1. Sanity check!

Suppose you have a biased coin, with outcomes H and T, with the probability of heads $\Pr[H] = \frac{3}{4}$ and the probability of tails $\Pr[T] = \frac{1}{4}$. Suppose you perform an experiment in which you toss the coin 3 times — an outcome of this experiment is (X_1, X_2, X_3), where $X_i \in \{H, T\}$.

a. What is the *sample space* for your experiment?

b. Which of the following are examples of *events*? Select all that apply.

(i) $\{(H,H,T),(H,H),(T)\}$
(ii) $\{(T,H,H),(H,T,H),(H,H,T),(H,H,H)\}$
(iii) $\{(T,T,T)\}$
(iv) $\{(T,T,T),(H,H,H)\}$
(v) $\{(T,H,T),(H,H,T)\}$

c. What is the probability of the outcome H,H,T?

d. What is the probability of the event that your outcome has exactly two heads?

2. Rain and Wind

The local weather channel just released a statistic for the months of November and December. It said that the probability that it would rain on a windy day is 0.3 and the probability that it would rain on a non-windy day is 0.8. The probability of a day being windy is 0.2. As a student in CS70, you are curious to play around with these numbers. Find the probability that

a. A given day is windy and rainy.

b. It rains on a given day.

c. Exactly one of two days is rainy. (Assume that the two days are independent.)

d. A non-rainy day is also non-windy.

3. Dice

Suppose we roll three fair 6-sided dice. Each one of the $6^3 = 216$ possible outcomes is assumed to be equally likely.

a. What is the probability that we get three distinct numbers?

b. Given that we got three distinct numbers, find the conditional probability that one of them was a six.

4. Shooting Range

You and your friend are at a shooting range. You ran out of bullets. Your friend still has two bullets left but magically lost his gun. Somehow you both agree to put the two bullets into your six-chambered revolver in successive order, spin the revolver, and then take turn shooting. Your first shot was a blank. You want your friend to shoot a blank too, should you spin the revolver again before you hand it to your friend?

5. Combinatorial proof

Prove the following identity by a combinatorial argument:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$