

1. Sanity check!

1. Is $f : \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(n) = n^2$ an injection (1-1)? Briefly justify.
2. Is $f : \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = x^3 + 1$ an surjection (onto)? Briefly justify.

2. Natural Numbers

Prove that for any integer $k \geq 1$, \mathbb{N}^k is countably infinite.

3. Bitstrings

Use diagonalization to show that set of all infinite-length bitstrings is not countable.

4. Countable Programs

Prove that the set of all programs is countably infinite.