Laplace Transform

$$L(1) = \frac{1}{s}$$

$$L(e^{at}) = \frac{1}{s-a}$$

$$L(e^{-at}) = \frac{1}{s+a}$$

$$L(t) = \frac{1}{s^2}$$

$$L(t^2) = \frac{2!}{s^3}$$

$$L(t^3) = \frac{3!}{s^4}$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

where n is a +ve integer

$$L(\sin at) = \frac{a}{s^2+a^2}$$

$$L(\cos at) = \frac{s}{s^2+a^2}$$

$$L(\sin hat) = \frac{a}{s^2-a^2}$$

$$L(\cos hat) = \frac{s}{s^2-a^2}$$

$$L(t\sin at) = \frac{2as}{(s^2+a^2)^2}$$

Inverse Laplace Transform

$$L^{-1}\left(\frac{1}{s}\right) = 1$$

$$L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$$

$$L^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$$

$$L^{-1}\left(\frac{1}{s^2}\right) = t$$

$$L^{-1}\left(\frac{2!}{s^3}\right) = t^2$$

$$L^{-1}\left(\frac{3!}{s^4}\right) = t^3$$

$$L^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$$

$$L^{-1}\left(\frac{a}{s^2+a^2}\right) = \sin at$$

$$L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$$

$$L^{-1}\left(\frac{a}{s^2-a^2}\right) = \sin hat$$

$$L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cos hat$$

$$L^{-1}\left(\frac{2as}{(s^2+a^2)^2}\right) = t\sin at$$

Problems :

1. Find $L^{-1}\left(\frac{1}{s-3}+s+\frac{s}{s^2-4}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s-3}+\frac{1}{s}+\frac{s}{s^2-4}\right)=L^{-1}\left(\frac{1}{s-3}\right)+L^{-1}(s)+L^{-1}\left(\frac{s}{s^2-4}\right)$$
$$=e^{3t}+1+\cos h2t$$
$$=e^{3t}+\cos h2t+1$$

2. Find $L^{-1}\left(\frac{1}{s^2}+\frac{1}{s+4}+\frac{1}{s^2+4}+\frac{s}{s^2-9}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s^2}+\frac{1}{s+4}+\frac{1}{s^2+4}+\frac{s}{s^2-9}\right)$$
$$=L^{-1}\left(\frac{1}{s^2}\right)+L^{-1}\left(\frac{1}{s+4}\right)+L^{-1}\left(\frac{1}{s^2+4}\right)+L^{-1}\left(\frac{s}{s^2-9}\right)$$
$$=t+e^{-4t}+\frac{\sin 2t}{2}+\cos h3t$$

3. Find $L^{-1}\left(\frac{1}{s}+\frac{2}{s^2}-\frac{3s}{s^2+4}+\frac{4}{s^2+16}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s}+\frac{2}{s^2}-\frac{3s}{s^2+4}+\frac{4}{s^2+16}\right)$$
$$=L^{-1}\left(\frac{1}{s}\right)+L^{-1}\left(\frac{2}{s^2}\right)-L^{-1}\left(\frac{3s}{s^2+4}\right)+L^{-1}\left(\frac{4}{s^2+16}\right)$$
$$=1+2t-3\cos 2t+\sin 4t$$

4. Find $L^{-1}\left(\frac{4}{s^6}-\frac{2}{s^{10}}+\frac{2}{s^2-9}+\frac{3s}{s^2+25}\right)$

Solution :

$$L^{-1}\left(\frac{4}{s^6}-\frac{2}{s^{10}}+\frac{2}{s^2-9}+\frac{3s}{s^2+25}\right)$$
$$=\frac{4}{5!}L^{-1}\left(\frac{5!}{s^6}\right)-\frac{2}{9!}L^{-1}\left(\frac{9!}{s^{10}}\right)+\frac{2}{3}L^{-1}\left(\frac{3}{s^2-9}\right)+3L^{-1}\left(\frac{s}{s^2+25}\right)$$
$$=\frac{1}{36}t^5-\frac{1}{181440}t^9+\frac{2}{3}\sin h3t+3\cos 5t$$

5. Find $L^{-1}\left(\frac{2}{s^5}-\frac{3}{s^4}+\frac{3}{s^2-3}+\frac{5}{s^2-100}+\frac{s}{s^2+10}\right)$

Solution :

$$L^{-1}\left(\frac{2}{s^5}-\frac{3}{s^4}+\frac{3}{s^2-3}+\frac{5}{s^2-100}+\frac{s}{s^2+10}\right)$$
$$=\frac{2}{4!}L^{-1}\left(\frac{4!}{s^5}\right)-\frac{3}{3!}L^{-1}\left(\frac{3!}{s^4}\right)+\frac{3}{\sqrt{3}}L^{-1}\left(\frac{\sqrt{3}}{s^2-\sqrt{3^2}}\right)+\ \frac{5}{10}L^{-1}\left(\frac{10}{s^2-100}\right)+L^{-1}\left(\frac{s}{s^2+10}\right)$$
$$=\frac{1}{12}t^4-\frac{1}{2}t^3\sqrt{3}\sin\sqrt{3}t+\frac{1}{2}\sin h10t+\cos\sqrt{10}t$$

6. Find $L^{-1}\left(\frac{5}{s^2-25}+\frac{4s}{s^2-16}+\frac{s}{s^2+9}+\frac{s}{s^2-25}\right)$

Solution :

$$L^{-1}\left(\frac{5}{s^2-25}+\frac{4s}{s^2-16}+\frac{s}{s^2+9}+\frac{s}{s^2-25}\right)$$
$$=L^{-1}\left(\frac{5}{s^2-25}\right)+4L^{-1}\left(\frac{s}{s^2-16}\right)+L^{-1}\left(\frac{s}{s^2+9}\right)+L^{-1}\left(\frac{s}{s^2-25}\right)$$
$$=\sin h5t+4\cos h4t+\cos 3t-\cos h5t$$

7. Find $L^{-1}\left(\frac{1}{2s+3}\right)$

Solution :

$$L^{-1}\left(\frac{1}{2s+3}\right)=\frac{1}{2}L^{-1}\left(\frac{1}{s+3/2}\right)$$
$$=\frac{1}{2}e^{-3/2t}$$

19. First Shiffting Property

(i) If $L^{-1}(F(s)) = f(t)$ then $L^{-1}(F(s-a)) = e^{at}L^{-1}(F(s))$

Proof:

We know that $L(f(t)) = F(s)$ then $L(e^{at}f(t)) = F(s-a)$

Hence $e^{at}f(t) = L^{-1}(F(s-a))$

$$e^{at}L^{-1}(F(s)) = L^{-1}(F(s-a))$$

1. Find $L^{-1}\left(\frac{1}{(s+1)^2}\right)$

Solution :

$$L^{-1}\left(\frac{1}{(s+1)^2}\right) = e^{-t}L^{-1}\left(\frac{1}{s^2}\right)$$
$$= e^{-t}t$$

2. Find $L^{-1}\left(\frac{1}{(s+1)^2+1}\right)$

Solution :

$$L^{-1}\left(\frac{1}{(s+1)^2+1}\right) = e^{-t}L^{-1}\left(\frac{1}{s^2+1}\right)$$
$$= e^{-t}\sin t$$

3. Find $L^{-1}\left(\frac{s-3}{(s-3)^2+4}\right)$

Solution :

$$L^{-1}\left(\frac{s-3}{(s-3)^2+4}\right) = e^{3t}L^{-1}\left(\frac{s}{s^2+4}\right)$$
$$= e^{3t}\cos 2t$$

4. Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

Solution :

$$L^{-1}\left(\frac{S}{(s+2)^2}\right) = L^{-1}\left(\frac{s+2-2}{(s+2)^2}\right)$$
$$= L^{-1}\left(\frac{s+2}{(s+2)^2} - \frac{2}{(s+2)^2}\right)$$
$$= L^{-1}\left(\frac{1}{(s+2)}\right) - 2L^{-1}\left(\frac{1}{(s+2)^2}\right)$$
$$= e^{-2t} - 2e^{-2t}\cdot t$$
$$= e^{-2t}(1-2t)$$

1. Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$

Solution :

Let $F'(s)=\frac{s}{(s^2+a^2)^2}$

$$\frac{d}{ds}F(s)=\frac{s}{(s^2+a^2)^2}$$

$$\therefore F(s)=\int\frac{s}{(s^2+a^2)^2}ds$$

Put $s^2+a^2=u$

$$2sds=du$$

$$\therefore \int\frac{s}{(s^2+a^2)^2}ds=\int\frac{\frac{du}{2}}{u^2}$$

$$=\frac{-1}{2u}=\frac{-1}{2(s^2+a^2)}$$

$$\therefore F(s)=\frac{-1}{2(s^2+a^2)}$$

We know that $L(F'(s))=-tL^{-1}(F(s))$

$$\therefore L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)=-tL^{-1}\left(\frac{-1}{2(s^2+a^2)}\right)$$

$$=\frac{t}{2}L^{-1}\left(\frac{1}{s^2+a^2}\right)$$

$$=\frac{t}{2}\frac{1}{a}L^{-1}\left(\frac{a}{s^2+a^2}\right)$$

$$=\frac{t}{2a}\sin at$$

2. Find $L^{-1}\left(\dfrac{s+3}{(s^2+6s+13)^2}\right)$

Solution :

Let $\left(\dfrac{s+3}{(s^2+6s+13)^2}\right) = F'(s)$

$$\frac{dF(s)}{ds} = \frac{s+3}{(s^2+6s+13)^2}$$

$$\therefore F(s) = \frac{(s+3)ds}{(s^2+6s+13)^2}$$

Put $s^2+6s+13=u$

$$(2s+6)ds=du$$

$$2(s+3)ds=du$$

(ie) $F(s)=\int\frac{\frac{du}{2}}{u^2}=\frac{-1}{2u}$

$$=\frac{-1}{2(s^2+6s+13)}$$

We know that $L^{-1}(F'(s))=-tL^{-1}(F(s))$

$$\therefore L^{-1}\left(\frac{s+3}{(s^2+6s+13)^2}\right)=-tL^{-1}\left(\frac{-1}{2(s^2+6s+13)}\right)$$

$$=\frac{t}{2}L^{-1}\left(\frac{-1}{(s^2+6s+13)}\right)$$

$$=\frac{t}{2}L^{-1}\left(\frac{1}{(s+3)^2+2^2}\right)$$

$$=\frac{t}{2}e^{-3t}L^{-1}\left(\frac{1}{s^2+2^2}\right)$$

$$=\frac{t}{2}e^{-3t}\frac{1}{2}L^{-1}\left(\frac{2}{s^2+2^2}\right)$$

$$=\frac{t}{4}e^{-3t}\sin 2t$$

3. Find $L^{-1}\left(\frac{2(s+1)}{(s^2+2s+2)^2}\right)$

Solution :

$$F'(s)=\frac{2(s+1)}{(s^2+2s+2)^2}$$

$$\frac{dF(s)}{ds}=\frac{2(s+1)}{(s^2+2s+2)^2}$$

$$F(s)=\int\frac{2(s+1)}{(s^2+2s+2)^2}ds$$

Put $s^2+2s+2=u$

$$(2s+2)ds=du$$

$$2(s+2)ds=du$$

$$\therefore F(s) = \int \frac{du}{u^2}$$

$$= \frac{-1}{u}$$

$$= \frac{-1}{s^2+2s+2}$$

$$\therefore L^{-1}\left(\frac{2(s+1)}{(s^2+2s+2)^2}\right) = -tL^{-1}\left(\frac{-1}{s^2+2s+2}\right)$$

$$= tL^{-1}\left(\frac{1}{s^2+2s+2}\right) = tL^{-1}\left(\frac{1}{(s+1)^2+1}\right)$$

$$= te^{-t}L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= te^{-t}\sin t$$

4. Find $L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right)$

Solution :

Let $F'(s) = \frac{s+2}{(s^2+4s+5)^2}$

Integrate both sides w.r.t 'S'

$$F'(s) = \frac{s+2}{(s^2+4s+5)^2}$$

$$\int F'(s) = \int \frac{(s+2)ds}{(s^2+4s+5)^2}$$

$$F(s) = \int \frac{(s+2)ds}{(s^2+4s+5)^2}$$

Let $y = s^2+4s+5$

$dy = (2s+4)ds$

$\frac{dy}{2} = (s+2)ds$

$$F(s) = \int \frac{dy/2}{y^2}$$

$$= \frac{1}{2}\int \frac{dy}{y^2}$$

$$= \frac{1}{2}\int y^{-2}dy$$

$$F(s) = \frac{1}{2}\left(\frac{y^{-2+1}}{-2+1}\right)$$

$$= \frac{-1}{2y}$$

-1

$$= \frac{-1}{2(s^2+4s+5)}$$

We know that

$$L^{-1}(F'(s)) = -tL^{-1}(F(s))$$

$$L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right) = -tL^{-1}\left(\frac{-1}{2(s^2+4s+5)}\right)$$

$$L^{-1}\left(\frac{s+2}{(s^2+4s+5)^2}\right) = \frac{t}{2}L^{-1}\left(\frac{1}{s^2+4s+5}\right)$$

$$= \frac{t}{2}L^{-1}\left(\frac{1}{(s+2)^2+1}\right)$$

$$= \frac{t}{2}e^{-2t}L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$= \frac{t}{2}e^{-2t}\sin t$$

1. Find $L^{-1}\left(\dfrac{s}{(s+2)^2+4}\right)$

Solution :

$$
\begin{aligned}
L^{-1}\left(\frac{s}{(s+2)^2+4}\right) &= L^{-1}\left(s\cdot\frac{1}{(s+2)^2+4}\right) \\
&= \frac{d}{dt}\left(\frac{1}{(s+2)^2+4}\right) \qquad \text{(using the above result)} \\
&= \frac{d}{dt}e^{-2t}L^{-1}\left(\frac{1}{s^2+4}\right) \\
&= \frac{d}{dt}e^{-2t}L^{-1}\left(\frac{1}{s^2+4}\right) \\
&= \frac{d}{dt}\left(e^{-2t}\frac{1}{2}\sin 2t\right) \\
&= \frac{1}{2}\left(2e^{-2t}\cos 2t+\sin 2te^{-2t}(-2)\right) \\
&= e^{-2t}(\cos 2t-\sin 2t)
\end{aligned}
$$

2. Find $L^{-1}\left(\frac{s}{(s+2)^2}\right)$

Solution :

$$
\begin{aligned}
L^{-1}\left(\frac{s}{(s+2)^2}\right) &= L^{-1}\left(\frac{s}{(s+2)^2}\right) \\
&= L^{-1}\left(s \cdot \frac{1}{(s+2)^2}\right) \\
&= \frac{d}{dt} L^{-1}\left(\frac{1}{(s+2)^2}\right) \\
&= \frac{d}{dt} e^{-2t} L^{-1}\left(\frac{1}{s^2}\right) \\
&= e^{-2t} + t(e^{-2t}(-2)) \\
&= e^{-2t}(1-2t)
\end{aligned}
$$

Aliter :

3. Find $L^{-1}\left(\dfrac{s^2}{(s^2+a^2)^2}\right)$

Solution :

$$L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right)=L^{-1}\left(s\cdot\frac{s}{(s^2+a^2)}\right)$$

$$=\frac{d}{dt}L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$$

$$=\frac{d}{dt}\left(\frac{t}{2a}\sin at\right) \qquad \text{(By the Previous Section 21.1 Problem No.1)}$$

$$=\frac{1}{2a}(at\cos at+\sin at)$$

4. Find $L^{-1}\left(\dfrac{s^2}{(s-1)^4}\right)$

Solution :

$$L^{-1}\left(\frac{s^2}{(s-1)^4}\right)=L^{-1}\left(s\cdot\frac{s}{(s-1)^4}\right)$$

$$=\frac{d}{dt}L^{-1}\left(\frac{s}{(s-1)^4}\right)$$

$$=\frac{d}{dt}L^{-1}\left(\frac{s-1+1}{(s-1)^4}\right)$$

$$=\frac{d}{dt}\left(L^{-1}\left(\frac{s-1}{(s-1)^4}\right)+L^{-1}\left(\frac{1}{(s-1)^4}\right)\right)$$

$$=\frac{d}{dt}\left(L^{-1}\left(\frac{1}{(s-1)^3}\right)+L^{-1}\left(\frac{1}{(s-1)^4}\right)\right)$$

$$=\frac{d}{dt}\left(e^tL^{-1}\left(\frac{1}{S^3}\right)+e^tL^{-1}\left(\frac{1}{S^4}\right)\right)$$

$$=\frac{d}{dt}\left(e^t\frac{t^2}{2}+e^t\frac{t^3}{6}\right)$$

$$=\frac{1}{2}\left(e^t2t+t^2e^t\right)+\frac{1}{6}\left(e^t3t^2+t^3e^t\right)$$

$$=te^t+e^tt^2+\frac{t^3e^t}{6}$$

5. Find $L^{-1}\left(\dfrac{s-3}{s^2+4s+13}\right)$

Solution :

$$\begin{aligned}
L^{-1}\left(\frac{s-3}{s^2+4s+13}\right) &= L^{-1}\left(\frac{s}{s^2+4s+13}\right) - L^{-1}\left(\frac{3}{s^2+4s+13}\right)\\
&= \frac{d}{dt}L^{-1}\left(\frac{1}{s^2+4s+13}\right) - 3L^{-1}\left(\frac{1}{s^2+4s+13}\right)\\
&= \frac{d}{dt}L^{-1}\left(\frac{1}{(s+2)^2+9}\right) - 3L^{-1}\left(\frac{1}{(s+2)^2+3^2}\right)\\
&= \frac{d}{dt}e^{-2t}L^{-1}\left(\frac{1}{s^2+3^2}\right) - 3e^{-2t}L^{-1}\left(\frac{1}{s^2+3^2}\right)\\
&= \frac{d}{dt}\left(e^{-2t}\frac{\sin 3t}{3}\right) - 3e^{-2t}\left(\frac{\sin 3t}{3}\right)\\
&= \frac{1}{3}(3e^{-2t}\cos 3t - 2\sin 3te^{-2t}) - e^{-2t}\sin 3t\\
&= e^{-2t}\cos 3t - \frac{5}{3}e^{-2t}\sin 3t
\end{aligned}$$

1. Find $L^{-1}\left(\frac{1}{s(s+1)}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s(s+1)}\right)=\int_0^t L^{-1}\left(\frac{1}{(s+1)}\right)dt \qquad \text{(by the above theorem)}$$

$$=\int_0^t e^{-t}dt$$

$$=\left(-e^{-t}\right)_0^t$$

$$=-\left(e^{-t}-1\right)$$

$$=1-e^{-t}$$

2. Find $L^{-1}\left(\frac{1}{s(s+2)^3}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s(s+2)^3}\right)=\int_0^t\left(\frac{1}{(s+2)^3}\right)dt$$

$$=\int_0^t e^{-2t}L^{-1}\left(\frac{1}{s^3}\right)dt$$

$$=\int_0^t \frac{e^{-2t}}{2}L^{-1}\left(\frac{2}{s^3}\right)dt$$

$$=\frac{1}{2}\int_0^t e^{-2t}t^2dt$$

$$=\frac{1}{2}\left[(t^2)\left(\frac{e^{-2t}}{-2}\right)-(2t)\left(\frac{e^{-2t}}{4}\right)+2\left(\frac{e^{-2t}}{-8}\right)\right]_0^t$$

$$\left[\because \int udv=uv-u'v_1+u''v_2\cdots\right]$$

$$=\frac{1}{2}\left[\frac{-t^2e^{-2t}}{2}-\frac{te^{-2t}}{2}-\frac{e^{-2t}}{4}+\frac{1}{4}\right]$$

$$=\frac{1}{2}\left[\frac{-e^{-2t}}{2}\left(t^2+t+\frac{1}{2}\right)+\frac{1}{4}\right]$$

$$=\frac{1}{8}\left(1-(2t^2+2t+1)e^{-2t}\right)$$

3. Find $L^{-1}\left(\dfrac{54}{s^3(s-3)}\right)$

Solution :

$$L^{-1}\left(\frac{54}{s^3(s-3)}\right)=54\int_0^t\int_0^t\int_0^t L^{-1}\left(\frac{1}{(s-3)}\right)dtdtdt$$

$$=54\int_0^t\int_0^t\int_0^t e^{3t}dtdtdt$$

$$=54\int_0^t\int_0^t\left(\frac{e^{3t}}{3}\right)_0^t dtdt$$

$$=18\int_0^t\int_0^t(e^{3t}-1)dtdt$$

$$=18\int_0^t\left(\frac{e^{3t}}{3}-t\right)_0^t dt$$

$$=18\int_0^t\left(\frac{e^{3t}}{3}-t\right)-\left(\frac{1}{3}-0\right)dt$$

$$=18\int_0^t\left(\frac{e^{3t}}{3}-t-\frac{1}{3}\right)dt$$

$$=18\left(\frac{e^{3t}}{9}-\frac{t^2}{2}-\frac{1}{3}t\right)_0^t$$

$$=18\left(\frac{e^{3t}}{9}-\frac{t^2}{2}-\frac{t}{3}-\frac{1}{9}\right)$$

$$=2e^{3t}-9t^2-6t-2$$

4. Find $L^{-1}\left(\dfrac{1}{s(s^2+a^2)}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)=\int_0^t L^{-1}\left(\frac{1}{s^2+a^2}\right)dt$$

$$=\int_0^t\frac{1}{a}L^{-1}\left(\frac{a}{s^2+a^2}\right)dt$$

$$=\frac{1}{a}\int_0^t\sin at dt$$

$$=\frac{1}{a}\left(\frac{-\cos at}{a}\right)_0^t$$

$$=\frac{-1}{a^2}(\cos at-1)$$

$$=\frac{+1}{a^2}(1-\cos at)$$

5. Find $L^{-1}\left(\dfrac{1}{(s^2+a^2)^2}\right)$

Solution :

$$L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right) = L^{-1}\left(\frac{s}{s(s^2+a^2)^2}\right)$$

$$= L^{-1}\left(\frac{1}{s}\cdot\frac{s}{(s^2+a^2)^2}\right)$$

$$= \int_0^t L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right) dt$$

$$= \int_0^t \frac{t\sin at}{2a}\, dt$$

$$= \frac{1}{2a}\left(t\left(\frac{-\cos at}{a}\right) - 1\left(\frac{-\sin at}{a^2}\right)\right)_0^t$$

$$= \frac{1}{2a}\left(\frac{-t\cos at}{a} + \frac{\sin at}{a^2}\right)$$

6. Find $L^{-1}\left(\dfrac{1}{s(s^2-2s+5)}\right)$

Solution :

$$L^{-1}\left(\frac{1}{s(s^2-2s+5)}\right)=L^{-1}\left(\frac{1}{s}\cdot\frac{1}{s^2-2s+5}\right)$$

$$=\int_0^t L^{-1}\left(\frac{1}{s^2-2s+5}\right)dt$$

$$=\int_0^t L^{-1}\left(\frac{1}{(s-1)^2+2^2}\right)dt$$

$$=\int_0^t e^t L^{-1}\left(\frac{1}{s^2+2^2}\right)dt$$

$$=\int_0^t e^t\,\frac{\sin 2t}{2}\,dt$$

$$=\frac{1}{2}\int_0^t e^t\sin 2t\;dt$$

$$=\frac{1}{2}\left[\frac{e^t}{1^2+2^2}(\sin 2t-2\cos 2t)\right]_0^t$$

$$=\frac{1}{10}\left[e^t\sin 2t-2e^t\cos 2t\right]_0^t$$

$$=\frac{1}{10}\left[e^t\sin 2t-2e^t\cos 2t-0+2\right]$$

$$=\frac{1}{10}\left[e^t\sin 2t-2e^t\cos 2t+2\right]$$

Find the inverse Laplace transform of the following functions.

1. $\dfrac{e^{-as}}{s^2},\ a>0$ Ans: $\begin{cases} 0 & \text{if } t<a \\ \dfrac{t-a}{1!} & \text{if } t>a \end{cases}$

2. $\dfrac{e^{-2s}-e^{-3s}}{s}$ Ans: $\begin{Bmatrix} 0 & \text{if } t<2 \\ 1 & \text{if } t>2 \end{Bmatrix} + \begin{Bmatrix} 0 & \text{if } t<3 \\ 1 & \text{if } t>3 \end{Bmatrix}$

3. $\dfrac{e^{-3s}}{s-2}$ Ans: $\begin{cases} 0 & \text{if } t<3 \\ e^{2(t-3)} & \text{if } t>3 \end{cases}$

4. $\dfrac{se^{-s}}{s^2+9}$ Ans: $\begin{cases} 0 & \text{if } t<1 \\ \cos 3(t-1) & \text{if } t>1 \end{cases}$

5. $\dfrac{1+e^{-\pi s}}{s^2+1}$ Ans: $\sin t + \begin{cases} 0 & \text{if } t<\pi \\ \sin(t-\pi) & \text{if } t>\pi \end{cases}$

6. $\dfrac{1}{(s+1)^3}$ Ans: $e^{-t}\dfrac{t^2}{2!}$

7. $\dfrac{s^2+2s+3}{s^3}$ Ans: $1+2t+\dfrac{t^2}{2!}$

8. $\dfrac{s}{(s-2)^4}$ Ans: $e^{2t}\dfrac{t^3}{3!}$

9. $\dfrac{2s+3}{s^2+4}$ Ans: $2\cos 2t + 6\sin 2t$

10. $\dfrac{s+6}{s^2-16}$ Ans: $\cos h4t + 24\sin h4t$

Exercise - 1 (h)

Find the inverse Laplace transform of the following functions.

1. $\dfrac{1}{s^2-6s+10}$ Ans: $e^{3t}\sin t$

2. $\dfrac{1}{s^2+8s+16}$ Ans: te^{-4t}

3. $\dfrac{3s-2}{s^2-4s+20}$ Ans: $3e^{2t}\cos 4t + e^{2t}\sin 4t$

4. $\dfrac{3s+7}{s^2-2s-3}$ Ans: $4e^{3t}-e^{-t}$

5. $\dfrac{s+a}{(s+a)^2+b^2}$ Ans: $e^{-at}(b\cos bt-(d-ca)\sin bt)$

6. $\dfrac{s}{(s-b)^2+a^2}$ Ans: $e^{bt}\cos at$

7. $\dfrac{s+1}{s^2+6s+25}$ Ans: $e^{-3t}\left(\cos 4t-\dfrac{1}{2}\sin 4t\right)$

8. $\dfrac{1}{s^2+8s+16}$ Ans: te^{-4t}

9. $\dfrac{s}{(s+3)^2}$ Ans: $e^{-3t}(1-2t)$

10. $\dfrac{s}{(s^2+1)^2}$ Ans: $\dfrac{t}{2}\sin t$

Exercise - 1(i)

Find the inverse Laplace transform of the following functions.

1. $\dfrac{s}{(s-4)^5}$ Ans: $\dfrac{e^{4t}t^3(4-3t)}{24}$

2. $\dfrac{1}{(s^2+9)^2}$ Ans: $\dfrac{\sin 3t-3t\cos 3t}{54}$

3. $\dfrac{s+2}{(s^2+4s+5)^2}$ Ans: $\dfrac{t}{2}e^{-2t}\sin t$

4. $\dfrac{s^2+2s}{(s^2+2s+2)^2}$ Ans: $te^{-t}\cos t$

5. $\dfrac{1}{s(s+2)^3}$ Ans: $\dfrac{1}{S}(1-(1+2t+2t^2)e^{-2t}$

6. $\dfrac{s^2-s+2}{s(s-3)(s+2)}$ Ans: $\dfrac{1}{3}+\dfrac{8}{15}e^{3t}+\dfrac{4}{5}e^{-2t}$

7. $\dfrac{2s-1}{s^2(s-1)^2}$ Ans: $t(e^t-1)$

8. $\dfrac{1}{s^2(s^2+a^2)^2}$ Ans: $\dfrac{at-\sin at}{a^3}$