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Throughput-Constrained Energy Efficiency Optimization for CSMA Networks

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***Abstract*—Carrier Sense Multiple Access (CSMA) has been widely applied to various kinds of wireless networks, such as Wi-Fi, to serve portable devices which are usually greedy in terms of throughput, but with finite battery budget. Accordingly, how to optimize the usage of finite battery budget to get the best possible throughput performance is of great importance. This paper aims to address this issue by focusing on a saturated CSMA network. Explicit expressions of maximum energy efficiency and the corresponding optimal backoff parameter with or without throughput constraint are derived. It is revealed that optimizing the energy efficiency leads to throughput performance degradation. With a stringent throughput constraint, the energy efficiency has to be sacrificed. The energy efficiency and the throughput can be optimized at the same time only in special cases, e.g., the network size is large. The analysis is verified by simulations and sheds important light on performance optimization of practical CSMA-based networks such as Wi-Fi 6 networks.**

***Index Terms*—CSMA, energy efficiency, throughput, optimization, Wi-Fi 6.**

I. Introduction

Carrier Sense Multiple Access (CSMA) is one of the most representative random access protocols. With CSMA, devices would sense the channel and transmit packets only if the channel is idle. Compared to Aloha, CSMA can achieve much better network throughput performance and therefore has been widely applied to various kinds of wireless networks, such as Wi-Fi networks and body sensor networks, for serving portable devices, e.g., sensors or smart phones [1]. However, the battery volume of those devices is usually finite, which imposes stringent requirement on the energy efficiency of the networks. It is therefore important to study how to optimize the energy efficiency of the CSMA network.

Extensive works have been done on the performance optimization of the CSMA network, while the major focus is on the network throughput. For instance, in [2]–[4], by modeling the channel traffic as a Poisson random variable, the maximum throughput of CSMA network was characterized. To achieve the maximum throughput, a device-level modeling approach was further established in [5], based on which the expressions

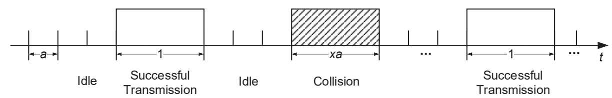

Fig. 1. The aggregate channel.

of optimal parameter setting were derived. Recently, a lot of studies have also been done to model and analyze the energy efficiency of CSMA networks. The energy efficiency expressions for non-persistent and p-persistent CSMA network were given in [6] and [7], respectively, and it was further proved in [7] that throughput and energy efficiency can be simultaneously optimized under the prerequisite that the power consumption during sensing and transmitting period was equal. Yet, this assumption does not hold in practical scenario. [8] introduced a new CSMA-based scheme that involved the transition of SLEEP and AWAKE states to enhance energy efficiency while producing optimized throughput.

Most of the above studies focus either on throughput optimization or energy efficiency optimization. A unified analysis of energy efficiency alongside throughput of the CSMA network remains unexplored. In practical CSMA networks, energy-hungry applications such as online games, multimedia voice and video streaming can also be throughput-intensive [9]. It is thus of great practical importance to study how to maximize the energy efficiency with/without the prerequisite of a throughput constraint, and how to characterize the trade-off between the energy efficiency optimization and the throughput optimization.

In this paper, we provide closed-form solutions to the above open questions. By exploiting the analytical framework for CSMA network proposed in [5], we derive the explicit expressions of the maximum energy efficiency and the corresponding optimal backoff parameter. With this, we reveal that there exists a crucial trade-off between energy efficiency and throughput, and a demanding throughput constraint will deteriorate energy efficiency. On this basis, for a given minimum required throughput, we derive the explicit expressions of the throughput-constrained maximum energy efficiency and the corresponding optimal backoff parameter. The analysis is verified by simulation results and further extended to Wi-Fi 6 networks. It is revealed that as the network size grows, by optimally tuning the backoff parameter, the trade-off between the energy efficiency optimization and the throughput optimization diminishes.

The remainder of this paper is organized as follows. Section II presents the system model and preliminary analysis. The

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maximum energy efficiency is proposed in Section III-A, and the trade-off analysis between energy efficiency and throughput is presented in Section III-B. The throughput-constrained maximum energy efficiency is presented in Section III-C. The above analysis will be applied to the Wi-Fi 6 network in Section IV. Concluding remarks are presented in Section V.

II. System Model and Preliminary Analysis

Consider an n-node slotted CSMA network where all the nodes transmit to a common receiver. As Fig. 1 illustrates, the time axis of the network is divided into multiple mini-slots, where the mini-slot length a is determined by the required sensing time of each node. Assume that each node always has packets to send, and accesses the channel at the beginning of each mini-slot if the channel is sensed idle. One packet is successfully transmitted if and only if there are no concurrent transmissions. Otherwise, collision occurs and transmission fails.

After the ith collision the access probability of each node becomes $q_i \in (0,1]$ for $i = 0,1,2,\ldots$. Let $q_i = q_0 Q(i)$, where $Q(i)$ is a monotonic non-increasing function of i with $Q(0) = 1$. A backoff scheme can then be characterized by the sequence of transmission probabilities $\{q_i\}_{i=0,\ldots,K}$. Without loss of generality, assume that the time interval of each successful transmission is one slot. Let x denote the number of mini-slots for each node to detect the collision. The collision-detection time can then be obtained as ax, and we assume $ax \leq 1$ [5].

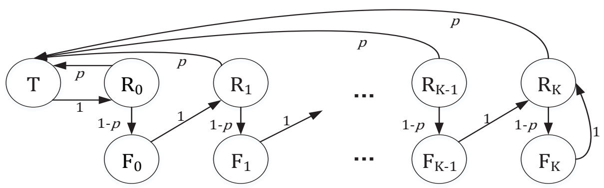


Fig. 2. Embedded Markov chain of the Markov renewal process.

To characterize the behavior of the HOL packet of each node, an ergodic discrete-time Markov renewal process has been established in [5], as shown in Fig. 2. In particular, each HOL packet could be in the following three states: 1) waiting (State R_i, $i = 0, \ldots, K$), 2) collision (State F_i, $i = 0, \ldots, K$) and 3) successful transmission (State T). An HOL packet moves from State R_i to State T if it is successfully transmitted, otherwise it will stay at State F_i till the collision ends and move on to R_{i+1}. Here i denotes the number of collisions experienced by the HOL packet and will increase until it reaches the cutoff phase K.

We assume nodes are battery-limited. Let E denote the amount of initial energy of each node, P_W denote the baseline power consumption of air interface of each node when it is not sending or receiving any data, P_T represent the power consumption of the interface during data transmission. An initial amount of energy can only support for a limited time. Accordingly, the objective of each node is to transmit as many packets as possible using the initial energy, which is also the focus in this paper.

III. Energy Efficiency Optimization

In this section, we aim to derive the energy efficiency of each node η, which is defined as the number of successfully transmitted packets with unit amount of energy, and is given by $\eta = \frac{M}{E}$, where M is the expected number of successfully transmitted packets by using the initial amount of energy. We can see that a large energy efficiency η implies that the initial energy of each node is used efficiently. Recall that each node's throughput, denoted by λ_{out}, indicates the average number of successfully-transmitted packets in each time slot for each node. M can then be obtained as the product of the throughput and the lasting time of each node supported by the initial energy E, denoted by T, as $M = \lambda_{\text{out}} T$. Appendix A shows that

$$\begin{aligned} T = &-anE\Big(1 + x - xp - (\tfrac{1}{a} - x)p\ln p\Big)/ \\ &\Big(an(xp - x - 1)P_W + \Big(((n-1)(1-ax)p - ax)P_W \\ &+ (ax(1-p) + p)P_T\Big)\ln p\Big). \end{aligned} \tag{1}$$

It can be further obtained that $\frac{\partial T}{\partial E} > 0$ and $\frac{\partial T}{\partial n} > 0$, which indicate that the lasting time T is an increasing function of E and n. Intuitively, each node can make more transmissions or hold the HOL packet longer before its energy runs out with larger initial energy E. When the number of nodes n is large, contention becomes intensive among nodes so that each node stays in waiting state longer, thus transmits less packages. As the baseline power consumption of air interface of each node when it is not sending or receiving any data P_W is smaller than the power consumption of the interface during data transmission P_T, we can see that the lasting time T would be large.

A. Maximum Energy Efficiency

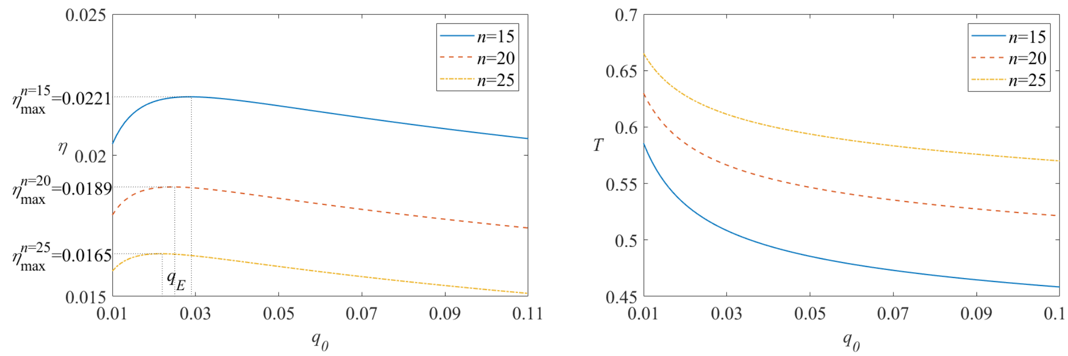
The throughput of each node λ_{out}, which is defined as the long-term fraction of each node's time for successful packet transmissions, has been obtained in [5] as

$$\lambda_{out} = \frac{1}{n} \cdot \frac{-p\ln p}{(1+x)a - (1-ax)p\ln p - axp}. \tag{2}$$

By combining (1) and (2), the energy efficiency of each node η can therefore be obtained as

$$\begin{aligned} \eta = &p\ln p/\Big(anxP_W p + ax(P_T - P_W)\ln p \\ &+ (1-ax)((n-1)P_W + P_T)p\ln p - (1+x)anP_W\Big). \end{aligned} \tag{3}$$

In this paper, we are interested in maximizing the energy efficiency η by tuning the initial transmission probability q_0, i.e., $\eta_{\max} = \max_{0<q_0\leq 1} \ \eta$. The following theorem presents the maximum energy efficiency η_{max} and the corresponding optimal transmission probability q_E.



(a) (b)

Fig. 3. (a) Energy efficiency of each node η versus the initial transmission probability q_0. (b) Lasting time T versus the initial transmission probability q_0. $E = 1$, $P_W = 1$, $P_T = 20$, $a = 0.1$, $x = 5$, $K = 6$, $Q(i) = 2^{-i}$.

Theorem 1. *The maximum energy efficiency is given by*

$$\begin{aligned}\eta_{max} =& p_E \ln p_E \Big/ \Big(anxP_W p_E + ax(P_T - P_W)\ln p_E \\ &+ (1-ax)((n-1)P_W + P_T)p_E \ln p_E \\ &- (1+x)anP_W \Big), \end{aligned} \tag{4}$$

where p_E is the single non-zero root of

$$anxP_W p + ax(P_T - P_W)\ln^2 p - (1+x)anP_W(1+\ln p) = 0. \tag{5}$$

η_{max} is achieved when the initial transmission probability q_0 is set to be

$$q_E = \frac{-\ln p_E}{n}\left[\sum_{i=0}^{K-1}\frac{p_E(1-p_E)^i}{Q(i)} + \frac{(1-p_E)^K}{Q(K)}\right]. \tag{6}$$

Proof. See Appendix B. ☐

Fig. 3 illustrates how the energy efficiency of each node η and the lasting time T vary with the initial transmission probability q_0. Intuitively, when q_0 is too small, e.g., $q_0 = 0.01$, each node would hold the HOL packet and seldom make transmission. Since the baseline power consumption of air interface of each node when it is not sending or receiving any data P_W is smaller than the power consumption of the interface during data transmission P_T, we can conclude that the lasting time T would be large. However, the energy efficiency η would be low because of the limited number of transmissions that each node makes in its life. On the other hand, when the initial transmission probability q_0 is too large, e.g., $q_0 = 0.1$, the energy efficiency η would be low as well because of the mounting contention among nodes. It can be clearly observed from Fig. 3 that only by optimally tuning the initial transmission probability q_0 according to Theorem 1 can the maximum energy efficiency η_{max} be achieved.

B. Trade-off between Energy Efficiency and Throughput

Besides η, the throughput λ_{out} is also an important performance indicator of the network. Therefore, we are interested in whether η and λ_{out} can be simultaneously optimized by tuning the initial transmission probability q_0.

Specifically, the maximum throughput of each node λ_{max} and the corresponding optimal transmission probability q_{Th} have been derived in [5] as

$$\lambda_{max} = \frac{1}{n}\cdot\frac{-p_{Th}\ln p_{Th}}{(1+x)a-(1-ax)p_{Th}\ln p_{Th}-axp_{Th}}, \tag{7}$$

and

$$q_{Th} = \frac{-\ln p_{Th}}{n}\left[\sum_{i=0}^{K-1}\frac{p_{Th}(1-p_{Th})^i}{Q(i)} + \frac{(1-p_{Th})^K}{Q(K)}\right], \tag{8}$$

respectively, where p_{Th} is the single non-zero root of

$$xp - (1+x)\ln p - x - 1 = 0. \tag{9}$$

The energy efficiency η_{Th} when λ_{out} is maximized, i.e., $q_0 = q_{Th}$, can therefore be obtained by substituting p_{Th} into (3) as

$$\begin{aligned}\eta_{Th} =& p_{Th}\ln p_{Th} \Big/ \Big(anxP_W p_{Th} + ax(P_T - P_W)\ln p_{Th} \\ &+ (1-ax)((n-1)P_W + P_T)p_{Th}\ln p_{Th} - (1+x)anP_W \Big). \end{aligned} \tag{10}$$

By comparing (5)–(9), we can see that q_{Th} differs with q_E and $\eta_{Th} \le \eta_{max}$. Note that η and λ_{out} can only be simultaneously maximized under several special cases, such as $P_T = P_W$, $x \to 0$ and $n \to \infty$, where (3) can be written as

$$\eta = \begin{cases} \frac{\lambda_{out}}{P_W}, & \text{if } P_T = P_W \text{ or } n \to \infty \\ \frac{1}{P_T + P_W(\frac{1}{\lambda_{out}} - 1)}, & \text{if } x \to 0. \end{cases} \tag{11}$$

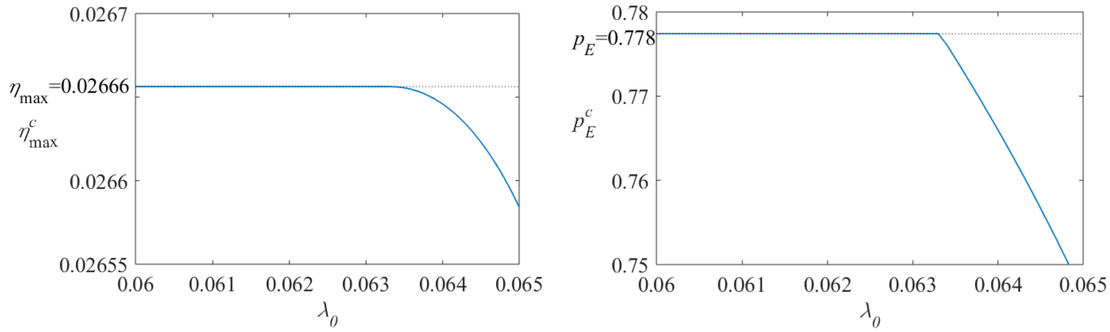
(11) indicates that η and λ_{out} are positively correlated, thus maximizing η is consistent with maximizing λ_{out}, vice versa.

Yet, in general, the trade-off between η and λ_{out} always exists and improving η may deteriorate λ_{out}. In the next subsection, we will study how to maximize η under the constraint of providing a guaranteed throughput, i.e., $\lambda_{out} \ge \lambda_0$.

C. Throughput-constrained Energy Efficiency Optimization

In this subsection, we have the following optimization problem

$$\begin{aligned}\eta_{\max}^c =& \max_{0<q_0\le 1} \eta \\ &s.t. \quad \lambda_{out} \ge \lambda_0, \end{aligned} \tag{12}$$



(a) (b)

Fig. 4. (a) Throughput-constrained maximum energy efficiency η^c_{max} versus the lower bound λ_0. (b) Corresponding successful transmission probability of the constrained maximum energy efficiency p^c_E versus λ_0. $P_W = 1$, $P_T = 20$, $a = 0.1$, $x = 5$, $n = 10$.

where $\lambda_0 \leq \lambda_{max}$.

The following theorem presents the throughput-constrained maximum energy efficiency η^c_{max} and the corresponding optimal transmission probability q^c_E.

Theorem 2. *The throughput-constrained maximum energy efficiency of each node η^c_{max} is given by*

$$\eta^c_{max} = p^c_E \ln p^c_E \Big/ \bigg(anxP_W p^c_E + ax(P_T - P_W)\ln p^c_E + (1-ax)((n-1)P_W + P_T)p^c_E \ln p^c_E - (1+x)anP_W \bigg), \tag{13}$$

where

$$p^c_E = \begin{cases} p_E, & \text{if } p_1 \leq p_E \leq p_2 \\ p_2, & \text{if } p_2 < p_E \leq 1. \end{cases} \tag{14}$$

p_1 and p_2 are the two non-zero roots of

$$\tfrac{1}{n} \cdot \tfrac{-p \ln p}{(1+x)a - (1-ax)p \ln p - axp} - \lambda_0 = 0 \tag{15}$$

and $p_1 \leq p_2$. η^c_{max} is achieved when the initial transmission probability q_0 is set to be

$$q^c_E = \tfrac{-\ln p^c_E}{n} \left[\sum_{i=0}^{K-1} \tfrac{p^c_E (1-p^c_E)^i}{Q(i)} + \tfrac{(1-p^c_E)^K}{Q(K)} \right]. \tag{16}$$

Proof. See Appendix C. □

Fig. 4 illustrates how the throughput-constrained maximum energy efficiency η^c_{max} and the corresponding successful transmission probability p^c_E vary with the throughput constraint lower bound λ_0, respectively. Intuitively, when λ_0 is small, e.g., $\lambda_0 = 0.062$, constraint of the throughput λ_{out} is loose so p_E satisfies $p_E \in [p_1, p_2]$. η^c_{max} is achieved when $p^c_E = p_E$ and we have $\eta^c_{max} = \eta_{max}$. When λ_0 is large, e.g., $\lambda_0 = 0.064$, the constraint of λ_{out} becomes demanding and p^c_E shifts to $p_2 < p_E$ and $\eta^c_{max} < \eta_{max}$ as shown in Fig. 4.

IV. Case Study: Wi-Fi 6 Networks

So far, we have demonstrated how to optimize the energy efficiency η for a general CSMA network with or without the throughput constraint. The CSMA mechanism has been broadly applied to several different types of practical networks, among which the Wi-Fi 6 network is a typical example. In this section, we will demonstrate how the above analysis can be applied to Wi-Fi 6 network.

The time axis of a Wi-Fi 6 network is divided into multiple mini time slots with length of σ [10]. As such, the length of each mini time slot a and the collision-detection time x can be written as

$$a = \sigma/\tau_T \text{ and } x = \tau_F/\sigma, \tag{17}$$

respectively. The mean holding time τ_F and τ_T (in the unit of seconds) of an HOL packet in States $F_i, i = 0, ..., K$ and T are given by [11] as

$$\begin{cases} \tau_F = \text{PHY header} + (\text{MAC header} + \text{PL})/R_D + \text{DIFS} \\ \tau_T = \text{PHY header} + (\text{MAC header} + \text{PL})/R_D \\ \qquad + \text{ACK}/R_B + \text{DIFS} + \text{SIFS}. \end{cases} \tag{18}$$

It has been indicated in Theorem 1 and 2 that to maximize the energy efficiency η of each node, the initial transmission probability should be carefully tuned. According to [10], the transmission probability of a State-R_i, $i = 0, ..., K$ HOL packet in Wi-Fi 6 network is given by

$$q_i = \tfrac{2}{1+W_i}, \tag{19}$$

where $W_i = W \cdot m^i$ is the backoff window size at state $R_i, i = 0, ..., K$, W is the initial backoff window size and $m > 1$ is the backoff factor. Therefore, to optimize the energy efficiency of a Wi-Fi 6 network, its initial backoff window size should be properly set. In particular, by combining (6) and (19), the optimal initial backoff window size for energy efficiency maximization without throughput constraint W_E can be obtained as

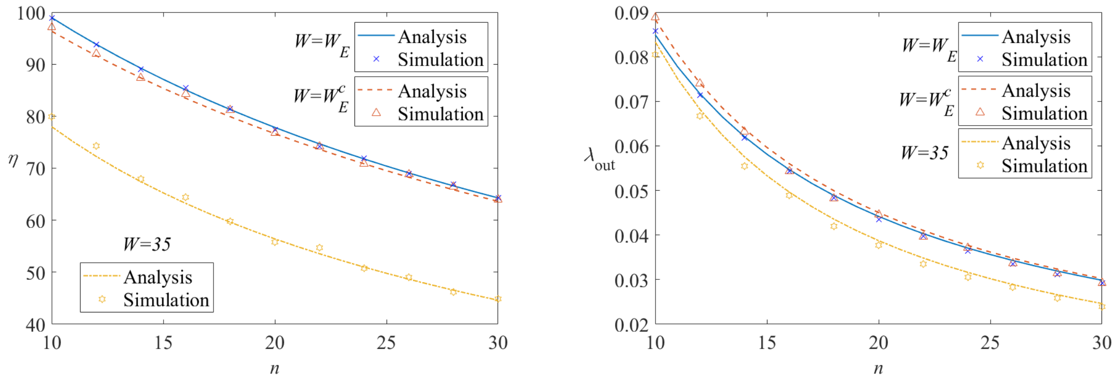
$$W_E = \tfrac{(-\frac{2n}{\ln p_E} - 1)(2p_E - 1)}{p_E - 2^K (1-p_E)^{K+1}} \tag{20}$$

with $m = 2$. Similarly, the throughput-constrained optimal initial backoff window size W^c_E can be obtained as

$$W^c_E = \tfrac{(-\frac{2n}{\ln p^c_E} - 1)(2p^c_E - 1)}{p^c_E - 2^K (1-p^c_E)^{K+1}} \tag{21}$$

by combining (16) and (19).

Let us now conduct simulations to verify the preceding theoretical analysis based on the simulation program of IEEE 802.11 network in [10], and take the system parameters adopted in the IEEE 802.11ax standard (i.e., the standard of



(a) (b)

Fig. 5. (a) Energy efficiency η versus number of nodes n. (b) Throughput λ_{out} versus number of nodes n. $P_W = 0.1W$, $P_T = 3W$, $\tau_F = 2.37 \times 10^{-3}$s, $\tau_T = 2.45 \times 10^{-3}$s, $K = 6$, $m = 2$, $\lambda_0 = 90\%\lambda_{max}$.

TABLE I
PARAMETER SETTING [12]

PHY header	20 μs
MAC Header	36 B
ACK	14 B+PHY header
DIFS	34 μs
SIFS	16 μs
Slot Time σ	9 μs
Packet Payload PL	2048 B
Data Rate R_D	7.2 Mbps
Basic Rate R_B	6 Mbps

Wi-Fi 6) as an example. Table I lists the typical values of key system parameters of the IEEE 802.11ax standard [12]. The simulated trajectory in Fig. 5a clearly shows that the energy efficiency η is optimized with $W = W_E$, compared to the case in which W is fixed, e.g., $W = 35$. Under the throughput constraint, the maximum energy efficiency η_{max} deteriorates to the throughput-constrained maximum energy efficiency η^c_{max} and can be achieved by setting W as W^c_E. As depicted in Fig. 5b, there exists a trade-off between the energy efficiency η and the throughput λ_{out}, and optimizing η can lead to a deteriorated λ_{out}. By setting W as W^c_E, the energy efficiency can be optimized under the constraint of a guaranteed throughput, compared with that when $W = 35$. It can also be concluded from Fig. 5a and Fig. 5b that when the number of nodes n becomes large, i.e. $n = 30$, trade-off between η and λ_{out} diminishes. It has been shown in (11) that η varies in positive correlation with λ_{out} when n is large, so that η and λ_{out} can be simultaneously optimized and the trade-off diminishes as a result. Simulation results in Fig. 5 well agree with the analysis.

V. Conclusion

In this paper, we provide closed-form solutions for an open issue on QoS guarantee in saturated CSMA networks: how to maximize the energy efficiency with/without the throughput constraint. Our analysis shows that optimally tuning the backoff parameter for optimizing the energy efficiency (*resp.* throughput) degrades the throughput (*resp.* energy efficiency) performance. Therefore, if a stringent throughput constraint is imposed, then the energy efficiency has to be sacrificed. We further apply the analysis to CSMA-based practical networks, i.e., Wi-Fi 6 networks. The simulation results demonstrate that to achieve the optimum energy efficiency while maintaining the throughput upon a certain level, it is of great necessity to adaptively tune the backoff window according to the system input parameters, such as the network size.

Appendix A
Derivation of (1)

As each node always has packets to send, it could be in the following states: 1) waiting state; 2) transmission state, i.e., it is transmitting packets. Let T_W and T_T denote the expected time for each node being in the waiting and transmission states during its lasting time, respectively, and we have

$$T_W + T_T = T \text{ and } P_W T_W + P_T T_T = E, \tag{22}$$

where P_W and P_T are the power consumption in the waiting and transmission state, respectively. Note that when each node is in the waiting state, its HOL packet could be in state $R_i, i = 0, ..., K$, and when each node is in transmission state, its HOL packet could be in state T and $F_i, i = 0, ..., K$. Based on the Markov renewal process in Section II, we have

$$\frac{T_W}{T_T} = \frac{\sum_{i=0}^{K} \tilde{\pi}_{R_i}}{\sum_{i=0}^{K} \tilde{\pi}_{F_i} + \tilde{\pi}_T} = \frac{\sum_{i=0}^{K} \pi_{R_i} \cdot \tau_{R_i}}{\sum_{i=0}^{K} \pi_{F_i} \cdot \tau_{F_i} + \pi_T \cdot \tau_T}. \tag{23}$$

Note that $\pi_{R_i}, \pi_{F_i}, \tau_{R_i}$, and $\tau_{F_i} i \in \{T, ..., K\}$ are given in (18)–(21) in [5] as functions of p (i.e., the steady-state probability of successful transmission of HOL packets), and the expression of p is given in (46) in [5].

By substituting (18)–(21) and (46) in [5] into (23), it can be obtained that

$$\frac{T_W}{T_T} = \frac{a(x+1-xp-(1/a-x)p\ln p)\cdot\frac{n}{-p\ln p}}{xa\cdot\frac{1-p}{p}+1} - 1. \tag{24}$$

(1) can then be obtained by combining (22) and (24).

Appendix B
Proof of Theorem 1

According to (3), it can be easily obtained that

$$\begin{aligned}\frac{\partial \eta}{\partial p} =& N_p / \Big(anxP_W p + (1-ax)((n-1)P_W + P_T)p\ln p \\ & + ax(P_T - P_W)\ln p - (1+x)anP_W \Big)^2, \end{aligned} \tag{25}$$

where N_p denotes the numerator of $\frac{\partial \eta}{\partial p}$, i.e.,

$$N_p = anxP_W p + ax(P_T - P_W)\ln^2 p - (1+x)anP_W(1+\ln p), \tag{26}$$

and $N_p = 0$ has the same non-zero roots as those of $\frac{\partial \eta}{\partial p} = 0$.

We have

$$\frac{\partial N_p}{\partial p} = -\frac{a}{p} \cdot \Bigg(2(P_W - P_T)x\ln p + nP_W(1 + x(1-p))\Bigg) < 0, \tag{27}$$

so N_p is a monotonically decreasing function of p. It can also be obtained from (26) that $\lim_{p\to 0} N_p > 0$ and $\lim_{p\to 1} N_p < 0$, base on which, we can come to the conclusion that N_p has a single non-zero root on $p \in (0,1)$ and η is a convex function of p. The maximum of η is achieved when $\frac{\partial \eta}{\partial p} = 0$, as long as $N_p = 0$. (4) is obtained by substituting p_E into (3) and (5) can be obtained by combining (26) and $N_p = 0$.

Under the constraint that each node always has packets to send as stated in Section II, it has been proved in [5] that (46) in [5] can be approximately written as (51) in [5] when p is small. (6) can be obtained by substituting p_E into (51) in [5].

Appendix C
Proof of Theorem 2

It can be obtained from (2) that

$$\frac{\partial \lambda_{out}}{\partial p} = \frac{N_l}{n(a(1+x-px)+(ax-1)p\ln p)^2}, \tag{28}$$

where N_l denotes the numerator of $\frac{\partial \lambda_{out}}{\partial p}$, i.e.,

$$N_l = a(xp - (1+x)\ln p - x - 1), \tag{29}$$

and $N_l = 0$ has the same non-zero roots as those of $\frac{\partial \lambda_{out}}{\partial p} = 0$.

We have

$$\frac{\partial N_l}{\partial p} = a(x - \frac{1+x}{p}) < 0, \tag{30}$$

so N_l is a monotonically decreasing function of p. It can also be obtained from (29) that $\lim_{p\to 0} N_l > 0$ and $\lim_{p\to 1} N_l < 0$, base on which, we can conclude that N_l has a single non-zero root on $p \in (0,1)$ and λ_{out} is a convex function of p on $p \in (0,1)$.

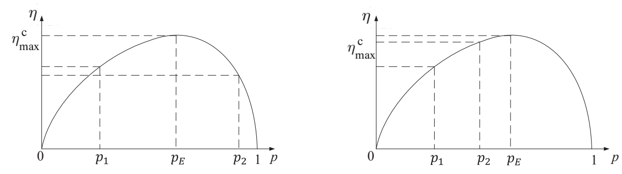
Let $p_1 \le p_2$ denote the two non-zero roots of $\lambda_{out} - \lambda_0 = 0$, and (15) can be obtained by substituting (2) into $\lambda_{out} - \lambda_0 = 0$. By combining (26) and (29), we can get

$$\frac{N_p}{nP_W} = N_l + \frac{ax}{n}(\frac{P_T}{P_W} - 1)\ln^2 p \ge N_l. \tag{31}$$

The single non-zero root of $N_p = 0$, denoted by p_E, is therefore larger than the single non-zero root of $N_l = 0$, which is denoted by p_{Th}, and we have $p_1 \le p_{Th} \le p_E$.

As $\lambda_0 \le \lambda_{max}$ and λ_{out} is a convex function of p, $\lambda_{out} \ge \lambda_0$ can only be satisfied when $p \in [p_1, p_2]$. The throughput-constrained optimization problem in (12) can therefore be written as

$$\eta^c_{\max} = \max_{0<q_0\le 1} \eta \qquad s.t. \quad p_1 \le p \le p_2. \tag{32}$$



(a) (b)

Fig. 6. (a) $p_E \in [p_1, p_2]$. (b) $p_2 < p_E \le 1$.

Let p^c_E denote the successful transmission probability at which the energy efficiency is optimized under the constraint in (32). To further determine p^c_E, let us consider the following scenarios.

1) If p_E in Theorem 1 satisfies $p_E \in [p_1, p_2]$, then as shown in Fig. 6a, the throughput-constrained maximum energy efficiency η^c_{max} can be achieved with $p^c_E = p_E$.
2) If $p_E \notin [p_1, p_2]$, then as shown in Fig. 6b, the throughput-constrained maximum energy efficiency η^c_{max} can be achieved with $p^c_E = p_2$.

(14) can be obtained by combining the above two cases.

(13) can be obtained by substituting p^c_E into (3) and (16) can be obtained by substituting p^c_E into (51) in [5].

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