

### 1. Is it a proposition?

- $2 + 2 = 4$
- $x + 2 = 4$
- Arnold Schwarzenegger is a handsome man.

### 2. Quantifiers

Let  $\mathbb{X} = \{\text{photos}\}$  and  $\mathbb{Y} = \{\text{humans}\}$ , which one of the following is equivalent to “Every photo is taken by some human”?

$(\forall x \in \mathbb{X})(\forall y \in \mathbb{Y})(x \text{ is taken by } y)$

$(\forall x \in \mathbb{X})(\exists y \in \mathbb{Y})(x \text{ is taken by } y)$

$(\exists x \in \mathbb{X})(\forall y \in \mathbb{Y})(x \text{ is taken by } y)$

$(\exists x \in \mathbb{X})(\exists y \in \mathbb{Y})(x \text{ is taken by } y)$

### 3. Direct Proofs

a) We call integer  $n$  an even number if and only if there exists an integer  $k$ , such that  $n = 2k$ . Prove that the negative of any even integer  $n$  is even.

b) Prove that the sum of any three consecutive integers is divisible by three.

#### 4. Proof by Contraposition

Let  $x$  and  $y$  be two positive integers. Prove that if  $x \times y < 36$  then  $x < 6$  or  $y < 6$ .