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CSc 301  HW#4

# Q1

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Homework 4

Q 1. $\int_0^2 \frac{1}{x+4}\,dx = \ln|x+4|\Big|_0^2 = \ln 6 - \ln 4 \approx \underline{0.405465}$

a) Using the Composite Trapezoidal rule:

$$(b-a)\left(\tfrac{1}{2}f(a) + \tfrac{1}{2}f(b)\right)$$
$$= (2-0)\left(\tfrac{1}{2}f(0) + \tfrac{1}{2}f(2)\right)$$
$$= 2\left(\tfrac{1}{2}\left(\tfrac{1}{4}\right) + \tfrac{1}{2}\left(\tfrac{1}{6}\right)\right)$$
$$= \tfrac{1}{4} + \tfrac{1}{6}$$
$$= \tfrac{10}{24}$$
$$= \tfrac{5}{12} \approx \underline{0.416667}$$

$|Error| = |0.416667 - 0.405465| = 0.011202 > 10^{-5}$

b) Using the Composite Simpson's rule:

$$(b-a)\left(\tfrac{1}{6}f(a) + \tfrac{2}{3}f\left(\tfrac{a+b}{2}\right) + \tfrac{1}{6}f(b)\right)$$
$$= (2-0)\left(\tfrac{1}{6}f(0) + \tfrac{2}{3}f\left(\tfrac{0+2}{2}\right) + \tfrac{1}{6}f(2)\right)$$
$$= 2\left(\tfrac{1}{6}\left(\tfrac{1}{4}\right) + \tfrac{2}{3}\left(\tfrac{1}{5}\right) + \tfrac{1}{6}\left(\tfrac{1}{6}\right)\right)$$
$$= \tfrac{1}{3}\left(\tfrac{1}{4}\right) + \tfrac{4}{3}\left(\tfrac{1}{5}\right) + \tfrac{1}{3}\left(\tfrac{1}{6}\right)$$
$$= \tfrac{1}{12} + \tfrac{4}{15} + \tfrac{1}{18}$$
$$= \tfrac{73}{180} \approx 0.405556$$

$|Error| = |0.405556 - 0.405465| = 0.000091 = 9.1 \times 10^{-5} > 10^{-5}$

c) Using the Composite Gaussian quadrature rule.

$$\int_a^b f(x)\,dx = f\left(\frac{b-a}{2}\left(-\frac{\sqrt{3}}{3}\right)+\frac{a+b}{2}\right)+f\left(\frac{b-a}{2}\left(\frac{\sqrt{3}}{3}\right)+\frac{a+b}{2}\right)$$

$$= f\left(\frac{2-0}{2}\left(-\frac{\sqrt{3}}{3}\right)+\frac{2+0}{2}\right)+f\left(\frac{2-0}{2}\left(\frac{\sqrt{3}}{3}\right)+\frac{2+0}{2}\right)$$

$$= f\left(1-\frac{\sqrt{3}}{3}\right)+f\left(1+\frac{\sqrt{3}}{3}\right)$$

$$= \frac{1}{5-\frac{\sqrt{3}}{3}}+\frac{1}{5+\frac{\sqrt{3}}{3}}$$

$$\approx 0.405405$$

$$|Error| = |0.405465 - 0.405405| = 5.96\times 10^{-5} > 10^{-5}$$

So, It needs at least 4 intervals to approximate to within $10^{-5}$

# Q2

```
% Script File: f(x)=sin(1/x)

close all
x = linspace(0,1,10);
y = sin(1./x);
for tol = [.01  .001]
   for m=3:2:9
      num0 = AdaptQNC('SpecHumps',0.1,2,m,tol);
   end
end
```

num0 = 28.3701

Thus, the number of subintervals is 28.3701.

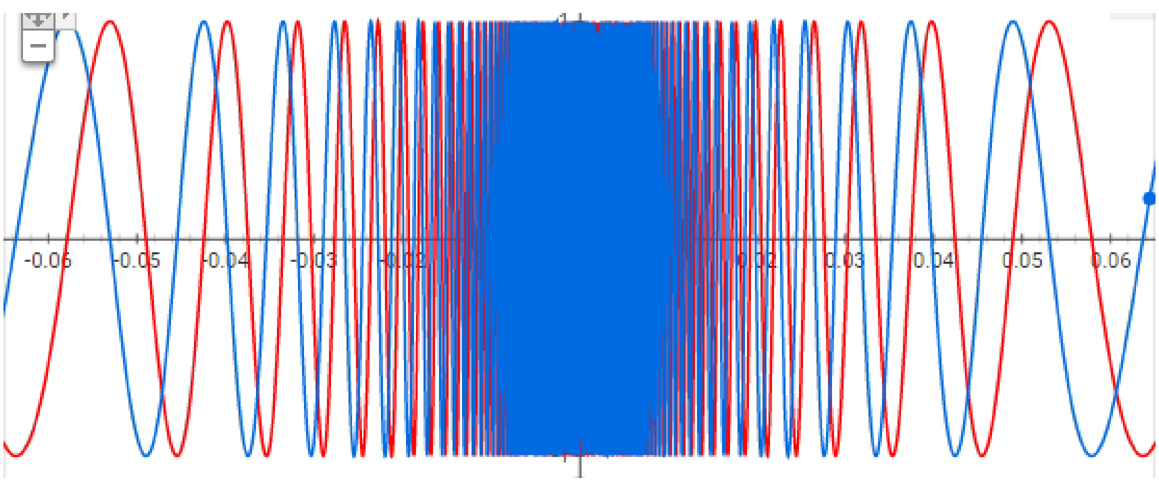
```
% Script File: f(x)=cos(1/x)

close all
x = linspace(0,1,10);
y = cos(1./x);
for tol = [.01  .001]
   for m=3:2:9
      num0 = AdaptQNC('SpecHumps',0.1,2,m,tol);
   end
end
```

num0 = 28.3701

Thus, the number of subintervals also is 28.3701.

We can see they are similar:



Q3

Q3 $$\int_{-1}^{1} f(x)\,dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

$\uparrow$ $p(x)$ $\qquad W^T \cdot f$ $\quad \langle W, f \rangle$

$$\int p(x) = \int a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$
$$= a_0 \int dx + a_1 \int x\,dx + a_2 \int x^2 dx + a_3 \int x^3 dx + \cdots$$

$W_1 \cdot f_1 + W_2 \cdot f_2 + W_3 \cdot f_3 + W_4 \cdot f_4$

$$\begin{cases} f(x) = 1 \\ f(x) = x \\ f(x) = x^2 \\ f(x) = x^3 \end{cases}$$

$$2 = \int_{-1}^{1} dx = a \cdot 1 + b \cdot 1 + c \cdot 0 + d \cdot 0$$
$$0 = \int_{-1}^{1} x\,dx = a(-1) + b \cdot 1 + c \cdot 1 + d \cdot 1$$
$$\frac{2}{3} = \frac{x^3}{3}\Big|_{-1}^{1} = \int_{-1}^{1} x^2 dx = a \cdot 1 + b \cdot 1 + c \cdot (-2) + d \cdot 2$$
$$0 = \int_{-1}^{1} x^3 dx = a \cdot (-1) + b \cdot 1 + c \cdot 3 + d \cdot 3$$

$$\begin{cases} a + b = 2 & (1) \\ -a + b + c + d = 0 & (2) \\ a + b - 2c + 2d = \frac{2}{3} & (3) \\ -a + b + 3c + 3d = 0 & (4) \end{cases}$$

(4) − (2),

$2c + 2d = 0$ (5)

(3) − (1),

$-2c + 2d = -\frac{4}{3}$ (6)

(5) + (6),

$4d = -\frac{4}{3}$

$\boxed{d = -\frac{1}{3}}$

then:

$2c + 2(-\frac{1}{3}) = 0$

$\boxed{c = \frac{1}{3}}$

In (2), $-a + b + \frac{1}{3} - \frac{1}{3} = 0$

$-a + b = 0$ (7)

Then, (1) + (7), $2b = 2$

$\boxed{b = 1}$

Hence, $a + 1 = 2$

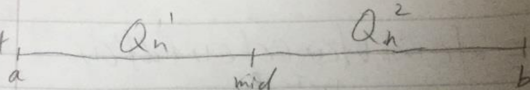
$\boxed{a = 1}$

Thus, the quadrature formula is

$$\int_{-1}^{1} f(x)\,dx = 1f(-1) + 1f(1) + \frac{1}{3}f'(1) - \frac{1}{3}f'(1)$$

# Q4

Q4




$Q_n = A_1 = Q_n^1$

$Q_{2n} = A_2 = Q_n^1 + Q_n^2$

$$\begin{cases} I = A_1 + E_1 & ① \\ I = A_2 + E_2 \cdot \frac{1}{2^{\alpha+1}} & ② \end{cases}$$

① − ②

$0 = A_1 - A_2 + E_1 - E_2 \frac{1}{2^{\alpha+1}}$

$\Rightarrow A_1 - A_2 \approx E\left(1 - \frac{1}{2^{\alpha+1}}\right)$

$E \approx \left| \frac{A_2 - A_1}{1 - \frac{1}{2^{\alpha+1}}} \right| < \delta$

$E \leq |A_2 - A_1|$

$E_2 = I - A_2$,

So, $|I - A_2| \leq |A_2 - A_1|$

$\Rightarrow |I - Q_{2n}| \leq |Q_{2n} - Q_n|$