Quantum Neural Network and Soft Quantum Computing

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A new paradigm of quantum computing, namely, soft quantum computing, is proposed for nonclassical computation using real world quantum systems with naturally occurring environment-induced decoherence and dissipation. As a specific example of soft quantum computing, we suggest a quantum neural network, where the neurons connect pairwise via the "controlled Kraus operations", hoping to pave an easier and more realistic way to quantum artificial intelligence and even to better understanding certain functioning of the human brain. Our quantum neuron model mimics as much as possible the realistic neurons and meanwhile, uses quantum laws for processing information. The quantum features of the noisy neural network are uncovered by the presence of quantum discord and by non-commutability of quantum operations. We believe that our model puts quantum computing into a wider context and inspires the hope to build a soft quantum computer much earlier than the standard one.

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Conventional (hard) computing is characterized by precision, certainty, and rigor. By contrast, "soft computing" [1, 2] is an approach to computing, which mimics the human thinking to learn and reason in an environment of imprecision, uncertainty, and partial truth, yet to achieve tractability, robustness, and low solution cost. The applications of soft computing cover various areas of fuzzy logic, neural networks, evolutionary computing, rough sets, and other similar techniques to address real world complexities. Another paradigm shift in computing is the exploitation of quantum computing, first introduced by Feynman in 1982. In its circuit-based model, quantum computing [3] consists of initializing input qubits, manipulating those qubits with a fixed sequence of quantum logic gates programmed by a specific quantum algorithm, and finally measuring all the output qubits. While recent years have witnessed remarkable progresses on physical implementations of quantum computing, there still exist a number of significant challenges in building a large-scale quantum computer. The main technical challenges include the high-precision initialization and readout of qubits well isolated from environment, high-precision quantum logic gates, and scalability of the physical qubits to a large-scale quantum computing device. Quantum error correction can correct errors occurred during computing, thereby allowing to overcome the decoherence effects caused by the noisy environment or faulty quantum logic gates. However, the required error rate for each gate is extremely low, typically 10^{-4}, which is very hard to fulfill for current quantum computing systems; for a topological approach to quantum computing with much better fault tolerance, see Refs. [4, 5].

A realistic quantum system is always characterized by non-unitary, faulty evolutions and coupled with the noisy and dissipative environment. The real system complexities in quantum domain call for a new paradigm of quantum computing, namely, soft (or natural) quantum computing, aiming at nonclassical computation using real world quantum systems with naturally occurring environment-induced decoherence and dissipation. Thus, by its very definition, soft quantum computing deals with nontrivial (i.e., classically intractable) quantum computing under the conditions of noisy and faulty quantum evolutions and measurements, while being tolerant to those effects that are detrimental for current quantum computing paradigm. Thus, unlike conventional (hard) quantum computing, soft quantum computing does not aim at universal computation, but certain specific computational tasks in an open quantum system.

The most important question that soft quantum computing attempts to address is whether or not this new paradigm shift in computing could help in a better understanding of certain functioning of the human brain ("quantum artificial intelligence"). Thus, in this work we propose a quantum neural network model as an illustration of soft quantum computing. The belief behind the proposal stems from the recent exciting discoveries on possible quantum mechanical effects in biological systems (for a review, see Ref. [6]). If photosynthesis [7] and avian navigation [8] can make use of quantum effects in certain manner, why the human brain—the most marvellous biological device—cannot utilize quantum laws to enhance its computing and reasoning power? Along this line, models of quantum cognition based on, e.g., neuronal microtubules [9] and nuclear spins [10, 11] were envisioned. Here, rather than focusing on physical implementations of quantum functioning of the brain, we are interested in the mathematical model of quantum neural network which works as a soft quantum computer. For a summary of existing quantum neural network models, see Ref. [12].

Mathematically, soft quantum computing starts with

n two-level quantum systems (qubits) coupled with their surrounding environment, and as such the initial state of the n qubits is a mixed state $\rho_{12\ldots n}$ in the computational basis $|0\rangle$ and $|1\rangle$. The evolution for such a soft quantum computer during computing is described by a completely positive Kraus map (denoted by a superoperator $\mathcal{O}$)

$$\mathcal{O}(\rho_{12\ldots n}) = \sum_k \hat{E}_k(t)\rho_{12\ldots n}\hat{E}_k^\dagger(t). \quad (1)$$

Here the quantum operations $\hat{E}_k$, which might be time-dependent, satisfy $\sum_k \hat{E}_k(t)\hat{E}_k^\dagger(t) \le I$, where the equality holds for a trace-preserving map. Important examples of the trace-preserving operations are projective measurements, unitary evolutions and partial tracing. After the noisy evolution, quantum measurement in the computational basis completes the soft quantum computing process. Below, we model neurons as noisy qubits; the network of such noisy qubits under noisy evolution and measurement is a particular model of soft quantum computing.

The human brain can be regarded as a neural network [2, 13] organized in a formidably sophisticated structure and has a huge number ($\sim 10^{11}$) of neurons. A drastically simplified drawing of a neuron is shown in Fig. 1a. A real neuron, as we now understand it, integrates hundreds or thousands of impinging signals through its dendrites. These signals result in a change of the internal cell potential in a complex pattern that depends on the excitatory or inhibitory nature of the synapses where the signals impinge on the cell body. The neuron outputs a signal through its axon to another neuron for processing in the form of an action potential only when its internal potential exceeds certain threshold. Note that a neuron has only a single axon, but a set of dendrites forming a tree-like structure.

Instead of modeling the neurons as in the conventional neural network models, or using the concepts borrowed from hard quantum computing [12], we model the jth neuron within the neural network as a basic unit of a soft quantum computer; the network can have any network architecture, which we do not specify below. As shown in Fig. 1b, in our quantum neural model each neuron has n_j inputs s_i $(i = 1, 2, ..., n_j)$ from possibly n_j other neurons, each of which "interacts" with the jth neuron. The interaction is characterized by a connection superoperator $\mathcal{W}_{ij}(t)$ instead of a connection matrix (or weighted pathways) w_{ij} in the conventional neural networks [2, 13]. Here we have shown the time-dependence of $\mathcal{W}_{ij}(t)$ explicitly. Then we assume that the jth neuron is a logic qubit, consisting of n_j physical qubits. The candidate of these physical qubits could be neuron's ion channels whose two conducting states (open and closed states) qualify them as the two-state systems. The detailed microscopic justification of physical qubits, which will be considered in future, is of course meaningful, but irrelevant here for our

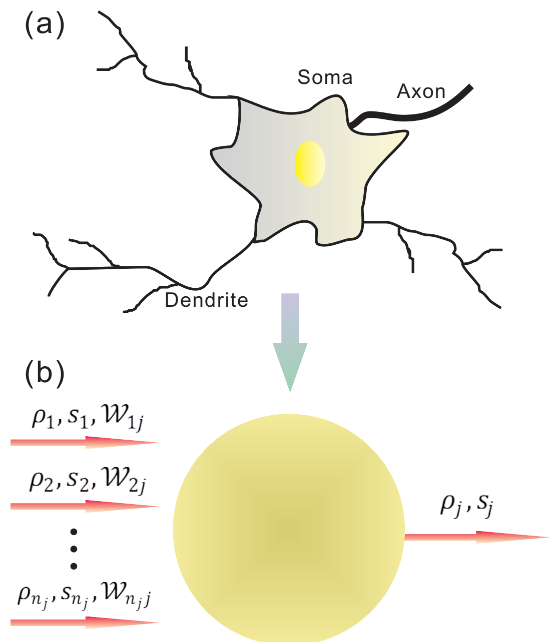

FIG. 1: A drastically simplified drawing of a neuron (a) and the soft quantum neuron model (b). In (a), the neuron integrates hundreds or thousands of impinging signals through its dendrites. After processing by the cell body, the neuron outputs a signal through its axon to another neuron for processing in the form of an action potential when its internal potential exceeds certain threshold.

mathematical formulation of the quantum neuron model. These n_j physical qubits are operated by n_j inputs via the connection superoperator $\mathcal{W}_{ij}(t)$. As in a real neuron integrating impinging signals and outputing a signal when the internal potential of the neuron exceeds certain threshold, here the resulting collective state of the n_j physical qubits represents the integration of impinging signals and determines the output state of the logic qubit. Thus, *we model the quantum neuron at each network node simultaneously as n_j physical qubits and as a logic qubit* such that it can interact with n_j impinging signals and output a *single* qubit state. In this way, the neuron model proposed above mimics as much as possible the realistic neurons and meanwhile, uses quantum laws for processing information.

The logic qubit is initially described by a density matrix ρ_j^{in}. By modeling each neuron as a logic qubit, we only consider binary signals, namely, $s_i = 0$ or 1. Whenever $s_i = 1$, corresponding to the case of a signal input, a quantum operation (here, a superoperator) $\mathcal{W}_{ij}$ will be acted upon ρ_j^{in}, while for $s_i = 0$ (zero input signal), nothing will happen to ρ_j^{in}. Obviously, such a conditional operation, called the "controlled Kraus operation" hereafter, is the generalization of the two-qubit controlled unitary operations in hard quantum computing. Meanwhile, each input neuron itself with input s_i is also a logic qubit in a mixed state ρ_i^{in}. As a result, the evolution of the whole system from the initial state $\bigotimes_{i=1}^{n_j} \rho_i^{in} \otimes \rho_j^{in}$ reads

$$\begin{aligned}\rho_{\{i\}j}^{out} &= \mathcal{T} \bigotimes_{i=1}^{n_j} \mathcal{O}_{ij}(\rho_i^{in} \otimes \rho_j^{in}) \\ &\equiv \mathcal{T} \bigotimes_{i=1}^{n_j} [(\mathcal{P}_{|0\rangle_i} \otimes \hat{I}_j + \mathcal{P}_{|1\rangle_i} \otimes \mathcal{W}_{ij}(t)](\rho_i^{in} \otimes \rho_j^{in}).\end{aligned} \tag{2}$$

Here the "superprojectors" $\mathcal{P}_{|s\rangle}$ are defined by $\mathcal{P}_{|s\rangle}\rho = |s\rangle \langle s| \rho |s\rangle \langle s|$, $\hat{I}$ is the identity operator, and $\mathcal{T}$ represents the time-ordering. Generally speaking, all $\mathcal{W}_{ij}(t)$ are acted upon the target neuron with specific temporal patterns and as such, the time-ordering of these actions is important as different $\mathcal{W}_{ij}(t)$ might be noncommutative. Note that, in addition to the conditional dynamical evolution in Eq. (2), each quantum neuron can also be subject to local noisy operations.

Here some remarks are in order. A neuron of the human brain normally has about 10^4 synapses on average. It is this large number of input neurons that activate the target neuron. According to modern neuroscience, the connection matrix w_{ij} is the mathematical abstraction of the synaptic efficacies of the inter-neuron synapses, while impinging signals mathematically represent action potentials. In classical neuron model, the activation of a real neuron is then determined by the internal integrated cell potential, or mathematically by the signal function and the internal firing threshold. For real neurons, as action potentials and cell potentials change much faster than changes in synaptic efficacies, neuron activations are a fast process, while the changes of synaptic efficacies are a slow one. In classical neural networks, the change of synaptic efficacies is implemented by a "learning algorithm" [2, 13] during a learning process in response to stable patterns of activity.

However, in the context of our current quantum neuron model, the above remarks should be looked from a different angle. Here a learning process is realized by a "quantum learning algorithm" (see, e.g., Ref. [14] for a review), i.e., a specific temporal pattern of quantum operations $\mathcal{W}_{ij}(t)$ depending on the states of input neurons. By using various quantum learning algorithms, the quantum neuron model opens a door to quantum manipulations of real neurons, artificial intelligence, and ultimately, the human brain. By sharp contrast, the existing quantum neural networks [12] model neurons as qubits in pure states and connection weights as usual quantum logic gates. However, the human brain is a highly open and decohering system. It is very unlikely to model the human brain as a well isolated quantum system. In our model, *the human brain is neither a classical computer (soft or hard) nor a standard quantum computer, but something in between, namely, a soft quantum computer*. By modeling neuron nodes as qubits in mixed states and their connections as connection superoperators, we are left to see to what extend our model could simulate and understand the real functioning of the human brain.

We used the superprojectors $\mathcal{P}_{|s\rangle}$ in Eq. (2) to select the computational basis as the input to the jth neuron. In doing so, we have actually implicitly assumed certain environment-induced decoherence [15] naturally occurring in the cell body to choose the preferred basis, namely, the computational basis. To be consistent for our quantum neuron model, we have to determine the output s_j of the jth neuron in the same way. The final state of the jth neuron can be obtained by tracing out all the input states, namely, $\rho_j^{out} = \mathrm{tr}_{\{i\}}\rho_{\{i\}j}^{out} = \mathcal{T}\prod_{i=1}^{n_j}[p_i\hat{I}_j + (1-p_i)\mathcal{W}_{ij}(t)]\rho_j^{in}$, where $p_i \equiv p_i(0) = \mathrm{tr}(|0\rangle_i \langle 0| \rho_i^{in})$ is the probability of $|0\rangle_i$ within ρ_i^{in}. The output state is then

$$\rho_j = (\mathcal{P}_{|0\rangle_j} + \mathcal{P}_{|1\rangle_j})\rho_j^{out}. \tag{3}$$

In other words, the output signal s_j of the jth neuron is

$$s_j = \begin{cases} 0 & \text{with probability } p_j(0) \\ 1 & \text{with probability } 1 - p_j(0) \end{cases} \tag{4}$$

This completes the specification of our quantum neural network model as a soft quantum computer. Interestingly, the above prescriptions on the input and output states in the preferred computational basis tacitly assumed each neuron in our model as a quantum self-measuring meter. This eliminates the ambiguity of designating signal functions of various forms in classical neural networks [13].

Physically, one could regard each input dendrite as a quantum channel [3], which carries out a quantum manipulation $\mathcal{W}_{ij}$ if and only if $s_i = 1$. The depolarizing channel can be represented by $\mathcal{W}_{dp}(\rho) = \sum_\mu M_\mu \rho M_\mu^\dagger$ with $M_0 = \sqrt{1-p}\hat{I}$ and $M_k = \sqrt{\frac{p}{3}}\sigma_k$ ($k = 1, 2, 3$; $0 \le p \le 1$), where σ_k are the Pauli operators. For a phase-damping channel $\mathcal{W}_{pd}(\rho) = \sum_\mu M_\mu \rho M_\mu^\dagger$, where $M_0 = \sqrt{1-p}\hat{I}$, $M_1 = \frac{\sqrt{p}}{2}(1+\sigma_3)$, and $M_2 = \frac{\sqrt{p}}{2}(1-\sigma_3)$, while for an amplitude-damping channel $\mathcal{W}_{ad}(\rho) = \sum_\mu M_\mu \rho M_\mu^\dagger$ one has two Kraus operators $M_0 = |0\rangle \langle 0| + \sqrt{1-p}\,|1\rangle \langle 1|$ and $M_1 = \sqrt{p}\,|0\rangle \langle 1|$.

Now it is ready to discuss some key features of the current quantum neural network or soft quantum computing. First we consider the simplest two-neuron case. For the two neurons in the initial states $\rho_1^{in} = p_1 |0\rangle_1 \langle 0| + (1-p_1) |1\rangle_1 \langle 1|$ ($p_1 \neq 0, 1$) and ρ_2^{in}, the action of a controlled Kraus operation $\mathcal{O}_{12}$ results in the final state

$$\begin{aligned}\rho_{12}^{out} &= (\mathcal{P}_{|0\rangle_1} \otimes \hat{I}_2 + \mathcal{P}_{|1\rangle_1} \otimes \mathcal{W}_{12})(\rho_1^{in} \otimes \rho_2^{in}) \\ &= p_1 |0\rangle_1 \langle 0| \otimes \rho_2^{in} + (1-p_1) |1\rangle_1 \langle 1| \otimes \mathcal{W}_{12}(\rho_2^{in}),\end{aligned} \tag{5}$$

where $\mathcal{W}_{12}$ represents a specific quantum channel. Is ρ_{12}^{out} nonclassically (or, quantum) correlated? Quantum correlations, if any, of ρ_{12}^{out} can be quantified by the quantum discord [16]. Any bipartite state is called fully classically correlated if it is of the form [17]

$$\rho_{12} = \sum_{i,j} p_{ij} \left|i\right\rangle_1 \left\langle i\right| \otimes \left|j\right\rangle_2 \left\langle j\right| ; \tag{6}$$

otherwise, it is quantum correlated. Here $\left|i\right\rangle_1$ and $\left|j\right\rangle_2$ are the orthonormal bases of the two parties, with nonnegative probabilities p_{ij}.

Obviously, for the two-neuron state ρ_{12}^{out} in Eq. (5) the first neuron becomes correlated with nonorthogonal states of the second neuron as far as $\mathcal{W}_{12}(\rho_2^{in})$ and ρ_2^{in} are nonorthogonal [18–20], namely, ρ_{12}^{out} has quantum correlations. In particular, Refs. [19, 20] show the creation of discord, from classically correlated two-qubit states, by applying an amplitude-damping process only on one of the qubits; for the phase-damping process, see Ref. [17]. Actually, ρ_{12}^{out} in Eq. (5) is the *classical-quantum state*, as dubbed in Ref. [19]—While for measurements on neuron-1 the discord is zero, measurements on neuron-2 in general lead to non-zero discord.

Thus, we reveal the first important feature of our neural network (a soft quantum computer). Namely, the neural network can develop quantum correlations although only mixed initial states and very noisy operations are available. Consequently, if soft quantum computing does mimic the working mechanism of the human brain, the brain is certainly quantum as there are quantum correlations among the neurons therein.

Now let us consider the three-neuron case. We are interested in two particular examples, where two controlled Kraus operations $\mathcal{O}_{13}$ and $\mathcal{O}_{23}$ are performed in different orders, namely, $\rho_{123}^{out} = \mathcal{O}_{13}\mathcal{O}_{23}(\rho_1^{in}\otimes\rho_2^{in}\otimes\rho_3^{in})$ and $\rho_{123}^{\prime out} = \mathcal{O}_{23}\mathcal{O}_{13}(\rho_1^{in}\otimes\rho_2^{in}\otimes\rho_3^{in})$. Assuming $\rho_1^{in} = p_1\left|0\right\rangle_1\left\langle 0\right| + (1-p_1)\left|1\right\rangle_1\left\langle 1\right|$ and $\rho_2^{in} = p_2\left|0\right\rangle_2\left\langle 0\right| + (1-p_2)\left|1\right\rangle_2\left\langle 1\right|$ yields

$$\begin{aligned}\rho_{123}^{out} &= p_1(1-p_2)\left|0\right\rangle_1\left\langle 0\right|\otimes\left|1\right\rangle_2\left\langle 1\right|\otimes\mathcal{W}_{23}(\rho_3^{in}) \\ &+ p_2(1-p_1)\left|1\right\rangle_1\left\langle 1\right|\otimes\left|0\right\rangle_2\left\langle 0\right|\otimes\mathcal{W}_{13}(\rho_3^{in}) \\ &+ p_1p_2\left|0\right\rangle_1\left\langle 0\right|\otimes\left|0\right\rangle_1\left\langle 0\right|\otimes\rho_3^{in} + (1-p_1) \\ &\times(1-p_2)\left|1\right\rangle_1\left\langle 1\right|\otimes\left|1\right\rangle_2\left\langle 1\right|\otimes\mathcal{W}_{13}\mathcal{W}_{23}(\rho_3^{in}).\end{aligned} \tag{7}$$

Meanwhile, it is easy to check that $\rho_{123}^{\prime out}$ can be obtained from ρ_{123}^{out} only by reversing the orders of $\mathcal{W}_{13}$ and $\mathcal{W}_{23}$ in the last term of Eq. (7). If $(1-p_1)(1-p_2)\neq 0$, then for noncommutative $\mathcal{W}_{13}$ and $\mathcal{W}_{23}$ the resulting states ρ_{123}^{out} and ρ_{123}^{out} are different from each other. The importance of the quantum operation orders is a quantum feature, which remains in our quantum neural network model. Using the national representative surveys (Gallup polls) and laboratory experiments, a recent result [21] reported an evidence supporting a quantum model of question order effects for human judgments. Whether or not our quantum neural network model is a quantum mechanical foundation for such a result is certainly an interesting future problem.

Open quantum systems have been considered for various applications in quantum information processing by engineering environment [22, 23] and in particular, for mixed-state quantum computing [24–27] or dissipative quantum computing [28, 29]. While soft quantum computing is no more powerful than the unitary circuit model as implied by the "dissipative quantum Church-Turing theorem" [29], there does exist a mixed-state quantum computing model (with a collection of qubits in the completely mixed state coupled to a single control qubit with nonzero purity), proposed by Knill and Laflamme [24] and known as deterministic quantum computation with one qubit (DQC1), which provides exponential speedup (for estimating the normalized-trace of a unitary matrix) over the best known classical algorithms. The possible role of quantum discord was considered, e.g., in Refs. [18, 30, 31], as a figure of merit for characterizing the nonclassical resources present in the DQC1, which itself is a special soft quantum computer. Thus, it is reasonable to expect that soft quantum computing has computational power between classical computation and usual quantum computation.

Finally, we give a brief remark on the physical implementation of soft quantum computing. Above discussions focus mainly on a special form of the conditional dynamical evolution as in Eq. (2); more general forms can of course be envisioned. A physical realization of $\mathcal{O}_{ij}$ is simple: The superprojectors $\mathcal{P}_{\left|s\right\rangle_i}$ correspond to a von Neumann measurement on neuron-i along the computational basis; conditionally on the measurement results, an identity operation $\hat{I}$ or $\mathcal{W}_{ij}(t)$ is acted upon neuron-j. Such a conditional dynamical evolution is experimentally friendly to implementation based on any quantum computing systems under current investigation, such as linear optics, superconducting qubits, atoms/ions, and quantum dots.

To summarize, we have proposed soft quantum computing as nonclassical computation using real world quantum systems with naturally occurring decoherence and dissipation induced by environment. A mathematical model of quantum neural network is suggested to illustrate soft quantum computing. Even using very simple and noisy operations [the controlled Kraus operations in Eq. (2)], the quantum features, such as the presence of quantum discord and non-commutability of quantum operations, remain in our model. As an experimentally friendly model, the neural network as proposed mimics as much as possible the realistic human brain and thus, paves the way to quantum artificial intelligence and to better understanding the working mechanism of the human brain. If the human brain does be a soft quantum computer, it utilizes quantum laws in such a fundamental way that it forms certainly a quantum correlated entity. This might be the reason why the human brain is much more powerful for certain tasks than classical computers, although we have to know more consequences and applications of the model to arrive at a conclusive result. While the soft quantum computer is much easier to build than the standard one, we need to do more works to understand what kinds of computational tasks it can execute and to find the corresponding efficient algorithms.

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