p8105_hw5_WL3011

Weiqi Liang

2024-11-13

Setup File

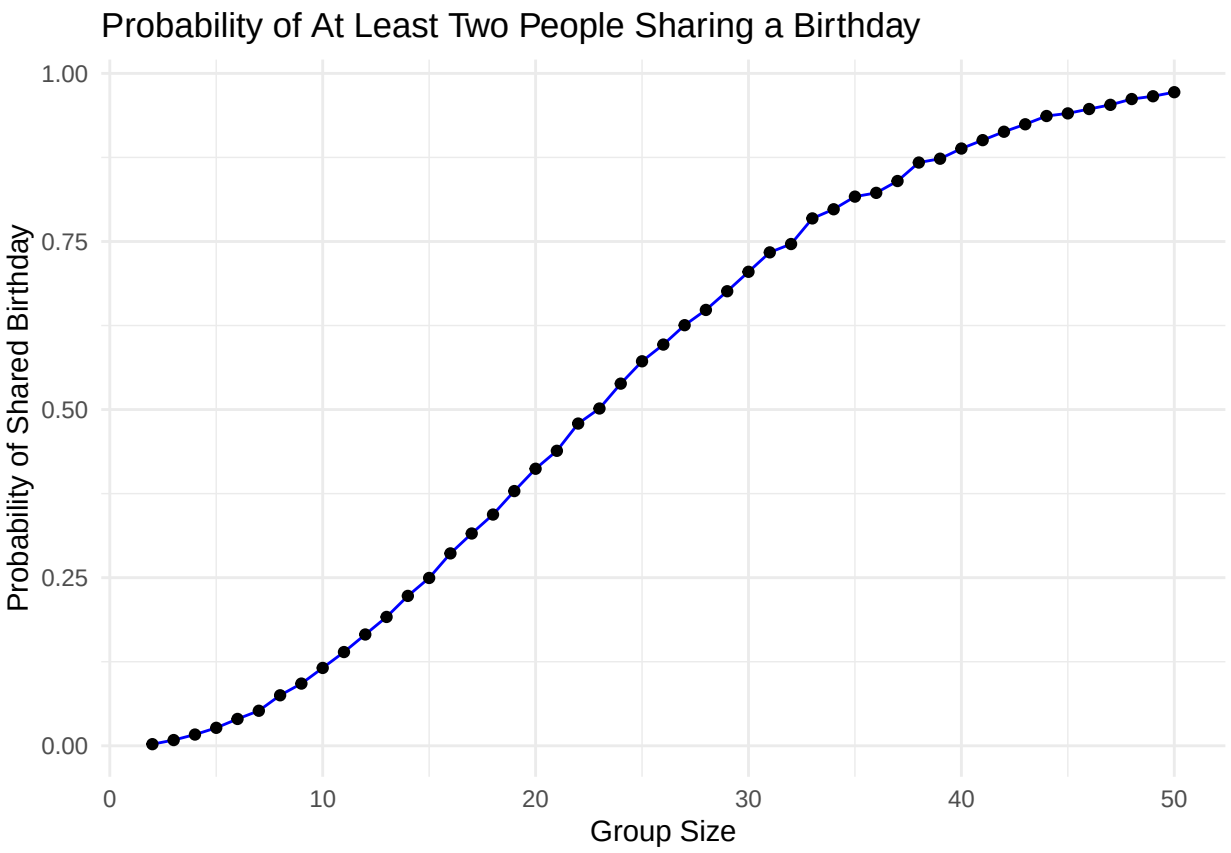
```
library(tidyverse)
library(ggplot2)
library(broom)
library(dplyr)
library(purrr)
set.seed(1)
```

Problem 1

```
bday_sim = function(n){

  #365 days, 10 people. Allow the same birthday
  bdays = sample(1:365, size = n, replace = TRUE)

  #check if anyone has the same birthday
  duplicate = length(unique(bdays)) < n

  return(duplicate)
}
```

```
sim_res =
  expand_grid(
    n = 2:50,
    iter = 1:10000
  ) |>
  mutate(res = map_lgl(n, bday_sim)) |>
  group_by(n) |>
  summarise(prob = mean(res))

sim_res |>
  ggplot(aes(x = n, y = prob)) +
  geom_line(color = "blue") +
  geom_point() +
  labs(
    x = "Group Size",
    y = "Probability of Shared Birthday",
```

```
    title = "Probability of At Least Two People Sharing a Birthday"
  ) +
  theme_minimal()
```



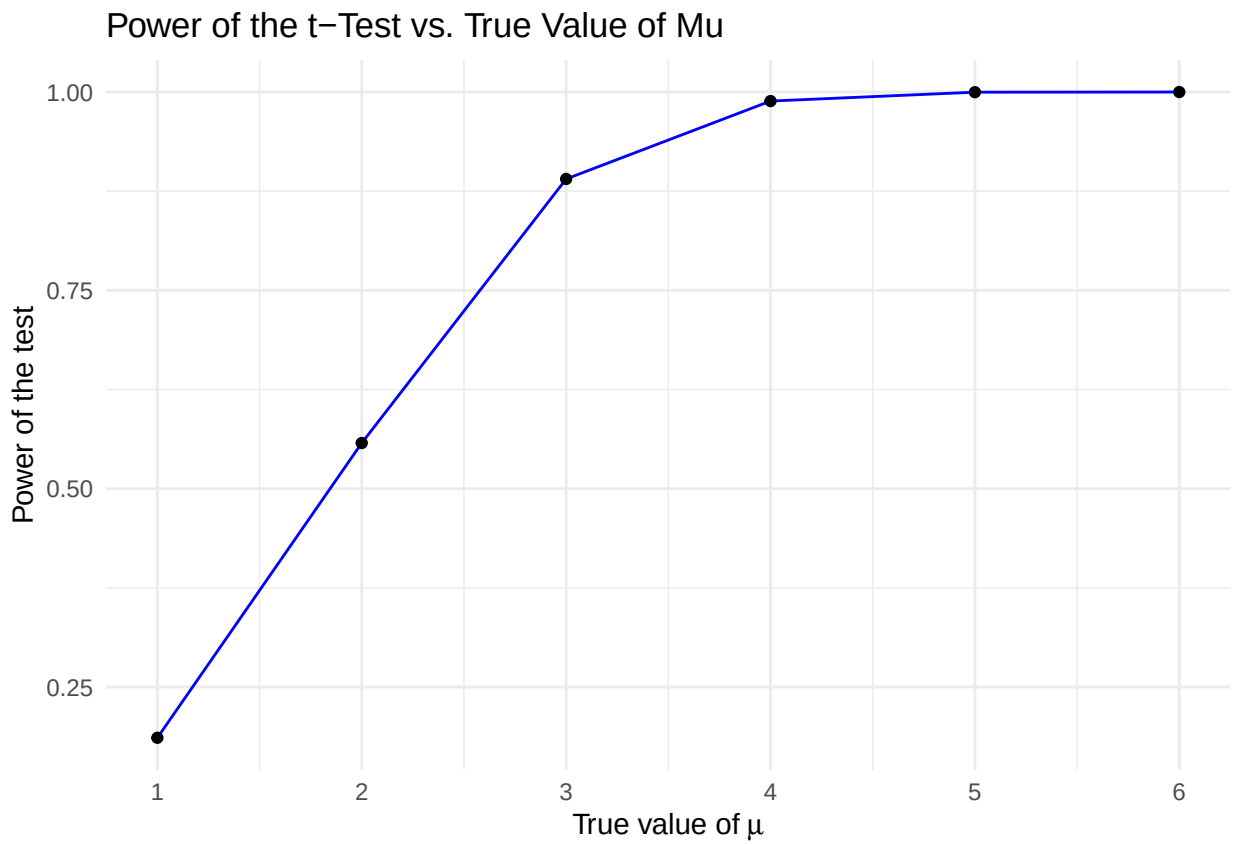
As the group size increases, the probability that at least two people share the same birthday increases rapidly. When the group size is about 23 people, the probability is closer to 0.5 (50%), meaning that in a group of 23 people, there is a 50% chance that at least two people will have the same birthday. When the group size approaches 50 people, the probability approaches 1 (100%), indicating that it is almost certain that at least two people in such a group will share a birthday.

Problem 2

```
# Simulation parameters
n = 30
sigma = 5
iter = 5000
alpha = 0.05
mu_values = 1:6

results = tibble(mu = numeric(), estimate = numeric())

for (mu in mu_values) {
  simulations =
    replicate(iter,
```

```
            {
    sample_data = rnorm(n, mean = mu, sd = sigma)
    test_result = t.test(sample_data, mu = 0)
    tidy(test_result) |>
      select(estimate, p.value) |>
      mutate(mu = mu)
            }, simplify = FALSE) |>
    bind_rows()

  results = bind_rows(results, simulations)
}
```

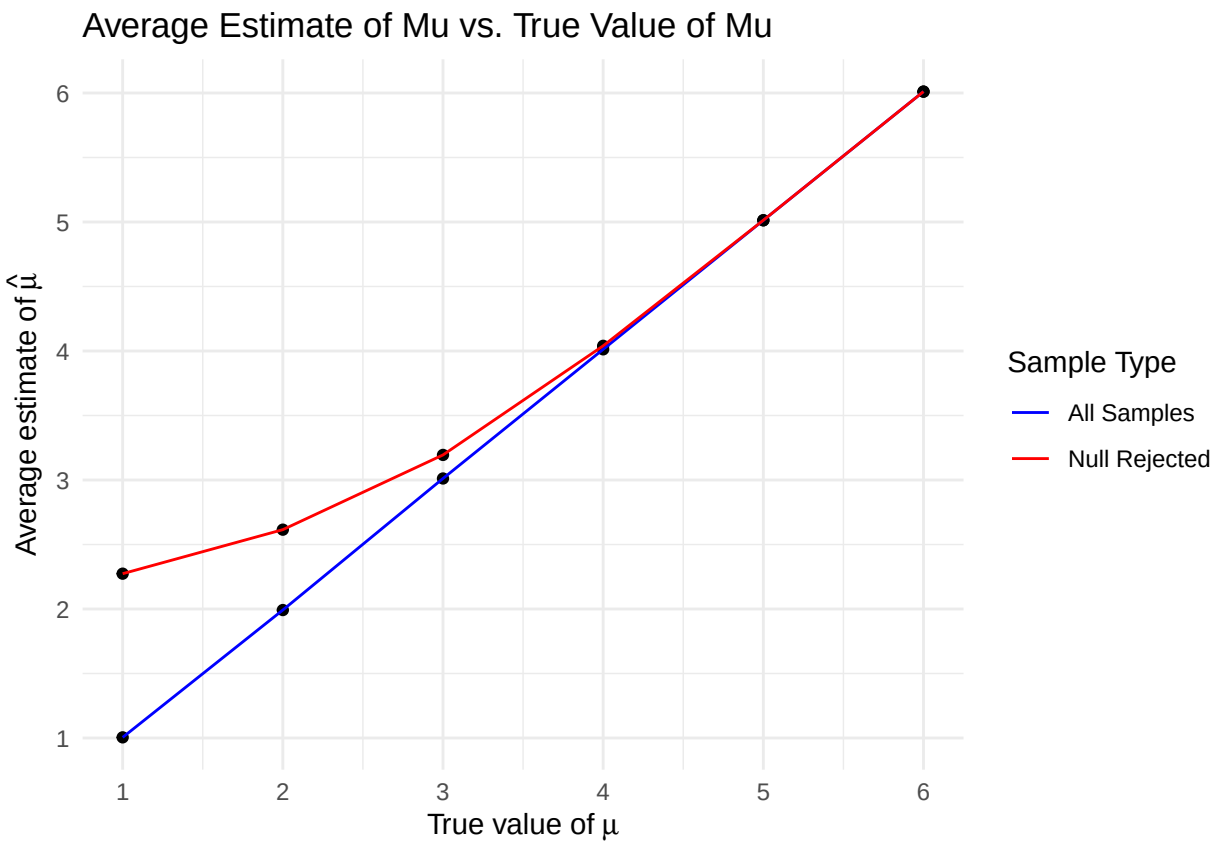
```
power_res = results |>
  group_by(mu) |>
  summarize(power = mean(p.value < alpha))

# Plot power vs true value of mu
ggplot(power_res, aes(x = mu, y = power)) +
  geom_line(color = "blue") +
  geom_point() +
  labs(
    x = expression("True value of" ~ mu),
    y = "Power of the test",
    title = "Power of the t-Test vs. True Value of Mu"
  ) +
  scale_x_continuous(
    breaks = seq(1, 6, by = 1)
  ) +
  theme_minimal()
```



This figure shows the relationship between statistical power of the t-test and the truth value of μ. When $\mu = 1$, the power is lower than 0.25, indicating that the test is difficult to detect the difference. With the increase of μ, the efficacy increases rapidly. When mu reaches 4 and above, the efficacy reaches 1 (100%), indicating that the test almost always correctly rejects the null hypothesis under these conditions.

```
estimate_data = results |>
  group_by(mu) |>
  summarize(
    avg_estimate_all = mean(estimate),
    avg_estimate_rejected = mean(estimate[p.value < alpha], na.rm = TRUE)
  )

# Plot average estimate of mu vs. true value of mu for all samples
ggplot(estimate_data, aes(x = mu)) +
  geom_point(aes(y = avg_estimate_all)) +
  geom_point(aes(y = avg_estimate_rejected)) +
  geom_line(aes(y = avg_estimate_all, color = "All Samples")) +
  geom_line(aes(y = avg_estimate_rejected, color = "Null Rejected")) +
  labs(
    x = expression("True value of" ~ mu),
    y = expression("Average estimate of" ~ hat(mu)),
    title = "Average Estimate of Mu vs. True Value of Mu"
  ) +
  scale_x_continuous(
    breaks = seq(1, 6, by = 1)
  ) +
```

```
  scale_y_continuous(
    breaks = seq(1, 6, by = 1)
  ) +
  scale_color_manual(name = "Sample Type", values = c("All Samples" = "blue", "Null Rejected" = "red")) +
  theme_minimal()
```



The sample average of $\hat{\mu}$ across tests for which the null is rejected approximately not equal to the true value of μ. When considering only samples that reject the null hypothesis, the average estimate $\hat{\mu}$ will generally deviate from the true value μ. This is because in the case where the null hypothesis is rejected, a larger $\hat{\mu}$ may be favored due to selection bias.

Problem 3

```
homicide_data = read_csv("./homicide-data.csv",
           na = c("NA", ".", "")) |>
           janitor::clean_names() |>
  mutate(city_state = paste(city, state, sep = ", ")) |>
  group_by(city_state) |>
  summarize(
    total_homicides = n(),
    unsolved_homicides = sum(disposition %in% c("Closed without arrest", "Open/No arrest"))
  )

baltimore_data =
```

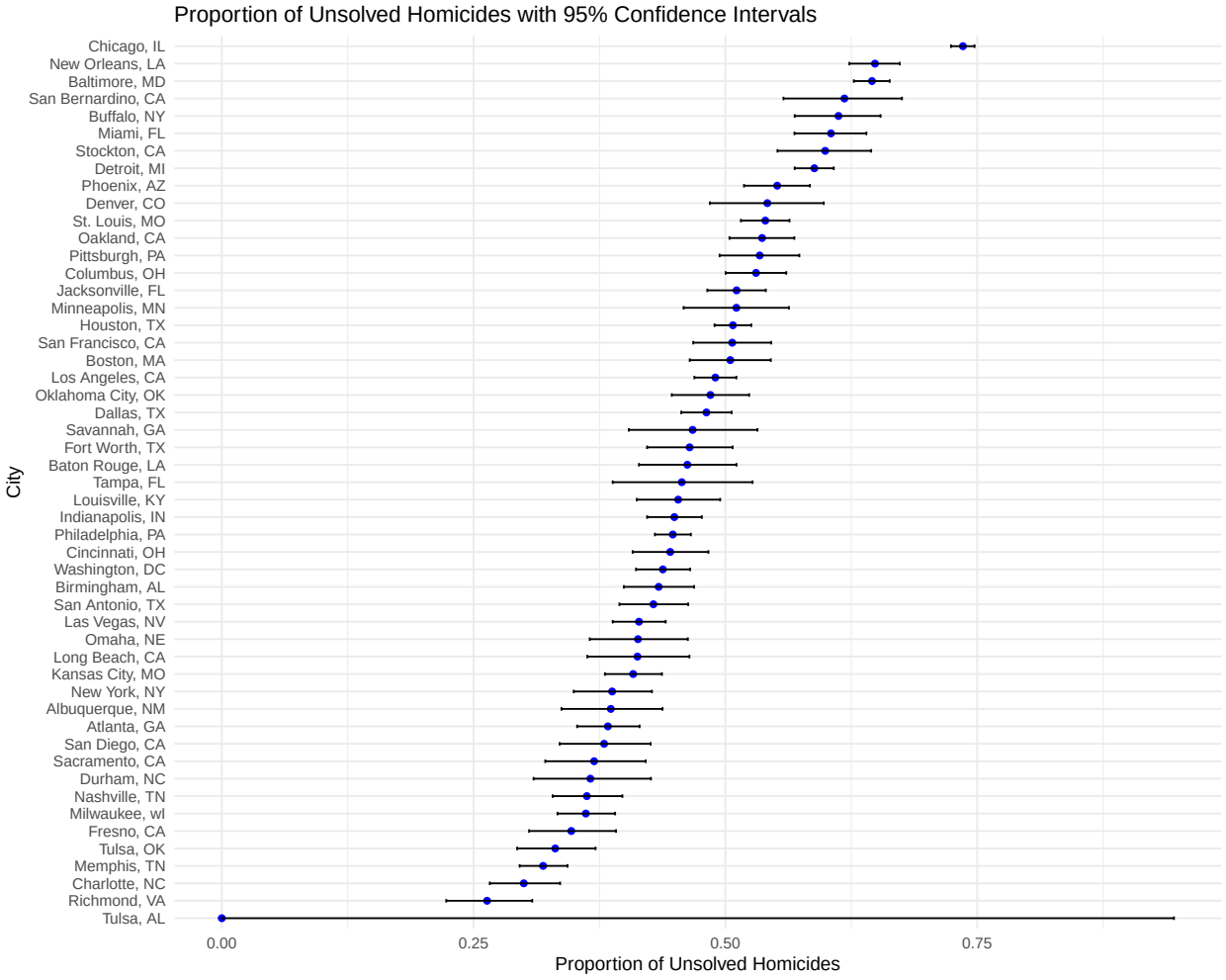
```
  homicide_data |>
  filter(city_state == "Baltimore, MD")

baltimore_prop_test =
  prop.test(
  baltimore_data |> pull(unsolved_homicides),
  baltimore_data |> pull(total_homicides)
)

baltimore_result =
  tidy(baltimore_prop_test) |>
  mutate(
    CI = paste0("(", round(conf.low, 4), ", ", round(conf.high, 4), ")")
  ) |>
  select(estimate, CI)
```

```
knitr::kable(baltimore_result)
```

estimate	CI
0.6455607	(0.6276, 0.6632)

```
city_result = homicide_data |>
  mutate(
    prop_test = map2(unsolved_homicides, total_homicides, ~ prop.test(.x, .y) |> tidy())
  ) |>
  unnest(prop_test) |>
  select(city_state, estimate, conf.low, conf.high)

city_result_table = city_result |>
  mutate(
    ci = paste0("(", round(conf.low, 4), ", ", round(conf.high, 4), ")")
  ) |>
  select(city_state, estimate, ci)
```

```
ggplot(city_result, aes(x = reorder(city_state, estimate), y = estimate)) +
  geom_point(color = "blue") +
  geom_errorbar(aes(ymin = conf.low, ymax = conf.high), width = 0.2) +
  labs(
    x = "City",
    y = "Proportion of Unsolved Homicides",
    title = "Proportion of Unsolved Homicides with 95% Confidence Intervals"
  ) +
  theme_minimal() +
  coord_flip()
```


Proportion of Unsolved Homicides with 95% Confidence Intervals
Chicago, IL
New Orleans, LA
Baltimore, MD
San Bernardino, CA
Buffalo, NY
Miami, FL
Stockton, CA
Detroit, MI
Phoenix, AZ
Denver, CO
St. Louis, MO
Oakland, CA
Pittsburgh, PA
Columbus, OH
Jacksonville, FL
Minneapolis, MN
Houston, TX
San Francisco, CA
Boston, MA
Los Angeles, CA
Oklahoma City, OK
Dallas, TX
Savannah, GA
Fort Worth, TX
Baton Rouge, LA
Tampa, FL
Louisville, KY
Indianapolis, IN
Philadelphia, PA
Cincinnati, OH
Washington, DC
Birmingham, AL
San Antonio, TX
Las Vegas, NV
Omaha, NE
Long Beach, CA
Kansas City, MO
New York, NY
Albuquerque, NM
Atlanta, GA
San Diego, CA
Sacramento, CA
Durham, NC
Nashville, TN
Milwaukee, wI
Fresno, CA
Tulsa, OK
Memphis, TN
Charlotte, NC
Richmond, VA
Tulsa, AL
City
0.00
0.25
0.50
0.75
Proportion of Unsolved Homicides