Effective Compilation of Higher-Order Programs

Roland Leißa Klaas Boesche Sebastian Hack Richard Membarth Arsène Pérard-Gayot Philipp Slusallek

http://compilers.cs.uni-saarland.de

https://github.com/AnyDSL/thorin

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Introduction

Intermediate Representations (IRs)

imperative languages C, Fortran, ...

- · instruction lists + CFGs
 - · LLVM
 - · GIMPLE (gcc)

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- graph-based IRs "sea of nodes" [Click95]
 - Java Hotspot
 - libFirm
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 - GIMPLE (gcc)
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functional languages Haskell, ML, ...

- λ-calculus
 - · Core (GHC)
 - · Lambda IR (OCaml)
 - · Continuation Passing Style (CPS) [Appel06]

```
struct Node {
    int data;
    Node* left;
    Node* right;
};
void post_order_visit(Node* n) {
    if (n->left)
        post_order_visit(n->left, f);
    if (n->right)
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    cout << n->data << endl;</pre>
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- 2. Higher-order Functions

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void sum(Node* n) {
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Compiling

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clang -03 -fno-exceptions

post_order_visit with clang -03 -fno-exceptions

```
ontry:

%_args.addr.i = alloca 122, align 4

%agg.tmp = alloca %'class.std::function', align 8

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%left = retelementate inhounds %atturct.Node, %attuct.Node %n. 864 0, 122 1
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   ".eds) - pateinesight inbunds Sainet Node, ; preds - Sd.; then.17, %_PREHimeticalFolicCKEXS1_exit, Sentry
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   f.end.if.end6_crit_edge: preds = %if.end %preds = %if.end %pre = getelementptr inbounds %"class.std::function", %"class.std::function" %if, 864 0, 122 0, 122 1 br label %if.end6
   . proc. vs. the participating behavior V structural fraction. Not assume that the process of the participating behavior v structural fraction. Not assume that the participation of the participation 
   50 - bitcast (1 (%minor.std:,Any_data", %minor.std:,Any_data", (32)" %_M_manager.i.H8 to <2 x (64 > store <2 x (64 > 64 > 65 < x (64 > 64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <2 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <3 x (64 > 65 ), align 8, than 111 ; Any_data", (32) ** %_M_manager.i.H8 to <3 x (64 > 65 ), 
   SHI * load II (N'usion.sid::_Asy_data", N'usion.sid::_Asy_data", N'usio
       them.122: preds = %_ZNStefunctionFviECCERES1_exit27
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   %call 121 = call zerost il %if(%'union.std::_Any_data'" dereferenceable(16) %_M_functor.130, %'union.std::_Any_data'" dereferenceable(16) %_M_functor.130, 122 2) #2 fr label %if:_end
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(and supplies flower starties, p. 192).

(blue distribution starties, p. 192).

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```

print with clang -03 -fno-exceptions

```
define void @_ZSprintP4Node(%struct.Node* nocapture readonly %n) #3 (
                So lead if [Simina.td:], Ang.data'', Simina.td:], Ang.data'', Ang.data'', Simina.td:], Ang.data'', Ang.data'', Ang.data'', Ang.data'', Ang.data'', Ang.data'', Ang.data'', Ang.data'', Ang.data'', A
                otny:
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30 - Sadi 12, 172 * S_exp, sligs 4, 1thus 114
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31 - Sadis 1, 1 - Sadis 13, 12 * Sadis 8, 1thus 114
31 - Sadis 1, 1 - Sadis 13, 12 * Sadis 8, 1thus 114
31 - Sadis 1, 1 - Sadis 13, 12 * Sadis 8, 1thus 114
31 - Sadis 1, 1 - Sadis 13, 12 * Sadis 8, 1thus 114
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define internal zeroszt i/ @_ZMS14_Function_basel_Ense_managerIZSprintFiNodeIX_618_M_managerEXSt9_Any_detaBESS_518_Manager_operation*("S'mains.txt::_Jen_etas" neceptorenecebbs(16) %_deta, "window.txt::_Jen_etas" neceptorenecebbs(16) %_deta."
                w.bb: ; pords = Nestry | ; pords
ns. bbl:

%1 = bbl:cast %'enion.std:_Ary_data'* %_dest to %'enion.std:_Ary_data'**

store %'enion.std:_Ary_data'* %_secre, %'enion.std:_Ary_data'** %1, align 8, ithea ill

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```

Working with higher-order Functions

 A Graph-Based Higher-Order Intermediate Representation Leißa, Köster, and Hack.
 CGO 2015

Working with higher-order Functions

- A Graph-Based Higher-Order Intermediate Representation Leißa, Köster, and Hack.
 CGO 2015
- Shallow Embedding of DSLs via Online Partial Evaluation Leißa, Boesche, Hack, Membarth, and Slusallek. GPCE 2015.

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void range(int a, int b,
       function<void(int)> f)
{
    if (a < b) {
       f(a);
        range(a+1, b, f);
void foo(int n) {
    range(0, n, [=] (int i) {
        use(i, n);
   });
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```
void (*f)(void* c, int i);
};
struct closure {
    closurebase base:
    int n;
};
void lambda(void* c, int i) {
    use(i, (closure* c)->n);
void range(int a, int b, void* c) {
    if (a < b) {
        ((closurebase*) c)->f(c, a);
        range(a+1, b, c);
void foo(int n) {
    closure c = {{&lambda}, n};
    range(0, n, &c);
}
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struct closurebase {

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    closurebase base:
void lambda(void* c, int i) {
    use(i, (closure* c)->n);
void range(int a, int b, void* c) {
        ((closurebase*) c)->f(c, a);
        range(a+1, b, c);
    closure c = {{&lambda}, n};
    range(0, n, &c);
```

```
void (*f)(void* c, int i);
                                  };
                                  struct closure {
                                      closurebase base:
                                      int n;
void range(int a, int b,
                                  };
       function<void(int)> f)
                                  void lambda(void* c, int i) {
    if (a < b) {
                                      use(i, (closure* c)->n);
        f(a);
        range(a+1, b, f);
                                  void range(int a, int b, void* c) {
                                      if (a < b) {
                                          ((closurebase*) c)->f(c, a);
void foo(int n) {
                                          range(a+1, b, c);
    range(0, n, [=] (int i) {
        use(i, n);
    });
                                  void foo(int n) {
                                      closure c = {{&lambda}, n};
                                      range(0, n, &c);
                                  }
```

```
};
                                  struct closure {
                                      closurebase base:
                                      int n;
void range(int a, int b,
                                  };
       function<void(int)> f)
                                  void lambda(void* c, int i) {
    if (a < b) {
                                      use(i, (closure* c)->n);
        f(a);
        range(a+1, b, f);
                                  void range(int a, int b, void* c) {
                                      if (a < b) {
                                          ((closurebase*) c)->f(c, a);
void foo(int n) {
                                          range(a+1, b, c);
    range(0, n, [=] (int i) {
        use(i, n);
    });
                                  void foo(int n) {
                                      closure c = {{&lambda}, n};
                                      range(0, n, &c);
                                  }
```

struct closurebase {

void (*f)(void* c, int i);

```
};
                                  struct closure {
                                      closurebase base:
                                      int n;
void range(int a, int b,
                                  };
       function<void(int)> f)
                                  void lambda(void* c, int i) {
    if (a < b) {
                                      use(i, (closure* c)->n);
        f(a);
        range(a+1, b, f);
                                  void range(int a, int b, void* c) {
                                      if (a < b) {
                                          ((closurebase*) c)->f(c, a);
void foo(int n) {
                                          range(a+1, b, c);
    range(0, n, [=] (int i) {
        use(i, n);
    });
                                  void foo(int n) {
                                      closure c = {{&lambda}, n};
                                      range(0, n, &c);
                                  }
```

```
void (*f)(void* c, int i);
```

 \cdot inline the call to the closure's function pointer

- inline the call to the closure's function pointer
- SSA-construct the closure struct
- dissolve the **struct** to scalar values (Scalar Replacement of Aggregates)

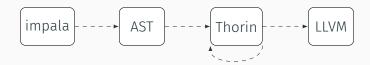
- inline the call to the closure's function pointer
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- inline the call to the closure's function pointer
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- usually works well for typical STL algorithms
- · fails for recursive higher-order functions like
 - · range
 - \cdot post_order_visit



- reimplement for every front-end
- taints the IR with implementation of higher-order functions
 - · bloats the IR
 - set of finely tuned analyses & transformations needed for optimization



- Thorin = higher-order + CPS + "sea of nodes"
- directly translate higher-order functions and calls to Thorin
- · keep higher-order functions till late during compilation
- powerful closure-elimination phase

```
int foo(int n) {
   int a;
   if (n==0) {
      a = 23;
   } else {
      a = 42;
   }
   return a;
}
```

```
int foo(int n) {
    int a;
    if (n==0) {
        a = 23;
    } else {
        a = 42;
    }
    return a;
}
```

```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = \( \phi(23 \) [then], 42 [else]);
    return a;
}
```

```
int foo(int n) {
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next:
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int foo(int n) {
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   goto next;
next:
   int a = \( \phi(23 \) [then], 42 [else]);
   return a;
}
```

```
foo(n: int, ret: int \rightarrow \bot) \rightarrow \bot:

let

then() \rightarrow \bot:

next(23)

else() \rightarrow \bot:

next(42)

next(a: int) \rightarrow \bot:

ret(a)

in

branch(n==0, then, else)
```

```
\begin{array}{l} \text{foo(n: int, ret: int} \rightarrow \bot) \rightarrow \bot: \\ \text{let} \\ \text{then()} \rightarrow \bot: \\ \text{next(23)} \\ \text{else()} \rightarrow \bot: \\ \text{next(42)} \\ \text{next(a: int)} \rightarrow \bot: \\ \text{ret(a)} \\ \text{in} \\ \text{branch(n==0, then, else)} \end{array}
```

```
foo(n: int, ret: int \rightarrow \bot) \rightarrow \bot:

let

then() \rightarrow \bot:

next(23)

else() \rightarrow \bot:

next(42)

next(a: int) \rightarrow \bot:

ret(a)

in

branch(n==0, then, else)
```

```
foo(n: int, ret: cn(int)):
           n = \sqrt{0}
         branch(*, then, else)
then():
         next(23)
else():
         next(42)
next(a: int):
         ret(a)
```

```
\begin{array}{l} \text{foo(n: int, ret: int} \rightarrow \bot) \rightarrow \bot: \\ \text{let} \\ \text{then()} \rightarrow \bot: \\ \text{next(23)} \\ \text{else()} \rightarrow \bot: \\ \text{next(42)} \\ \text{next(a: int)} \rightarrow \bot: \\ \text{ret(a)} \\ \text{in} \\ \text{branch(n==0, then, else)} \end{array}
```

```
foo(n: int, ret: cn(int)):
        branch(•, then, else)
then():
        next(23)
else():
        next(42)
next(a: int):
        ret(a
```

```
\begin{array}{l} \text{foo(n: int, ret: int} \rightarrow \bot) \rightarrow \bot: \\ \text{let} \\ \text{then()} \rightarrow \bot: \\ \text{next(23)} \\ \text{else()} \rightarrow \bot: \\ \text{next(42)} \\ \text{next(a: int)} \rightarrow \bot: \\ \text{ret(a)} \\ \text{in} \\ \text{branch(n==0, then, else)} \end{array}
```

```
foo(n: int, ret: cn(int)):
           n = \sqrt{0}
         branch(•, then, else)
them():
         next(23)
else():
         next(42)
next(a: int):
         ret(a)
```

```
\begin{array}{l} \text{foo(n: int, ret: int} \rightarrow \bot) \rightarrow \bot: \\ \text{let} \\ \text{then()} \rightarrow \bot: \\ \text{next(23)} \\ \text{else()} \rightarrow \bot: \\ \text{next(42)} \\ \text{next(a: int)} \rightarrow \bot: \\ \text{ret(a)} \\ \text{in} \\ \text{branch(n==0, then, else)} \end{array}
```

```
foo(n: int, ret: cn(int)):
        branch(•, then, else)
them():
        next(23)
else():
        next(42)
next(a: int):
        ret(a
```

Classic CPS vs Thorin

Classic CPS	Thorin

Classic CPS vs Thorin

Classic CPS	Thorin
let	graph edge (acyclic graph)
letrec	graph edge (<mark>cyclic</mark> graph)

Classic CPS vs Thorin

Classic CPS	Thorin
let	graph edge (acyclic graph)
letrec	graph edge (<mark>cyclic</mark> graph)
block nesting	implicit
name resolution	graph edge
name capture	-

SSA vs Thorin

```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = \phi(23 [then], 42 [else]);
    return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

SSA vs Thorin

```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = \phi(23 [then], 42 [else]);
    return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

parameter

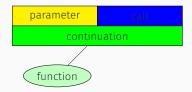
continuation

```
int foo(int n) { branch(n==0, then, else) then: goto next; else: goto next; next: int a = \phi(23 \text{ [then], } 42 \text{ [else]}); return a; }
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

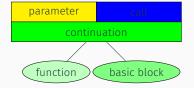
```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = \( \phi(23 \) [then], 42 [else]);
    return a;
}
```

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foo(n: int, ret: cn(int)):
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```



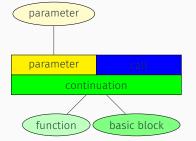
```
int foo(int n) {
   branch(n==0, then, else)
then:
   goto next;
else:
   goto next;
next:
   int a = φ(23 [then], 42 [else]);
   return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```



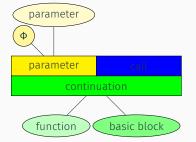
```
int foo(int n) {
  branch(n==0, then, else)
then:
  goto next;
else:
  goto next;
next:
  int a = \phi(23 [then], 42 [else]);
  return a;
}
```

```
foo(n: int, ret: cn(int)):
  branch(n==0, then, else)
then():
  next(23)
else():
  next(42)
next(a: int):
  ret(a)
```



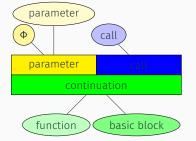
```
int foo(int n) {
  branch(n==0, then, else)
then:
  goto next;
else:
  goto next;
next:
  int a = o(23 [then], 42 [else]);
  return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```



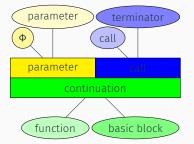
```
int foo(int n) {
  branch(n==0, then, else)
then:
  goto next;
else:
  goto next;
next:
  int a = o(23 [then], 42 [else]);
  return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```



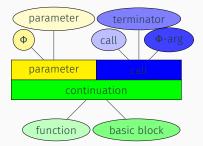
```
int foo(int n) {
  branch(n==0, then, else)
then:
  goto next;
else:
  goto next;
next:
  int a = \( \phi \) (23 [then], 42 [else]);
  return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(42)
ret(a: int):
    ret(a)
```



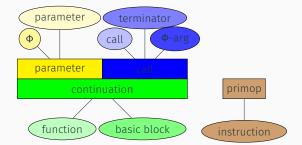
```
int foo(int n) {
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```



```
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then:
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  return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
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    next(42)
next(a: int):
    ret(a)
```



Lambda Mangling

```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = φ(23 [then], 42 [else]);
    return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

Thorin program in CFF if

```
int foo(int n) {
   branch(n==0, then, else)
then:
   goto next;
else:
   goto next;
next:
   int a = φ(23 [then], 42 [else]);
   return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
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else():
    next(42)
next(a: int):
    ret(a)
```

- Thorin program in CFF if
 - first-order continuation ⇒ basic block

```
int foo(int n) {
    branch(n==0, then, else)
then:
    goto next;
else:
    goto next;
next:
    int a = \phi(23 [then], 42 [else]);
    return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

- · Thorin program in CFF if
 - first-order continuation ⇒ basic block
 - top-level, continuation with "return" ⇒ function

```
int foo(int n) {
   branch(n==0, then, else)
then:
   goto next;
else:
   goto next;
next:
   int a = φ(23 [then], 42 [else]);
   return a;
}
```

```
foo(n: int, ret: cn(int)):
    branch(n==0, then, else)
then():
    next(23)
else():
    next(42)
next(a: int):
    ret(a)
```

- · Thorin program in CFF if
 - first-order continuation ⇒ basic block
 - top-level, continuation with "return" ⇒ function
- straightforward to translate to SSA form [Kelsey95]
- · no closures needed

```
void range(int a, int b, function<void(int)> f) {
   //...
   range(a+1, b, f);
}
```

```
range(a: int, b: int, f: cn(int, cn()), ret: cn()):
    /*
    * ...
    */
    range(a+1, b, f, ret)
```

- · recursion-free or
- tail-recursive

```
void range(int a, int b, function<void(int)> f) {
    //...
    range(a+1, b, f);
}
```

```
range(a: int, b: int, f: cn(int, cn()), ret: cn()):
    /*
    * ...
    */
   range(a+1, b, f, ret)
```

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```

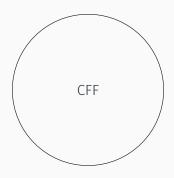
```
range(a: int, b: int, f: cn(int, cn()), ret: cn()):
    /*
    * ...
    */
    range(a+1, b, f, ret)
```

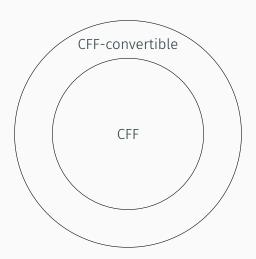
- · recursion-free or
- tail-recursive

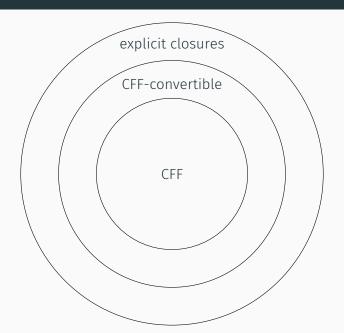
```
void range(int a, int b, function<void(int)> f) {
   //...
   range(a+1, b, f);
}
```

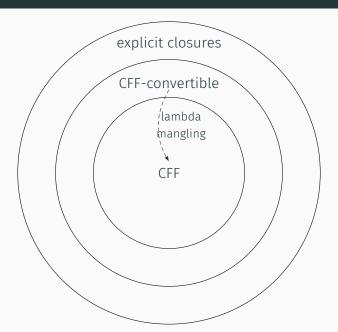
```
range(a: int, b: int, f: cn(int, cn()), ret: cn()):
    /*
    * ...
    */
    range(a+1, b, f, ret)
```

- · recursion-free or
- tail-recursive









- · (partial) inlining
- (partial) outlining

- · (partial) inlining
- · (partial) outlining
- · clone basic blocks/functions

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- · (partial) outlining
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- · loop peeling
- loop unrolling

- · (partial) inlining
- · (partial) outlining
- clone basic blocks/functions
- · loop peeling
- · loop unrolling
- tail-recursion elimination
- ...

Impala

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
    if n.right != nil {
        post order visit(n.right, f);
    f(n.data)
fn print(n: &Node) -> () {
    post_order_visit(n, |d| {
        println(d);
    });
```

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
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fn print(n: &Node) -> () {
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```

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    post_order_visit(n, |d| {
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fn print(n: &Node) -> () {
    post order visit(n, |d| {
        println(d);
    });
```

Impala - for Syntax

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
    if n.right != nil {
        post order visit(n.right, f);
    f(n.data)
fn print(n: &Node) -> () {
    for d in post order visit(n) {
        println(d);
```

Impala - sum

```
fn sum(n: &Node) -> () {
    let mut result = 0;

    for d in post_order_visit(n) {
        result += d
    }

    println(result);
}
```

Impala - sum

```
fn sum(n: &Node) -> () {
    let mut result = 0;

    for d in post_order_visit(n) {
        result += d
    }

    println(result);
}
```

Impala - sum

```
fn sum(n: &Node) -> () {
   let mut result = 0;

   for d in post_order_visit(n) {
      result += d
   }

   println(result);
}
```

```
fn sum(n: &Node) -> () {
   let mut result = 0;
   post_order_visit(n, |d| {
        if d == 23 {
          return()
       result += d
    println(result);
```

```
fn sum(n: &Node) -> () {
   let mut result = 0;
    post_order_visit(n, |d| {
        if d == 23 {
           return()
        result += d
    println(result);
```

```
fn sum(n: &Node) -> () {
   let mut result = 0;
    post_order_visit(n, |d| {
        if d == 23 {
           return()
        result += d
    println(result);
```

```
fn sum(n: &Node) -> () {
   let mut result = 0;
    post_order_visit(n, |d| {
        if d == 23 {
            return()
        result += d
    println(result);
```

Impala - continue is the new return

```
fn sum(n: &Node) -> () {
   let mut result = 0;
    for d in post order visit(n) {
        if d == 23 {
            continue()
        result += d
    println(result);
```

Impala - Give me a break, please!

```
fn sum(n: &Node) -> () {
   let mut result = 0;
    for d in post order visit(n) {
        if d == 23 {
           break()
        result += d
    println(result);
```

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
    if n.right != nil {
        post order visit(n.right, f);
    f(n.data)
fn print(n: &Node) -> () {
    for d in post order visit(n) {
        println(result);
```

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
    if n.right != nil {
        post order visit(n.right, f);
    f(n.data)
fn print(n: &Node) -> () {
    for d in post order visit(n) {
        println(result);
```

```
fn post_order_visit(n: &Node, f: fn(int) -> ()) -> () {
    if n.left != nil {
        post_order_visit(n.left, f);
    if n.right != nil {
        post order visit(n.right, f);
    f(n.data)
fn print(n: &Node) -> () {
    for d in post order visit(n) {
        println(result);
```

```
define internal void @post order visit 392(%Node* %n 394) {
post order visit 392 start:
  br label %post order visit
post order visit:
 %0 = getelementptr inbounds %0, %Node* %n_394, i32 0, i32 1
 %1 = load %Node*, %Node** %0
 %2 = icmp ne %Node* %1, null
  br i1 %2, label %if then, label %if else
if then:
  call void @post_order_visit_392(%Node* %1)
  br label %next
if else:
 br label %next
; ...
```

```
define internal void @post order visit 392(%Node* %n 394) {
post order visit 392 start:
  br label %post order visit
post order visit:
 %0 = getelementptr inbounds %0, %Node* %n_394, i32 0, i32 1
 %1 = load %Node*, %Node** %0
 %2 = icmp ne %Node* %1, null
  br i1 %2, label %if then, label %if else
if then:
  call void @post_order_visit_392(%Node* %1)
  br label %next
if else:
 br label %next
; ...
```

```
define internal void @post order visit 392(%Node* %n 394) {
post order visit 392 start:
  br label %post order visit
post order visit:
 %0 = getelementptr inbounds %0, %Node* %n_394, i32 0, i32 1
 %1 = load %Node*, %Node** %0
 %2 = icmp ne %Node* %1, null
 br i1 %2, label %if then, label %if else
if then:
  call void @post_order_visit_392(%Node* %1)
  br label %next
if else:
 br label %next
; ...
```

```
define internal void @post order visit 392(%Node* %n 394) {
post order visit 392 start:
  br label %post order visit
post order visit:
 %0 = getelementptr inbounds %0, %Node* %n_394, i32 0, i32 1
 %1 = load %Node*, %Node** %0
 %2 = icmp ne %Node* %1, null
  br i1 %2, label %if then, label %if else
if then:
 call void @post_order_visit_392(%Node* %1)
  br label %next
if else:
 br label %next
; ...
```

```
; ...
next:
 %3 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 2
 %4 = load %Node*, %Node** %3
 %5 = icmp ne %Node* %4, null
  br i1 %5, label %if then2, label %if else1
if then2:
  call void @post_order_visit_392(%Node* %4)
  br label %next3
if else1:
 br label %next3
next3:
 %6 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 0
 %7 = load i32, i32 * %6
 call void @println(i32 %7)
 ret void
```

```
; ...
next:
 %3 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 2
 %4 = load %Node*, %Node** %3
 %5 = icmp ne %Node* %4, null
 br i1 %5, label %if_then2, label %if else1
if then2:
  call void @post_order_visit_392(%Node* %4)
  br label %next3
if else1:
 br label %next3
next3:
 %6 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 0
 %7 = load i32, i32 * %6
 call void @println(i32 %7)
 ret void
```

```
; ...
next:
 %3 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 2
 %4 = load %Node*, %Node** %3
 %5 = icmp ne %Node* %4, null
  br i1 %5, label %if_then2, label %if_else1
if then2:
 call void @post_order_visit_392(%Node* %4)
  br label %next3
if else1:
 br label %next3
next3:
 %6 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 0
 %7 = load i32, i32 * %6
 call void @println(i32 %7)
 ret void
```

```
; ...
next:
 %3 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 2
 %4 = load %Node*, %Node** %3
 %5 = icmp ne %Node* %4, null
  br i1 %5, label %if_then2, label %if_else1
if then2:
  call void @post_order_visit_392(%Node* %4)
  br label %next3
if else1:
 br label %next3
next3:
 %6 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 0
 %7 = load i32, i32* %6
 call void @println(i32 %7)
 ret void
```

```
; ...
next:
 %3 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 2
 %4 = load %Node*, %Node** %3
 %5 = icmp ne %Node* %4, null
  br i1 %5, label %if_then2, label %if_else1
if then2:
  call void @post_order_visit_392(%Node* %4)
  br label %next3
if else1:
 br label %next3
next3:
 %6 = getelementptr inbounds %0, %Node* %n 394, i32 0, i32 0
 %7 = load i32, i32 * %6
 call void @println(i32 %7)
  ret void
```

Evaluation

Benchmarks – The Computer Language Benchmark Game¹

	runtime in ms	
	С	Impala
aobench	1.220	1.357
fannkuch-redux	27.137	28.070
fasta	2.313	1.517
mandelbrot	2.143	2.113
meteor-contest	0.047	0.043
n-body	5.497	6.130
pidigits	0.710	0.763
regex	6.477	6.470
reverse-complement	1.090	1.220
spectral-norm	4.423	4.480

https://benchmarksgame.alioth.debian.org/

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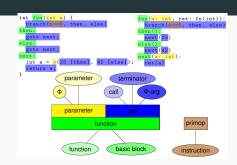
- · high-order IR does not "hurt" performance
- · all closures removed

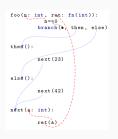
https://benchmarksgame.alioth.debian.org/

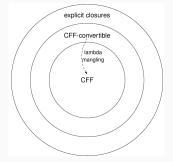
Summary

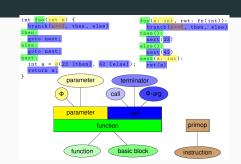


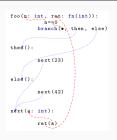


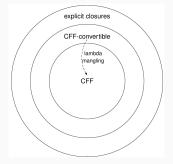


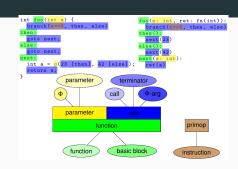












Thank you! Questions?