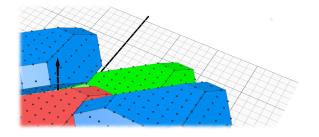
### Analyzing and Optimizing your Loops with Polly

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#### State of Variables

#### **Program**

```
for (i = 0; i <= n; i++)
  for (j = 0; j <= i; j++)
    S(i,j);</pre>
```

# State of Variables n = 4, i = 0, j = 0

#### **Statement Instances Executed**

S(0,0)

# State of Variables n = 4, i = 1, j = 0

# for $(i = 0; i \le n; i++)$

S(i,j);

for  $(j = 0; j \le i; j++)$ 

#### State of Variables n = 4, i = 1, j = 1

S(i,j);

for  $(j = 0; j \le i; j++)$ 

### State of Variables

$$n=4$$
,  $i=2$ ,  $j=0$ 

State of Variables 
$$n = 4$$
,  $i = 2$ ,  $j = 1$ 

**State of Variables** Program 
$$n = 4$$
,  $i = 2$ ,  $j = 2$ 

$$S(2,0), S(2,1), S(2,2)$$
  
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

S(i,j);

for  $(j = 0; j \le i; j++)$ 

## State of Variables

$$n=4,\,i=3,\,j=0$$

State of Variables 
$$n = 4, i = 3, j = 1$$

$$S(3,0), S(3,1), \\ S(2,0), S(2,1), S(2,2) \\ S(1,0), S(1,1) \\ S(0,0)$$

State of Variables 
$$n = 4, i = 3, j = 2$$

$$S(3,0), S(3,1), S(3,2), \\ S(2,0), S(2,1), S(2,2) \\ S(1,0), S(1,1) \\ S(0,0)$$

# State of Variables n = 4, i = 3, j = 3

$$S(3,0), S(3,1), S(3,2), S(3,3)$$
  
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

# State of Variables n = 4, i = 4, j = 0

## Statement Instances Executed

S(4,0),

S(3,0), S(3,1), S(3,2), S(3,3)

S(2,0), S(2,1), S(2,2)

S(1,0), S(1,1) S(0,0)

# State of Variables n = 4, i = 4, j = 1

## State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 2$ 

# for (i = 0; i <= n; i++) for (j = 0; j <= i; j++) S(i,j);</pre>

$$S(4,0), S(4,1), S(4,2), \\ S(3,0), S(3,1), S(3,2), S(3,3)$$

## State of Variables n = 4, i = 4, i = 3

#### **Statement Instances Executed**

S(1,0), S(1,1)S(0,0)

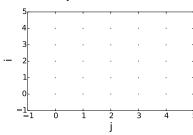
S(i,j);

# for (i = 0; i <= n; i++) for (j = 0; j <= i; j++)

#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 4$ 

#### Iteration space

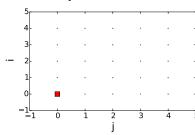


#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 4$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### Iteration space

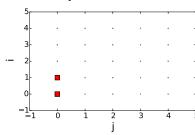


#### State of Variables

$$n = 4$$
,  $i = 0$ ,  $j = 0$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### **Iteration space**

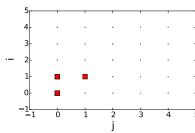


#### State of Variables

$$n = 4$$
,  $i = 1$ ,  $j = 0$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### **Iteration space**

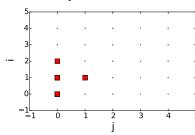


#### State of Variables

$$n = 4$$
,  $i = 1$ ,  $j = 1$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### Iteration space

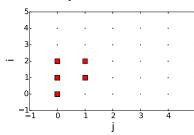


#### State of Variables

$$n = 4$$
,  $i = 2$ ,  $j = 0$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### Iteration space

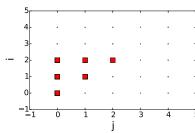


#### State of Variables

$$n = 4$$
,  $i = 2$ ,  $j = 1$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### Iteration space

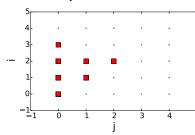


#### State of Variables

$$n = 4$$
,  $i = 2$ ,  $j = 2$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### Iteration space

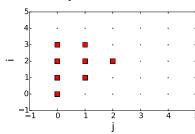


#### State of Variables

$$n = 4$$
,  $i = 3$ ,  $j = 0$ 

$$S(4,0)$$
,  $S(4,1)$ ,  $S(4,2)$ ,  $S(4,3)$ ,  $S(4,4)$   
 $S(3,0)$ ,  $S(3,1)$ ,  $S(3,2)$ ,  $S(3,3)$   
 $S(2,0)$ ,  $S(2,1)$ ,  $S(2,2)$   
 $S(1,0)$ ,  $S(1,1)$   
 $S(0,0)$ 

#### Iteration space

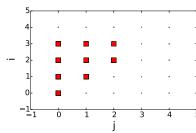


#### State of Variables

$$n = 4$$
,  $i = 3$ ,  $j = 1$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

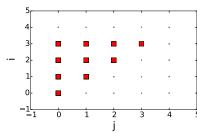
#### **Iteration space**



#### State of Variables

$$n = 4$$
,  $i = 3$ ,  $j = 2$ 

#### Iteration space

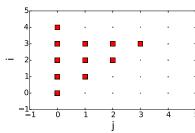


#### State of Variables

$$n = 4$$
,  $i = 3$ ,  $j = 3$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

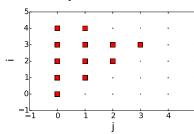
#### Iteration space



#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 0$ 

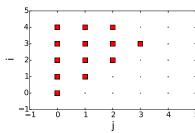
#### **Iteration space**



#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 1$ 

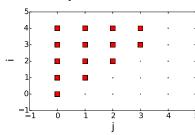
#### **Iteration space**



#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 2$ 

#### Iteration space

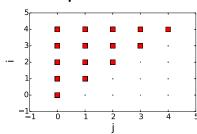


#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 3$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

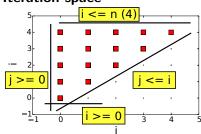
#### Iteration space



#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 4$ 

#### **Iteration space**

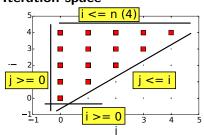


#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 4$ 

$$S(4,0), S(4,1), S(4,2), S(4,3), S(4,4)$$
  
 $S(3,0), S(3,1), S(3,2), S(3,3)$   
 $S(2,0), S(2,1), S(2,2)$   
 $S(1,0), S(1,1)$   
 $S(0,0)$ 

#### **Iteration space**



#### State of Variables

$$n = 4$$
,  $i = 4$ ,  $j = 4$ 

$$= \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}$$

## Schedule: Original

#### Model

$$\mathcal{I}_{S} = \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}$$
  
$$\Theta_{S} = \{S(i,j) \to \frac{(i,j)}{s}\}$$

#### Code

```
for (i = 0; i <= n; i++)
  for (j = 0; j <= i; j++)
    S(i,j);</pre>
```

# Schedule: Original

#### Model

$$\mathcal{I}_{S} = \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}$$
  
$$\Theta_{S} = \{S(i,j) \to \frac{(i,j)}{(i,j)}\}$$

```
for (c0 = 0; c0 <= n; c0++)
for (c1 = 0; c1 <= c0; c1++)
S(c0,c1);
```

## Schedule: Interchanged

#### Model

$$\mathcal{I}_{S} = \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}$$
  
$$\Theta_{S} = \{S(i,j) \to \frac{(j,i)}{s}\}$$

```
for (c0 = 0; c0 <= n; c0 += 1)
for (c1 = c0; c1 <= n; c1 += 1)
S(c1, c0);
```

## Schedule: Strip-Mined

#### Model

$$\mathcal{I}_{S} = \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}$$
  
$$\Theta_{S} = \{S(i,j) \to \frac{(\lfloor i/4 \rfloor, j, i \bmod 4)}{(\lfloor i/4 \rfloor, j, i \bmod 4)}\}$$

```
for (c0 = 0; c0 <= floord(n, 4); c0 += 1)
  for (c1 = 0; c1 <= min(n, 4 * c0 + 3); c1 += 1)
    for (c2 = max(0, -4 * c0 + c1);
        c2 <= min(3, n - 4 * c0); c2 += 1)
        S(4 * c0 + c2, c1);</pre>
```

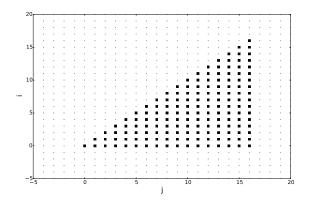
## Schedule: Tiled

#### Model

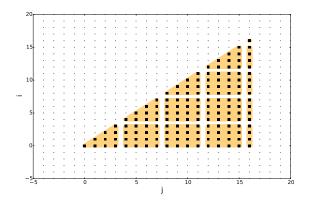
```
\mathcal{I}_{S} = \{S(i,j) \mid 0 \le i \le n \land 0 \le j \le i\}
\Theta_{S} = \{S(i,j) \to \frac{\lfloor i/4 \rfloor, \lfloor j/4 \rfloor, i \mod 4, j \mod 4}{}\}
```

```
// Tiles
for (c0 = 0; c0 <= floord(n, 4); c0 += 1)
  for (c1 = 0; c1 <= c0; c1 += 1)
    // Iterations
  for (c2 = 0; c2 <= min(3, n - 4 * c0); c2 += 1)
    for (c3 = 0; c3 <= min(3, 4 * c0 - 4 * c1 + c2);</pre>
```

# Tiling illustrated



# Tiling illustrated



# Polly

## Get Polly

- Install Polly
  http://polly.grosser.es/get\_started.html
- ► Load Polly into clang (or gcc, opt, ...)
  alias clang clang -Xclang -load -Xclang LLVMPolly.so
- Default behaviour preserved
- Enable Polly optionally

## Optimizing with Polly

```
for (int i = 0; i < N; i++)
for (int j = 0; j < M; j++) {
   C[i][j] = 0;
   for (int k = 0; k < K; k++)
        C[i][j] += A[i][k] + B[k][j];
}</pre>
```

# Optimizing with Polly

```
for (int i = 0; i < N; i++)
  for (int j = 0; j < M; j++) {
    C[i][j] = 0;
    for (int k = 0; k < K; k++)
      C[i][j] += A[i][k] + B[k][j];
  }
$ clang -03 gemm.c -o gemm.clang
$ time ./gemm.clang
real 0m15.336
```

## Optimizing with Polly

```
for (int i = 0; i < N; i++)
  for (int j = 0; j < M; j++) {
    C[i][j] = 0;
    for (int k = 0; k < K; k++)
      C[i][j] += A[i][k] + B[k][j];
  }
$ clang -03 gemm.c -o gemm.clang
$ time ./gemm.clang
real 0m15.336
$ clang -03 gemm.c -o gemm.polly -mllvm -polly
$ time ./gemm.polly
real 0m2.144s
```

## LLVM's Loop Optimization Infrastructure

### **Loop Analysis**

- Natural Loop Detection
- Scalar Evolution
- (Region Info)

## Simple Loop Transformations

- Loop Simplify
- Loop Rotation
- Induction Variable Simplification
- Loop Invariant Code Motion
- Loop Unroll
- Loop Unswitch
- ► Loop Strength Reduction

# Classical Loop Transformations

- Loop Interchange (not part of -O3)
- ► Loop Distribution (not part of -O3)

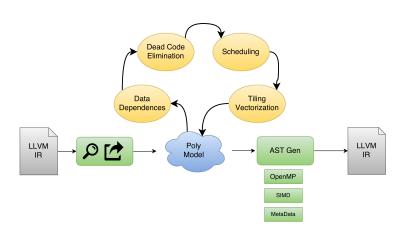
#### Vectorization

- Loop Vectorization
- SLP vectorization
- BB vectorizer (outdated)

#### Other

Loop Reroll

# The Polly Architecture



# Report detected scops: -Rpass-analysis=polly

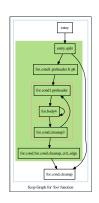
```
void foo(long T, float A[][1024]) {
1
     for (long t = 0; t < T; t++)
2
       for (long i = 1; i < 1024 - 1; i++)
3
         A[t+1][i] += A[t][i+1] + A[t][i-1];
   }
   $ polly-clang-opt -03 -mllvm -polly -Rpass-analysis=polly scop.c
   scop.c:2:3: remark: SCoP begins here. [-Rpass-analysis=polly-scops]
     for (long t = 0; t < T; t++)
   scop.c:4:50: remark: SCoP ends here. [-Rpass-analysis=polly-scops]
         A[t+1][i] += A[t][i+1] + A[t][i-1]:
```

## Report problems: -Rpass-missed=polly

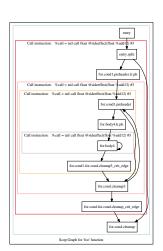
```
float sideeffect(float):
   void foo(long T, long N, float A[][N]) {
     for (long t = 0; t < T; t++)
3
       for (long i = 1; i < N - 1; i++)
         A[t+1][i] += sideeffect(A[t][i+1] + A[t][i-1]);
   $polly-clang-opt -c -03 -mllvm -polly -Rpass-missed=polly missed.c
   missed.c:3:5: remark: The following errors keep this region
                         from being a Scop.
         [-Rpass-missed=polly-detect]
       for (long i = 1; i < N - 1; i++)
   missed.c:5:20: remark: This function call cannot be handled.
                          Try to inline it.
         [-Rpass-missed=polly-detect]
         A[t+1][i] += sideeffect(A[t][i+1] + A[t][i-1]);
```

# Highlight SCoPs in CFG

-polly-show



-polly-view-all



## The Polyhedral Representation (-debug-only=polly-scops)

```
for (long t = 0; t < T; t++)
  for (long i = 1; i < 1024 - 1; i++)
    A[t+1][i] += A[t][i+1] + A[t][i-1]:
Domain :=
    [T] \rightarrow \{ Stmt_for_body4[i0, i1] : 0 \le i0 \le T \text{ and } 0 \le i1 \le 1021 \};
Schedule :=
    [T] -> { Stmt_for_body4[i0, i1] -> [i0, i1] };
ReadAccess :=
                    [Reduction Type: NONE] [Scalar: 0]
    [T] -> { Stmt_for_body4[i0, i1] -> MemRef_A[i0, 2 + i1] };
ReadAccess :=
                    [Reduction Type: NONE] [Scalar: 0]
    [T] -> { Stmt_for_body4[i0, i1] -> MemRef_A[i0, i1] };
ReadAccess :=
                    [Reduction Type: NONE] [Scalar: 0]
    [T] -> { Stmt_for_body4[i0, i1] -> MemRef_A[1 + i0, 1 + i1] };
MustWriteAccess := [Reduction Type: NONE] [Scalar: 0]
    [T] -> { Stmt_for_body4[i0, i1] -> MemRef_A[1 + i0, 1 + i1] };
```

# The Generated AST: (-debug-only=polly-ast)

```
for (long t = 0; t < T; t++)
  for (long i = 1; i < 1024 - 1; i++)
    A[t+1][i] += A[t][i+1] + A[t][i-1];
if (1)
 for (c0 = 0; c0 < T; c0+=1)
    for (c1 = c0; c1 \le c0+1021; c1+=1)
       Stmt_for_body4(c0, -c0 + c1);
else
 { /* original code */ }
```

## Loops

▶ counted
 for (i=0; i < n / 13; i+=2)</pre>

## Loops

counted

```
for (i=0; i < n / 13; i+=2)
```

Presburger Expressions

```
for (i=0; i<22 \&\& i>n; i+=2)
```

## Loops

counted
for (i=0; i < n / 13; i+=2)</pre>

- ▶ Presburger Expressions for (i=0; i<22 && i>n; i+=2)
- Multiple back-edges/exit-edges break; continue;

## Loops

counted
for (i=0; i < n / 13; i+=2)</pre>

▶ Presburger Expressions for (i=0; i<22 && i>n; i+=2)

- Multiple back-edges/exit-edges break; continue;
- ▶ do..while, while

## Loops

#### counted

- for (i=0; i < n / 13; i+=2)
- ▶ Presburger Expressions for (i=0; i<22 && i>n; i+=2)
- Multiple back-edges/exit-edges break; continue;
- ▶ do..while, while

#### **Conditions**

▶ Presburger Conditions
if (5\*i+b <= 13 12 > b)

## Loops

#### counted

- ▶ Presburger Expressions for (i=0; i<22 && i>n; i+=2)
- Multiple back-edges/exit-edges break; continue;
- ▶ do..while, while

#### Conditions

- ▶ Presburger Conditions
  if (5\*i+b <= 13 12 > b)
  - Data-dependent
    if (B[i]) A[i] = A[i]/B[i]

## Loops

- counted
  - for (i=0; i < n / 13; i+=2)
- ▶ Presburger Expressions
  for (i=0; i<22 && i>n; i+=2)
- Multiple back-edges/exit-edges break; continue;
- ▶ do..while, while

#### Conditions

- ▶ Presburger Conditions
  if (5\*i+b <= 13 12 > b)
- ▶ Data-dependent
  if (B[i]) A[i] = A[i]/B[i]
- Unstructured control flow goto;

## Loops

#### counted

- ▶ Presburger Expressions for (i=0; i<22 && i>n; i+=2)
- Multiple back-edges/exit-edges break; continue;
- ▶ do..while, while

#### **Conditions**

- ▶ Presburger Conditions
  if (5\*i+b <= 13 12 > b)
- Data-dependent
  if (B[i]) A[i] = A[i]/B[i]
- Unstructured control flow goto;

### **Arrays**

- ► Multi-dimensionality: A[][n][m] / A[][10][100]
- Keywords: restrict

#### Calls

- Memory intrinsics: memset/memmove/memcpy
- Approximated behaviour: read-none/read-only/pointer-arguments-only

## Examples: Valid SCoPs

### do..while loop

```
int i = 0;
do {
  int b = 2 * i;
  int c = b * 3 + 5 * i;
  A[c] = i;
  i++;
} while (i < N);</pre>
```

## pointer loop

```
int A[1024]
int *B;
while(B < &A[1024]) {
  *B = i;
  ++B;
}</pre>
```

# Profitability Heuristics

## Polly's default policy: No regressions

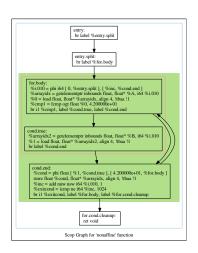
- Minimal compile time increase
- No spurious run-time changes

#### Rules:

- Bail out as early as possible
  - At least two loops (or one very big one)
  - At least one read access
- Only change IR if Polly did something beneficial
  - Performed Schedule Transformation
  - Added alias run-time check

Can be overwritten by: -polly-process-unprofitable

### Non-affine Statements



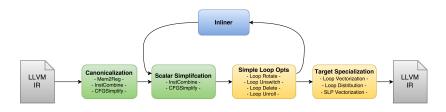
## Schedule Optimizer: -polly-opt-isl

Roman Garev (outer-loop vectorization)

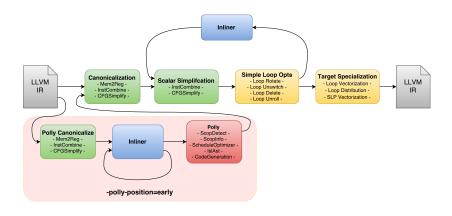
- Schedule using a Pluto style LP to maximize:
  - Data locality
  - Parallelism
  - Tilability
- Post-scheduling optimizations
  - ► Tile innermost tileable band
  - Strip-mine innermost parallel loop for SIMDization

**Implementation:** isl\_schedule

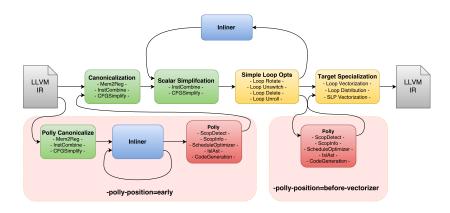
## LLVM Pass Pipeline



## LLVM Pass Pipeline



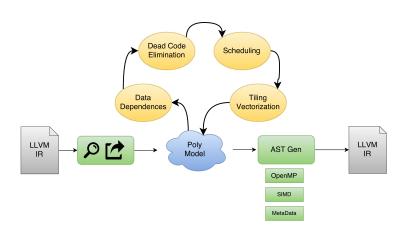
## LLVM Pass Pipeline



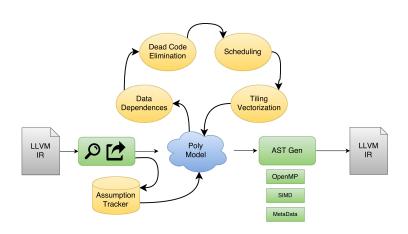
# Auto Parallelization: (-mllvm) -polly-parallel (-lgomp)

- Run outer-most parallel loop with OpenMP
- Directly emit calls to libgomp (gcc's OpenMP library)
- Execution can be controlled by setting OMP environment variables:
  - OMP\_SCHEDULE=static,dynamic,guided,auto
  - OMP\_NUM\_TRHEADS=< num > or (-mllvm)-polly-num-threads=< num >

# Optimistic Assumption Tracking



# Optimistic Assumption Tracking



## Assumption tracking in Polly

## Assumption tracking in Polly

```
void oddEvenCopy(int N, int M, float A[][M]) {
     for (int i = 0; i < M; i++)
                                                        \Rightarrow 15s
        for (int j = 0; j < N; j++)
          A[2 * j][i] = A[2 * j + 1][i];
                \Downarrow Clearly beneficial loop interchange \Downarrow
void oddEvenCopy(int N, int M, float A[][M]) {
     for (int j = 0; j < N; j++)
                                                        \Rightarrow 2s
        for (int i = 0; i < M; i++)
         A[2 * j][i] = A[2 * j + 1][i];
```

### ... is not always obvious to the compiler

### ... is not always obvious to the compiler

- ▶ Interchange only allowed if  $M \le 20000$  (or N < 0)
- ▶ ..., but code with M = 20001 is well defined.

# Be optimistic - Optimize for the common case

- 1. Take & collect assumptions
- 2. Simplify
- 3. Verify dynamically

#### Run-time alias checks

#### Run-time alias checks

```
 \begin{array}{c} \text{void aliasChecks(long n, long m,} \\ & \text{float A[],} \\ & \text{float B[][m])} \; \{ \\ \text{for (long i = 0; i < n; i++)} \\ & \text{for (long j = 0; j < m; j++)} \\ & \text{A[i] += B[i][j];} \end{array} \end{array} \\ \begin{array}{c} \text{if $(\&B[n-1][m] <= \&A[0]$} \\ || \&A[n] <= \&B[0][0]) \\ \text{for (int c0 = 0; c0 < n; c0 += 1)} \\ \text{for (int c1 = 0; c1 < m; c1 += 1)} \\ \text{Stmt\_for\_body4(c0, c1);} \\ \text{else} \\ \{ \text{/* original code */} \} \\ \end{array}
```

## Possibly Invariant Loads

```
void mayLoad(int *s0, int *s1) {
  for (int i = 0; i < *s0; i++)
    for (int j = 0; j < *s1; j++)
    ...
}</pre>
```

### Possibly Invariant Loads

# **Check Hoisting**

```
for (int i = 0; i < N; i++) {
  for (int j = 0; j < N; j++)
    A[i][j] = B[i][j];

if (DebugLevel > 5)
    printf("Column \%d copied\n", i)
}
```

# Check Hoisting

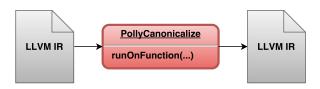
```
for (int i = 0; i < N; i++) {
  for (int j = 0; j < N; j++)
    A[i][j] = B[i][j];
    if (DebugLevel > 5)
      printf("Column \%d copied\n", i)
}
```

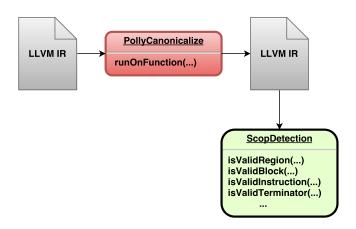
```
if (DebugLevel <= 5) {
    #pragma parallel
    for (int i = 0; i < N; i++)
        #pragma simd
    for (int j = 0; j < N; j++)
        A[i][j] = B[i][j];
} else {
    /* . . */</pre>
```

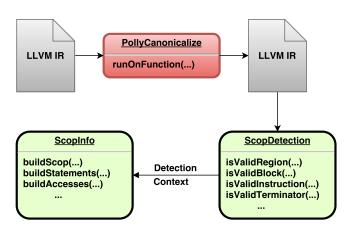
## User provided assumptions

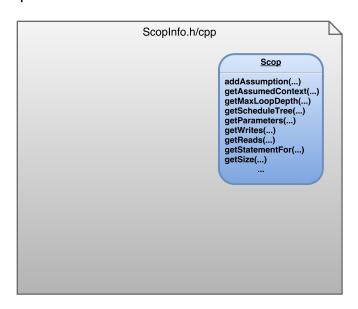
## User provided assumptions

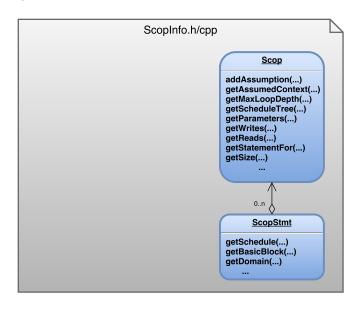
### User provided assumptions II

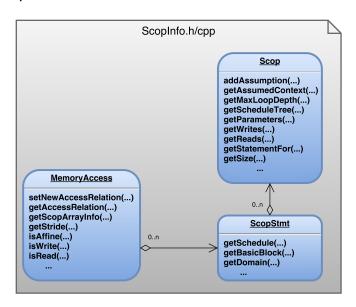


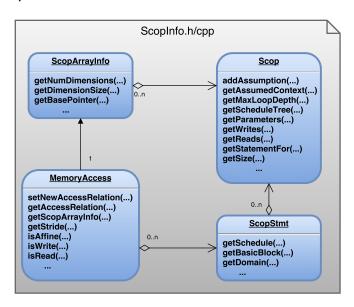




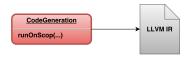


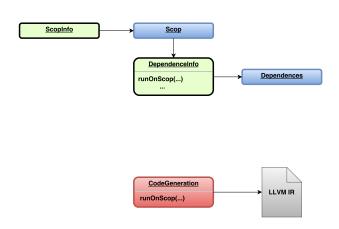


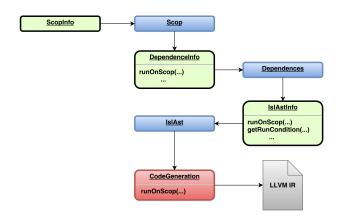


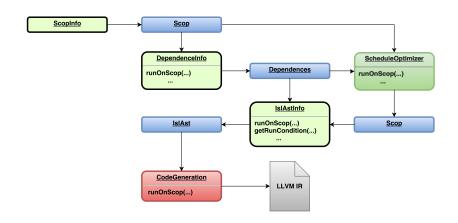


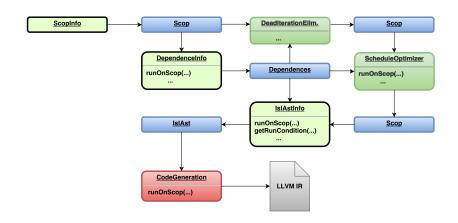












```
void compute_rhs() {
 int i, j, k, m;
 double rho inv. uijk. up1. um1. vijk. vp1. vm1. wijk. wp1. wm1:
 if (timeron) timer_start(t_rhs);
 for (k = 0; k \le grid points[2]-1; k++) {
   for (j = 0; j <= grid_points[1]-1; j++) {
     for (i = 0: i <= grid points[0]-1: i++) {
       rho_inv = 1.0/u[k][j][i][0];
       rho_i[k][j][i] = rho_inv;
       us[k][j][i] = u[k][j][i][1] * rho_inv;
        vs[k][j][i] = u[k][j][i][2] * rho_inv;
        ws[k][j][i] = u[k][j][i][3] * rho_inv;
        square[k][j][i] = 0.5* (
            u[k][i][i][1]*u[k][i][i][1] +
            u[k][i][i][2]*u[k][i][i][2] +
            u[k][j][i][3]*u[k][j][i][3] ) * rho_inv;
       qs[k][j][i] = square[k][j][i] * rho_inv;
   }
```

```
for (k = 0; k \le grid_points[2]-1; k++) {
 for (j = 0; j <= grid_points[1]-1; j++) {
    for (i = 0; i <= grid_points[0]-1; i++) {
     for (m = 0; m < 5; m++) {
        rhs[k][j][i][m] = forcing[k][j][i][m];
 }
if (timeron) timer_start(t_rhsx);
for (k = 1; k <= grid_points[2]-2; k++) {
 for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      uijk = us[k][j][i];
     up1 = us[k][j][i+1];
     um1 = us[k][j][i-1];
      rhs[k][j][i][0] = rhs[k][j][i][0] + dx1tx1 *
        (u[k][i][i+1][0] - 2.0*u[k][i][i][0] +
          u[k][i][i-1][0]) -
        tx2 * (u[k][j][i+1][1] - u[k][j][i-1][1]);
```

```
rhs[k][j][i][1] = rhs[k][j][i][1] + dx2tx1 *
  (u[k][i][i+1][1] - 2.0*u[k][i][i][1] +
   u[k][i][i-1][1]) +
 xxcon2*con43 * (up1 - 2.0*uijk + um1) -
 tx2 * (u[k][j][i+1][1]*up1 -
     u[k][i][i-1][1]*um1 +
      (u[k][j][i+1][4]- square[k][j][i+1]-
      u[k][j][i-1][4]+ square[k][j][i-1])* c2);
rhs[k][j][i][2] = rhs[k][j][i][2] + dx3tx1 *
  (u[k][i][i+1][2] - 2.0*u[k][i][i][2] +
  u[k][i][i-1][2]) +
 xxcon2 * (vs[k][j][i+1] - 2.0*vs[k][j][i] +
     vs[k][i][i-1]) -
 tx2 * (u[k][j][i+1][2]*up1 - u[k][j][i-1][2]*um1);
rhs[k][j][i][3] = rhs[k][j][i][3] + dx4tx1 *
 (u[k][i][i+1][3] - 2.0*u[k][i][i][3] +
   u[k][i][i-1][3]) +
 xxcon2 * (ws[k][j][i+1] - 2.0*ws[k][j][i] +
     ws[k][i][i-1]) -
 tx2 * (u[k][j][i+1][3]*up1 - u[k][j][i-1][3]*um1);
 /* ≈300 more lines of similar code */
```

<sup>&</sup>lt;sup>a</sup>Sanyam and Yew, PLDI 15

+ 6× speedup for 8 threads/cores <sup>a</sup>

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- Possible variant loop bounds

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- Possible execution of non-pure calls

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- + 6× speedup for 8 threads/cores <sup>a</sup>
- Possible variant loop bounds
- Possible out-of-bound accesses
- Possible execution of non-pure calls
- Possible integer under/overflows complicate loop bounds

<sup>&</sup>lt;sup>a</sup>Sanyam and Yew, PLDI 15

```
rhs.c:419:1: remark: No-overflows assumption: [grid_points, grid_points', grid_points'', timeron] ->
{: (grid_points >= 3 and grid_points' >= 3 and grid_points'') >= -2147483643) or (grid_points >= 3 and grid_points' >= -2147483643 and grid_points' >= -2147483644) or (grid_points <= 2 and grid_points >= -2147483643 and grid_points' >= 3 and grid_points' >= -2147483644 (grid_points <= 2 and grid_points >= -2147483644 and grid_points' >= 2 and grid_points' >= -2147483644 (grid_points' -2-2147483644 and grid_points' >= 3 and grid_points' >= -2147483644 (grid_points' = -2147483644 and grid_points' >= 3 and grid_points' >= -2147483644 (grid_points' >= -2147483644 and grid_points' >= 3 and grid_points' >= -2147483644 and grid_points' >= -
```

```
__builtin_assume(grid_points[0] >= -2147483643 && grid_points[1] >= -2147483643 && grid_points[2] >= -2147483643);
```

```
float BlkSchlsEqEuroNoDiv(float sptprice, float strike, float rate,
                          float volatility, float time, int otype) {
    float xD1, xD2, xDen, d1, d2, FutureValueX, NofXd1, NofXd2, NegNofXd1,
          NegNofXd2, Price;
    xD1 = rate + volatility * volatility: * 0.5:
    xD1 = xD1 * time;
    xD1 = xD1 + log( sptprice / strike );
    xDen = volatility * sqrt(time):
    xD1 = xD1 / xDen:
    xD2 = xD1 - xDen;
    d1 = xD1:
    d2 = xD2:
    NofXd1 = CNDF( d1 );
    NofXd2 = CNDF(d2);
    FutureValueX = strike * ( exp( -(rate)*(time) ) ):
    if (otype == 0) {
        Price = (sptprice * NofXd1) - (FutureValueX * NofXd2);
    } else {
       NegNofXd1 = (1.0 - NofXd1):
       NegNofXd2 = (1.0 - NofXd2);
       Price = (FutureValueX * NegNofXd2) - (sptprice * NegNofXd1):
    return Price;
```

 $+2.9\times$  speedup for manual parallelization on a quad-core i7

- $+2.9\times$  speedup for manual parallelization on a quad-core i7
- +2.8× speedup for automatic parallelization on a quad-core i7

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- Possible aliasing

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- +2.8× speedup for automatic parallelization on a quad-core i7
- Possible aliasing
- Possible execution of non-pure calls

```
int bs_thread(void *tid_ptr) {
    int tid = *(int *)tid_ptr;
    int start = tid * (numOptions / nThreads);
    int end = start + (numOptions / nThreads);
   for (int j = 0; j < NUM_RUNS; j++)</pre>
        for (int i = start; i < end; i++)
          prices[i] = BlkSchlsEqEuroNoDiv(sptprice[i], strike[i], r
                                           volatility[i], otime[i],
   return 0;
```

- $+2.9\times$  speedup for manual parallelization on a quad-core i7  $+2.8\times$  speedup for automatic parallelization on a quad-core i7
- Possible aliasing
- Possible execution of non-pure calls
- Possible execution of dead-iterations (0 <= j < NUM\_RUNS 1)</li>

```
int bs_thread(void *tid_ptr) {
      int tid = *(int *)tid_ptr;
      int start = tid * (numOptions / nThreads);
      int end = start + (numOptions / nThreads);
      for (int j = 0; j < NUM_RUNS; j++)</pre>
          for (int i = start; i < end; i++)
            prices[i] = BlkSchlsEqEuroNoDiv(sptprice[i], strike[i], r
                                              volatility[i], otime[i],
      return 0;
+2.9\times speedup for manual parallelization on a quad-core i7
```

- 2.9 × speedup for manual parametrzation on a quad-core in
- $+2.8 \times$  speedup for automatic parallelization on a quad-core i7
- $+6.5\times$  speedup for sequential execution (native input)
- Possible aliasing
- Possible execution of non-pure calls
- Possible execution of dead-iterations (0 <= j < NUM\_RUNS 1)

### The Polly Loop Optimizer

- High-level loop manipulation framework for LLVM
- Generic loop modeling based on "Semantic SCoPs"
- Optimistic assumptions in case of insufficient static knowledge
- Fast compile-time
- Open and welcoming community (we try at least)
- Industry/Research Partnership through pollylabs.org

# Thank you!

```
for (k = 1; k \le M; k++) {
 mc[k] = mpp[k - 1] + tpmm[k - 1]:
 if ((sc = ip[k - 1] + tpim[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = dpp[k - 1] + tpdm[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = xmb + bp[k]) > mc[k]) mc[k] = sc:
 mc[k] += ms[k]:
 if (mc[k] < -INFTY) mc[k] = -INFTY;
 dc[k] = dc[k - 1] + tpdd[k - 1];
 if ((sc = mc[k - 1] + tpmd[k - 1]) > dc[k]) dc[k] = sc;
 if (dc[k] < -INFTY) dc[k] = -INFTY;
 if (k < M) {
   ic[k] = mpp[k] + tpmi[k];
   if ((sc = ip[k] + tpii[k]) > ic[k]) ic[k] = sc;
   ic[k] += is[k]:
   if (ic[k] < -INFTY) ic[k] = -INFTY;
```

```
#pragma clang loop vectorize(enable)
for (k = 1; k \le M; k++) {
 mc[k] = mpp[k - 1] + tpmm[k - 1]:
 if ((sc = ip[k - 1] + tpim[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = dpp[k - 1] + tpdm[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = xmb + bp[k]) > mc[k]) mc[k] = sc;
 mc[k] += ms[k]:
 if (mc[k] < -INFTY) mc[k] = -INFTY;
for (k = 1; k \le M; k++) {
 dc[k] = dc[k - 1] + tpdd[k - 1];
 if ((sc = mc[k - 1] + tpmd[k - 1]) > dc[k]) dc[k] = sc;
 if (dc[k] < -INFTY) dc[k] = -INFTY:
 if (k < M) {
   ic[k] = mpp[k] + tpmi[k];
   if ((sc = ip[k] + tpii[k]) > ic[k]) ic[k] = sc;
   ic[k] += is[k]:
   if (ic[k] < -INFTY) ic[k] = -INFTY;
```

+ up to 30% speedup

```
for (k = 1; k \le M; k++) {
 mc[k] = mpp[k - 1] + tpmm[k - 1]:
 if ((sc = ip[k - 1] + tpim[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = dpp[k - 1] + tpdm[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = xmb + bp[k]) > mc[k]) mc[k] = sc;
 mc[k] += ms[k]:
 if (mc[k] < -INFTY) mc[k] = -INFTY;
 dc[k] = dc[k - 1] + tpdd[k - 1];
 if ((sc = mc[k - 1] + tpmd[k - 1]) > dc[k]) dc[k] = sc;
 if (dc[k] < -INFTY) dc[k] = -INFTY:
#pragma clang loop vectorize(enable)
for (k = 1; k \le M; k++) {
 if (k < M) {
   ic[k] = mpp[k] + tpmi[k];
   if ((sc = ip[k] + tpii[k]) > ic[k]) ic[k] = sc;
   ic[k] += is[k]:
   if (ic[k] < -INFTY) ic[k] = -INFTY;
```

+ up to 30% speedup

```
#pragma clang loop vectorize(enable)
for (k = 1; k \le M; k++) {
 mc[k] = mpp[k - 1] + tpmm[k - 1]:
 if ((sc = ip[k - 1] + tpim[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = dpp[k - 1] + tpdm[k - 1]) > mc[k]) mc[k] = sc;
 if ((sc = xmb + bp[k]) > mc[k]) mc[k] = sc:
 mc[k] += ms[k]:
 if (mc[k] < -INFTY) mc[k] = -INFTY;
for (k = 1; k \le M; k++) {
 dc[k] = dc[k - 1] + tpdd[k - 1];
 if ((sc = mc[k - 1] + tpmd[k - 1]) > dc[k]) dc[k] = sc;
 if (dc[k] < -INFTY) dc[k] = -INFTY:
#pragma clang loop vectorize(enable)
for (k = 1; k \le M; k++) {
 if (k < M) {
   ic[k] = mpp[k] + tpmi[k];
   if ((sc = ip[k] + tpii[k]) > ic[k]) ic[k] = sc;
   ic[k] += is[k]:
   if (ic[k] < -INFTY) ic[k] = -INFTY;</pre>
```

+ up to 50% speedup

1 vectorized loop  $\implies$  + up to 30% speedup