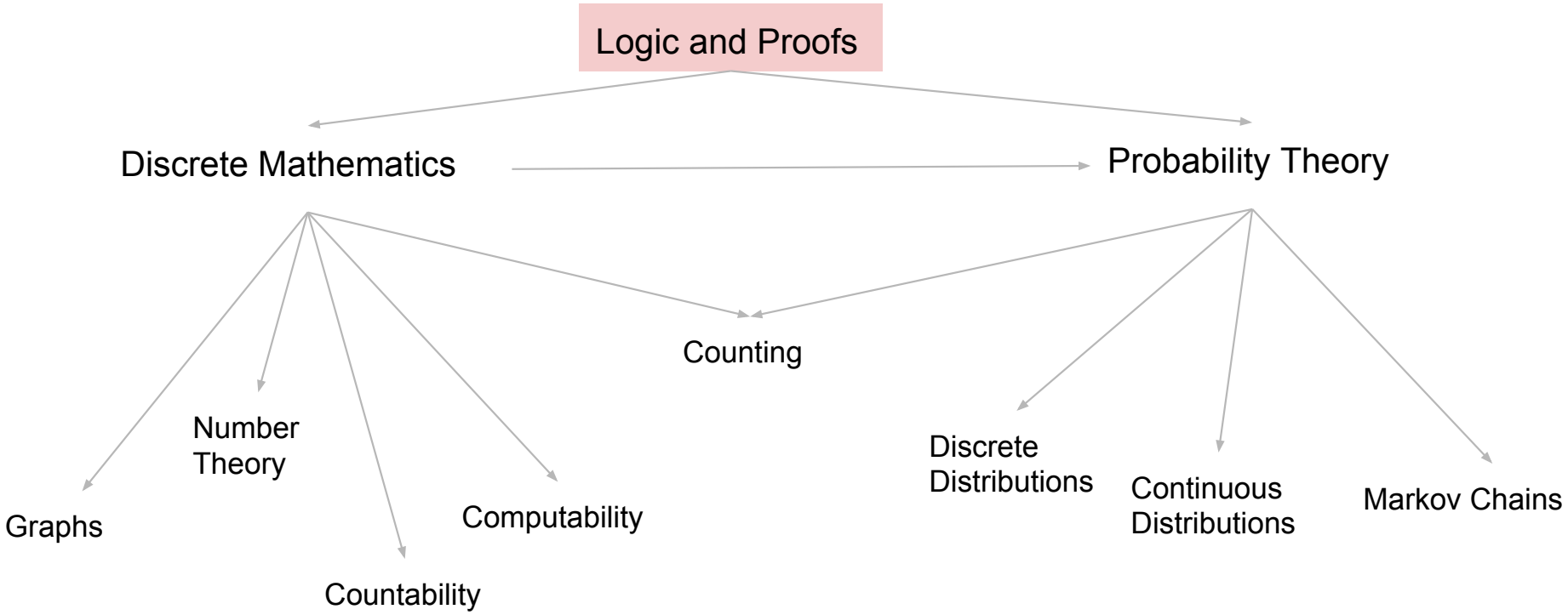
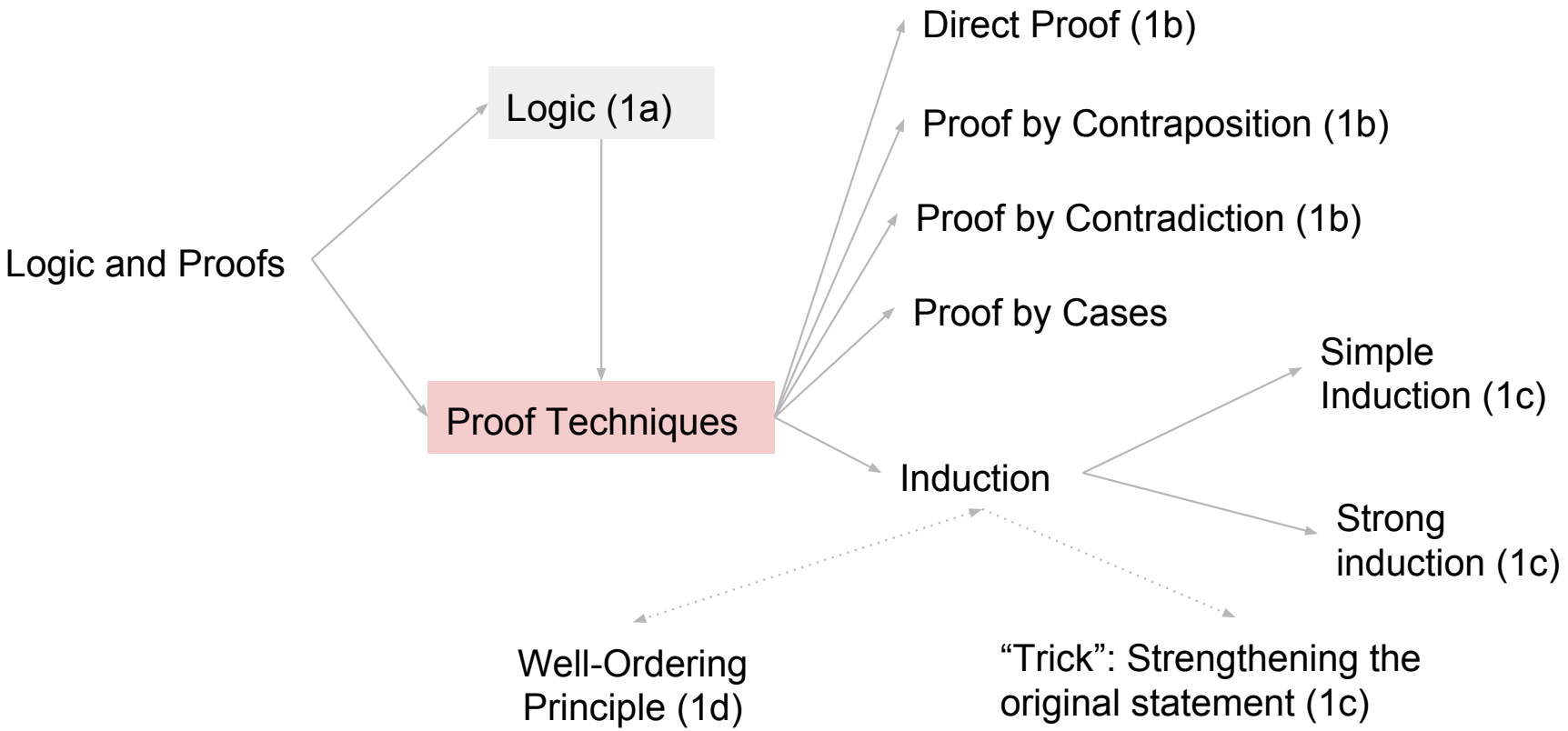
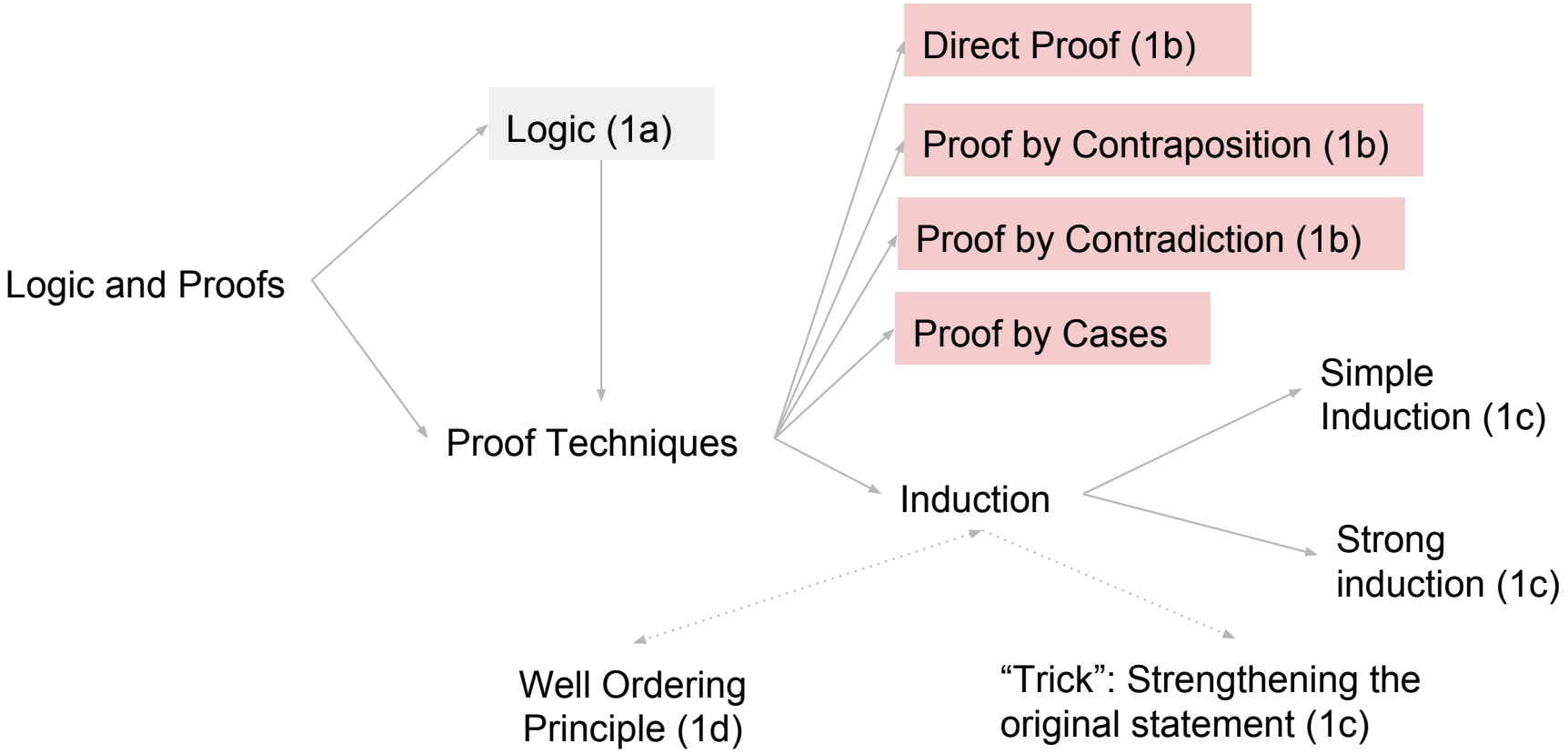
CS70 Map


Logic and Proofs
Discrete Mathematics
Probability Theory
Counting
Number Theory
Graphs
Countability
Computability
Discrete Distributions
Continuous Distributions
Markov Chains

CS70 Map: Logic and Proofs



CS70 Map: Logic and Proofs



Q1: Logic (Review)

Logical Symbols: Negation ($\neg$), AND ($\wedge$), OR ($\vee$)

Quantifiers: Existential Quantifier ($\exists$), Universal Quantifier ($\forall$)

Implications: $(P \Rightarrow Q) \equiv (\neg P \vee Q)$

Equivalence ($\equiv$): You can think of it as iff ($\Leftrightarrow$)[1]

[1] $\equiv$ and $\Leftrightarrow$ aren't exactly the same, but we wouldn't be too strict about this detail in this course. Checkout this. In general, when you proofs, use $\Leftrightarrow$ instead of $\equiv$ if both directions hold.

Q2: Contraposition

Goal: To prove $P \Rightarrow Q$

Approach:

Assume $\neg Q$.

...

Therefore $\neg P$.

Conclusion: $(\neg Q \Rightarrow \neg P) \equiv (P \Rightarrow Q)$

Q3: Perfect Square (Direct Proof)

Goal: To prove $P \Rightarrow Q$

Approach:

Assume P.

...

Therefore Q.

Conclusion: $P \Rightarrow Q$

Q4: Number of Friends (Contradiction)

Goal: To prove P.

Approach:

Assume $\neg$ P.

... R ...

... $\neg$ R.

Conclusion: $\neg$ P$\Rightarrow$ (R $\wedge$ $\neg$ R). Contradiction. Thus, P.

Side Note: Contradiction, Contraposition ???

contraposition ≈ direct proof + contradiction. This is what I meant:

Goal: To prove $P \Rightarrow Q$.

Approach:

Assume P.

    Assume $\neg Q$.

    … $\neg P$.

    Thus Q.

Conclusion: $P \Rightarrow Q$.

Side Note: Contradiction, Contraposition ???

Contraposition:

Goal: To prove P ⇒ Q.

Approach:

Assume P.

> Assume ¬ Q.
>
> … ¬ P. ¬ Q ⇒ ¬ P
>
> Thus Q.

Conclusion: P ⇒ Q.

Side Note: Contradiction, Contraposition ???

Direct Proof:

Goal: To prove P ⇒ Q.

Approach:

Assume P.

> Assume ¬ Q.
>
> … ¬ P.
>
> Thus Q.

Conclusion: P ⇒ Q.

Side Note: Contradiction, Contraposition ???

Contradiction:

Goal: To prove P ⇒ Q.

Approach:

Assume P. Goal inherent from direct proof: Want to prove Q.

Assume ¬ Q.

… ¬ P. This contradicts with the assumption P.

Thus Q.

Conclusion: P ⇒ Q.

So, understand the proof techniques, not just remember the names.

Q5: Fermat's Contradiction

Try it on your own :)

Extra Practice: Proof by Cases

Prove that if n is an integer, then $3n^2 + n + 14$ is even.