CS 70 Discrete Mathematics and Probability Theory
Spring 2015 Vazirani Discussion 1W

1. Truth tables

Use truth tables to show the following identities (note that the first two are known as *De Morgan's Laws*):

1. $\neg(A \vee B) \equiv \neg A \wedge \neg B$.
2. $\neg(A \wedge B) \equiv \neg A \vee \neg B$.
3. $A \iff B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$.
4. $(A \Rightarrow (B \Rightarrow C)) \vee (B \Rightarrow (A \wedge C)) \equiv \neg A \vee \neg B \vee C$.

2. Writing in propositional logic

For each of the following sentences, translate the sentence into propositional logic using the notation introduced in class, and write its negation.

1. The square of a nonzero integer is positive.
2. There are no integer solutions to the equation $x^2 - y^2 = 10$.
3. There is one and only one real solution to the equation $x^3 + x + 1 = 0$.
4. For any two distinct real numbers, we can find a rational number in between them.

3. Implication

Which of the following implications are true? Give a counterexample for each false assertion.

1. $\forall x \forall y\, P(x,y)$ implies $\forall y \forall x\, P(x,y)$.
2. $\exists x \exists y\, P(x,y)$ implies $\exists y \exists x\, P(x,y)$.
3. $\forall x \exists y\, P(x,y)$ implies $\exists y \forall x\, P(x,y)$.
4. $\exists x \forall y\, P(x,y)$ implies $\forall y \exists x\, P(x,y)$.

4. Proof by contraposition

Let x be a positive real number. Prove that if x is irrational (i.e., not a rational number), then $\sqrt{x}$ is also irrational.

5. Proof by cases

A *perfect square* is an integer n of the form $n = m^2$ for some integer m. Prove that every odd perfect square is of the form $8k+1$ for some integer k.