CS70–Spring 2013 — Homework 9

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Collaborators: None

1.Introductions

1. Suppose the starting person is A_0, then we keep going and form a path starting from A_0 to A_k. We claim that from A_k, the next must be either A_0 or a new person A_{k+1}, it cannot be himself or anyone A_i, where $i \in (0, k]$

   If it's for person in A_i, A_{i-1} already holds a card of his name, and the card has only one, which is a contradiction.

2. Always $\frac{1}{n}$.

   To distribute n cards to n people, we have $n!$ ways doing so.
   Suppose person x is in a cycle of group of size k. $x \to A_0 \to A_1 \to \ldots \to A_{k-1}$. So to choose these $k-1$ extra people to form a cycle of size k, there are $\binom{n-1}{k-1}$ ways. This cycle can be arranged in order in $(k-1)!$ ways. And the rest of the $n-k$ cards and $n-k$ people have $(n-k)!$ permutations to arrange.
   So the probability is

   $$P = \frac{\binom{n-1}{k-1}(k-1)!(n-k)!}{n!} = \frac{1}{n}$$

   (Note: Another way of doing so is you can try $k = 1, 2, 3, \ldots, n$, you will see the probability is $\frac{n-1}{n} \times \frac{n-2}{n-1} \times \frac{n-3}{n-2} \times \ldots \times \frac{1}{n-k+1} = \frac{1}{n}$ for a given k),

2. Round the clock

3. Fair bet?

Two cases, one is 1 number repeat 3 times, another is 2 number repeat twice. So number of ways for exactly 4 different number is

$$\binom{6}{3} \times 6 \times 5 \times 4 \times 3 + \frac{\binom{6}{2} \times \binom{4}{2} \times 6 \times 5 \times 4 \times 3}{2} = 7200 + 16200 = 23400$$

$$P = \frac{23400}{6^6} = 0.5015 > 0.5$$

It's not a fair game, but I can accept it since it's good for me

4. Box of marbles

1.

$$\begin{aligned} P[X = B] &= P[Box1 \cap X = B] + P[Box2 \cap X = B] \\ &= \frac{1}{2} \times \frac{1}{10} + \frac{1}{2} + \frac{1}{2} \\ &= \frac{3}{10} \end{aligned}$$

2.

$$\begin{aligned} P[Box1 \mid X = B] &= \frac{P[Box1 \cap X = B]}{P[X = B]} \\ &= \frac{\frac{1}{20}}{\frac{3}{10}} \\ &= \frac{1}{6} \end{aligned}$$

3.

$$\begin{aligned} P[Second = B] &= P[First = B \cap Second = B] + P[First = R \cap Second = B] \\ &= \frac{100}{1000} \times \frac{99}{999} + \frac{900}{1000} \times \frac{100}{999} \\ &= \frac{1}{10} \end{aligned}$$

5. Disease diagnosis

$$\begin{aligned} P(\text{H1N1}) &= 0.01 \\ P(\text{flu}) &= 0.1 \\ P(\text{neither}) &= 0.89 \\ P(\text{fever}|\text{H1N1}) &= 1 \\ P(\text{fever}|\text{flu}) &= 0.3 \\ P(\text{fever}|\text{neither}) &= 0.02 \end{aligned}$$

$$P(\text{H1N1}|\text{fever}) = \frac{P(\text{H1N1} \cap \text{fever})}{P(\text{fever})}$$

So we need to calculate $P(fever)$

$$\begin{aligned} P(\text{fever}) &= P(\text{H1N1} \cap \text{fever}) \ + P(\text{flu} \cap \text{fever}) + P(\text{neither} \cap \text{fever}) \\ &= 0.01 \times 1 + 0.01 \times 0.3 + 0.89 \times 0.2 \\ &= 0.05978 \end{aligned}$$

So

$$\begin{aligned}
P(\text{H1N1}|\text{fever}) &= \frac{0.01 \times 1}{0.05978} \approx 0.173 \\
P(\text{flu}|\text{fever}) &= \frac{0.1 \times 0.3}{0.05978} \approx 0.52 \\
P(\text{neither}|\text{fever}) &= \frac{0.89 \times 0.02}{0.05978} \approx 0.308
\end{aligned}$$

It's most likely to have flu, then neither, H1N1 the least.

6. Futurama Question

$$\begin{aligned}
P(\text{pos}|\text{bne}) &= 0.8 \\
P(\text{pos}|\text{nbne}) &= \frac{1}{9} \\
P(\text{bne}) &= \frac{1}{10} \\
P(\text{nbne}) &= \frac{9}{10}
\end{aligned}$$

$$\begin{aligned}
P(\text{bne}|\text{pos}) = \frac{P(\text{bne} \cap \text{pos})}{P(\text{pos})} &= \frac{P(\text{pos}|\text{bne}) \times P(\text{bne})}{P(\text{pos} \cap \text{bne}) + P(\text{pos} \cap \text{nbne})} \\
&= \frac{0.8 \times \frac{1}{10}}{0.8 \times \frac{1}{10} + \frac{1}{9} \times \frac{9}{10}} \\
&= 0.44
\end{aligned}$$

7. The Simpson's Question

The Professor made a mistake. Probability is not enough, he needs to consider the actual number of drama, comedy in TV series and movies.

Category	**Drama**	**Comedy**
TV Shows	1 bad, 0 good	9 bad, 1 good
Movies	501 bad, 499 good	1 bad, 1 good

We can see

$$\begin{aligned}
P[\text{good Drama } |\text{TV}] &= 0 \\
P[\text{good Comedy } | \text{ TV}] &= \frac{1}{10}
\end{aligned}$$

$$\begin{aligned}
P[\text{good Drama } |\text{Movies}] &= \frac{1}{2} \\
P[\text{good Comedy } |\text{Movies}] &= \frac{499}{1000}
\end{aligned}$$

We can see

$$P[\text{good Drama}|\text{TV}] < P[\text{good Comedy}|\text{TV}]$$
$$P[\text{good Drama}|\text{Movies}] < P[\text{good Comedy}|\text{Movies}]$$

However,

$$P[\text{good Drama}] = \frac{2}{12} = \frac{1}{6}$$
$$P[\text{good Comedy}] = \frac{499}{1001}$$

Apparently, $P[\text{good Drama}] > P[\text{good Comedy}]$

(Note: From HW9 solutions this question)