dt

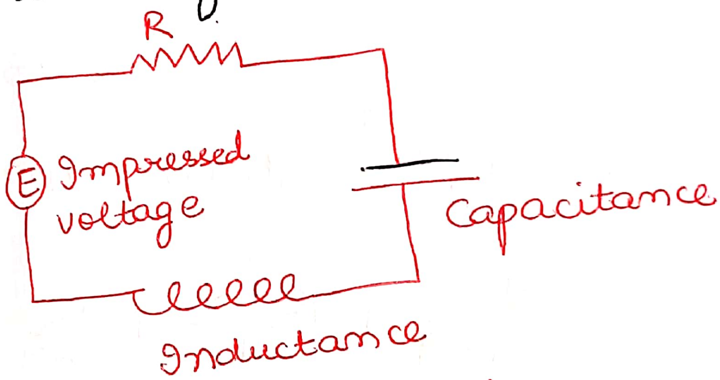
Sol
$$x = c_1 e^{t} + c_2 e^{-t} - 1$$
$$y = c_1 e^{t} - c_2 e^{-t} - 1.$$

) $\frac{dx}{dt} = 2y - 1$, $\frac{dy}{dt} = 1 + 2x$

Sol
$$x(t) = c_1 e^{2t} + c_2 e^{-2t} - \frac{1}{2}$$
$$y(t) = c_1 e^{2t} - c_2 e^{-2t} + \frac{1}{2}$$

RLC- Circuit

An RLC-series circuit consists of a resistor, a conductor, a capacitor and an emf as shown in the figure



RLC - circuit

Using the Kirchhoff's law i.e sum of the voltage drops across the three elements inductor, resistor and capacitance equal to the external source E. Thus the RLC - circuit is modeled by

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = E(t) \quad \text{—} \ (1)$$

which contains two independent Q and I

Since $I = \frac{dQ}{dt}$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = E(t) \quad -(2).$$

Differentiate (1) wrt t

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = \frac{dE}{dt}.$$

Q A circuit consists of an inductance of 0.05 henrys, a resistance of 5 ohms and a condensor of capacitance 4×10^{-4} farad. If $Q = I = 0$ when $t=0$ find $Q(t)$ and $I(t)$ when there is a constant emf of 110 volts.

Sol $L = 0.05$, $R = 5$, $C = 4\times10^{-4}$, $E = 110$

The differential equation of RLC - circuit

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

$$0.05\frac{d^2Q}{dt^2} + 5\frac{dQ}{dt} + \frac{Q}{4\times10^{-4}} = 110$$

or $$\frac{d^2Q}{dt^2} + 100\frac{dQ}{dt} + 50{,}000\,Q = 2200$$

$$m^2 + 100m + 50{,}000 = 0$$

$$m = -50 \pm 50\sqrt{19}\,i.$$

$$C_F = e^{-50t}\left(A\cos 50\sqrt{19}t + B\sin 50\sqrt{19}t\right)$$

$$Q_{sp} = \frac{1}{D^2 + 100D + 50{,}000} 2200 = \frac{2200}{50{,}000} = \frac{11}{250}$$

$$Q = e^{-50t}\left(A\cos \omega t + B\sin \omega t\right) + \frac{11}{250}$$

$\omega = 50\sqrt{19}$ $\qquad Q = 0$ at $t = 0$.

$$0 = A + \frac{11}{250} \qquad A = \frac{-11}{250}.$$

Differentiating Q w.r.t

$$I(t) = \frac{dQ}{dt} = e^{-50t}\left(-\omega A\sin \omega t + B\omega \cos \omega t\right)$$

$$- 50 e^{-50t}\left(A\cos \omega t + B\sin \omega t\right)$$

Since $I = 0$ at $t = 0$, we have.

$$0 = B\omega - 50A \quad \text{or} \quad B = \frac{50A}{\omega}$$

$$I(t) = e^{-50t} \big($$

$$I(t) = \frac{44}{\sqrt{19}} e^{-50t} \sin 50\sqrt{19}\, t.$$