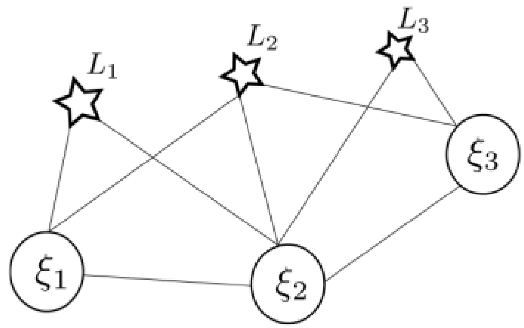
1.信息矩阵相关

1.1 绘制上述系统的信息矩阵Λ

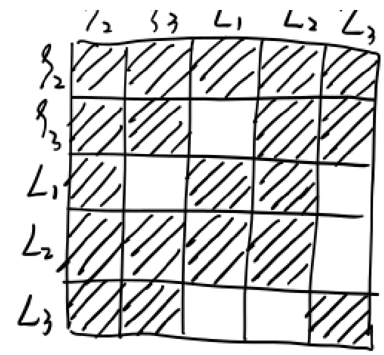


1.1 信息矩阵 Λ

	ξ_1	ξ_2	ξ_3	L_1	L_2	L_3
ξ_1	3	1		1	1	
ξ_2	1	5	1	1	1	1
ξ_3		1	3		1	1
L_1	1	1		2		
L_2	1	1	1		3	
L_3		1	1			2

1.2 绘制相机ξ_1被marg后的信息矩阵Λ'

$$\Lambda' = \Lambda_{bb} - \Lambda_{ba}\Lambda_{aa}^{-1}\Lambda_{ab} =$$



$$\begin{array}{|c|c|}\hline \Lambda_{aa} & \Lambda_{ab} \\ \hline \Lambda_{ba} & \Lambda_{bb} \\ \hline \end{array}$$

Λ_{bb} :

$$\begin{array}{|c|c|c|c|c|}\hline 5 & 1 & 1 & 1 & 1 \\ \hline 1 & 3 & & 1 & 1 \\ \hline 1 & & 2 & & \\ \hline 1 & 1 & & 3 & \\ \hline 1 & 1 & & & 2 \\ \hline \end{array}$$

Λ_{ab} :

$$\begin{array}{|c|c|c|c|c|}\hline 1 & & 1 & 1 & \\ \hline \end{array}$$

Λ_{ba} :

$$\begin{array}{|c|}\hline 1 \\ \hline \\ \hline 1 \\ \hline 1 \\ \hline \\ \hline \end{array}$$

Λ_{aa} :

$$\begin{array}{|c|}\hline 3 \\ \hline \end{array}$$

2.证明信息矩阵与协方差的逆之间的关系

Fisher Information

Fisher information is a quantity associated with parametric families of probability distributions. Let X be a set of outcomes and for each parameter θ in some set $\Theta \subset R^d$ let $p_\theta(x)$ be the distribution over X associated with θ. The *Fisher information* for the family $P = \{p_\theta : \theta \in \Theta\}$ is the matrix valued function where the entry[1] at the ith row and jth column is

$$I_{i,j}(\theta) = \mathbb{E}_X \left[(D_i \log p_\theta(X)) \left(D_j \log p_\theta(X) \right) \right]$$

where the expectation is over the random variable X drawn from the distribution p_θ, and D_i denotes the partial derivative $\frac{\partial}{\partial \theta_i}$. The Fisher information is always symmetric and positive semi-definite and can be seen as measuring the "sensitivity" of the *log likelihood* $\log p_\theta(x)$ on the outcomes in a neighbourhood of θ.

… and the Hessian of log likelihood

The result that had me puzzled for some time was the "obvious" fact that

$$I_{i,j}(\theta) = -\mathbb{E}_X \left[D_{i,j} \log p_\theta(X) \right]$$

where $D_{i,j}$ denotes the second-order partial derivative $\frac{\partial^2}{\partial \theta_i \partial \theta_j}$. What this says is that the Fisher information is closely related to the curvature of the log likelihood function, as measured by its *Hessian* — that is, the matrix of its second derivatives $H[\log p_\theta(x)] = (D_{i,j} \log p_\theta(x))_{i,j=1}^d$.

After much head-scratching, I realised that the "trick" I was missing was the observation that (under some mild conditions) the second derivatives and integrals can be switched so

$$\int_X D_{i,j} p_\theta(X)\, dx = D_{i,j} \int_X p_\theta(X)\, dx = D_{i,j} 1 = 0$$

since each p_θ is a distribution.

With the above identity in hand, establishing the relationship between Fisher information and the Hessian of log likelihood is just an application of the chain and product rules and noting that $D_i \log p_\theta(x) = \frac{D_i p_\theta(x)}{p_\theta(x)}$. Thus,

$$D_{i,j} \log p_\theta(x) = D_i \left(\frac{D_j p_\theta(x)}{p_\theta(x)} \right) = \frac{D_{i,j} p_\theta(x)}{p_\theta(x)} - \frac{D_i p_\theta(x)}{p_\theta(x)} \frac{D_j p_\theta(x)}{p_\theta(x)}.$$

Taking expectations and using the aforementioned trick gives the result since $\mathbb{E}_X \left[\frac{D_{i,j} p_\theta(x)}{p_\theta(x)} \right] = \int_X D_{i,j} p_\theta(x)\, dx = 0$.

信息矩阵与对数似然函数的负的Hessian矩阵相等，并且在高斯分布的假设下，Hessian of

Negative Log Likelihood即Inverse of Covariance Matrix

3.零空间

```
H.block(i * 6, i * 6, 6, 6) += jacobian_Ti.transpose() * jacobian_Ti;
/// 请补充完整作业信息矩阵块的计算
H.block(i * 6, poseNums * 6 + j * 3, 6, 3) += jacobian_Ti.transpose() * jacobian_Pj;
H.block(poseNums * 6 + j * 3, i * 6, 3, 6) += jacobian_Pj.transpose() * jacobian_Ti;
H.block(poseNums * 6 + j * 3, poseNums * 6 + j * 3, 3, 3) +=
    jacobian_Pj.transpose() * jacobian_Pj;
```

```
 0.00520788
 0.00502341
  0.0048434
 0.00451083
  0.0042627
 0.00386223
 0.00351651
 0.00302963
 0.00253459
 0.00230246
 0.00172459
0.000422374
3.21708e-17
2.06732e-17
1.43188e-17
7.66992e-18
6.08423e-18
6.05715e-18
3.94363e-18
xwl@xwl-Inspiron-15-7000-Gaming:~/Documents/VSLAM-f
```

奇异值最后7维接近0，即零空间维度为7