# Stochastic Optimization for Green Unmanned Aerial Communication Systems with Solar Energy

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Abstract—Unmanned Aerial wireless Communication Systems (UASs), featuring the low cost and flexible deployment of unmanned aerial vehicles (UAVs), have attracted intensive attention recently to provide wireless communication services in some specific scenarios, e.g., disaster areas and temporary hotspots. Nonetheless, due to UAVs' limited on-board energy storage, the provisioning of wireless communications can deplete their carried energy, consequently landing on the ground. To mitigate this issue, harvesting solar energy to power UAVs is a promising alternative solution. However, the essential dynamics of solar energy can seriously affect communication performance in UASs. In this paper, we explore dynamic solar energy to supply a UAS with an aerial base station (BS) and aim to minimize the long-term time-averaged energy consumption of the UAS. Particularly, we formulate a Long-term time-averaged Energy Consumption minimization problem (LEC) by jointly taking into account transmission power and data rate. Considering that LEC is time-coupling nonlinear programming (NLP), we reformulate a relaxed online optimization problem, called STP (single-time slot problem), by employing Lyapunov optimization theory. Then, we develop a joint power and rate control algorithm to solve STP. Particularly, we theoretically show that the proposed algorithm can achieve (D/V + C)-approximation and guarantee stability. Extensive simulation results have shown the performance gain, in terms of stability and throughput, achieved by the proposed algorithm.

*Index Terms*—Unmanned aerial systems, renewable energy, green communications, stochastic optimization.

# I. INTRODUCTION

Terrestrial communication infrastructure plays a critical role in providing people with reliable and high-quality wireless communication services [1]. Nonetheless, it is very challenging to deploy terrestrial communication infrastructure promptly and economically in some scenarios, such as disaster areas and temporary hotspots. For example, it is not practical to build terrestrial communication infrastructure for a temporary hotspot. To address the issue, Unmanned Aerial Vehicle (UAV) has been considered as a promising alternative to provide wireless communication services in a timely manner. Specifically, UAVs are equipped with transceivers, thus being able

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to support wireless communications. In particular, UAVs are usually high-maneuverability and low-cost. As a result, they can be deployed to some areas flexibly to provide on-demand communication services.

However, energy is a critical concern in the Unmanned Aerial wireless communication System (UAS). Due to the limited capacity of on-board batteries, the provisioning of wireless communications can deplete UAVs' carried energy very quickly. Thus, UAVs must land on the ground to replenish the exhausted batteries, which consequently interrupts ongoing wireless communications. Therefore, the energy capacity of UAVs has become a bottleneck of providing ubiquitous, reliable, and high-quality wireless communications.

To break the bottleneck, industry and academia have considered utilizing solar energy to power UAVs. Specifically, UAVs are equipped with solar panels to harvest solar energy, thus replenishing on-board batteries. In so doing, we can mitigate the energy issue in the UAS significantly and hence prolong the time of UAV hovering in the sky. A real-world example of a solar-powered UAS is HAPSMobile Sunglider [2]. On the other hand, some researchers have been concerned about offering wireless communications by exploiting solar-powered UASs [3]-[5]. For instance, Dwivedi et al. proposed and implemented an approach to determine the energy optimal dynamic attitude for a solar-powered aircraft by using the finite-time sliding-mode approach [3]. Sekander et al. investigated a statistical model of the harvested renewable energy and developed a harvest-store-consume architecture for UAV-based communications [4]. Luo et al. studied an energy consumption rate minimization problem by taking into account routing, data rate, and transmission power in a solar-powered UAV relaying system [5]. However, the communication performance can be greatly affected by the essential dynamics of the harvested solar energy, which is largely ignored by previous studies.

In this paper, we propose a stochastic approach to optimize the energy consumption of providing wireless communications in a UAS with dynamic solar energy. Specifically, we first formulate a Long-term time-averaged Energy Consumption minimization problem, called LEC, by jointly considering transmission power and rate. Then, since stochastic opti-

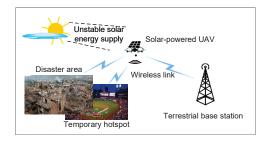


Fig. 1. A typical UAS used in some scenarios.

mization problem LEC is a time-coupling, non-convex, and nonlinear programming, thus in general NP-hard, we employ the Lyapunov optimization theory to reformulate LEC as a relaxed online optimization problem, which is called STP. To solve STP, we develop a joint power and rate control algorithm that is proved to be able to achieve (D/V+C)-approximation. Besides, we also theoretically prove that our developed algorithm can guarantee the strong stability of the UAS. Extensive simulation results have been provided to demonstrate the efficacy of our proposed algorithm.

The rest of the paper is organized as follows. We present the considered system model in Section II. Section III describes the problem formulation, the solution to the formulated problem, and the theoretic performance analysis. We subsequently show simulation results in Section IV, and finally conclude the paper in Section V.

#### II. SYSTEM MODEL

We consider downlink communications in a simple but typical UAS with a solar-powered aerial base station (BS) and multiple mobile users (MUs), as shown in Fig. 1. Specifically, the aerial BS is a UAV equipped with a transceiver. It first receives data from the terrestrial BS via a backhaul link and then provides downlink wireless communications to a set  $\mathcal{M} = \{1, 2, \cdots, M\}$  of MUs. Besides, the aerial BS is also equipped with solar panels that harvest solar energy, and the harvested energy is stored at on-board batteries [3].

We consider employing an orthogonal frequency division multiple access (OFDMA) method to assign orthogonal channels to MUs, in order to avoid channel interference. Moreover, we consider the UAS as a time-slotted communication system with index t corresponding to a slot  $(t-1,t] \cdot T_0$ , where  $T_0$  is the duration of one slot.

#### A. Power Control

In practice, the transmission power of the UAV transmitting data over a channel is subject to

$$0 \le p_{l_m^T}^T \le P_{mx}^{UAV}, \forall m \in \mathcal{M}, \tag{1}$$

where  $l_m^t$  is the channel between the aerial BS and MU m at slot t,  $p_{l_m}^T$  is the transmission power of the aerial BS transmitting data to MU m over channel  $l_m^t$  at slot t, and  $P_{mx}^{UAV}$  is the maximum transmission power supported by the

aerial BS. In addition, we can further find the constraint of the node-level transmission power as follows.

$$0 \le \sum_{m \in \mathcal{M}} p_{l_{t_m}}^T \le P_{mx}^{UAV}. \tag{2}$$

# B. Data Transmission under the Channel Capacity Constraint

The aerial BS transmits data to MUs over wireless channels. Let  $R_{l_m^t}$  denote the data rate of the aerial BS transmitting data to MU m over the channel. In practical UASs, to transmit data via a channel, the data rate is subject to

$$R_{l_m^t} \le C_{l_m^t},\tag{3}$$

where  $C_{l_m^t}$  is the channel capacity. Based on the Shannon theory [6], we have  $C_{l_m^t} = W_{l_m^t} \log_2(1 + p_{l_m^t}^T g_{l_m^t}/(N_0 W_{l_m^t}))$ , where  $g_{l_m^t}$  is the channel gain of channel  $l_m^t$ ,  $N_0$  is the power spectrum density of noise at receivers, and  $W_{l_m^t}$  is the bandwidth of channel  $l_m^t$ . On the other hand, the data rate  $R_{l_m^t}$  needs to satisfy the minimum data rate required to guarantee the QoS (quality of service) of the MUs. Hence, we have

$$R_{l_m^t} \ge R_m^0, \tag{4}$$

where  ${\cal R}_m^0$  is the minimum data rate required by MU m.

#### C. Solar Power Harvesting

According to the result of [7], the harvested solar power can be given by  $P^R(t) = \eta P^{rec}(t)$ , where  $\eta$  is the harvesting efficiency of the solar panels and  $P^{rec}(t)$  is the received solar power determined by the absorption coefficient related to the solar radiation intensity and cloud thickness. Thus, the harvested solar energy at slot t is given by  $E^S(t) = P^R(t)T_0$ . In actual scenarios, the solar radiation intensity and the thickness of a cloud usually follow a stochastic process [8]. As a result, the solar power arrived at solar panels is stochastic as well. Inspired by [3], we assume that  $P^{rec}(t)$  follows an independent and identically distributed (i.i.d.) stochastic process with the range  $[0, P^{rec}_{mx}]$ , and the harvested solar energy at slot t, denoted by  $E^S(t)$ , is bounded by

$$E^S(t) \le \eta P_{mx}^{rec} T_0. \tag{5}$$

# D. Energy Consumption

We consider that the energy consumption of the UAV in the UAS is two-fold: 1) keeping UAV hovering in the sky and 2) performing wireless communications. Let  $P^{pro}$  denote the UAV's propulsion power consumed for keeping the UAV hovering in the sky, which is considered as a constant. On the other hand, the UAV transmits data to M MUs, which consumes the transmission power as  $P_t^T = \sum_{m \in \mathcal{M}} p_{l_m}^T$ . Besides, the UAV consumes energy for receiving data from the terrestrial BS, which costs the receiving power as  $\sum_{m \in \mathcal{M}} P_m^R$ . As a result, the UAV's energy consumption during a slot can be given by

$$E^{T}(t) = (P^{pro} + \sum_{m \in \mathcal{M}} P_{l_{m}}^{T} + \sum_{m \in \mathcal{M}} P_{m}^{R}) T_{0}.$$
 (6)

Denote by E(t) the energy backlog at the end of slot t. Based on the amount of harvested and consumed energy, we can find the energy backlog at the end of slot t as follows.

$$E(t) = E(t-1) + E^{S}(t) - E^{T}(t).$$
(7)

In practice, the energy backlog is subject to the energy capacity of the on-board batteries carried by the UAV, denoted by  $E_{mx}$ , and the minimum residual energy threshold, denoted by  $E_{th}$ . Thus, we have

$$E_{th} \le E(t) \le E_{mx}.\tag{8}$$

# E. System Stability

Due to the dynamics of the harvested solar energy, we need to guarantee the stability of the UAS to provide MUs with reliable communication services. Without loss of generality, according to [9], we define the strong stability of a discretetime process energy backlog E(t) as follows.

**Definition 1** (Strong stability). A discrete time process E(t)is strongly stable if:

$$\lim \sup_{\tau \to \infty} (1/\tau) \sum_{t=0}^{\tau-1} \mathbb{E}\{|E(t)|\} < \infty. \tag{9}$$

# III. LONG-TERM TIME-AVERAGED ENERGY CONSUMPTION MINIMIZATION

## A. Problem Formulation

We consider minimizing the long-term time-averaged energy consumption while satisfying the required QoS and guaranteeing the system stability without requiring any prior knowledge of communication parameters. Based on the above analysis, we have the long-term time-averaged energy consumption of the UAV as follows:

$$\bar{E} = \lim_{T \to \infty} (1/T) \sum_{t=0}^{T-1} \mathbb{E} \{ E^T(t) \}.$$
 (10)

Therefore, we can formulate LEC as follows:

**LEC:** 
$$\min_{\mathbf{P}(t),\mathbf{R}(t)} \bar{E}$$
 (11) s.t.  $(1) - (5), (8), (9),$ 

where  $\mathbf{P}(t) = \{p_{l_m^t}^T, \forall m \in \mathcal{M}\}$  and  $\mathbf{R}(t) = \{R_{l_m^t}, \forall m \in \mathcal{M}\}$  $\mathcal{M}$ }. Since  $E^S(t)$  in LEC follows a stochastic process, LEC is a stochastic optimization problem. In particular, LEC is a timecoupling NLP problem, which in general is NP-hard, making it even more challenging to solve.

#### B. Lyapunov-based Stochastic Optimization

To solve LEC, we break the time coupling and reformulate a relaxed online optimization problem by employing Lyapunov optimization theory. Particularly, the reformulated one is an online energy consumption minimization problem that can be solved based on the current system state only.

Since the queue maintained in the UAS is E(t), we can define a Lyapunov function [9] as

$$L(E(t)) \triangleq \frac{1}{2}(E(t)). \tag{12}$$

This is a scalar measure to indicate the congestion of the energy queue in the UAS. L(E(t)) being small implies that the queue backlog is small. Besides, we also define the one-slot conditional Lyapunov drift as

$$\Delta(E(t)) \triangleq \mathbb{E}\{L(E(t+1)) - L(E(t))|E(t)\}. \tag{13}$$

Minimizing  $\Delta(E(t))$  at each time slot, we can achieve meanrate stable by pushing the energy backlog E(t) towards a lower congestion state. According to the result of [9], if a queue E(t)is mean-rate stable, then E(t) will be strongly stable. Hence, (9) can be guaranteed by minimizing D(E(t)).

Considering that our objective is to provide communication services with the required QoS to the most MUs, and take control action to limit  $\Delta(E(t))$ , we minimize the following drift-plus-penalty (DPP) function:

$$D(E(t)) = \Delta(E(t)) + V \mathbb{E}\{E^{T}(t)|E(t)\},$$
 (14)

where V is a non-negative parameter representing the importance weight on how much we emphasize on the energy consumption minimization. We focus on optimizing transmission power in  $E^{T}(t)$  to minimize D(E(t)) while  $P^{pro}$ and  $P_m^R$  in  $E^T(t)$  are constants. Thus, we relax  $E^T(t)$  as  $E^{T}(t) = \sum_{m \in \mathcal{M}} P_{l_{m}^{t}}^{T} T_{0}.$  We can have the following lemma.

**Lemma 1.** Given  $\Delta(E(t))$  defined in (13), we have

$$D(E(t)) = \Delta(E(t)) + V\mathbb{E}\{E^{T}(t)|E(t)\} \le A + U(t), (15)$$

where A is a constant, i.e.,

$$A = (1/2)(\eta P_{mx}^{\text{rec}} T_0)^2. \tag{16}$$

U(t) is only related to the variables  $P_{l_t}^T$  is of the transmission power allocated to link  $l_m^t$  at slot t, i.e.,

$$U(t) = (1/2)\mathbb{E}\{(\sum_{m \in \mathcal{M}} P_{l_m}^T T_0)^2 - 2\sum_{m \in \mathcal{M}} P_{l_m}^T T_0(E^S(t) - E(t) + V) | E(t)\}.$$
(17)

*Proof.* Due to (5) and (8), we can obtain  $2E(t)E^S(t) \ge 0$  and  $(E^S(t))^2 \leq (\eta P_{mx}^{\text{rec}} T_0)^2$ . As a result, we can further have

$$(1/2)\mathbb{E}\{(E^S(t))^2 - 2E(t)E^S(t)|E(t)\} \le (1/2)(\eta P_{mx}^{\text{rec}} T_0)^2.$$
(18)

Substituting (18) into (14) yields

$$D(E(t)) \le (1/2)(\eta P_{mx}^{\text{rec}} T_0)^2 + (1/2)\mathbb{E}\{(E^T(t))^2 -2E^T(t)E^S(t) - 2E(t)E^T(t) + VE^T(t)|E(t)\}.$$
(19)

Thus, the proof of Lemma 1 is concluded. 

Our objective is to minimize the right-hand side of (15) at each slot based on the previous residual energy and the radio channels. Because A is a constant, we aim to minimize U(t). We now exploit the concept of opportunistically minimizing an expectation [9], which is to minimize

(12) 
$$U'(t) = \frac{T_0^2}{2} \left( \sum_{m \in \mathcal{M}} P_{l_m^t}^T \right)^2 + T_0(V/T_0 - E(t) - E^S(t)) \sum_{m \in \mathcal{M}} P_{l_m^t}^T.$$

Therefore, we can reformulate LEC as a single-time slot problem (STP):

STP: 
$$\min_{\mathbf{P}(t),\mathbf{R}(t)} U'(t),$$
  
s.t.  $(1) - (5), (8).$ 

STP is still an NLP problem that is hard to solve.

### C. Online Power and Rate Control Algorithm

To find the transmission power and data rate, we design a joint power and rate control algorithm to solve STP. Specifically, we first arrive at the following theorem about the constraint (3) in STP.

**Theorem 1.** The original problem LEC can obtain an optimal solution only when  $R_{l_m^t} = C_{l_m^t}, \forall m \in \mathcal{M}$ , holds.

Proof. We exploit a contradiction method to prove Theorem 1. Specifically, suppose that there exists at least one channel  $l_0$  at a slot  $t_0$  that satisfies  $C_{l_0^{t_0}} > R_{l_0^{t_0}}$ . Denote the optimal objective value of LEC by  $E^* = \lim_{T \to \infty} (1/T) \sum_{t=0}^{T-1} \mathbb{E}\{E^{T*}(t)\}$ . Recall that the channel capacity can be represented as  $C_{l_0^{t_0}} = W_{l_0^{t_0}} \log_2(1 + p_{l_0^{t_0}}^T g_{l_0^{t_0}}/(N_0 W_{l_0^{t_0}}))$ . Hence, if  $C_{l_0^{t_0}} > R_{l_0^{t_0}}$ , then  $p_{l_0^{t_0}}^{T*} > N_0 W_{l_0^{t_0}}(2^{R_{l_0^{t_0}}/W_{l_0^{t_0}}} - 1)/g_{l_0^{t_0}}$ . Considering that  $R_{l_0^{t_0}}$  and  $C_{l_0^{t_0}}$  are non-coupling, there must exist an positive constant  $\Delta$  satisfying  $p_{l_0^{t_0}}^{T*} - \Delta = N_0 W_{l_0^{t_0}}(2^{R_{l_0^{t_0}}/W_{l_0^{t_0}}} - 1)/g_{l_0^{t_0}}$ . Correspondingly, if the transmission power of channel  $l_0$  is set to be  $p_{l_0}^{T*}(t_0) - \Delta$  in slot  $t_0$ , a lower objective  $E^{\dagger} = \lim_{T \to \infty} (1/T) \sum_{t=0}^{T-1} \mathbb{E}\{E^{T*}(t) - \Delta\}$  than  $E^*$  can exist. This result implies a contradiction with the assumption. The proof is concluded.

According to Theorem 1, we can replace the inequality constraint in (3) by constraint  $R_{l_m^t} = C_{l_m^t}, \quad \forall m \in \mathcal{M}$ . As a result, we can obtain  $R_{l_m^t} = W_{l_m^t} \log_2(1 + p_{l_m^t}^T g_{l_m^t}/(N_0 W_{l_m^t}))$ . Thus, we can replace variable  $\mathbf{P}(t)$  by  $\mathbf{R}(t)$  to have

$$p_{l_m^T}^T = R_{l_m^+}^+ = N_0 W_{l_m^t} (2^{R_{l_m^t}/W_{l_m^t}} - 1)/g_{l_m^t},$$
 (20)

where  $R^+_{l^t_m}$  is a non-linear and non-decreasing function of  $R_{l^t_m}$  that equals  $p^T_{l^t_m}$ . Furthermore, the node-level transmission power consumption is given by  $P^T(t) = \sum_{m \in \mathcal{M}} R^+_{l^t_m}$ . Thus, U'(t) can be rewritten as

$$U^{r}(p) = \frac{1}{2} T_{0}^{2} \left( \sum_{m \in \mathcal{M}} R_{l_{m}^{+}}^{+} \right)^{2} + T_{0}(V/T_{0} - P^{R}(t)T_{0} - E(t))$$

$$\sum_{m \in \mathcal{M}} R_{l_{m}^{+}}^{+}$$

Besides, constraints (1) and (2) in STP can be written respectively as

$$0 \le R_{l_m^+}^+ \le P_{mx}^{UAV},\tag{21}$$

and

$$\sum_{m \in \mathcal{M}} R_{l_m^+}^+ \le P_{mx}^{UAV}. \tag{22}$$

Therefore, STP can be equivalently written as

STP-1: 
$$\min_{\mathbf{P}(t)} U^r(p),$$
  
s.t.  $(4), (5), (8), (21), (22).$ 

We can observe that STP-1 is a convex optimization problem with respect to  $R^+_{l^+_m}$ . Due to the non-decreasing characteristic of  $R^+_{l^+_m}$  in  $U^r(p)$ , the optimal solution is unique in the feasible solution space. Hence, the optimal solution can be obtained by performing the derivative of  $U^r(p)$  with respect to  $R^+_{l^+_m}$  equal to zero, i.e., the optimal solution  $R^{+*}_{l^+_m} = P^R(t) - V/T_0^2 + E(t)/T_0$ .

# D. Theoretic Analysis

1) Lyapunov Optimality: To find the optimality, we first define a performance gap that is the difference between the long-term time-averaged energy consumption achieved by STP and an optimal Offline Optimization Problem called OOP. Particularly, OOP has complete information about the UAS, and solving OOP can give us the optimal solution at each slot. To be more specific, OOP (offline optimization problem) is formulated as follows:

**OOP:** 
$$\min_{\mathbf{P}(t),\mathbf{R}(t)} E^{T}(t),$$
  
s.t.  $(1) - (5), (8), (9),$ 

where P(t) and R(t) are respectively the transmission power and rate vectors determined by the current network status in slot t. In the following, we will present the theoretical analysis of the performance gap.

Assume that after solving STP, we can obtain the long-term time-averaged energy consumption by  $E^{stp} = \lim_{\tau \to \infty} (1/\tau) \sum_{t=0}^{\tau-1} E^{T*}(t)$ . Let  $E^{oop}$  be the long-term time-averaged energy consumption achieved by solving OOP. Consider that solving STP achieves a C-approximation, where C is non-negative. That means, for all t, we have

$$E^{T*}(t) \le E^{oop} + C. \tag{23}$$

**Theorem 2.** (Optimality) For all t > 0, we have

$$\sup E^{stp} \le E^{oop} + D/V + C,\tag{24}$$

where D is a finite constant related to the bound of input and output queues of E(t).

*Proof.* Based on  $0 \le E^T(t) \le P_{mx}^{UAV}T_0$  and  $0 \le E^S(t) \le \eta P_{mx}^{rec}T_0$ , we can have an upper bound of Lyapunov drift for queue E(t), denoted by D, i.e.,  $\Delta(E(t)) \le D = (1/2)(P_{mx}^{UAV}T_0)^2$ . Then, the DPP function, i.e., (14), is bounded by

$$\Delta(E(\tau)) + V\mathbb{E}\{E^T(\tau)|E(\tau)\} \le D + V\mathbb{E}\{E^{T*}(\tau)|E(\tau)\},$$

Substituting (23) into the above inequation yields

$$\Delta(E(\tau)) + V\mathbb{E}\{E^{T*}(\tau)|E(\tau)\} \le D + VC + VE^{oop}.$$

Taking expectations of both sides of the above inequation, summing the telescoping series over  $\tau \in \{0, 1, \dots, t-1\}$ , and dividing by Vt, we can have

$$(1/Vt)(\mathbb{E}\{L(E(t))\} - \mathbb{E}\{L(E(0))\}) + (1/t)\sum_{\tau=0}^{t-1}\mathbb{E}\{E^{T*}(t)|E(\tau)\}) \le E^{oop} + D/V + C.$$

Taking a lim sup of both sides yields:

$$\limsup_{t \to \infty} (1/t) \sum_{\tau=0}^{t-1} \mathbb{E}\{E^{T*}(t) | E(\tau)\}\} \le E^{oop} + D/V + C,$$

According to Theorem 2, we can find that increasing V can reduce the performance gap of long-term time-averaged energy consumption achieved by solving STP and OOP.

2) Energy Stability: The energy stability of the backlog queue E(t) can be guaranteed after solving STP-1 by employing our proposed solution, which is detailed in the following.

**Theorem 3.** (Stability) Solving our optimization problem SPT can guarantee that the queue E(t) is strongly stable, i.e., E(t) is bounded by a finite constant for all t.

*Proof.* For energy backlog queue E(t), we have  $E(t+1)=E(t)-E^T(t)+E^S(t)$ , and summing the telescoping series of it over  $\tau \in \{0,1,\cdots,t-1\}$  yields  $E(t)=E(0)-t(E^T(t)-E^S(t))$ . The transmission energy consumption  $E^T(t)$  and harvested solar energy  $E^S(t)$  is limited by  $P_{mx}^{rec}T_0$ , respectively. Then, we have

$$E(t) \le E(0) + tT_0 P_{m_T}^{rec},$$
 (25)

for any t. The right-hand side of (25) is a finite constant when initial energy backlog E(0) is finite. Besides, due to (8), we have  $E(t) \geq E_{th}$ . Thus, E(t) is bounded by a finite constant for any t, which implies E(t) is strongly stable [9]. Hence, the proof of Theorem 3 is concluded.

# IV. SIMULATION RESULTS

## A. Simulation Settings

Inspired by the urban-macro with aerial vehicle scenario presented in [10], we conduct simulations in a 3D urban scenario where a UAV is deployed at the place with the altitude ranged from 22.5 m to 100 m, and the horizontal coordinate of its hovering center is (0, 0). The UAV receives data from a BS and transmits data to 10 MUs. These MUs are randomly distributed at positions with coordinates ([-750, -500], [500, 750], 0) and ([500, 750], [-750, -500], 0), respectively. The UAV's maximum transmission power is set to be  $P_{mx}^{UAV} = 1$  Watt. For an MU, its required minimum data rate is randomly set by choosing a value from [2, 6] Mbps. For the radio channel, the noise power spectrum density at receivers and the bandwidth allocated to each channel are set to be  $N_0 = -170$  dBm/Hz and  $W_l = 1$  MHz. Inspired by [10], the path loss (in dB) is modeled by PL = 1

 $\begin{array}{l} P_{LOS} \times (28 + 22 \log_{10}(d) + 20 \log_{10}(f_c)) + P_{NLOS} \times -17.5 + \\ (46 - 7 \log_{10}(h)) \log_{10}(d) + 20 \log_{10}(40\pi f_c/3), \text{ where } P_{LOS} = \\ d_1/d_{2d} + \exp(-d_{2d}/d_2)(1 - d_1/d_{2d}) \text{ and } P_{NLOS} = 1 - P_{LOS}. \\ d_{2d} \text{ (in meters) is the horizontal distance between the ground node and the UAV, } d_1 = \max(460 \log_{10}(h) - 700, 18) \text{ ($h$ is the altitude of the UAV), and } d_2 = 4300 \log_{10}(h) - 3800. \\ \text{Besides, the carrier frequency } f_c = 2 \text{ GHz.} \end{array}$ 

According to the previous work in [11], the maximum amount of harvested solar power and solar energy harvesting efficiency is set to be  $P^{rec}_{mx}=1$  Watt and  $\eta=0.9$ , respectively. The harvested solar power,  $P^{rec}(t)$ , follows an i.i.d. in the range of [0, 1] Watt. Based on the result of [12], the maximum-endurance hovering speed is set to be 10 m/s, and the hovering radius is assumed to be 5 m. For the convenience of analysis, the period of each slot is set to be 1 second, and we only measure the transmission power consumption of the UAV in the simulation examples. We conduct simulations up to 500 times to average the simulation results by employing the Monte Carlo simulation method.

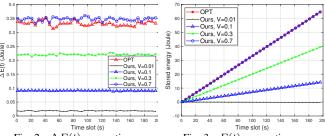
## B. Performance Improvement of Our Proposed Algorithm

In this simulation example, we measure the energy stability and the throughput improvement of our proposed algorithm. To further illustrate the efficacy, we compare the energy achieved by our proposed algorithm with that by solving OOP, which is called OPT (optimization). Besides, we change the value of parameter V in the simulation examples to explore its impact.

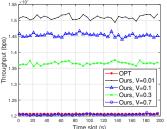
1) Stability: To measure the energy stability improved by our proposed algorithm, we define  $\Delta E(t) = |E(t+1) - E(t)|$ , and show how  $\Delta E(t)$  changes with time in Fig. 2. Besides, we show how E(t) changes with time in Fig. 3. From these two figures, we can observe that our proposed algorithm outperforms OPT in terms of energy stability. Besides, the energy system will be more stable when V is smaller. For example, in Fig. 2,  $\Delta E(t)$  is about 0.018 J when V = 0.01, while  $\Delta E(t)$  are about 0.092 J and 0.217 J when V=0.1and V = 0.3, respectively. Moreover, the performance of our proposed algorithm and OPT becomes similar when V is large enough. For example,  $\Delta E(t)$  is about 0.33 J when employing OPT, while  $\Delta E(t)$  is about 0.34 J when V = 0.7. The reason is that V is defined as the relative importance of minimizing energy consumption compared with the queue stability of E(t). These observations verify the superiority of our proposed algorithm for guaranteeing energy stability.

On the other hand, we find from Fig. 3 that the stored energy along with time becomes lower when the energy system is more stable. The reason is that when the UAV harvests more solar energy in one slot, it can consume more transmission energy to ensure stability, i.e., the balance between energy harvesting and energy consumption.

2) Throughput: Since the UAV consumes more energy for transmission when it harvests more solar energy by using our proposed algorithm, we further examine the throughput







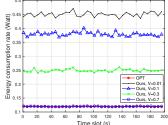


Fig. 2.  $\Delta E(t)$  versus time. Fig. 3. E(t) versus time.

Fig. 4. Throughput versus time.

Transmission energy con-Fig. 5. sumption rate versus time.

improvement of our proposed algorithm in Fig 4. From Fig 4, we observe that the throughput is improved when employing our proposed algorithm. For example, the average throughput is about 15.15 Mbps when using our proposed algorithm under V = 0.01, while the average throughput is about 12.07 Mbps by employing OPT. This observation validates the above result in Sec IV-B1.

## C. Performance Gap

To explore the performance gap between our proposed algorithm and the one solving OOP, we examine the energy consumption rate by varying the predefined parameter V. Fig 5 reveals the effect of V on the energy consumption rate by employing our proposed algorithm. Specifically, we can observe that when increasing V, the energy consumption rate will approximate the optimal value, i.e., the performance gap will be decreasing, and the energy consumption rates achieved by our proposed algorithm are almost the same with OPT when setting V = 0.7. This observation is consistent with the theoretic performance analysis.

# V. CONCLUSIONS

In this paper, we have studied the problem of the longterm time-averaged energy consumption minimization in a UAS with an aerial BS powered by dynamic solar energy. Specifically, we have jointly considered both transmission power and data rate to formulate a time-coupling energy consumption minimization problem subject to MUs' required QoS and strong stability. Since the formulated problem is non-convex, nonlinear, and time-coupling, we have employed Lyapunov optimization theory to reformulate a relaxed online optimization problem, called STP, to break the time coupling. To find the solution to STP, we have designed a joint power and rate control algorithm. In particular, we have theoretically proved that our proposed algorithm can achieve a (D/V+C)approximation and guarantee strong stability. We have conducted extensive simulations, and simulation results show the performance gain, in terms of stability and throughput, achieved by our proposed algorithm.

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