CS 70 Spring 2015 — Discrete Mathematics and Probability Theory — Vazirani — Discussion 8W

1. **Sanity check!**

   Prove that the halting problem is undecidable by writing a self-referential program. You can assume you have some subroutine TESTHALT(P,x) that returns true if P halts on input x and false otherwise. Try not to look at the notes while you do this!

2. **Self Reference**

   Here we learn about self-reference through a paradox dubbed the "Grelling-Nelson Paradox". We define two types of adjectives:

   1. An adjective is called **autological** if it describes itself. For example "short" is autological, since the word "short" is a short word. "English", "unhyphenated" and "pentasyllabic" are also autological.
   2. An adjective is called **heterological** if it does not describe itself. Hence "long" is a heterological word (because it is not a long word), as are "unwritten" and "monosyllabic".

   Now, it seems that all adjectives must either be autological or heterological, because either it describes itself, or it doesn't. Let's see if this is truly the case.

   a. Is "heterological" a heterological word?

   b. Is "autological" an autological word?

3. **Dead-Code Problem**

   Here we try to show the dead-code problem is undecidable. The problem is as follows: given a program P, an input x, and a line n in P, does P on input x ever execute line n?

4. Undecidability

Here we use reductions to prove that certain problems are undecidable.

a. The totality problem is defined as follows: A program F is said to be **total** if $F(x)$ is defined for all x. Assume there exists a procedure TOTAL that takes an input program P and outputs 'Yes' if P halts on all inputs and 'No' otherwise. Argue that this means we could solve the halting problem.

b. What does this mean about our assumption that TOTAL exists? Is the totality problem decidable?

c. The equivalence problem is defined as follows: Given two programs P and Q, do they compute the same function? (is $P(x) = Q(x)$ for all x?). Prove that the equivalence problem is undecidable by reducing to the totality problem.