CSc 301: Numerical Issues in Scientific Programming

Homework Assignment 4

1. Determine the number of intervals required to approximate

$$\int_0^2 \frac{1}{x+4}\, dx$$

to within 10^{-5} and compute the approximation.

   a. Using the Composite Trapezoidal rule;
   b. Using the Composite Simpson's rule;
   c. Using the Composite Gaussian quadrature rule.

2. Plot $f(x) = \sin(1/x)$ and $g(x) = \cos(1/x)$ on $[0.1\,, 2]$. Use Adaptive quadrature to approximate the integrals

$$\int_{0.1}^{2} f(x)\, dx \text{ and } \int_{0.1}^{2} g(x)\, dx$$

to within 10^{-3}. Find the number of subintervals used for each function. Are they similar? Explain.

3. Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)\, dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that is exact for all polynomials of degree less then or equal to 3.

4. P 4.3.6 page 155.

P4.3.6 Let Q_n be the equal spacing composite trapezoidal rule:

$$Q_n = h\left(\frac{1}{2}f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(x_n)\right) \qquad h = \frac{b-a}{n-1},$$

where $x = linspace(a, b, n)$ and we assume that $n \geq 2$. Assume that there is a constant C (independent of n), such that

$$I = \int_a^b f(x)fx = Q_n + Ch^2.$$

(a) Give an expression for $|I - Q_{2n}|$ in terms of $|Q_{2n} - Q_n|$. (b) Write an *efficient* script that computes Q_{2k+1}, where k is the smallest positive integer so that $|I - Q_{2k+1}|$ is smaller than a given positive tolerance `tol`. You may assume that such a k exists. You may assume that the integrand function is available in `f.m` and that it accepts vector arguments.