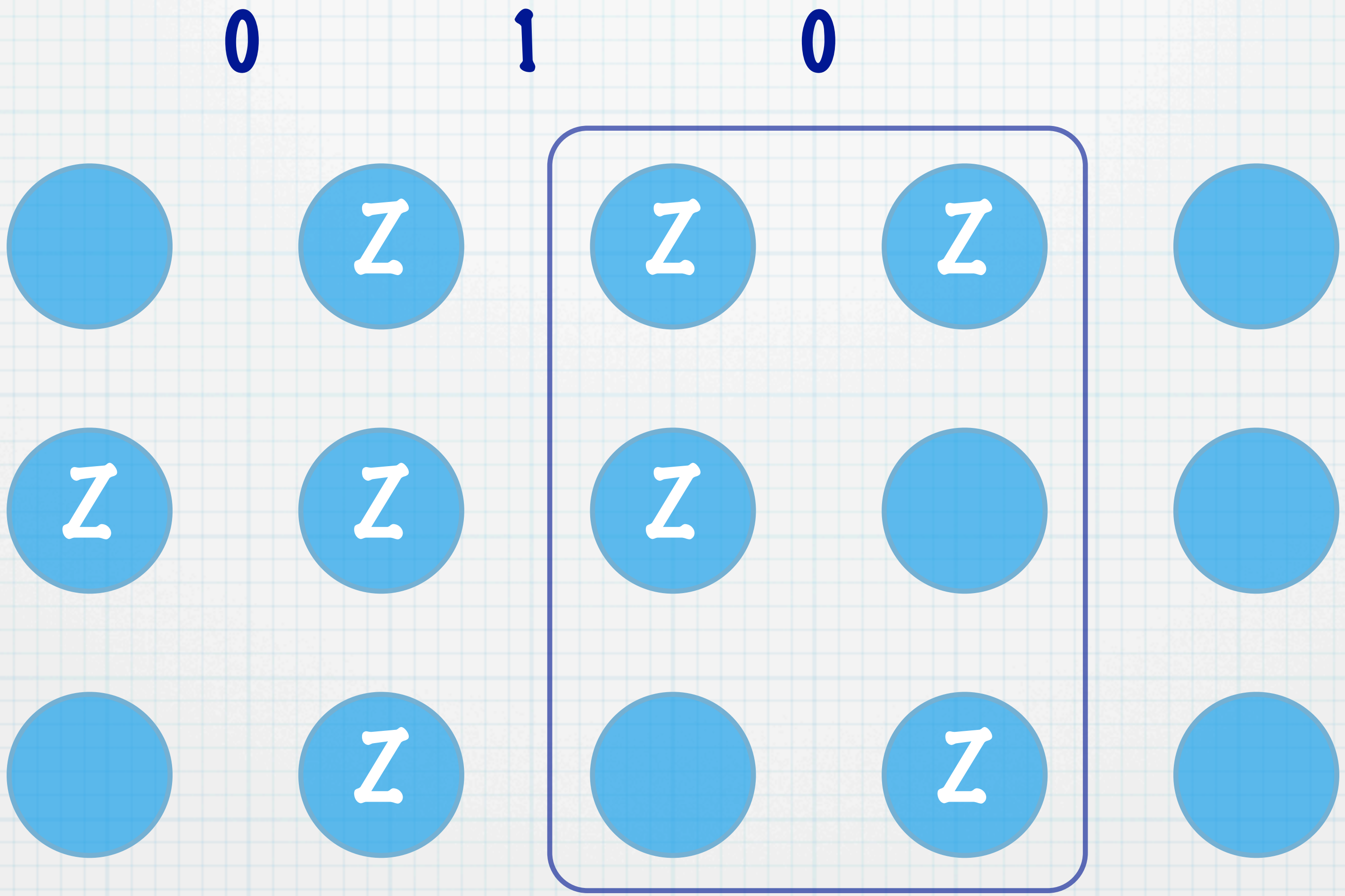

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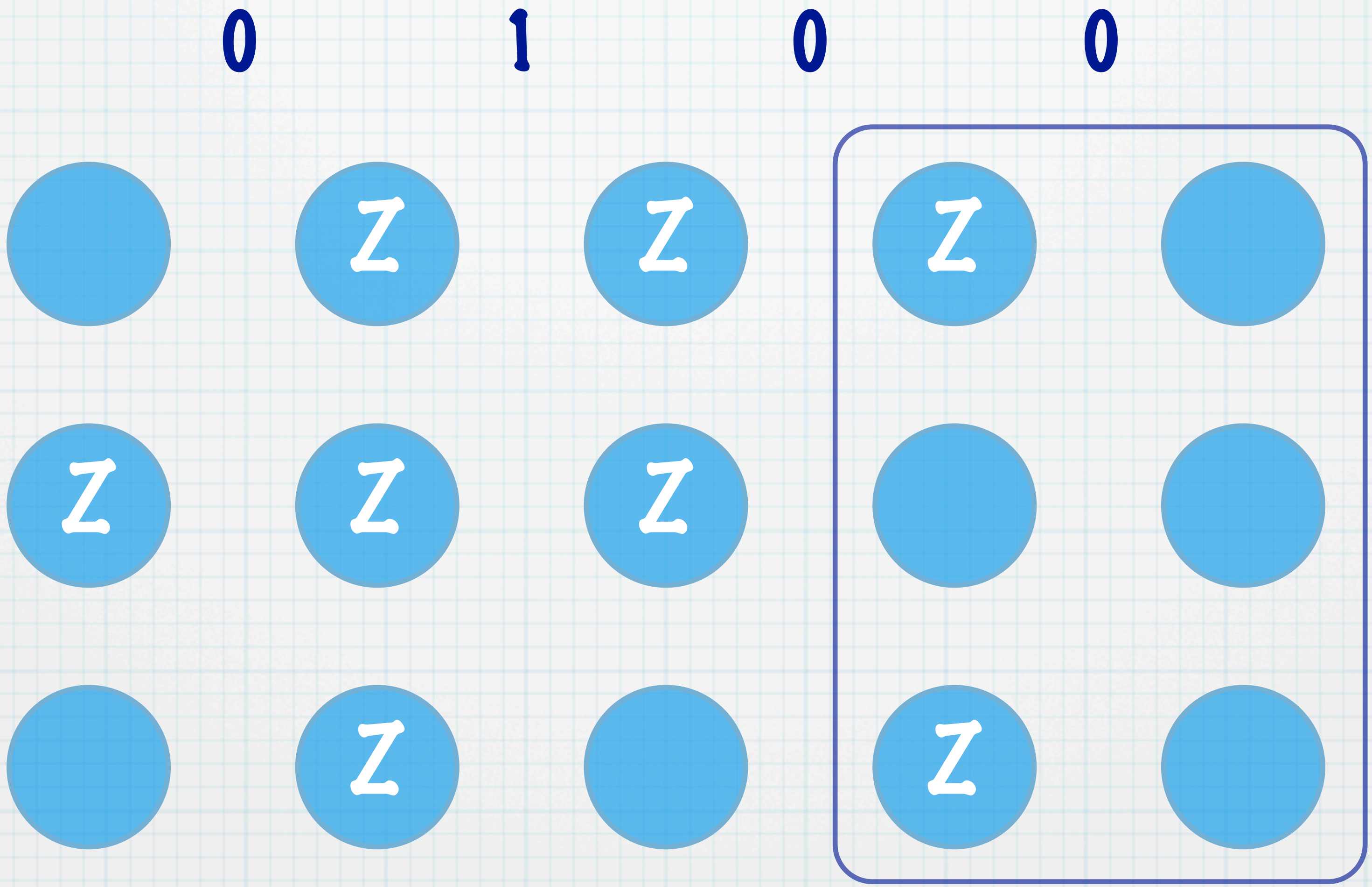

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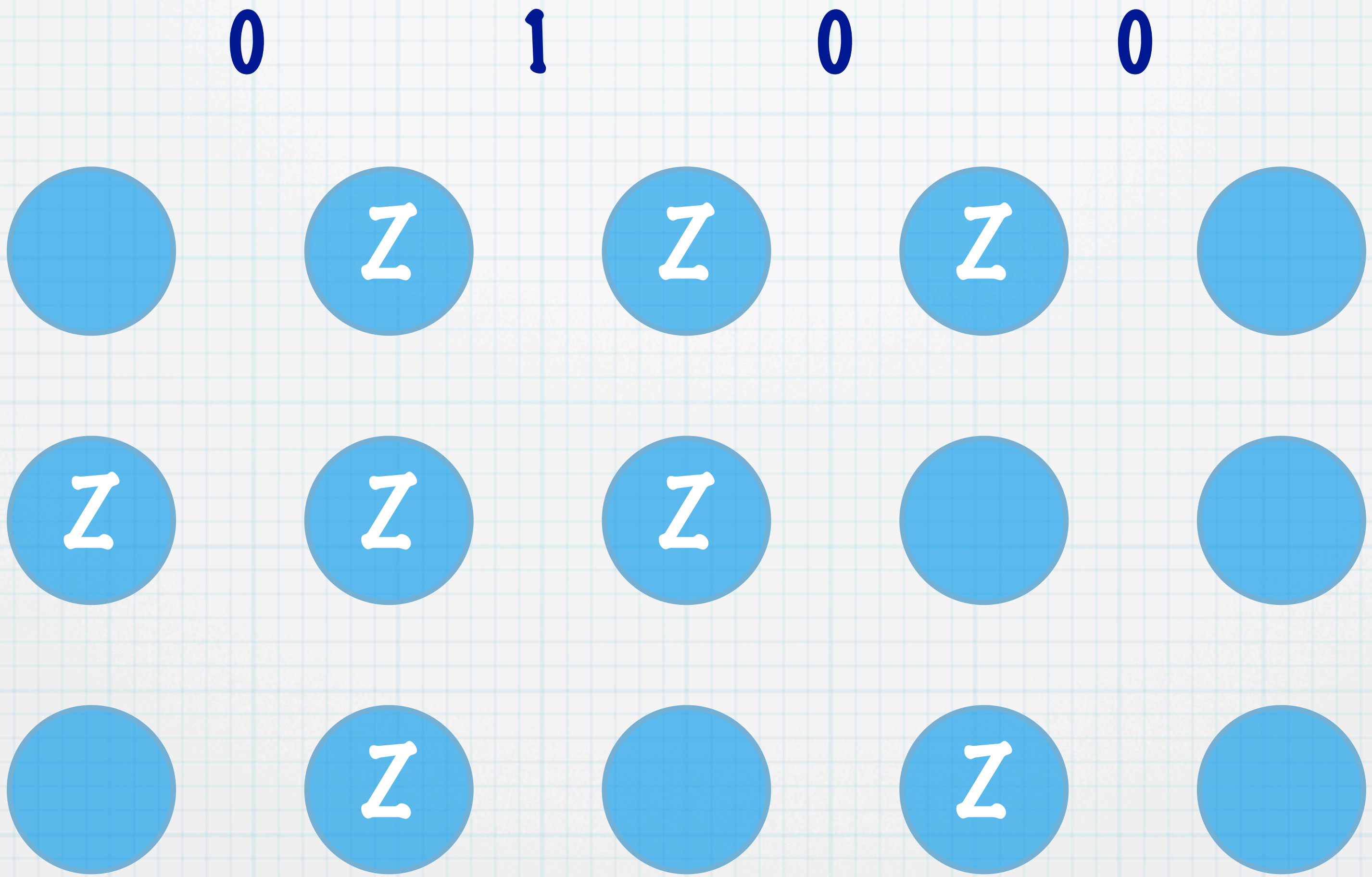

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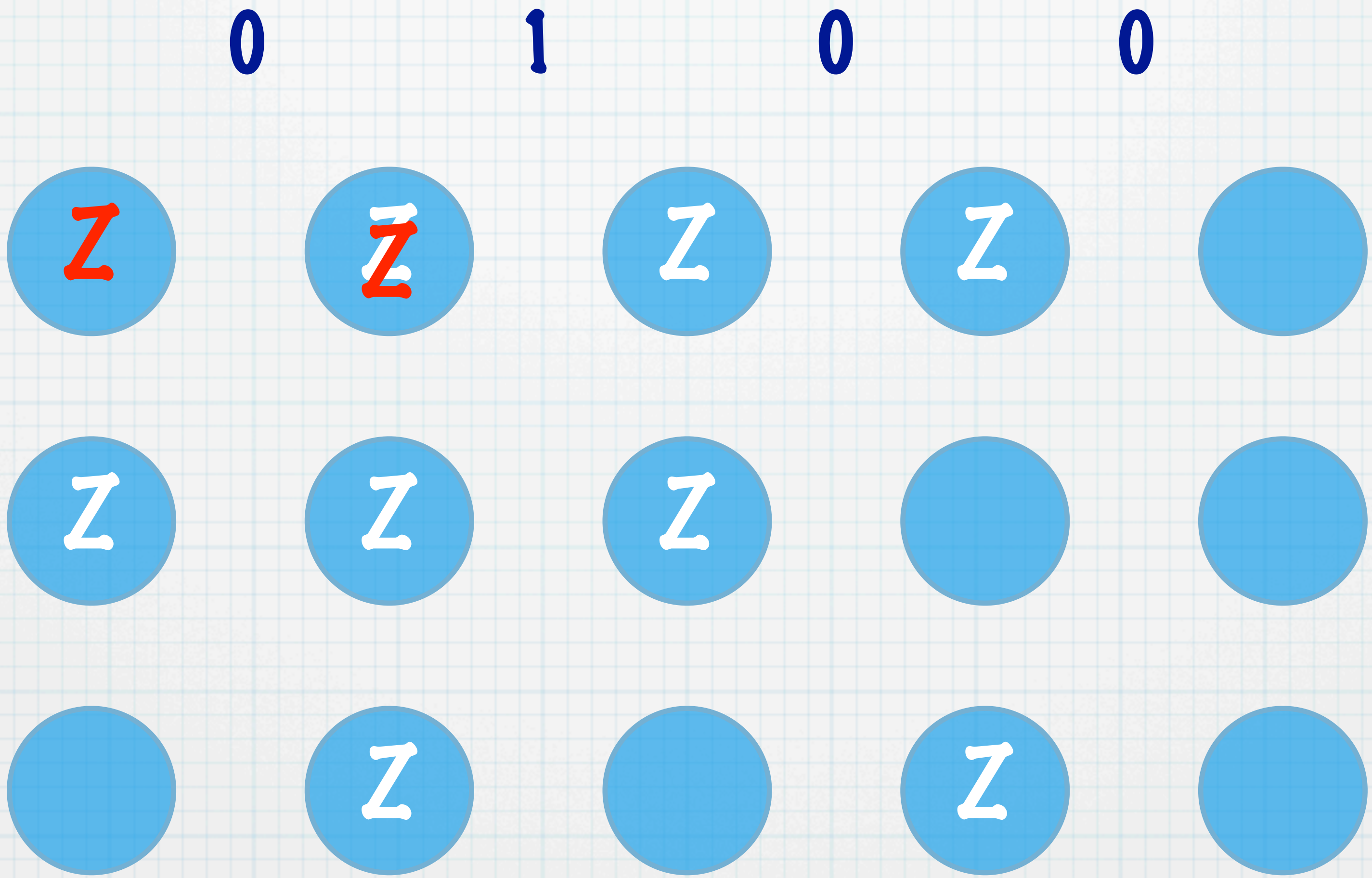
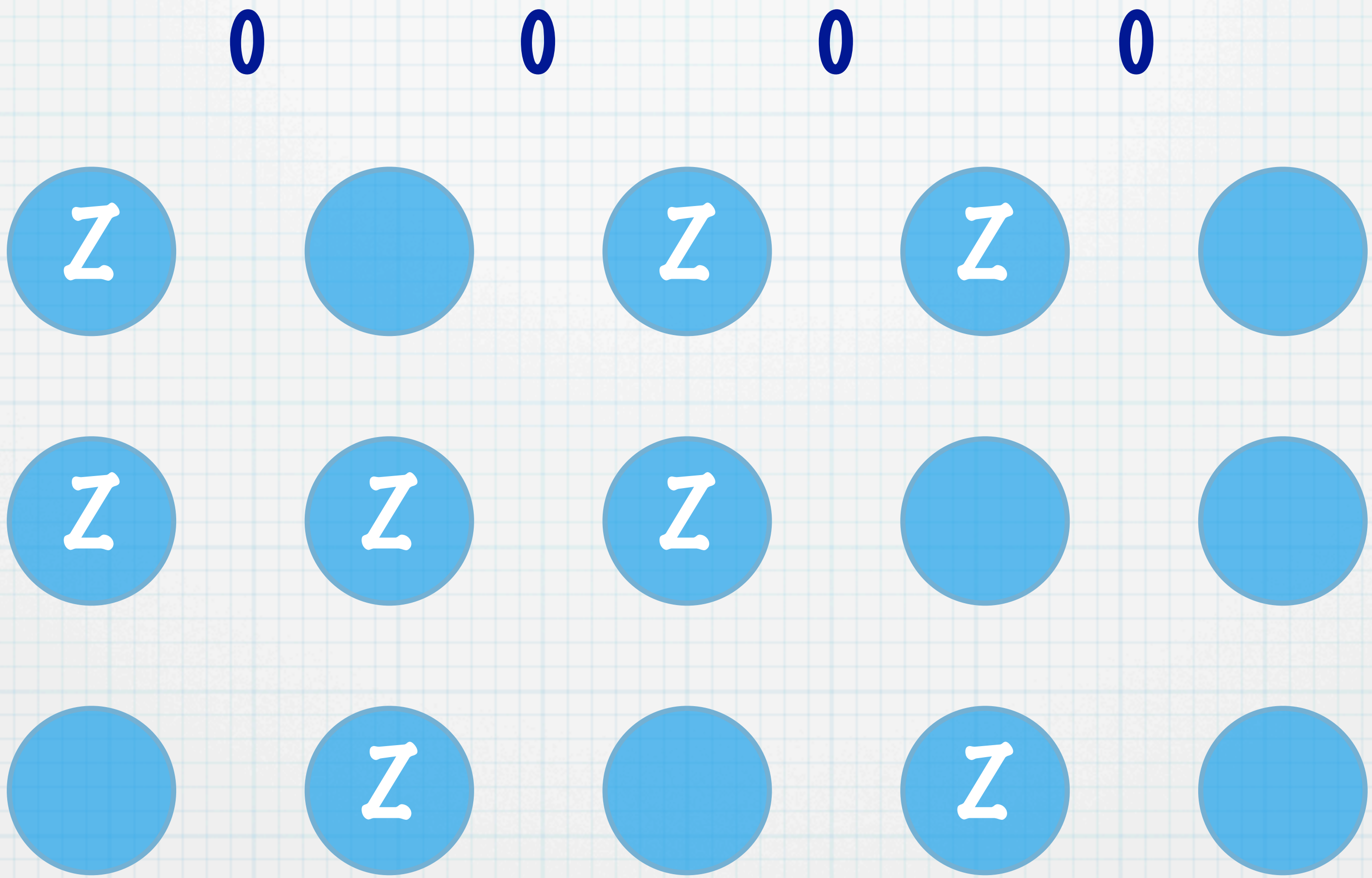


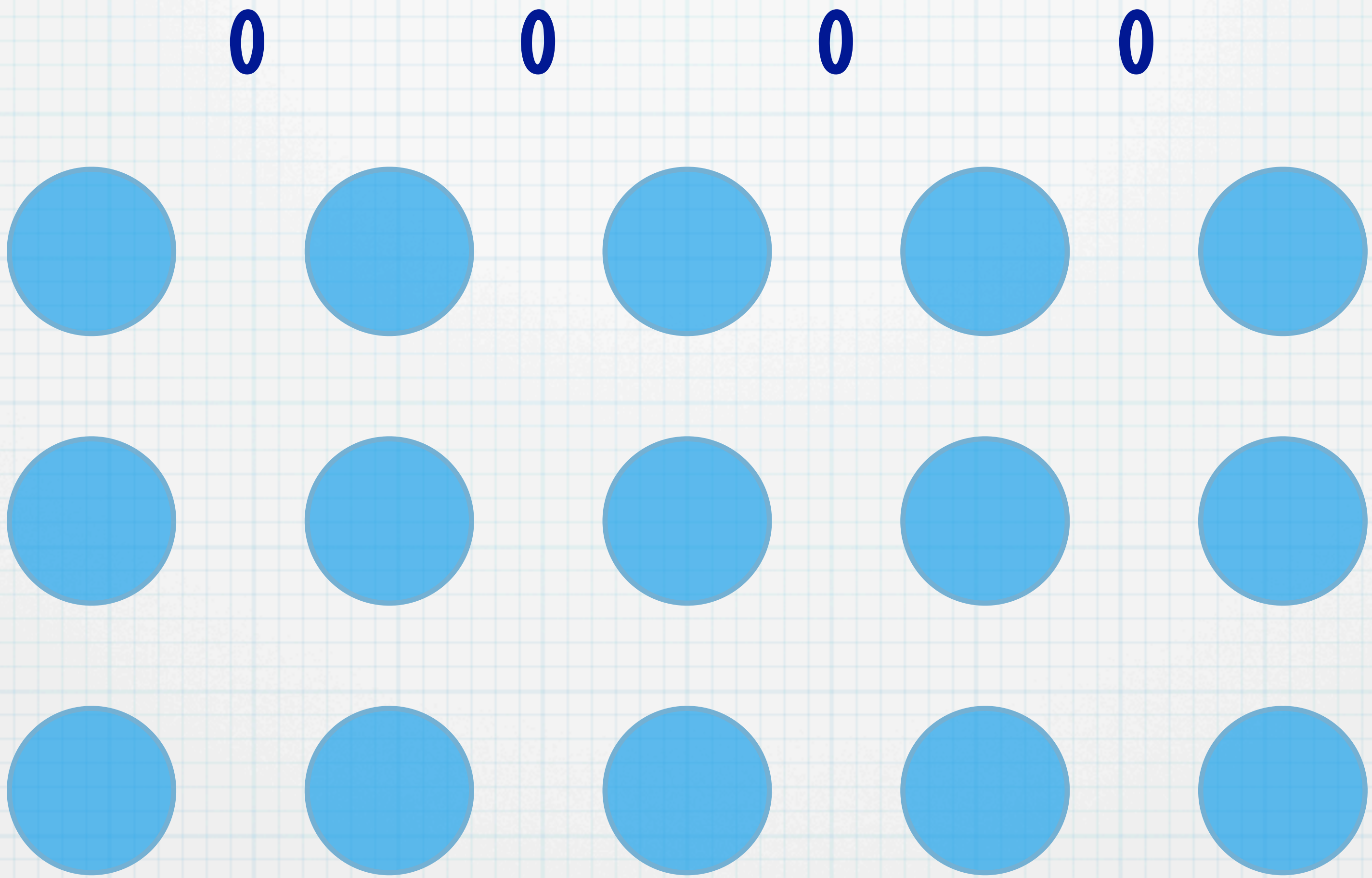
Apply correction

Error correction



Gauge-equivalent to no error

Error correction



Gauge-equivalent to no error

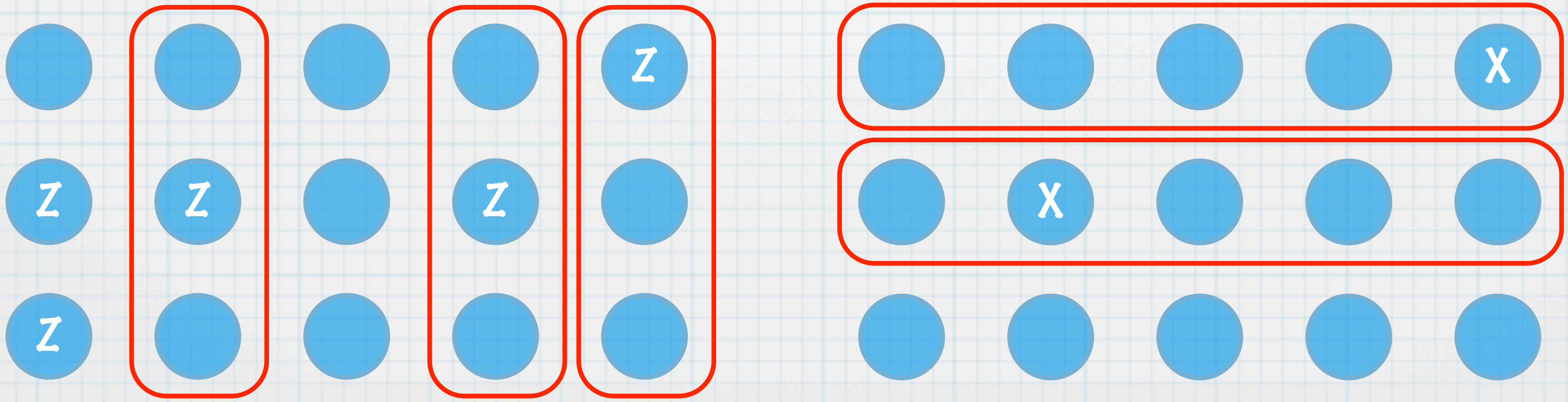
Error correction

Error correction will fail if:

- More than half of <u>columns</u> have odd number of Z errors

OR

- More than half of <u>rows</u> have odd number of X errors

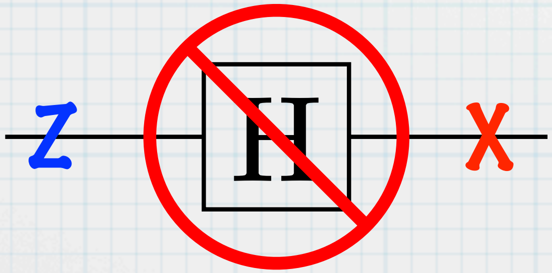


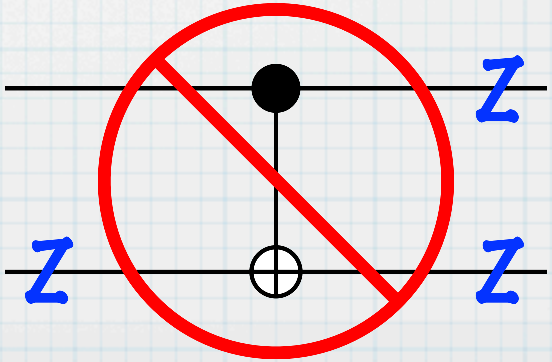
Fault-tolerant gadgets

- Asymmetric Bacon-Shor codes offer more power to treat Z errors vs X errors
- Goal: design fault-tolerant gadgets to do the same
- Key ideas:
  - Bias-compatible gate set
  - Teleported CNOT gate
  - Magic state distillation

Gate set

- Want Z errors to be more common than X errors
- Gates should not transform Z errors into X errors
- Avoid cascading errors in gates





Gate set

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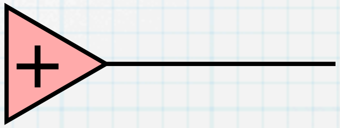
$$\mathcal{P}_{|+\rangle} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) =$$



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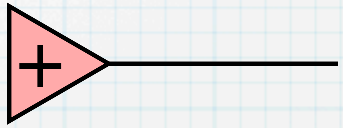
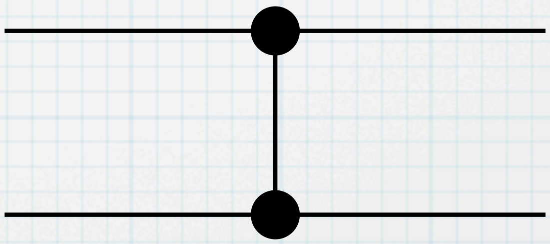


$$CZ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} =$$



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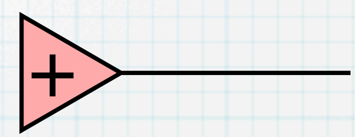

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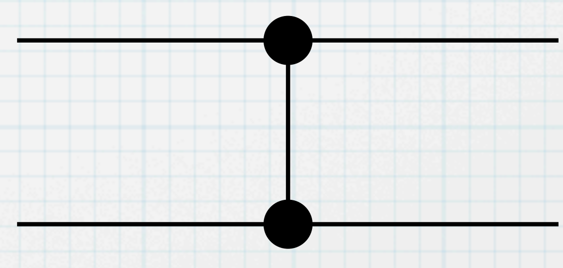

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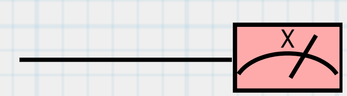

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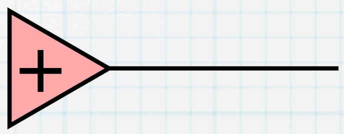

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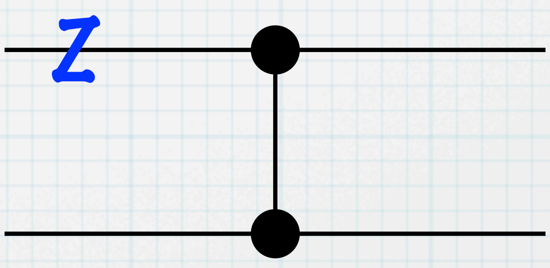

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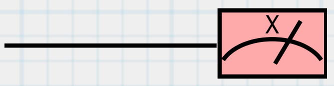

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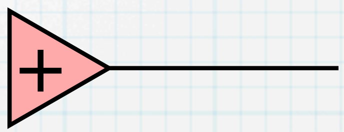

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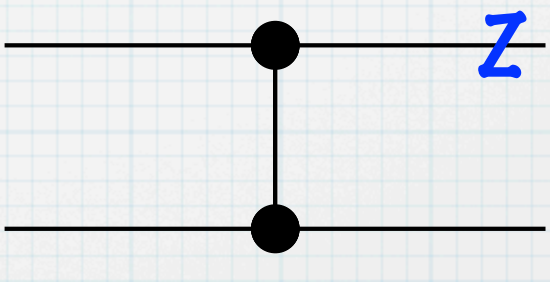

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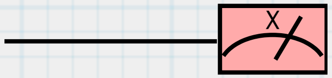

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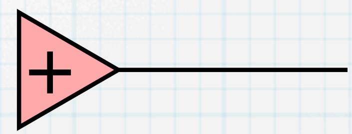

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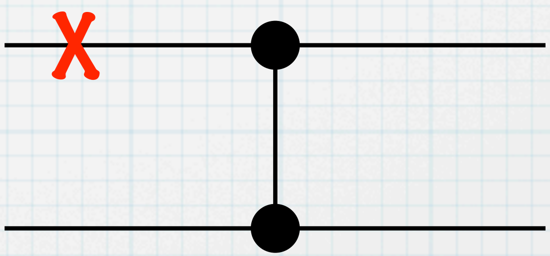

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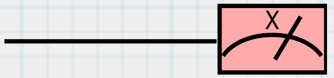

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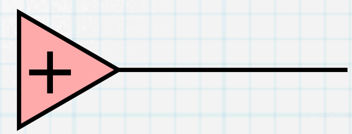

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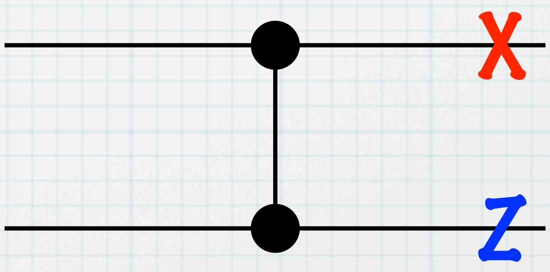

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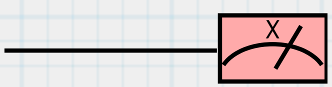

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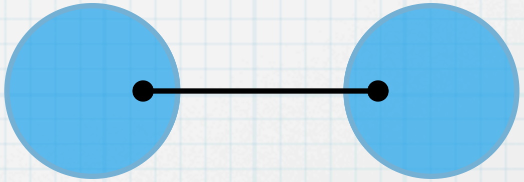

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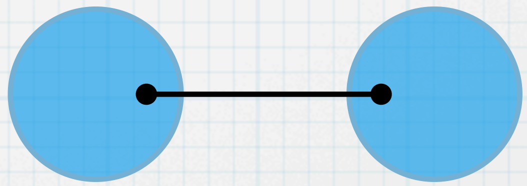

$$\mathcal{M}_X =$$

Universality

biased noise

$$\mathcal{G}_{\mathrm{fund}}$$
$$||$$
$$\{\mathrm{CZ}, \mathcal{P}_{|+\rangle}, \mathcal{M}_X\}$$

Universality

biased noise *weak, balanced noise*

$$\mathcal{G}_{\text{fund}} \xrightarrow{\text{Bacon-Shor code}} \mathcal{G}_{\text{CSS}}$$

$$\mathcal{G}_{\text{fund}} = \{\text{CZ}, \mathcal{P}_{|+\rangle}, \mathcal{M}_X\}$$

$$\mathcal{G}_{\text{CSS}} = \{\text{CNOT}, \mathcal{P}_{|+\rangle}, \mathcal{P}_{|0\rangle}, \mathcal{M}_X, \mathcal{M}_Z\}$$

Universality

biased noise *weak, balanced noise* *arbitrarily weak noise*

$$\mathcal{G}_{\mathrm{fund}} \xrightarrow[\text{Bacon-Shor code}]{} \mathcal{G}_{\mathrm{CSS}} \xrightarrow[\text{Concatenated code}]{}$$

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biased noise — *weak, balanced noise* — *arbitrarily weak noise*

$$\mathcal{G}_{\text{fund}} \xrightarrow{\text{Bacon-Shor code}} \mathcal{G}_{\text{CSS}} \xrightarrow{\text{Concatenated code}} \mathcal{G}_{\text{universal}}$$

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$$\{\mathcal{P}_{|+i\rangle}, \mathcal{P}_{|T\rangle}\} \xrightarrow{\text{State injection and distillation}} \mathcal{G}_{\text{universal}}$$

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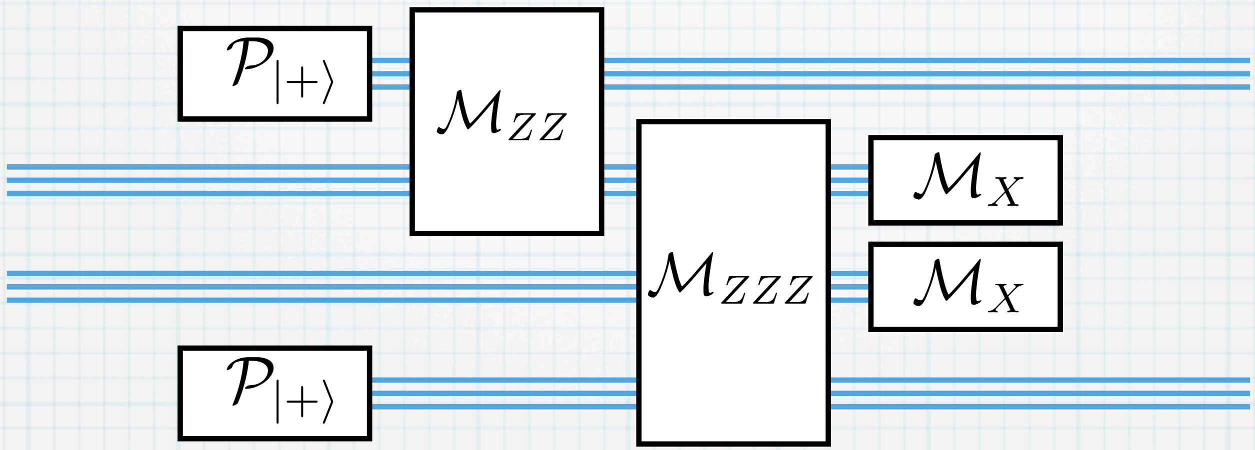
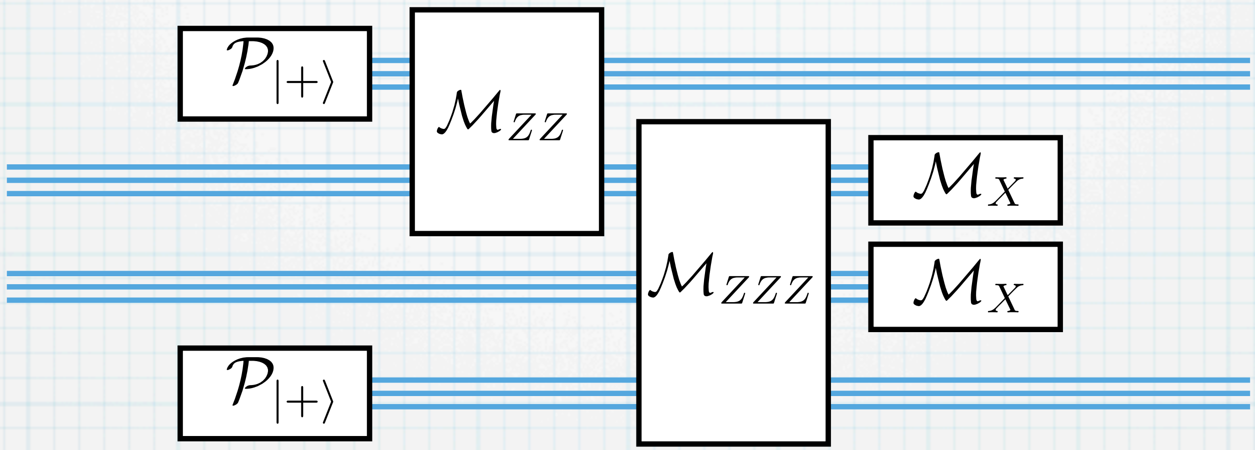
Teleported CNOT



$$|10\rangle \to |0100\rangle + |0101\rangle + |1100\rangle + |1101\rangle$$

- If outcomes of Z measurements are 0, perform exactly CNOT. Otherwise different by local Pauli's
- Error correction by teleportation as well

Teleported CNOT



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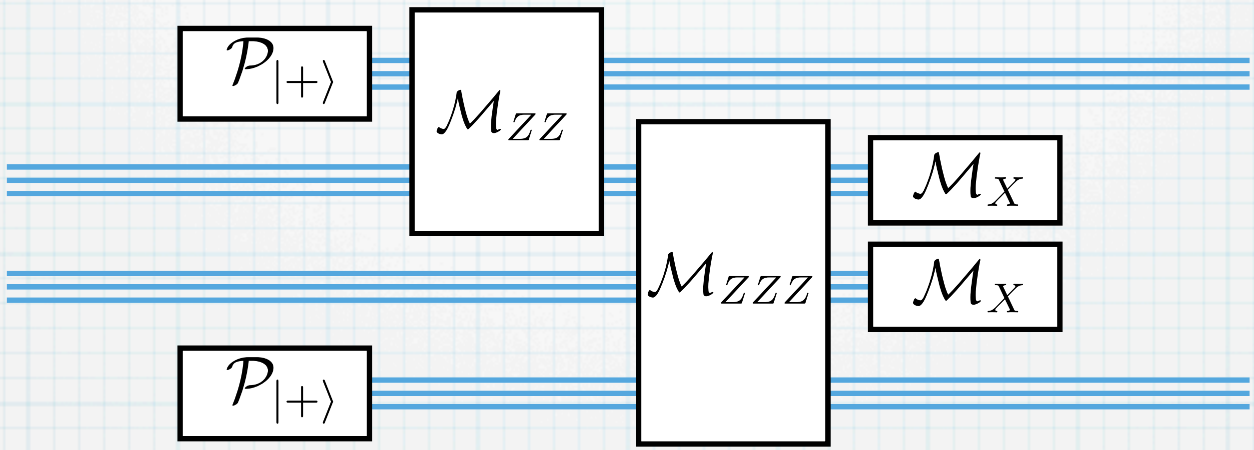

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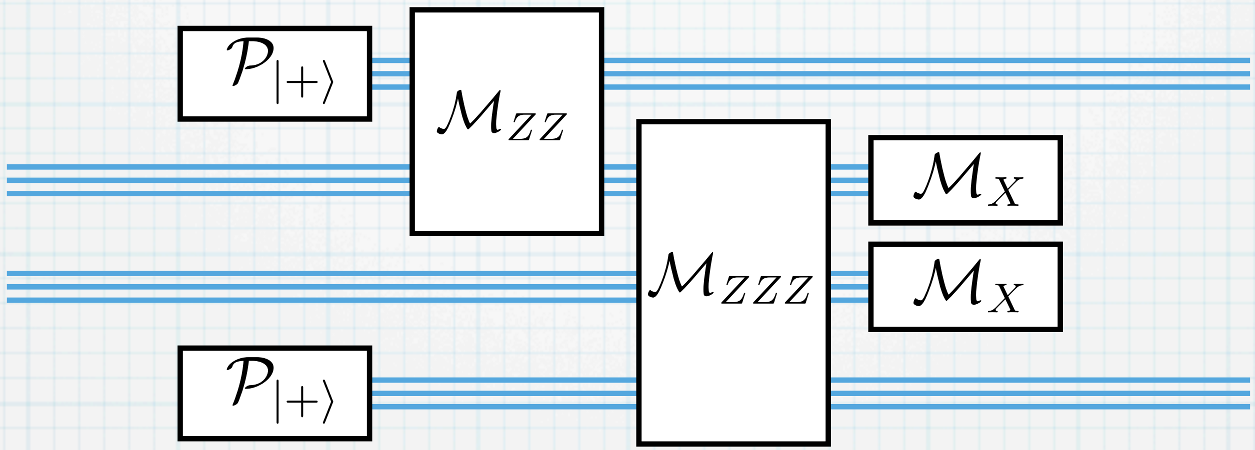

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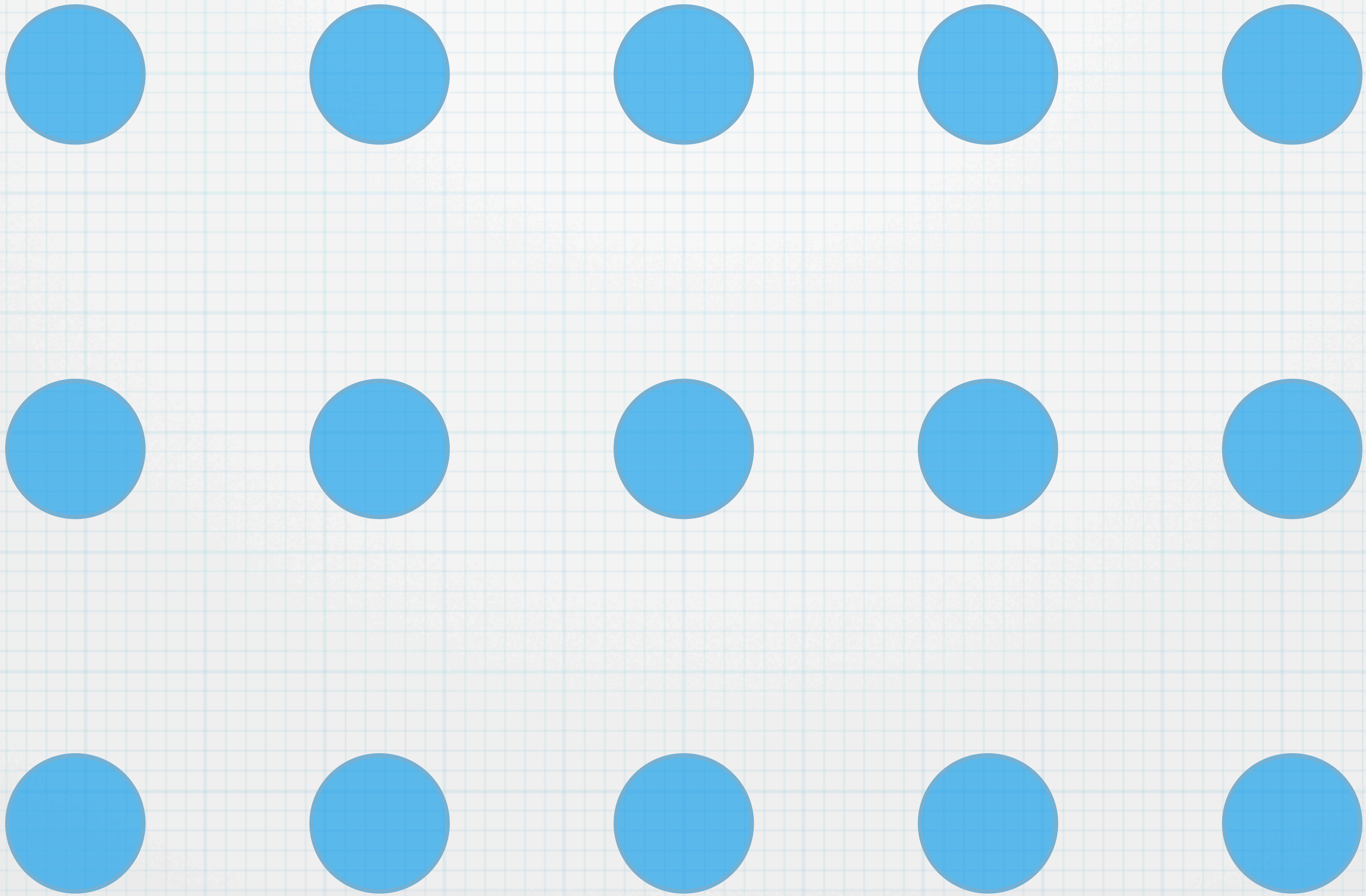
$|+\rangle$ Preparation



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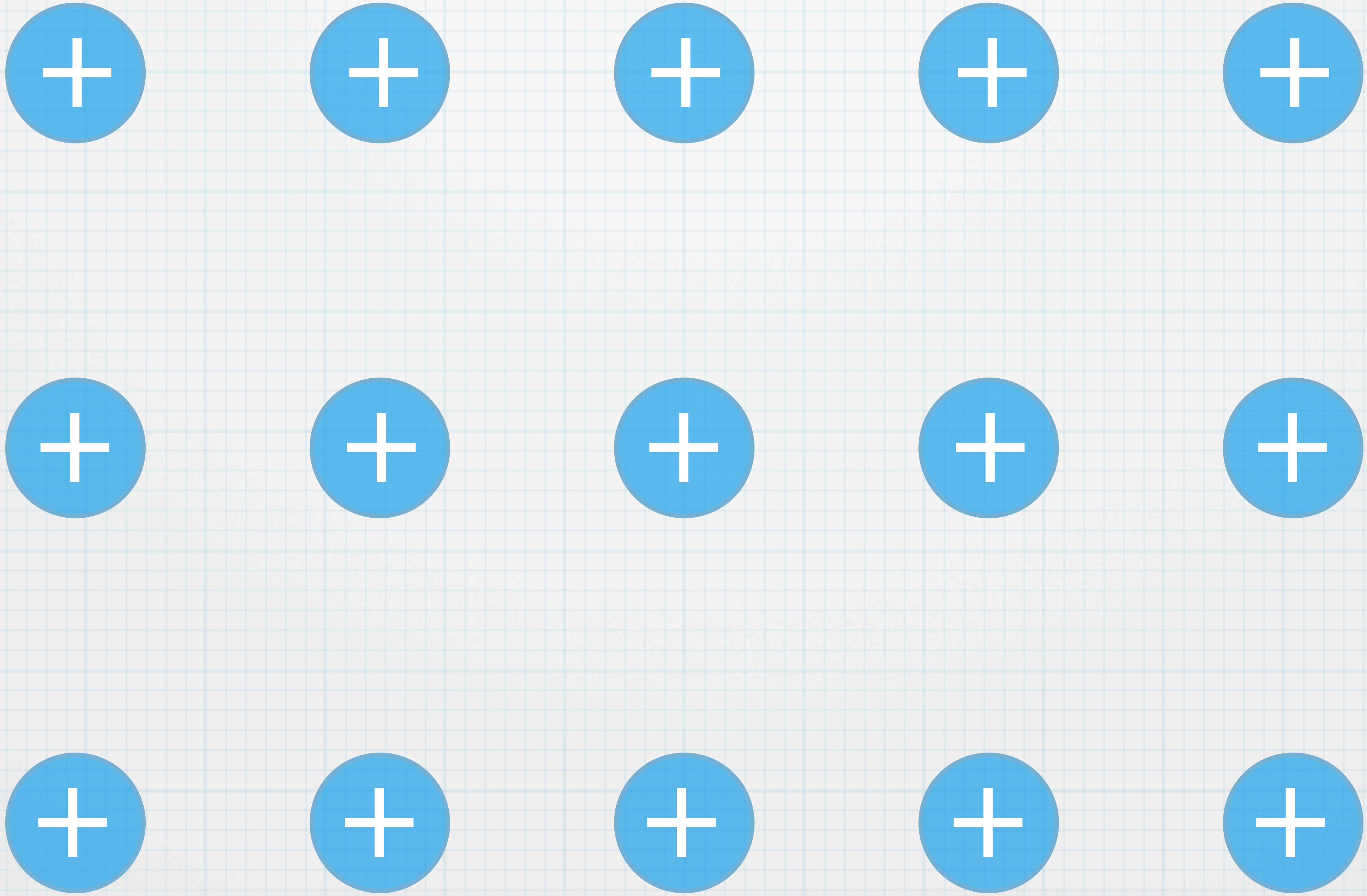

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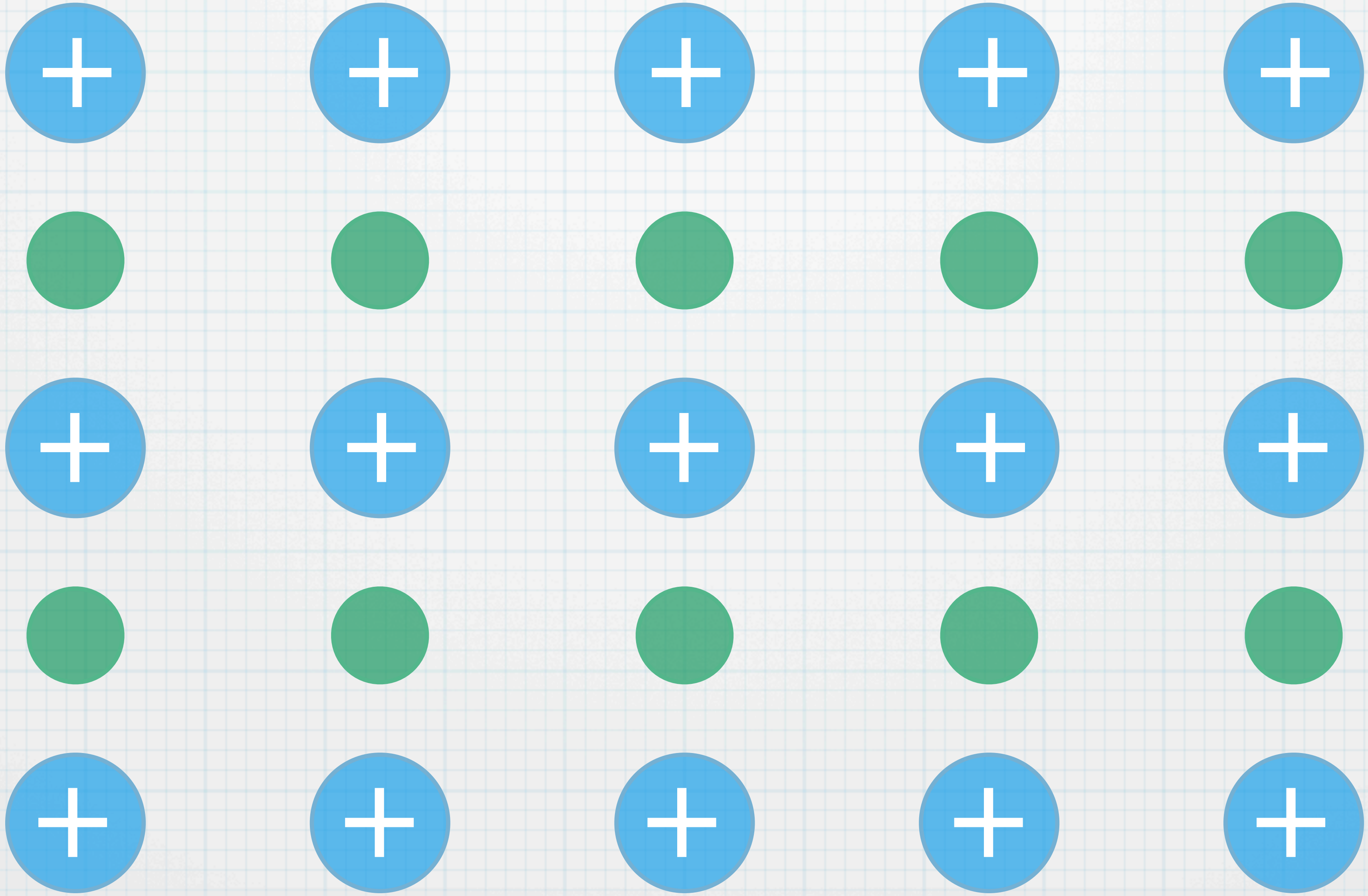

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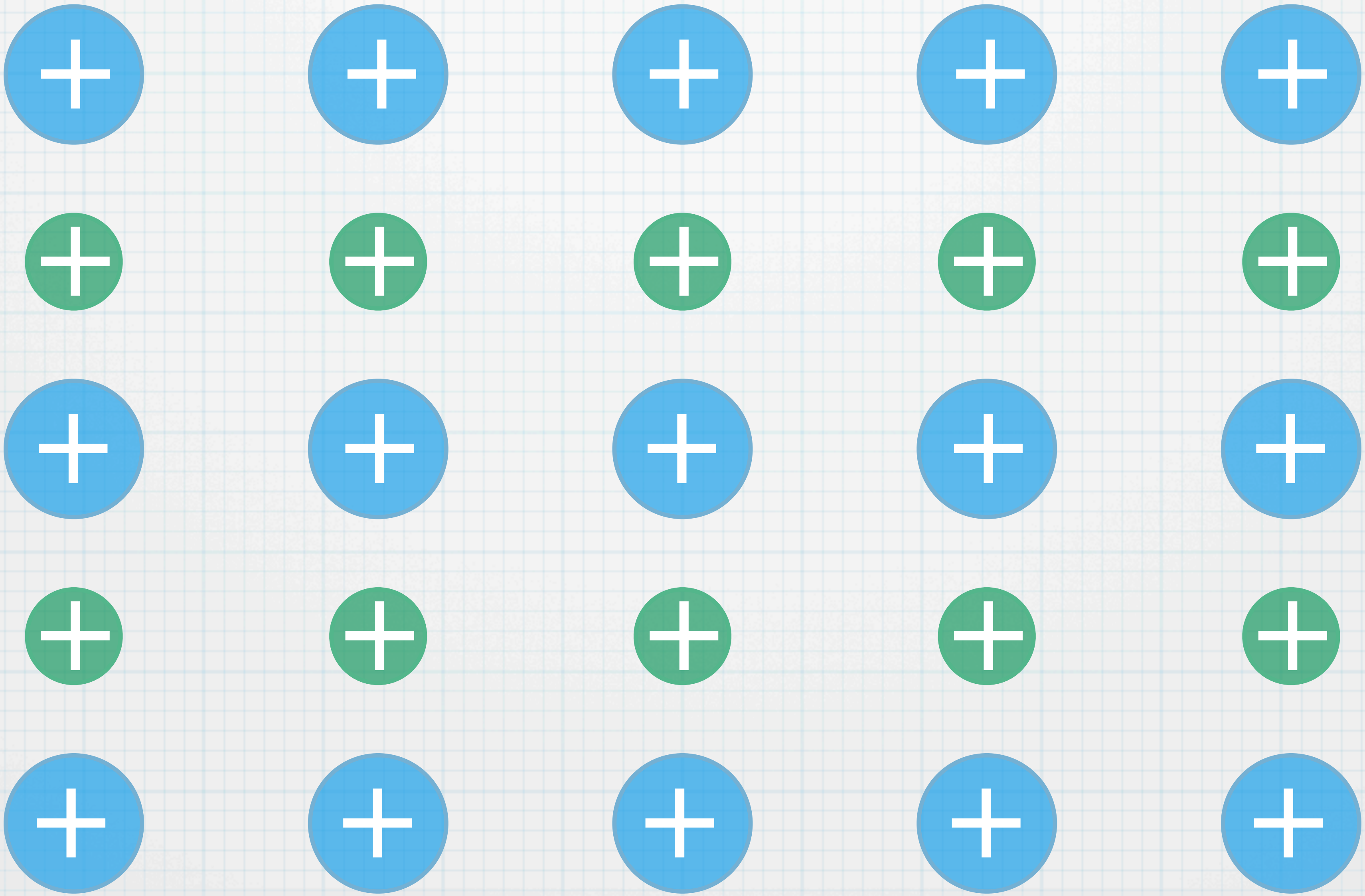
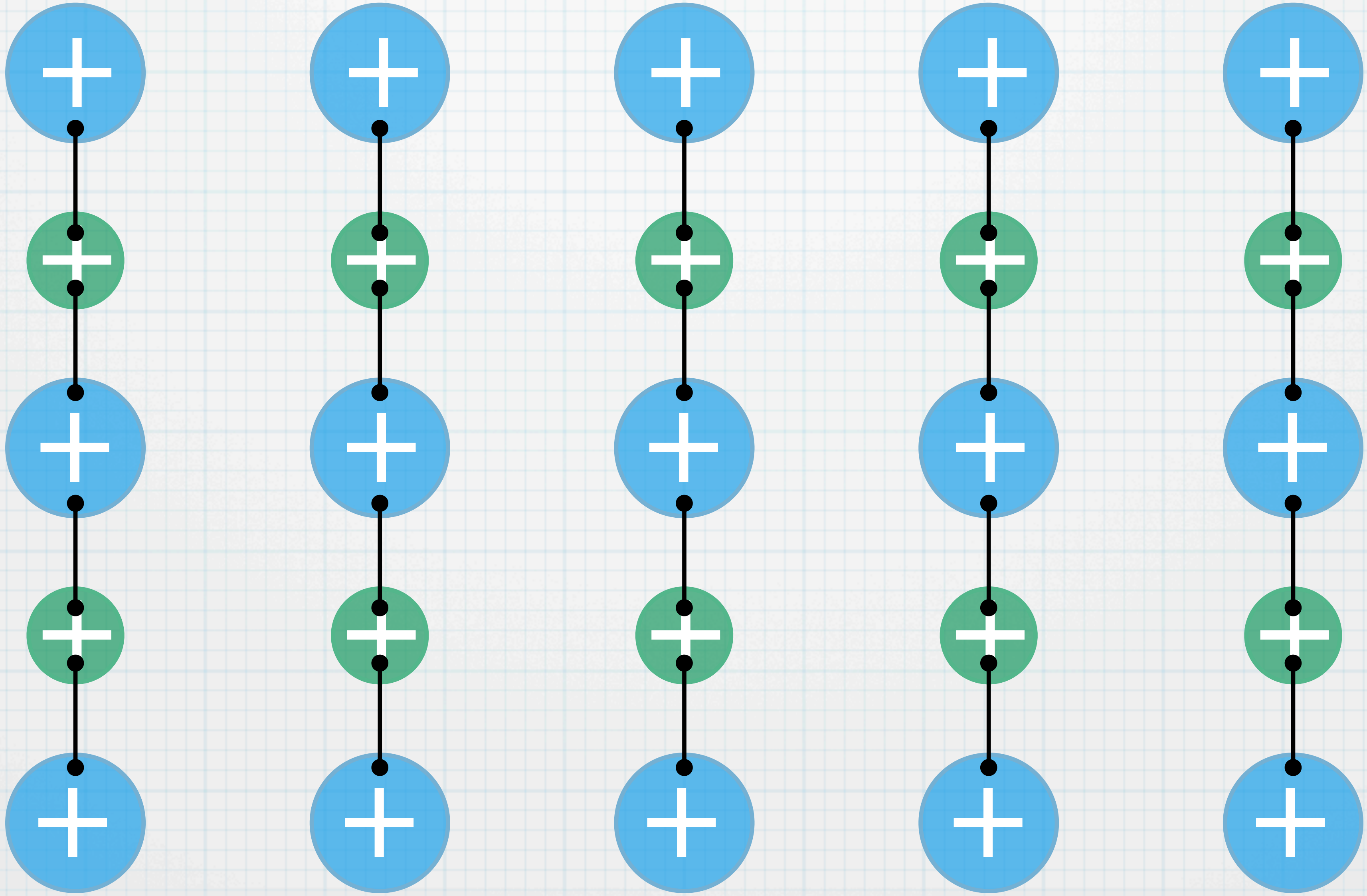


$|+\rangle$ Preparation



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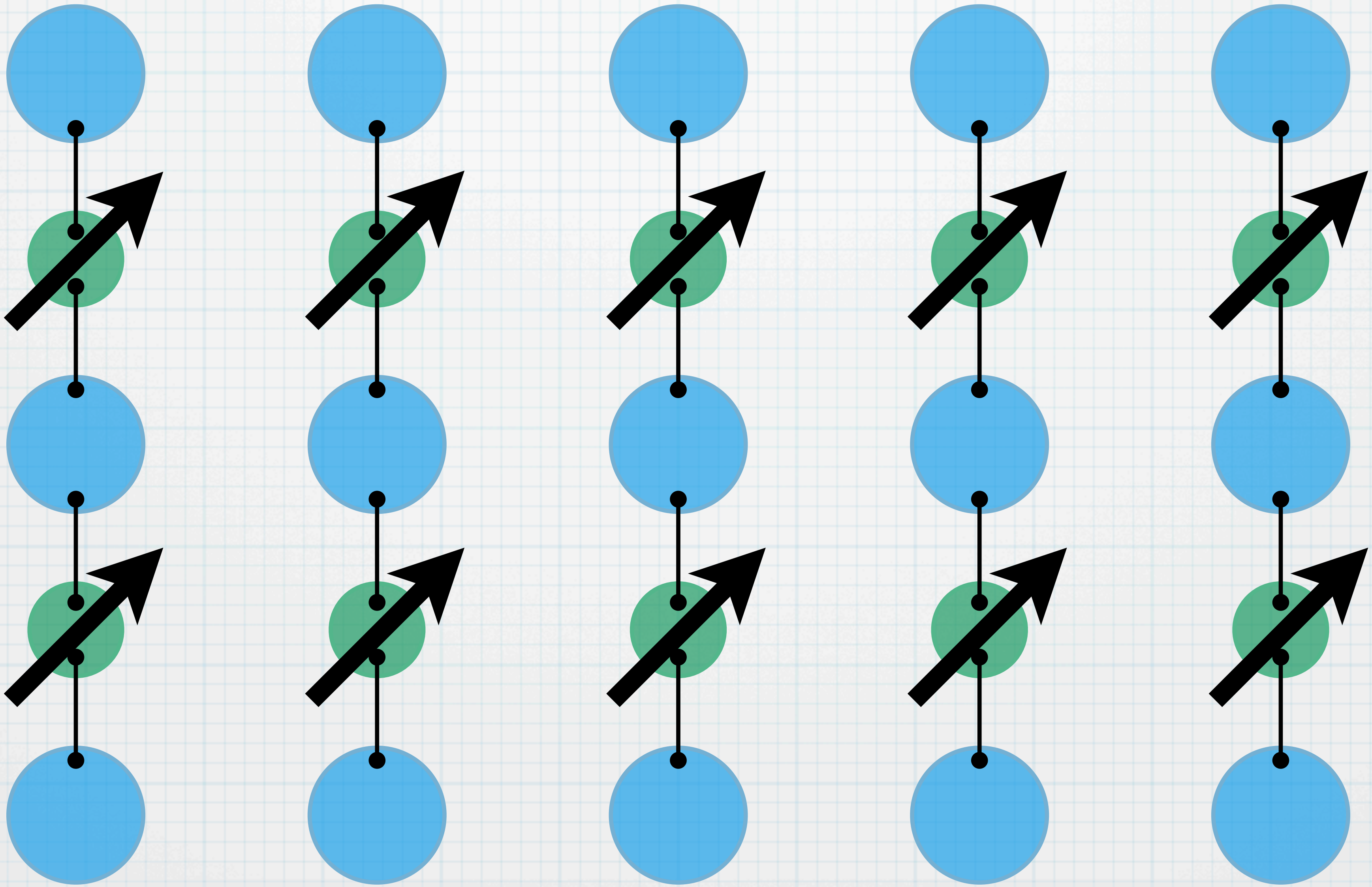

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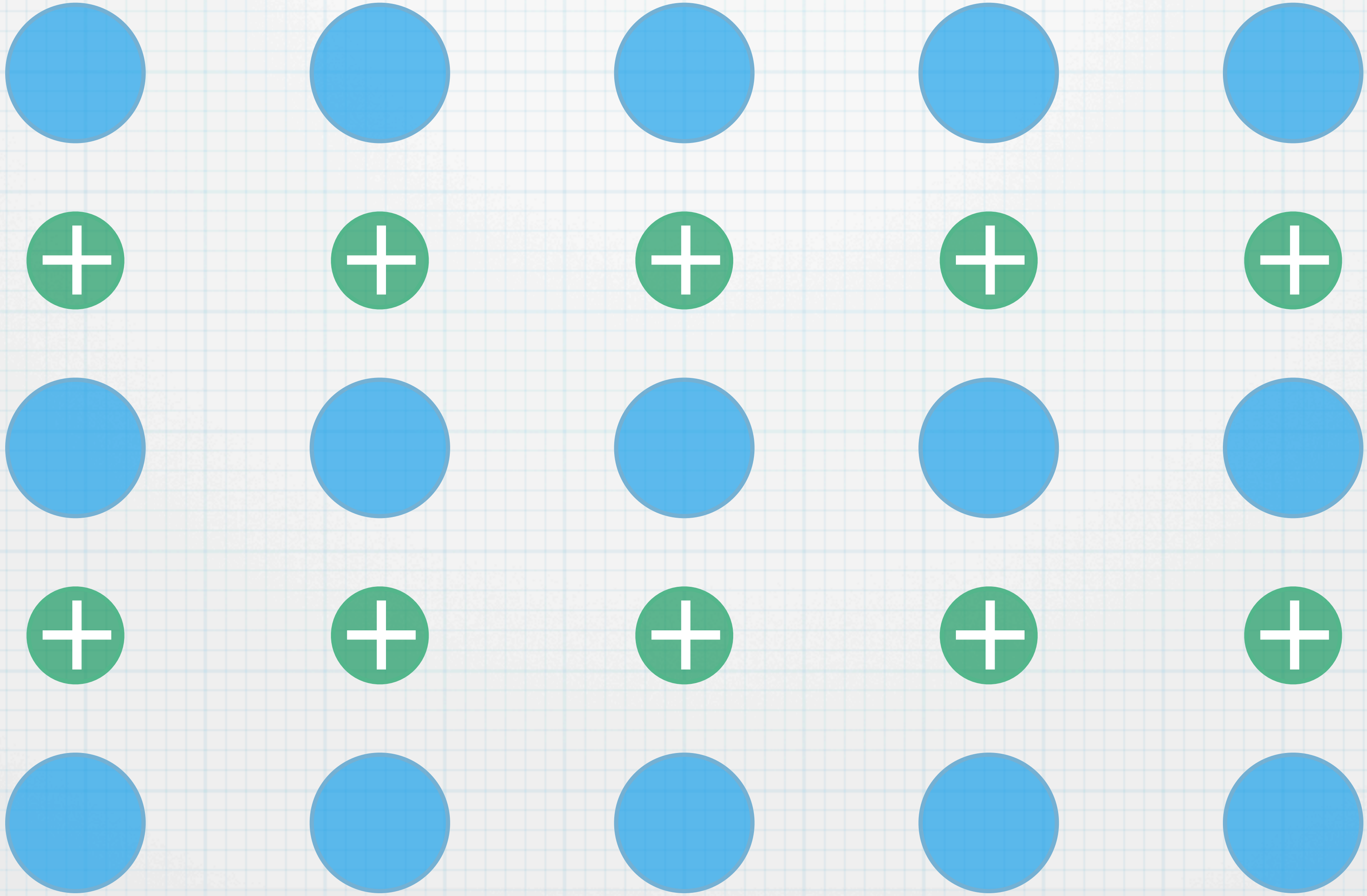

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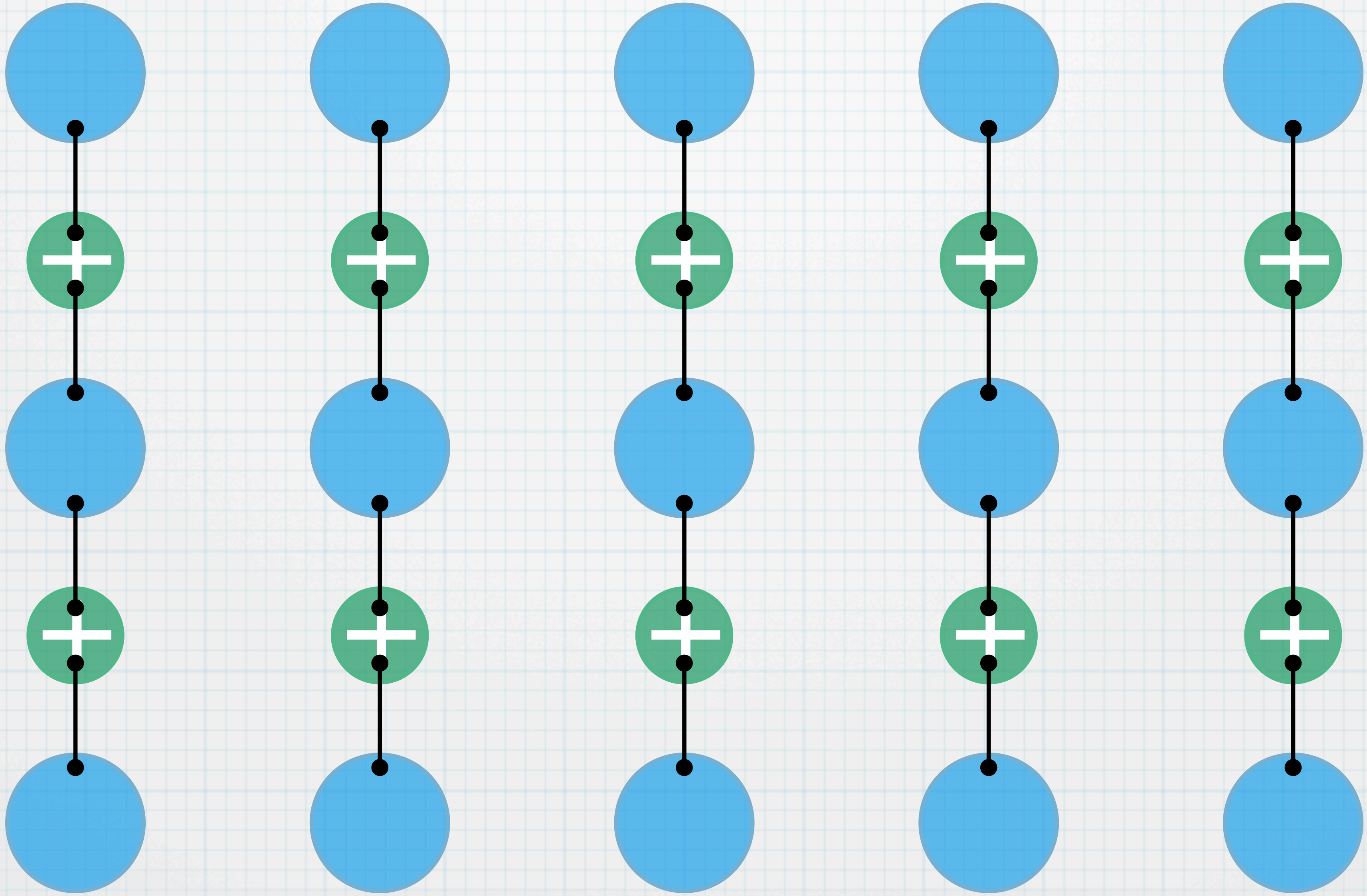
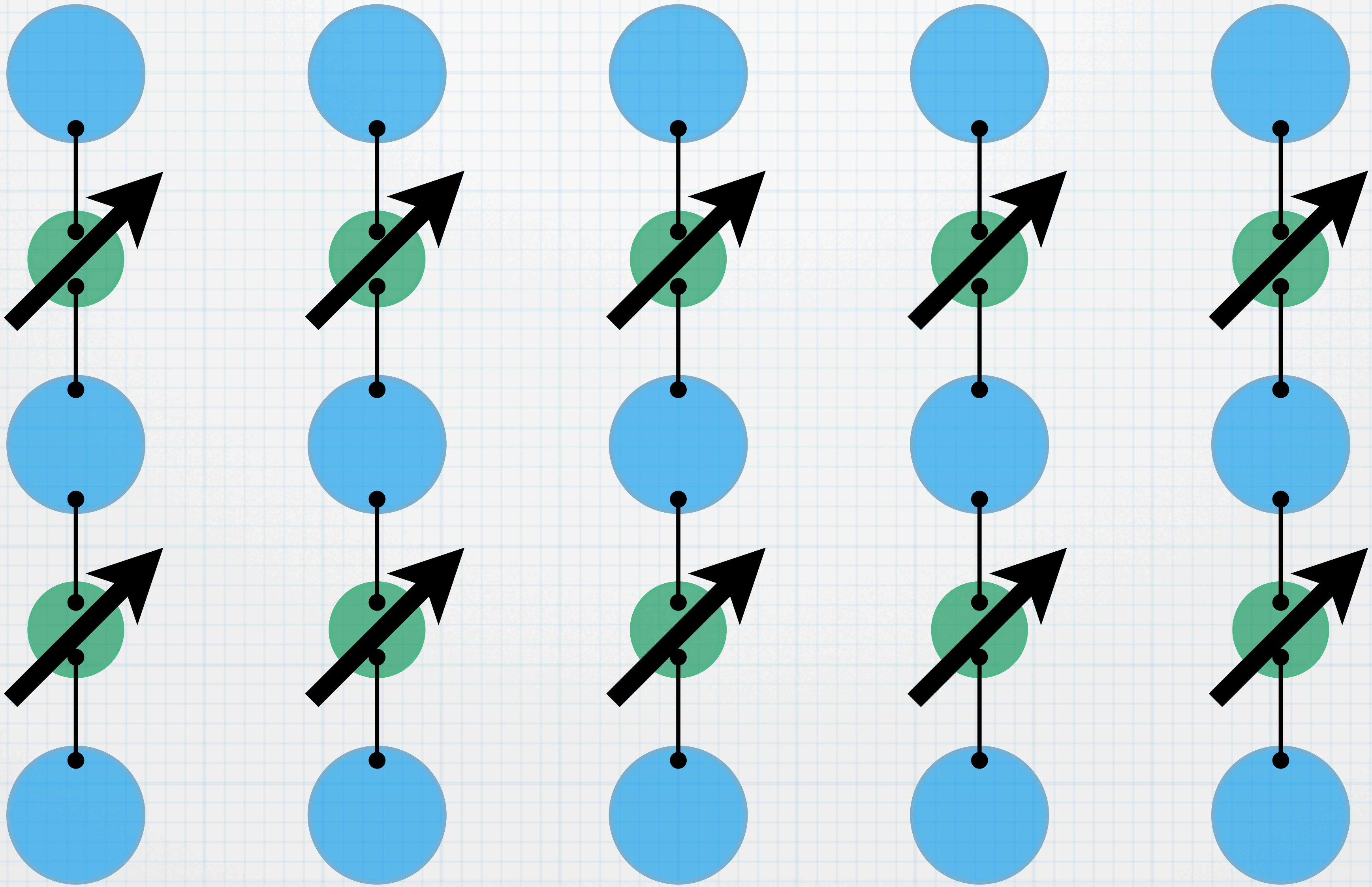
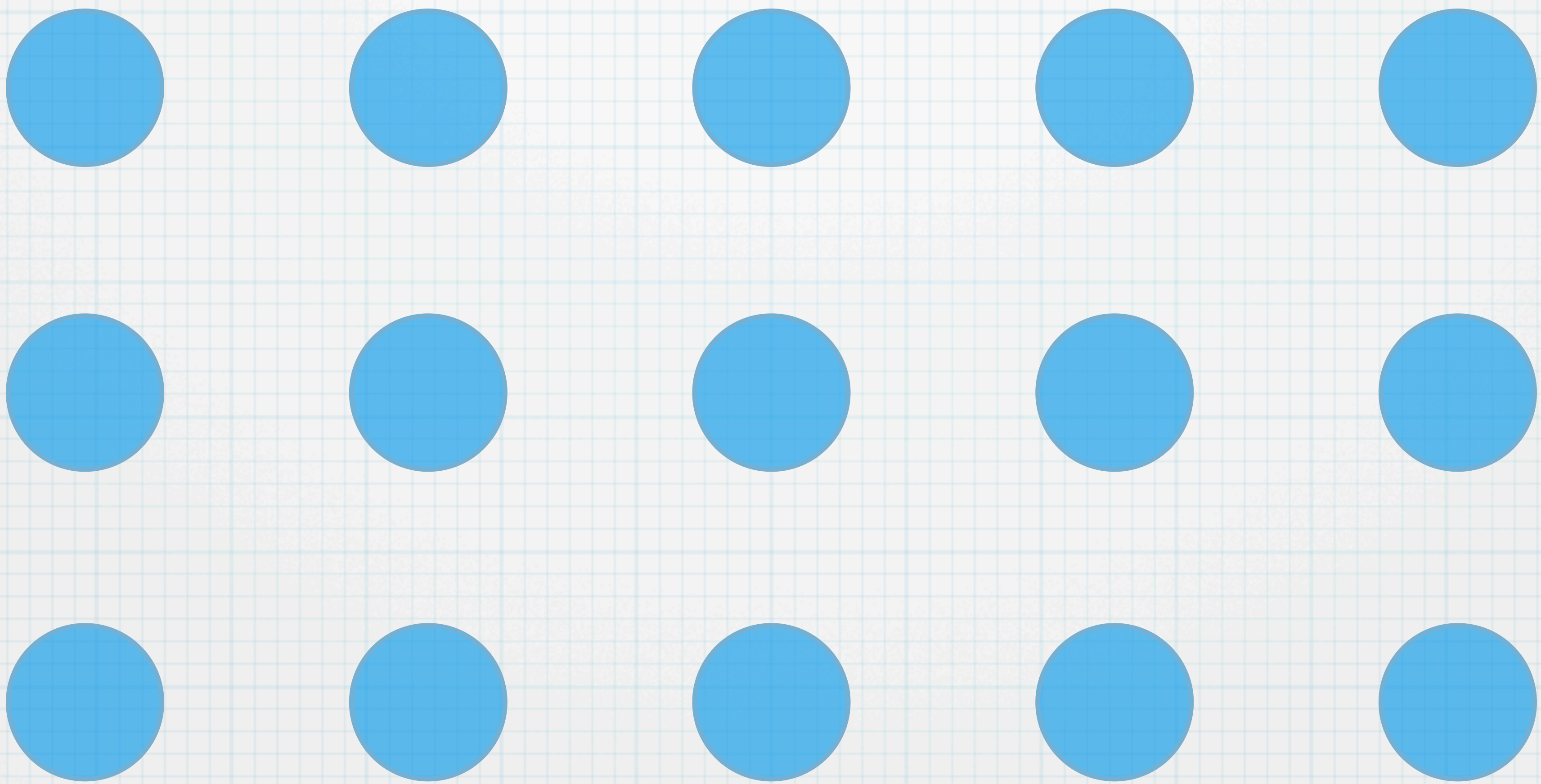


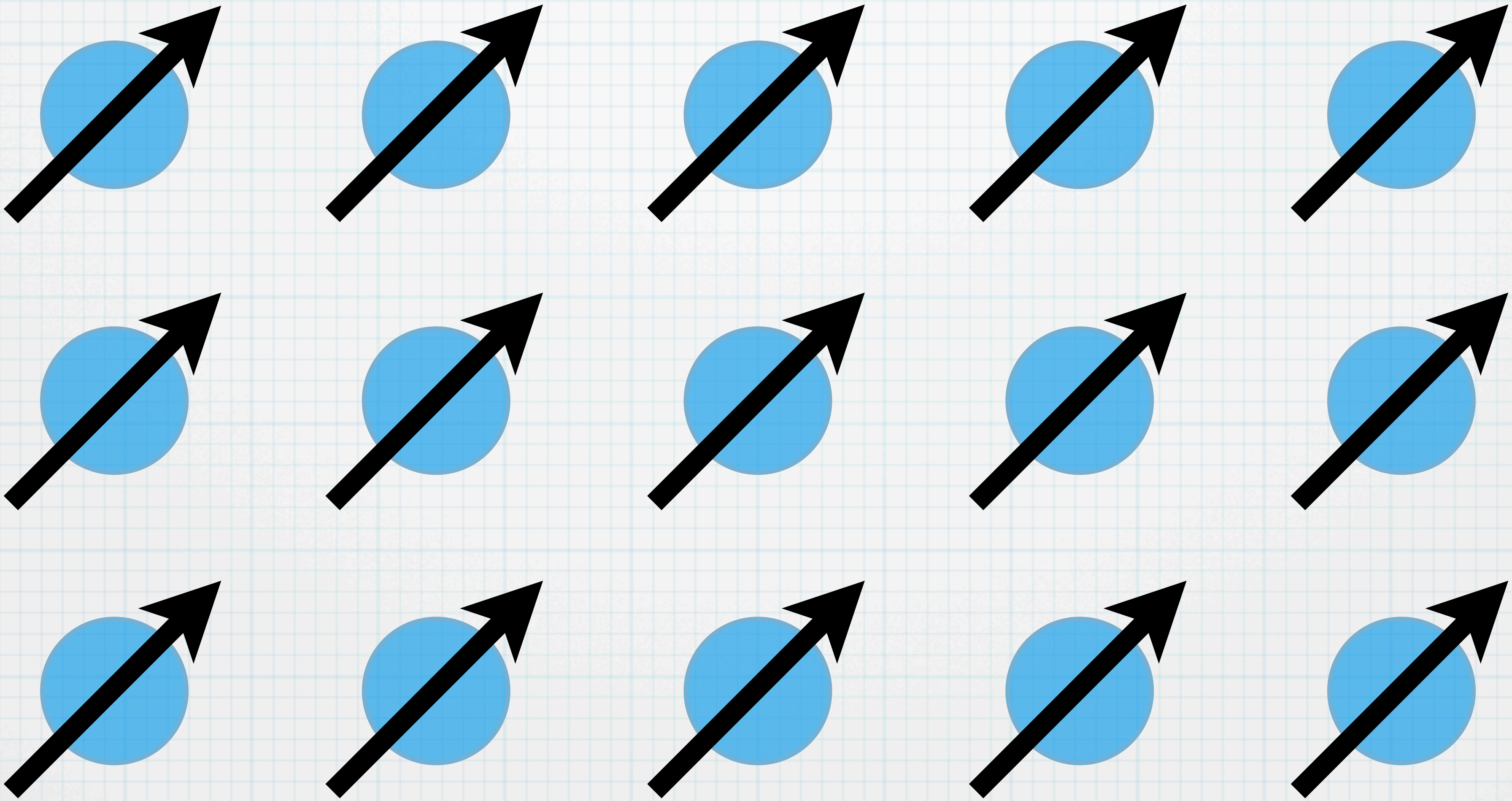
$|+\rangle$ Preparation



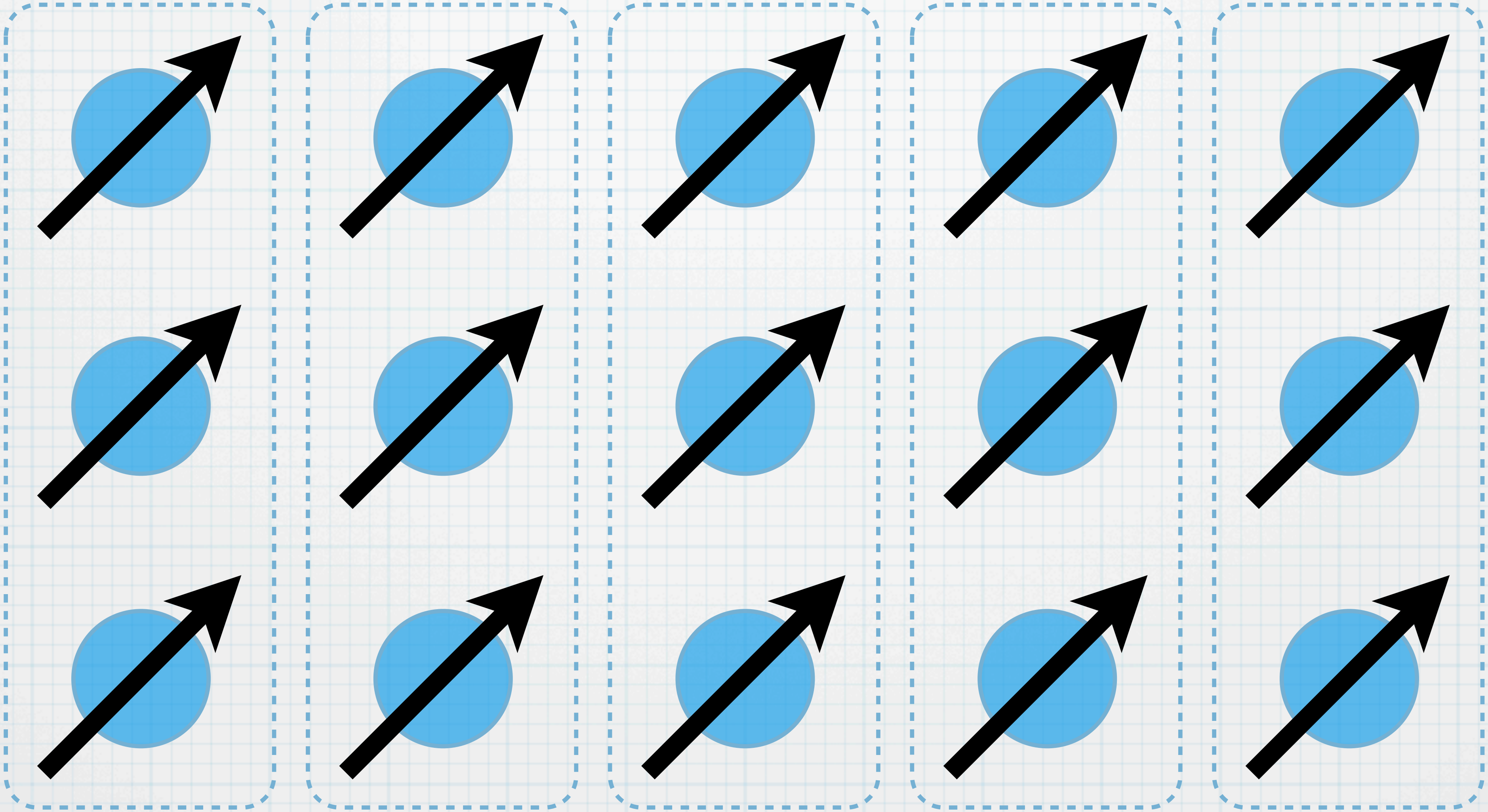
X Measurement



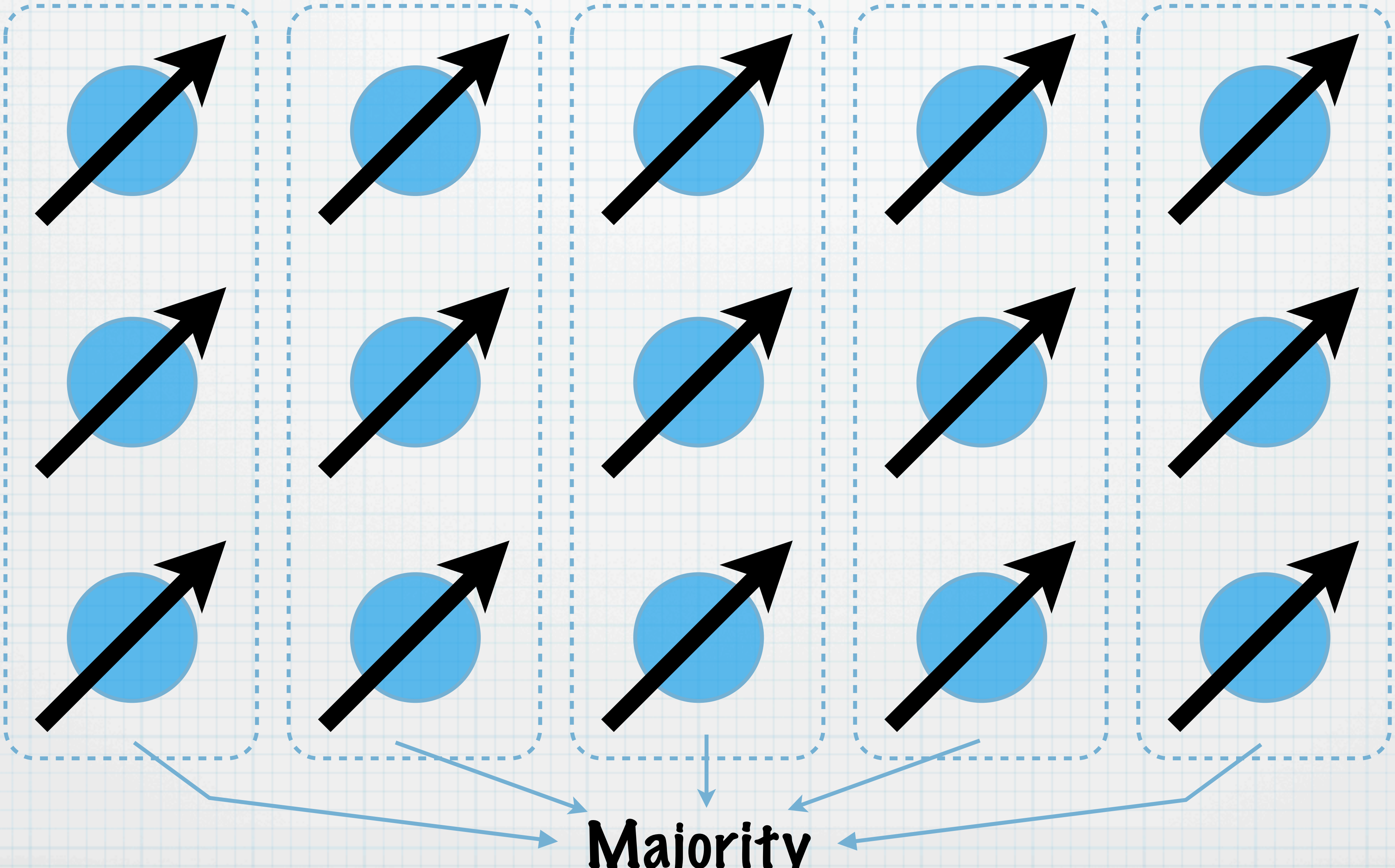
X Measurement



X Measurement



X Measurement



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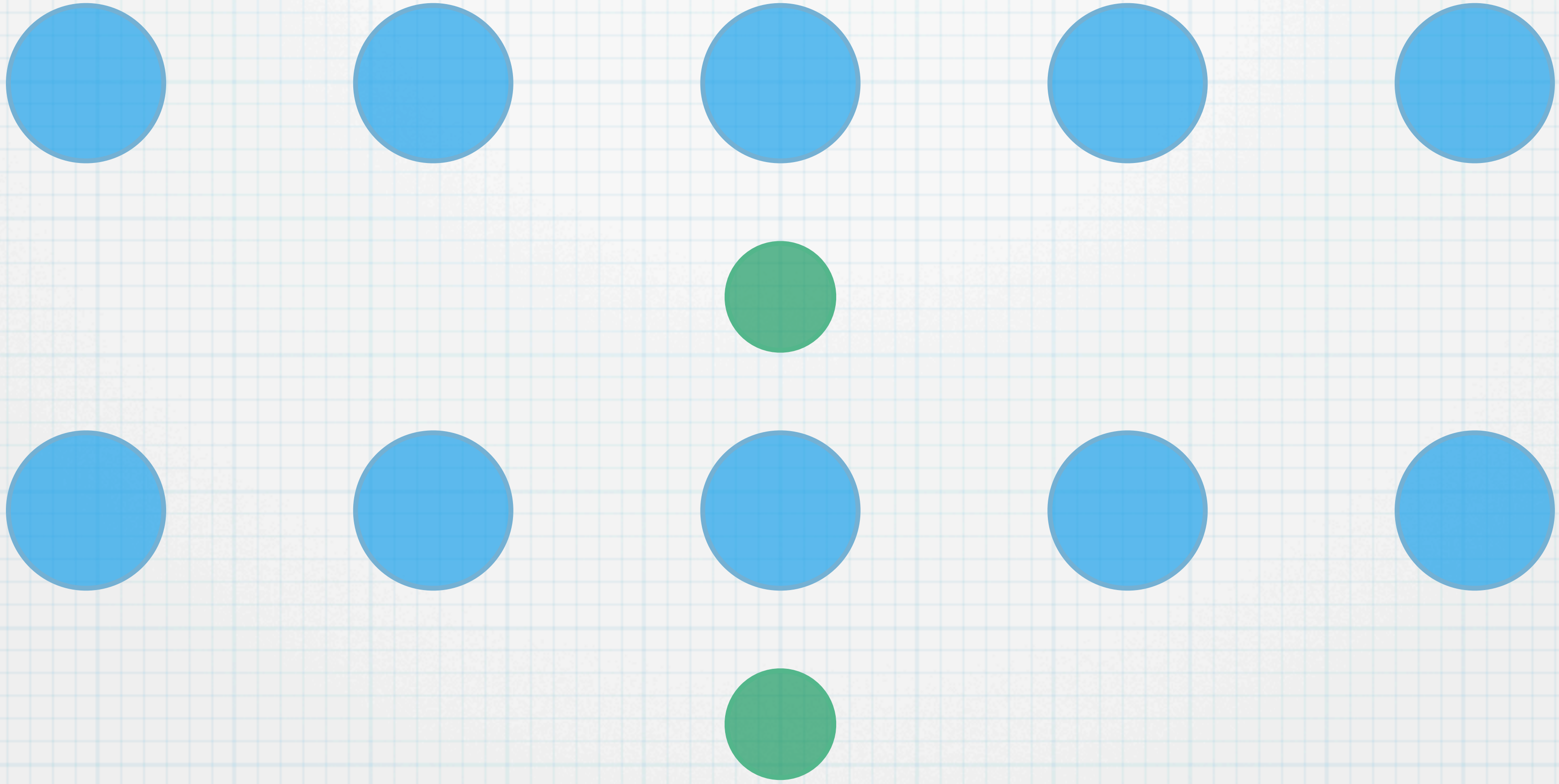

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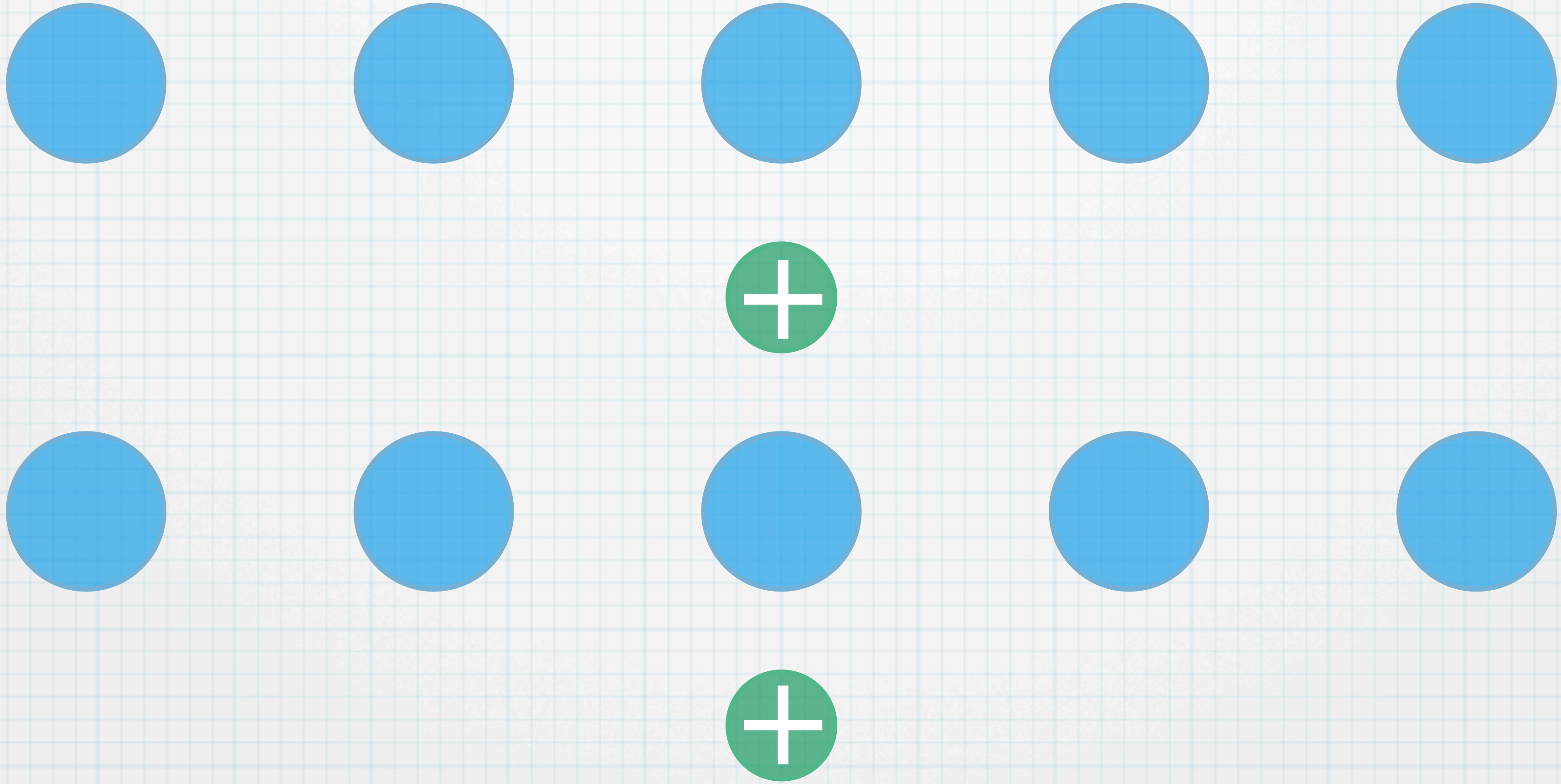

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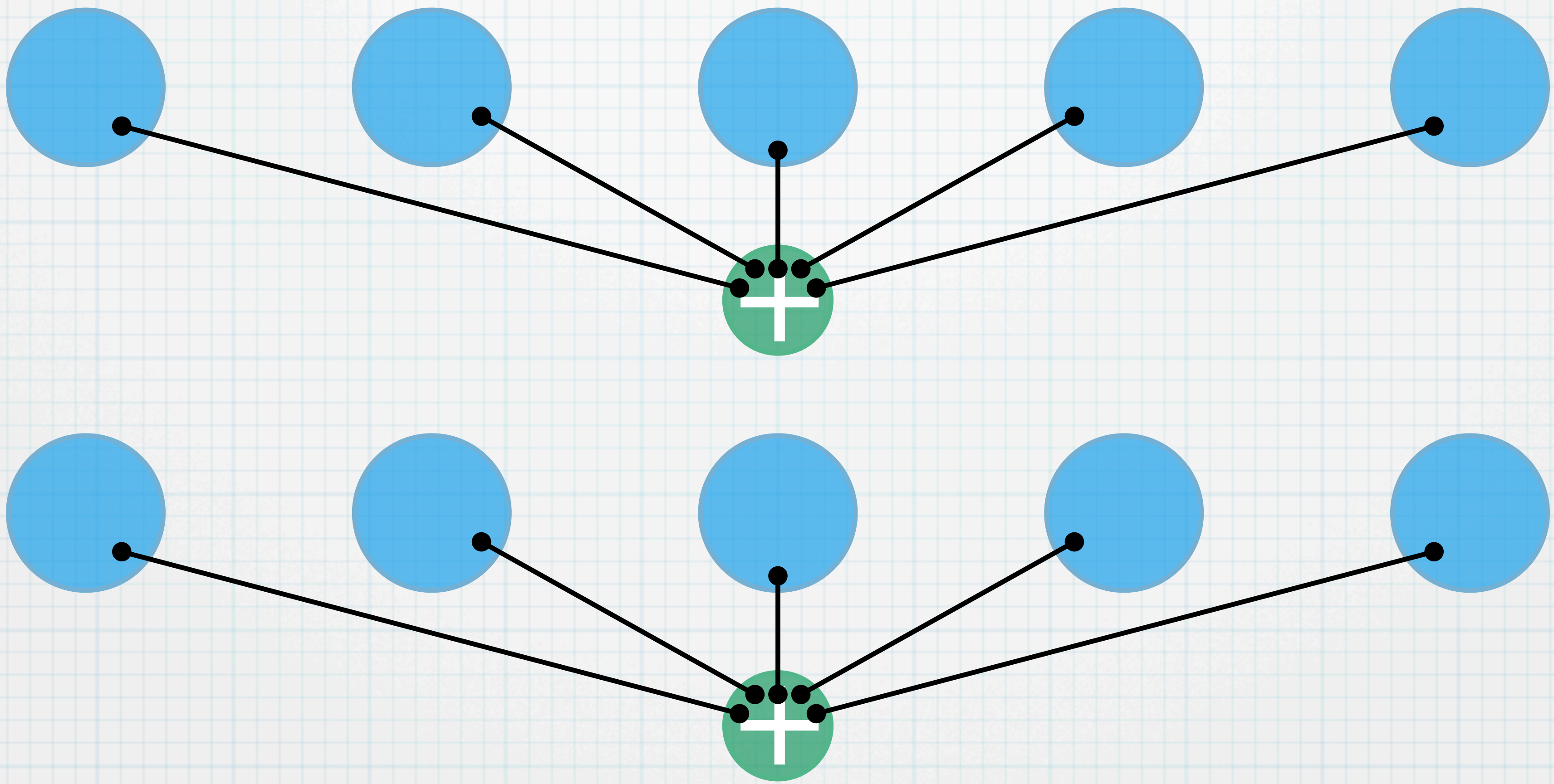

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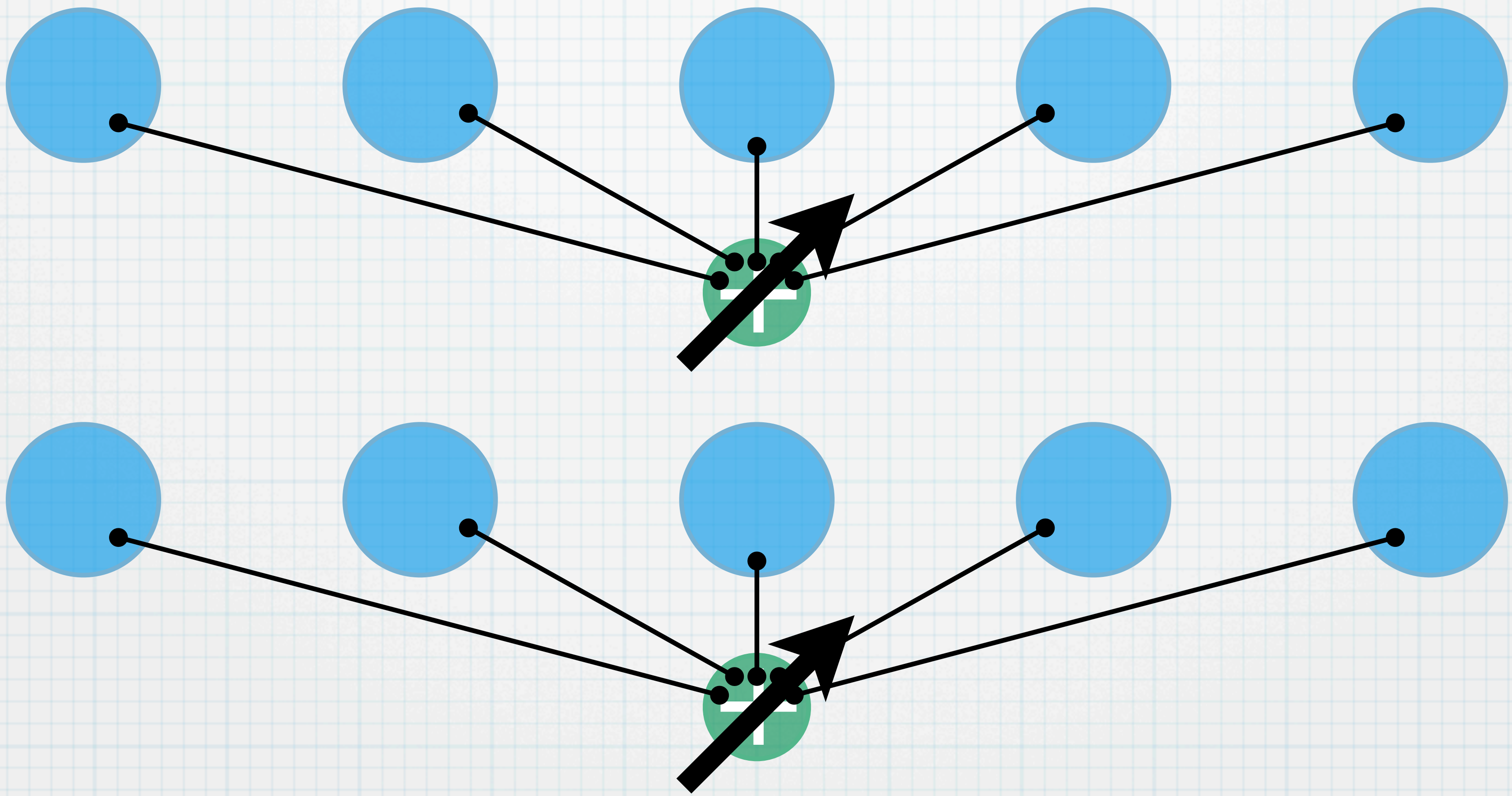
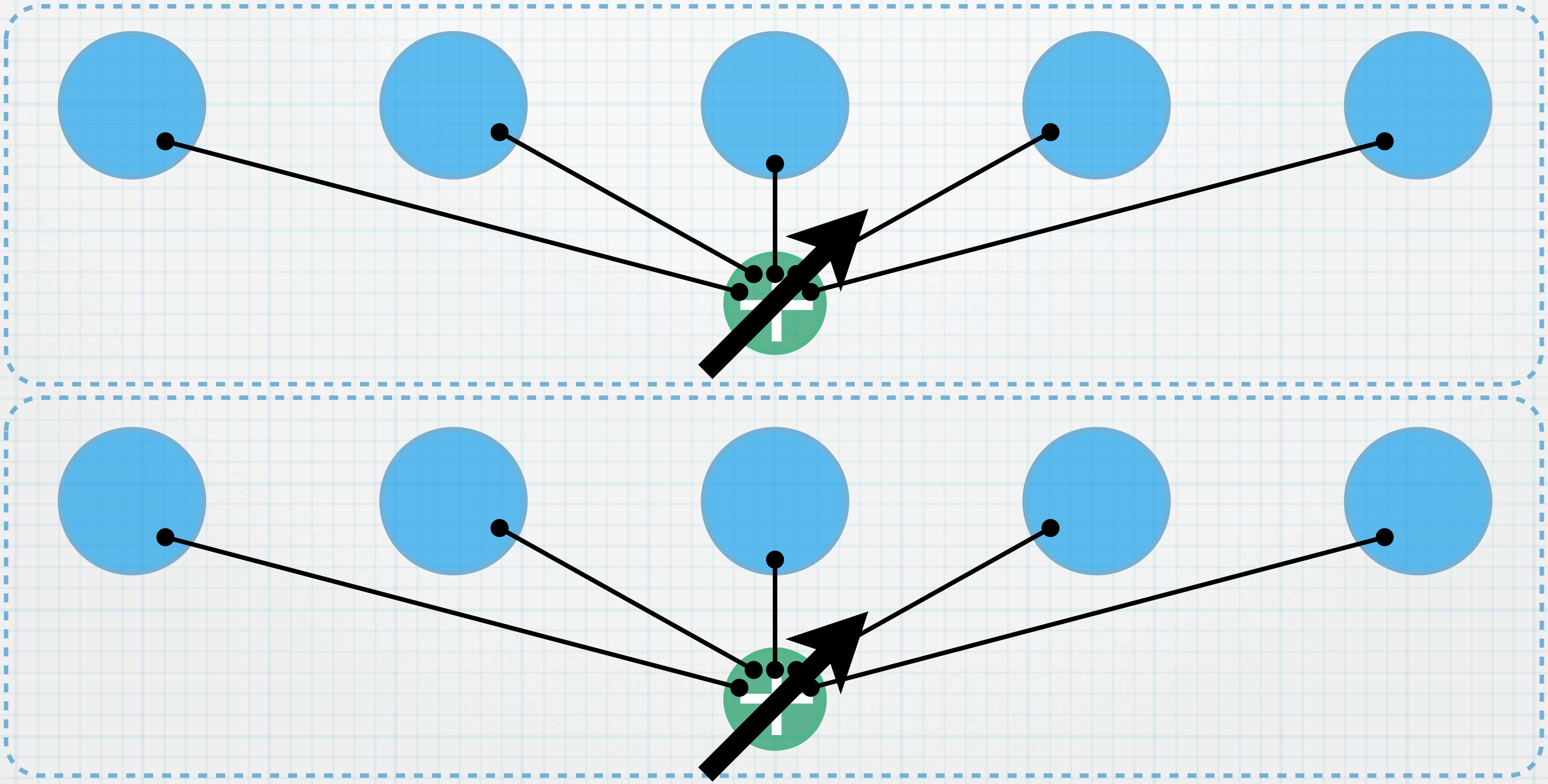


Z Measurement



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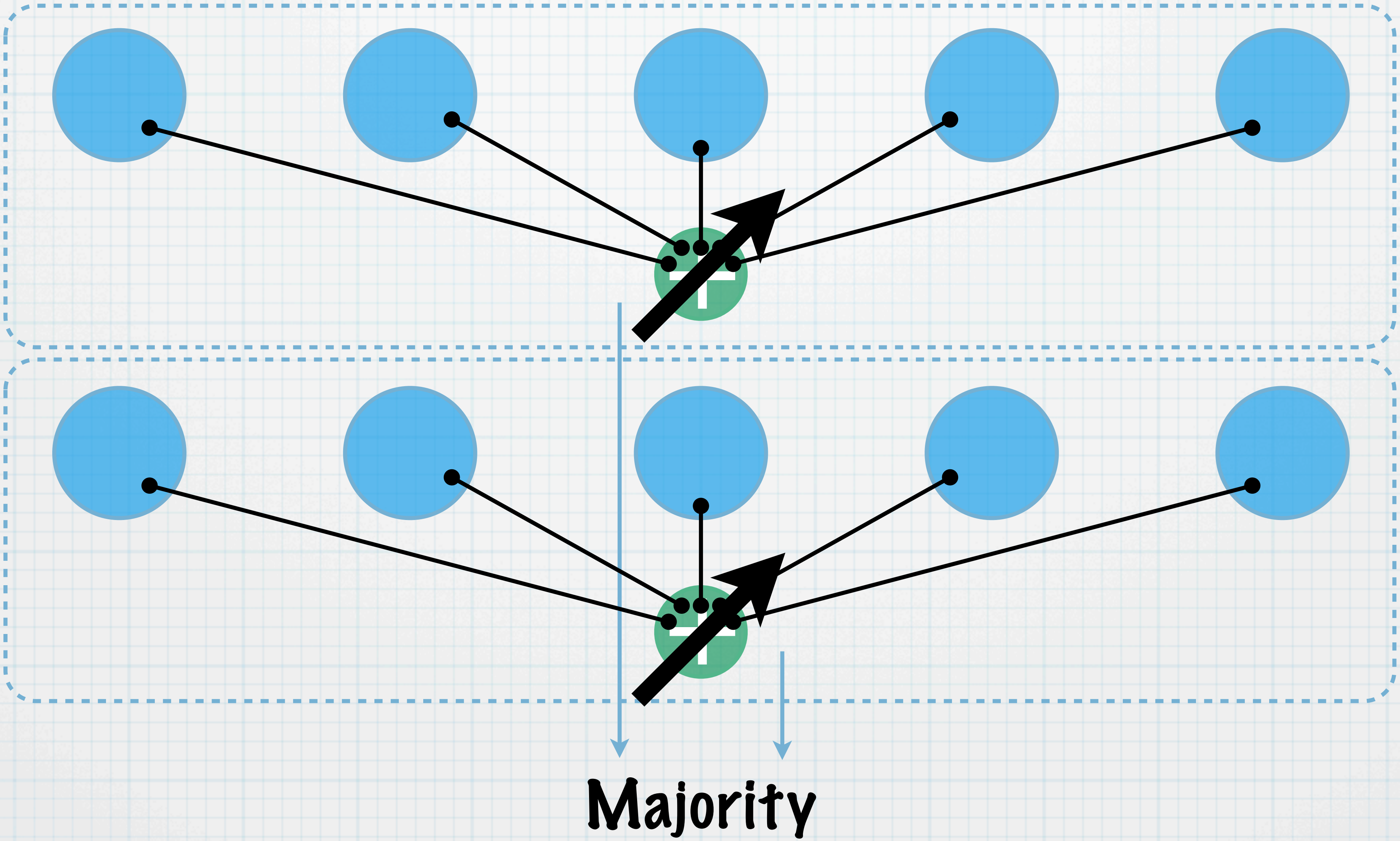
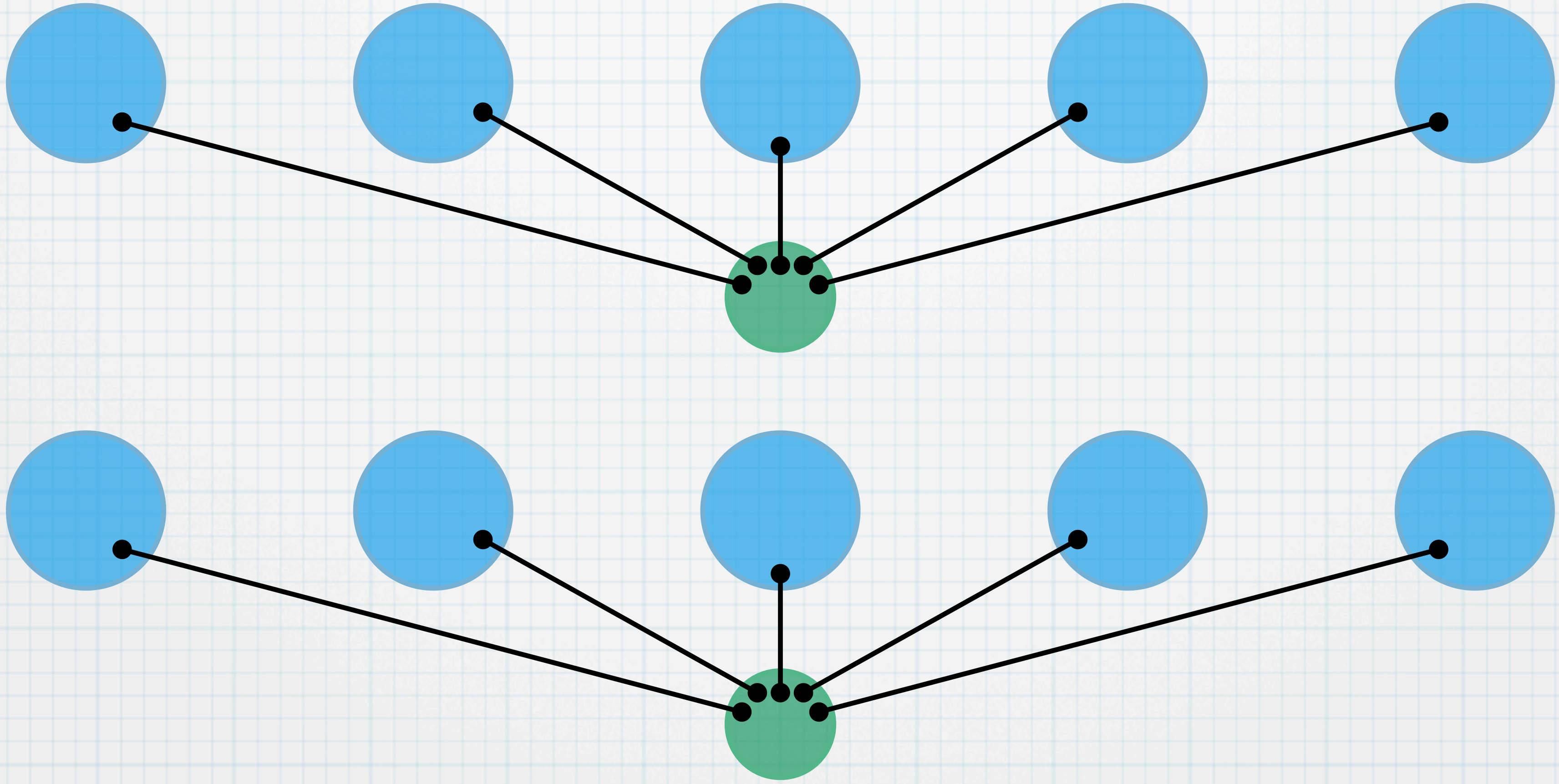
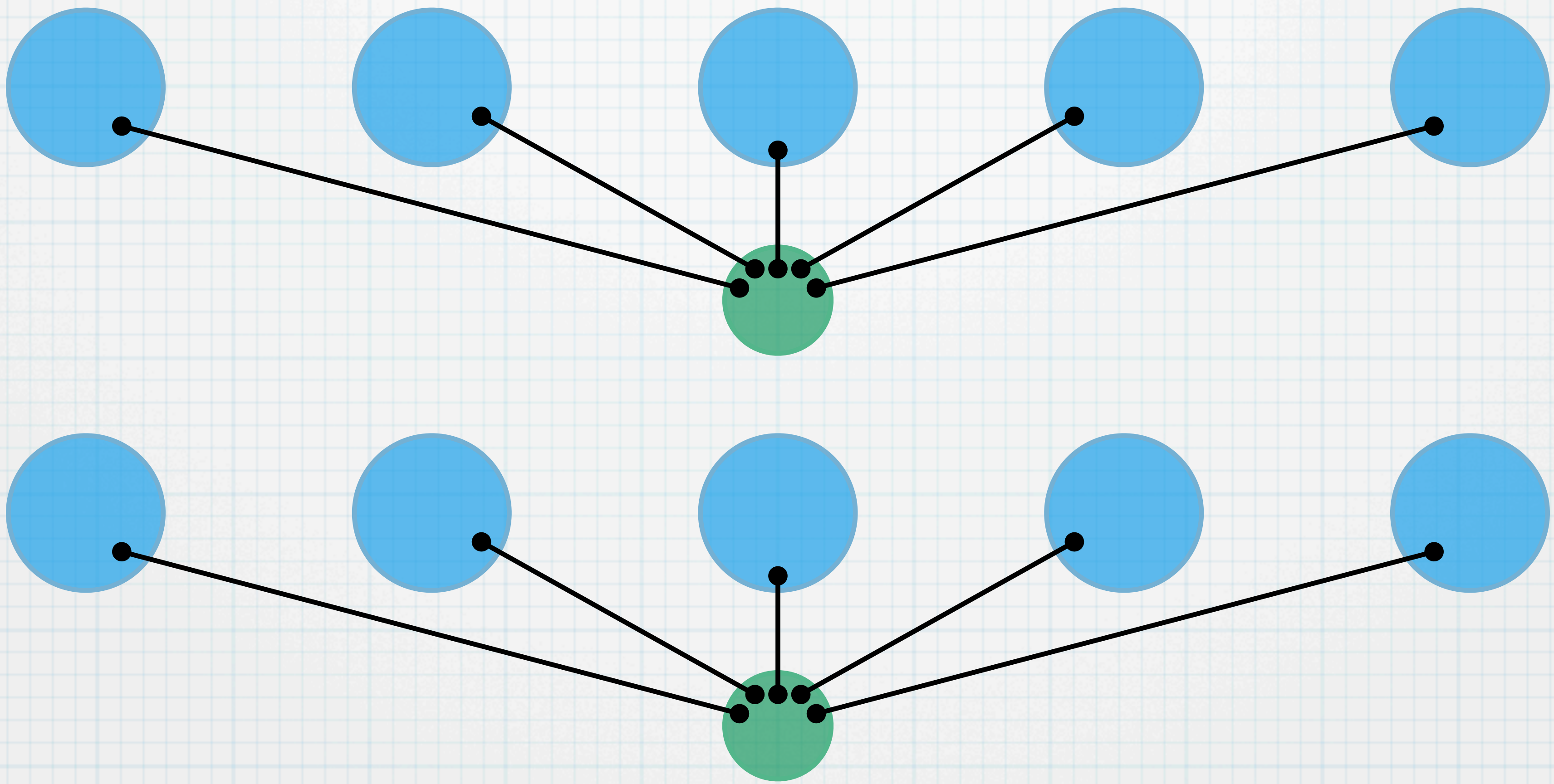


Z Measurement

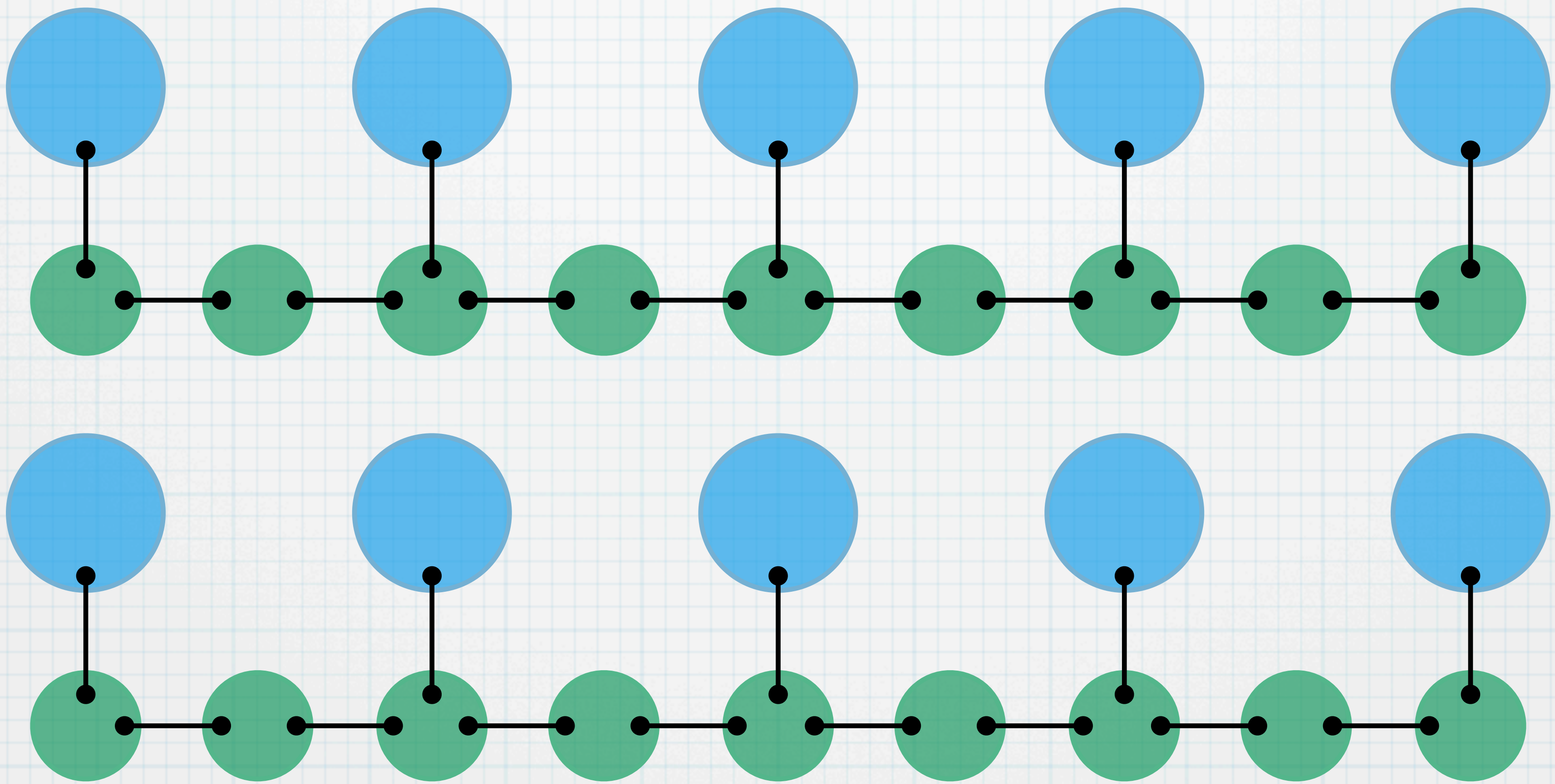


Z Measurement

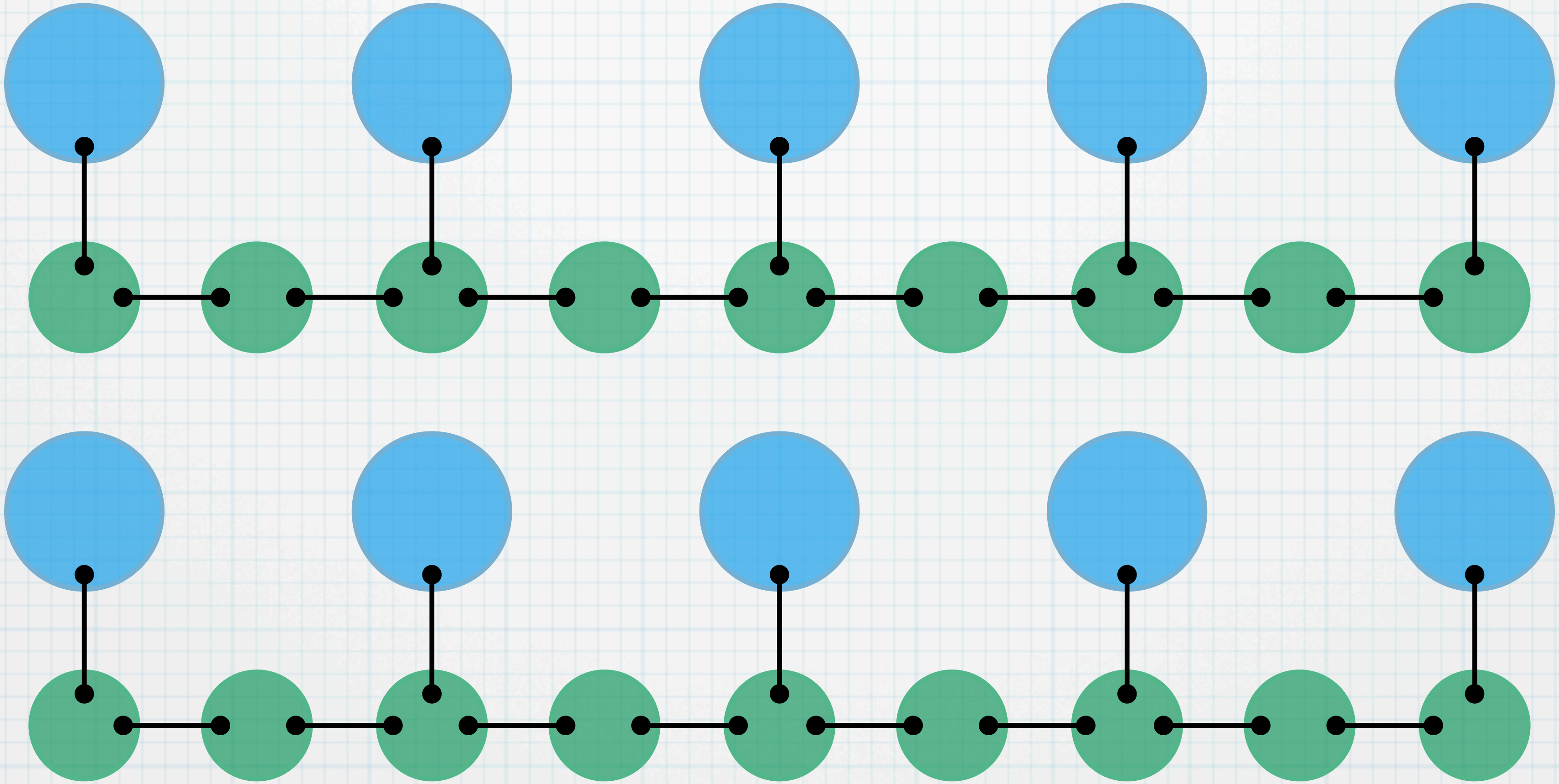


Not fault-tolerant!

Z Measurement

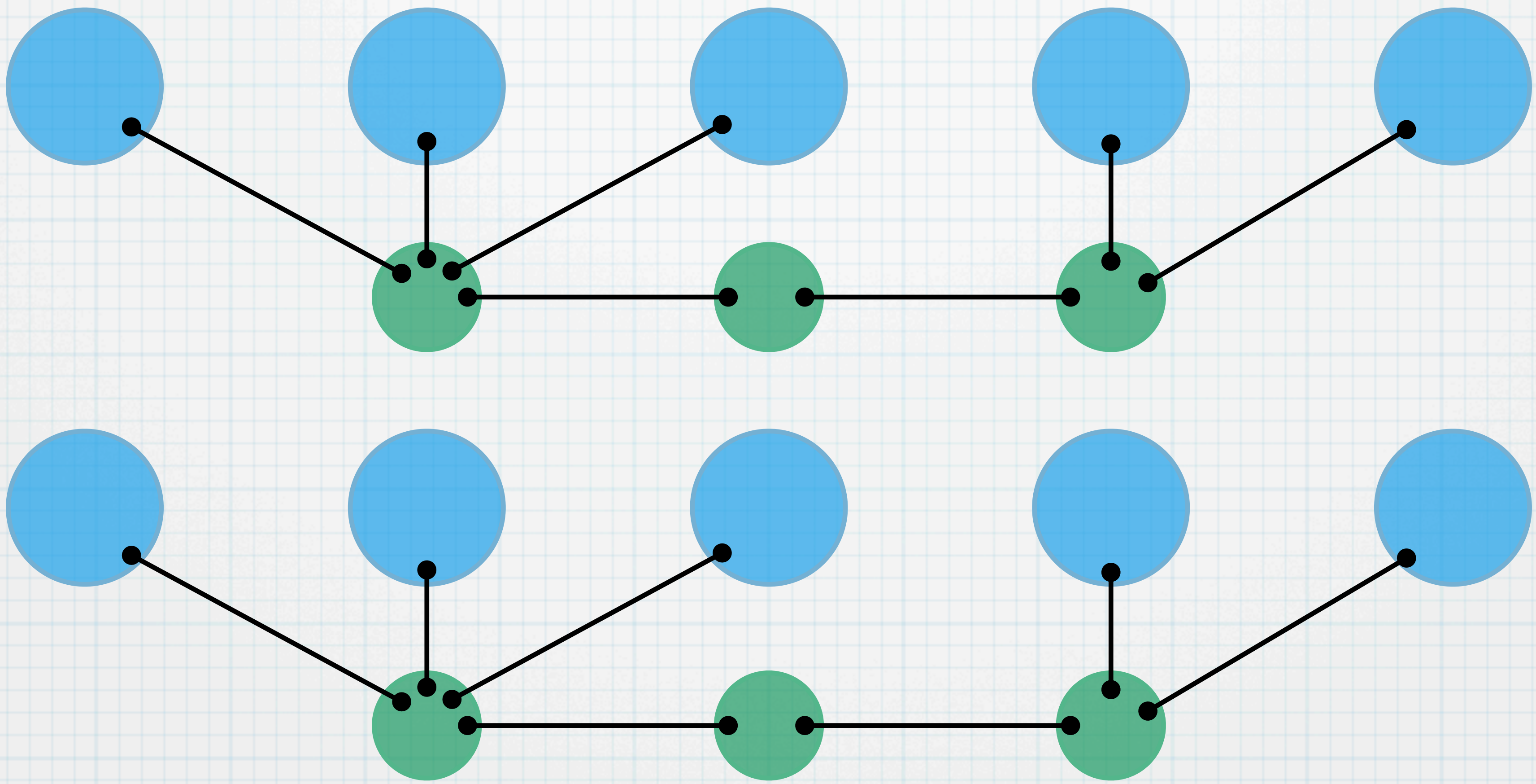


Z Measurement

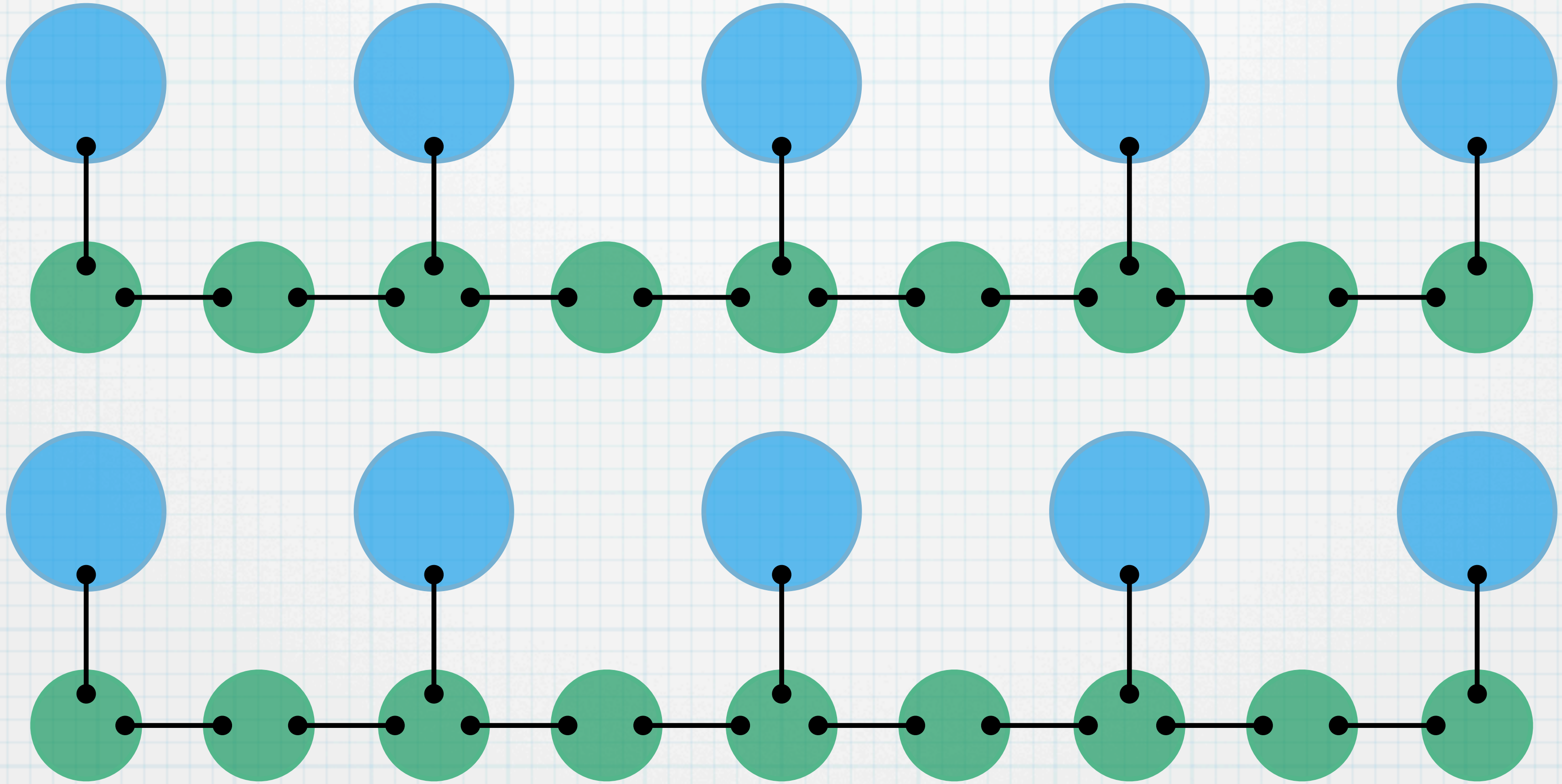


Cat states are large!

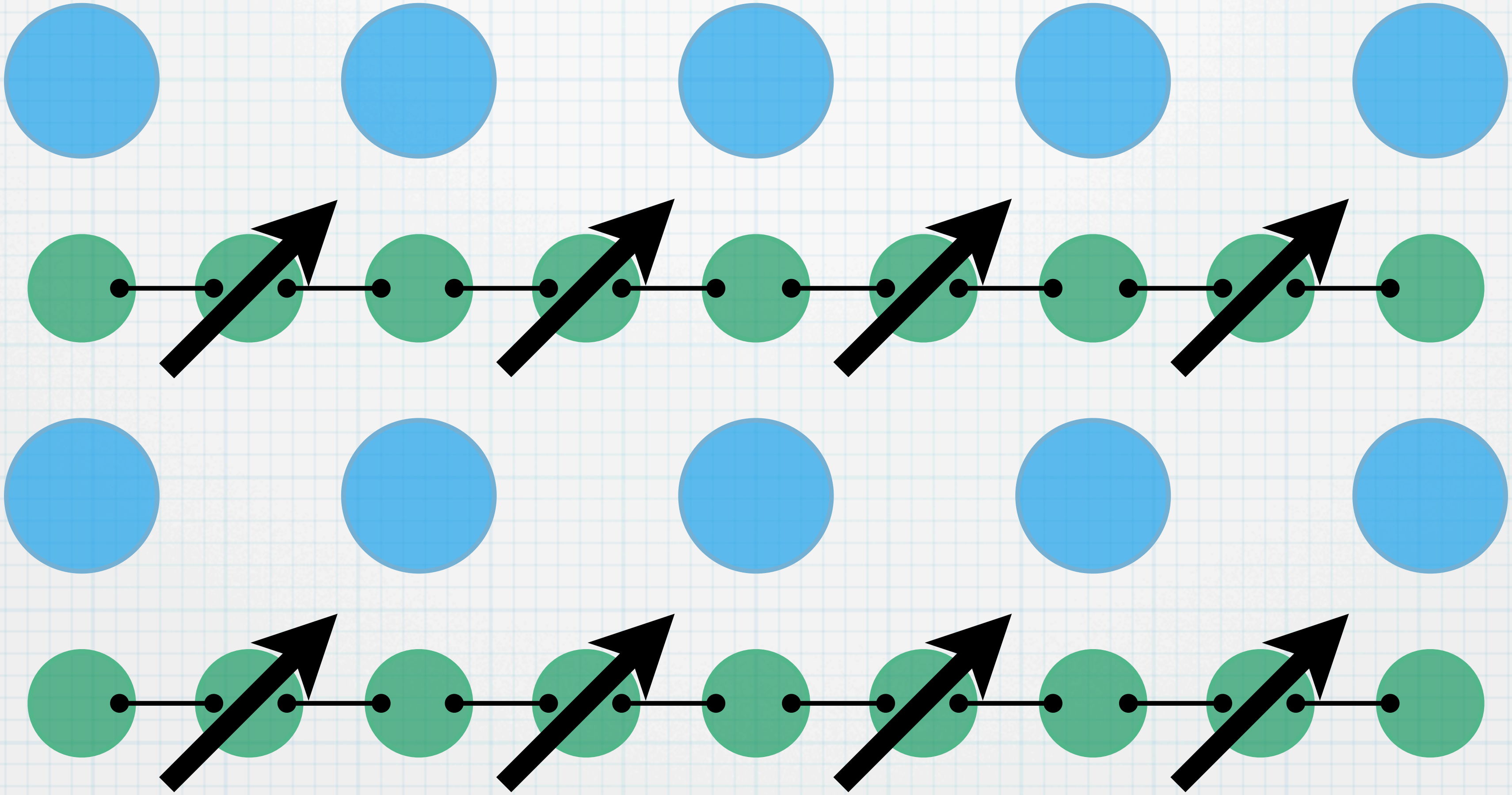
Z Measurement



Z Measurement



Z Measurement



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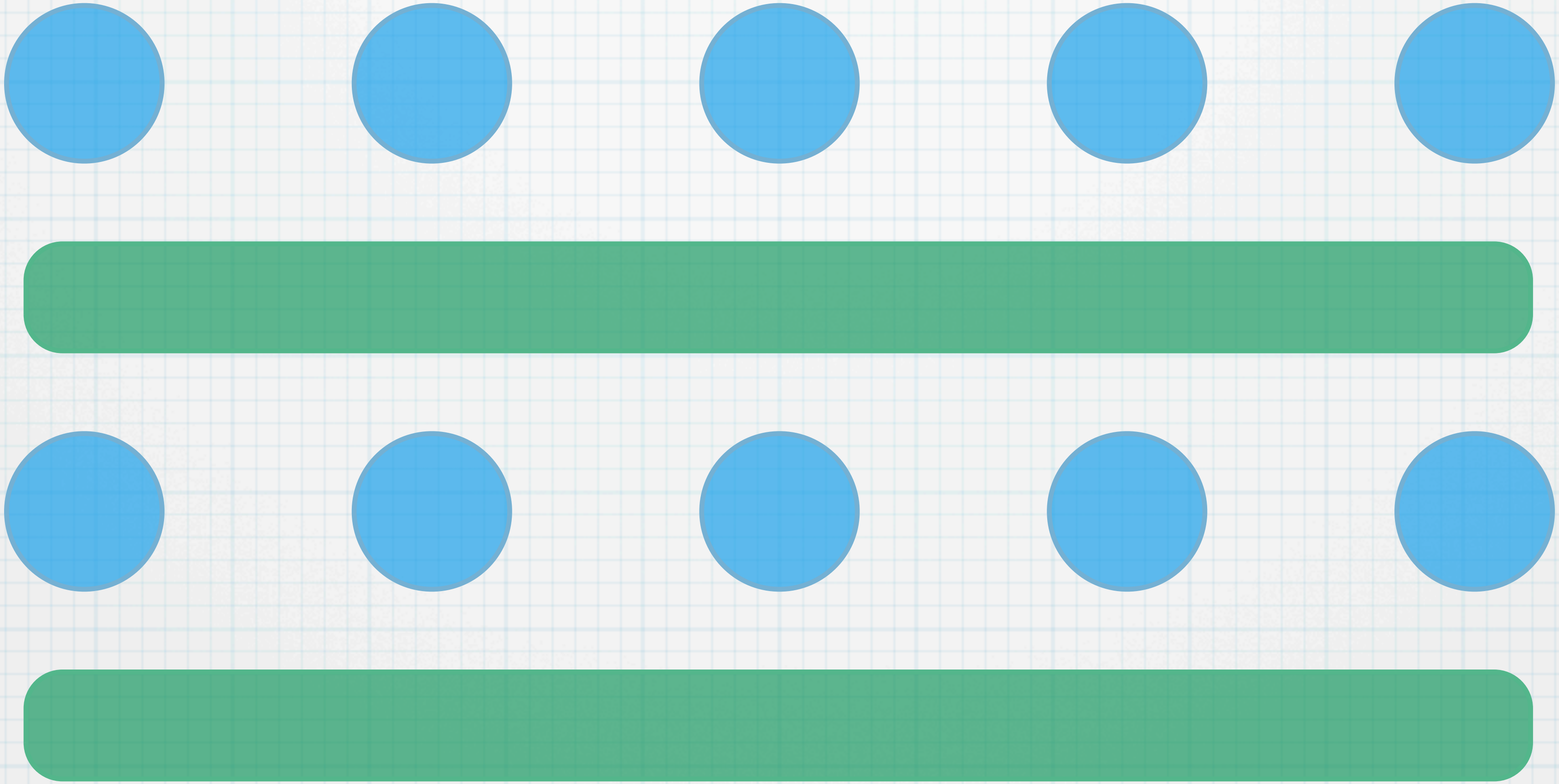
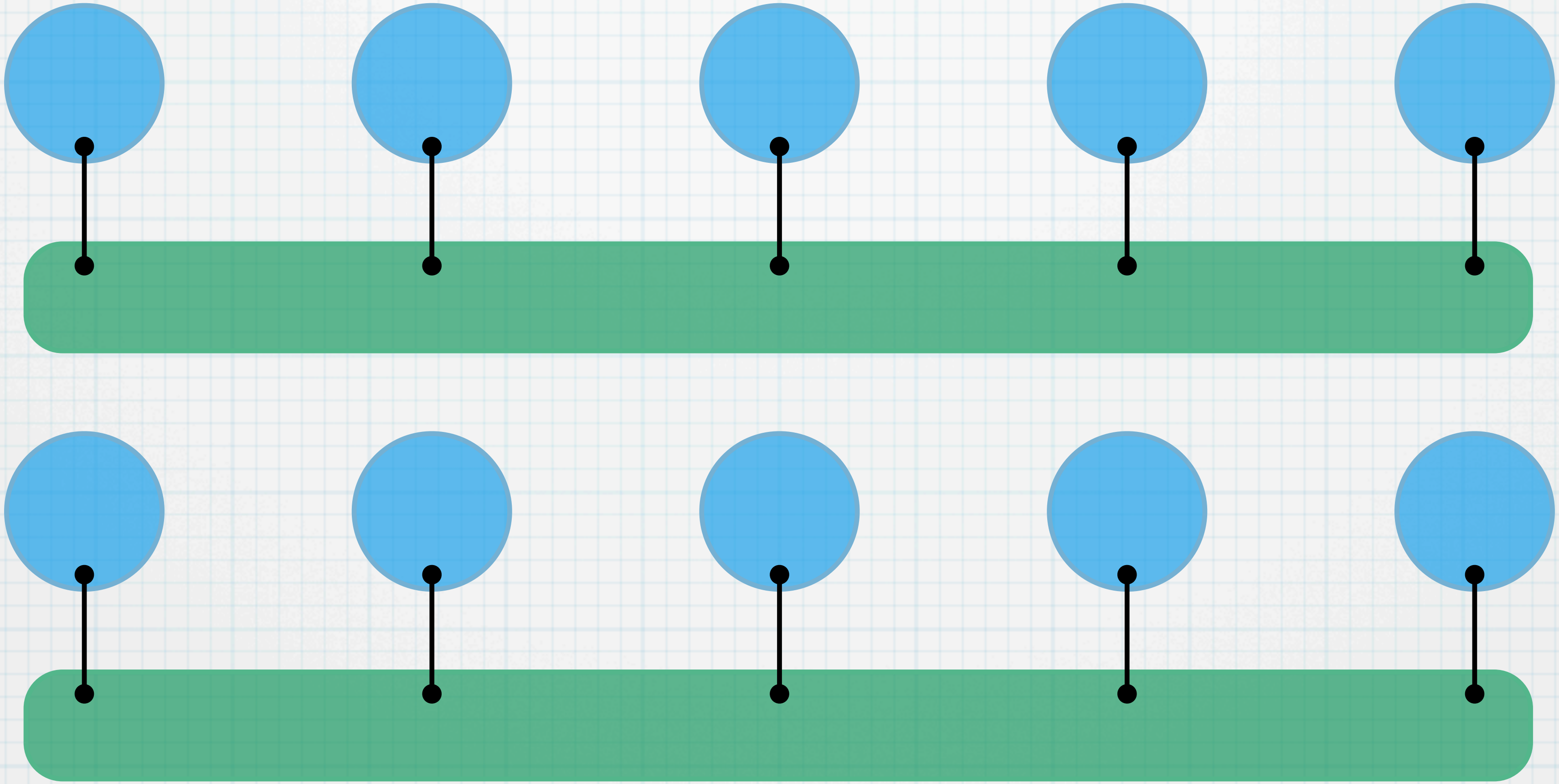
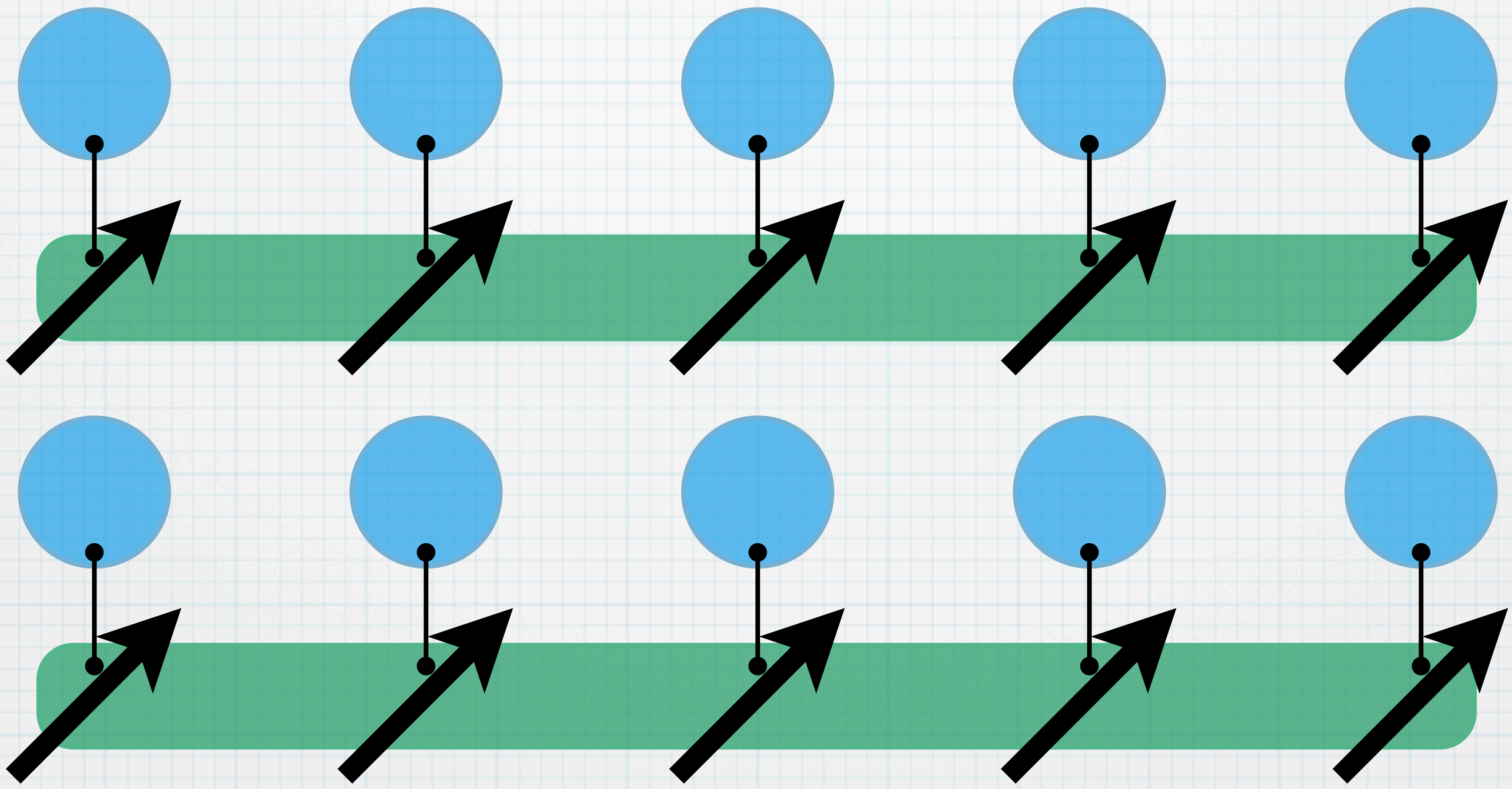


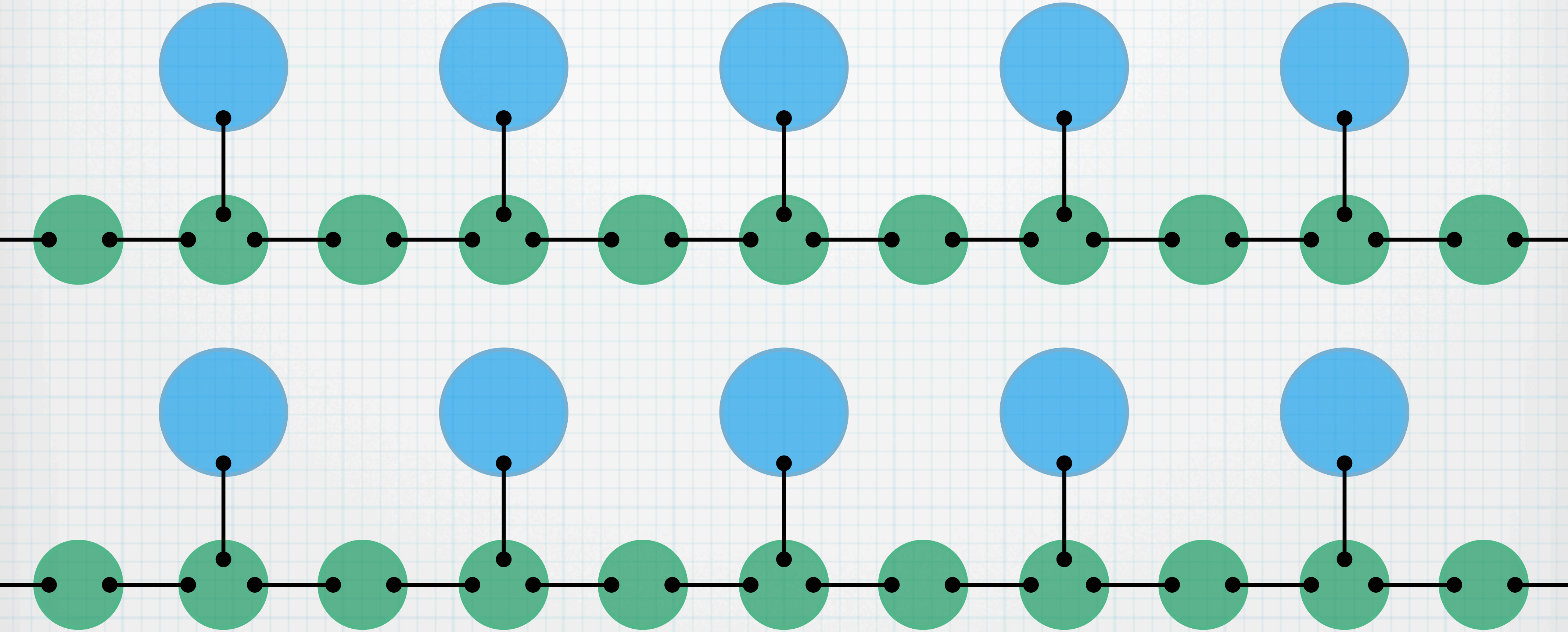
Z Measurement



Z Measurement



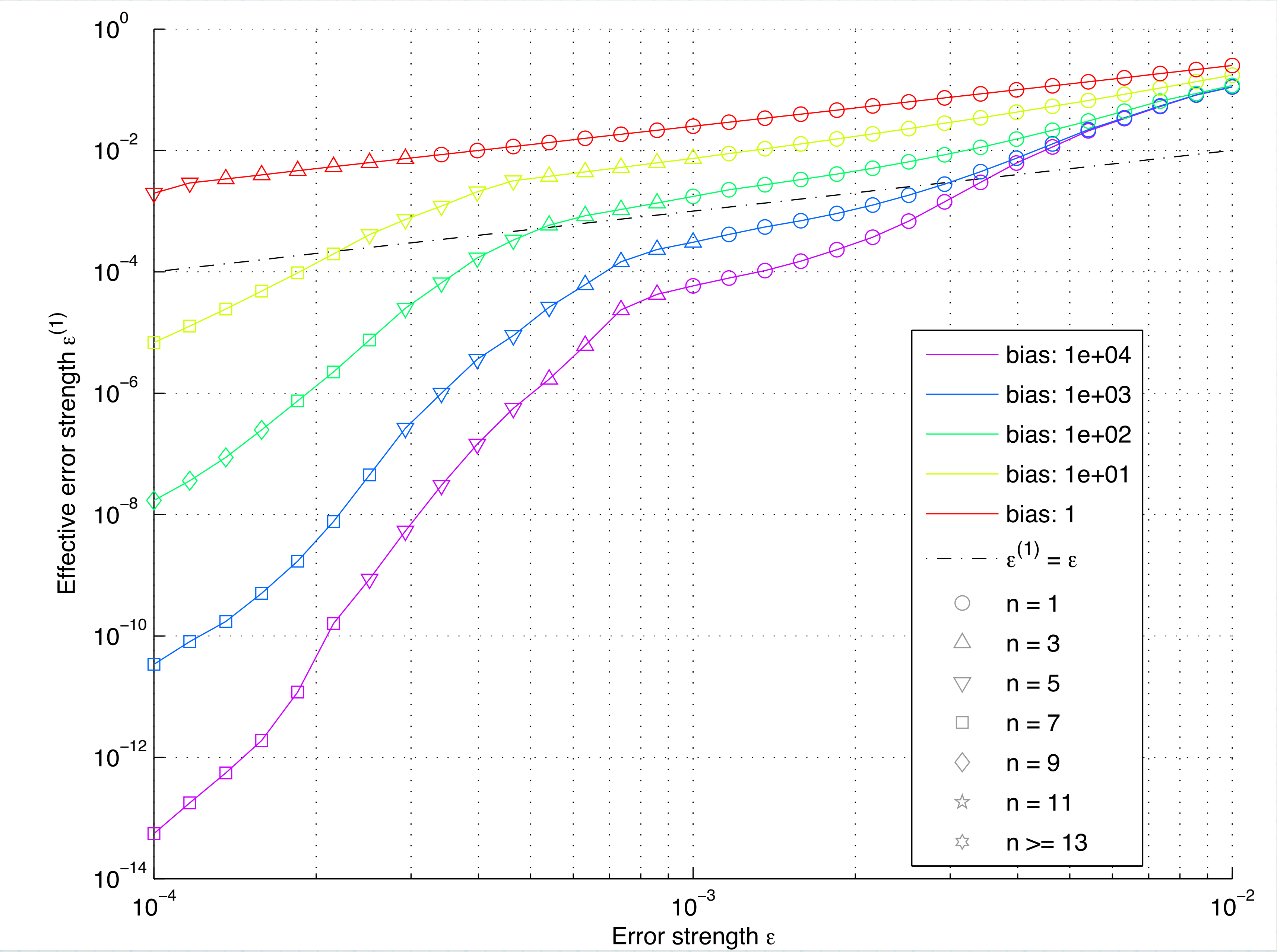
ZZ and ZZZ Measurement



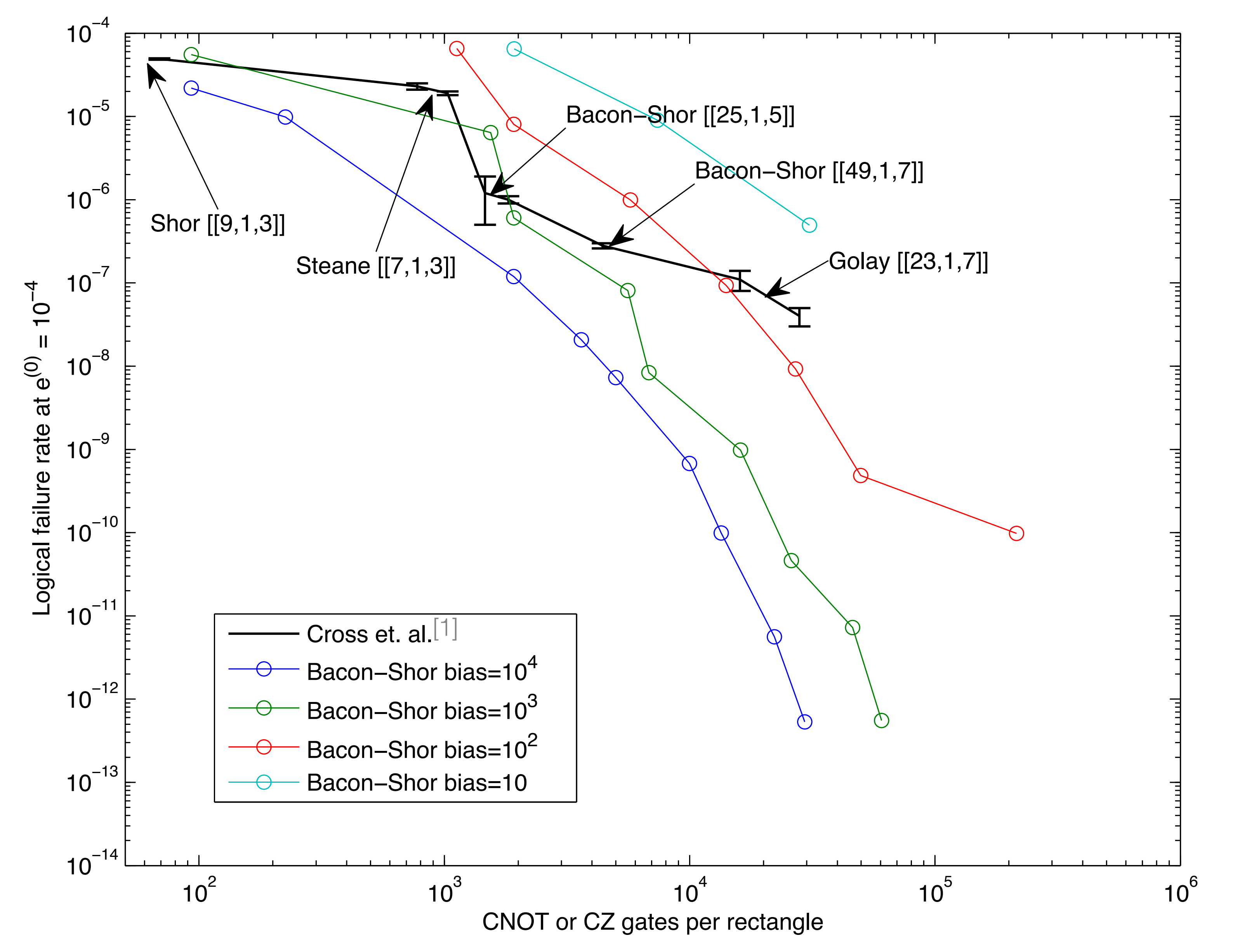
Error Analysis

- Analyze noise under local stochastic biased noise model with error rates ε (dephasing) and ε' (arbitrary)
- Define bias $b = \varepsilon/\varepsilon'$
- A key difficulty is ensuring that cat states are prepared correctly
- Arrive at an analytic upper bound on effective noise strength for given code block size (n,m) and measurement repetition rates
- Brute-force search for best parameters

Error performance



Resource requirements



[1] Cross et. al., arXiv:0711.1556v2 (1995)

Summary

- Designed fault-tolerant gadgets for asymmetric Bacon-Shor codes
- Provable upper bound on the error rate
- Achieve significant reduction in error strength for a modest number of gates
- Possible to layout qubits and gates in a geometrically local fashion

Thank you!