Optimal Design for UAV-Assisted Energy Constrained Communication: Joint Power Control and Continuous Trajectory Design

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ICC 2022 - IEEE International Conference on Communications | 978-1-5386-8347-7/22/$31.00 ©2022 IEEE | DOI: 10.1109/ICC45855.2022.9838255

***Abstract*—For unmanned aerial vehicle (UAV)-assisted wireless networks, the continuous trajectory designs generally suffer from infinite number of variables of the continuous UAV trajectory. In this paper, to avoid unexpected trajectory approximation and overcome the difficulty in obtaining an accurate continuous trajectory, we aim at characterizing an analytical optimal solution for jointly designing the resource allocation and continuous UAV trajectory. We focus on a scenario with UAV at a fixed altitude being deployed to assist the wireless communication with a ground user. With limited energy accessible for the wireless transmissions, we construct a throughput maximization problem for jointly optimizing the continuous transmit power and the UAV's continuous trajectory. Via duality analysis, we obtain the features of the optimal power control and successfully convert the dual problem to a series of pure trajectory design problems, which can be optimally addressed based on a mechanical equivalence approach. Afterwards, we accordingly propose an algorithm for optimally solving the dual problem, from which the optimal joint solution can be analytically constructed in a closed form. Finally, we also verify our proposed algorithm and confirm the optimality of the obtained solution via simulations.**

***Index Terms*—Unmanned aerial vehicles (UAVs), energy constrained communication, continuous trajectory design, joint optimization, artificial potential field (APF).**

I. Introduction

Unmanned aerial vehicles (UAVs), thanks to their high deployment flexibility, have already been widely implemented in copious military and civil scenarios [1], for instance, in object tracking, emergency rescue, photography and detection. Likewise, as an essential promoting component for 5G and 6G network, UAV is also capable of assisting wireless communications and contributes to a better system performance than the conventional territorial network [2]. The produced air-to-ground channels have generally a much higher probability to be dominated by line-of-sight (LoS) links, such that a high-quality communication can be anticipated in UAV-assisted networks. Among the existing research works, the design for UAV deployment position has been well investigated respectively for coverage enlargement [3] and throughput enhancement [4]. Besides, UAV can also flexibly fly rather closer to served users to avoid the large path loss in signal propagation. In the literature, via optimizing UAV trajectory, significant performance improvement has been achieved with respect to the channel capacity [5] and the energy efficiency [6]. Combining with wireless power transfer (WPT) technologies, the UAV trajectory design has also been efficiently addressed in [7] in WPT networks.

Moreover, in wireless communications, the limited radio resources generally force the system to operate in a more efficient manner. In UAV-assisted wireless networks, since the time-varying UAV position has lead to dramatically varying channel states, a resource allocation jointly designed with UAV trajectory will be highly recommended. Thus, numerous research works are motivated to study the joint optimization of UAV trajectory respectively with precoding process [8], beamforming [9] and power control [10]. However, almost all the existing works regarding UAV trajectory design have either adopted a trajectory quantization in time scale for the simplicity of design [8]–[10], or applied an approximated trajectory structure [7], both of which have made it impossible to obtain the optimal continuous trajectory. Fortunately, in our previous work [11], we have proposed a novel approach for UAV trajectory design, in which the trajectory design problem is completely equivalent to a mechanical problem. Via optimizing the continuous rope shape in the mechanical problem according to physical principles, the continuous optimal UAV trajectory is for the first time obtained in a closed form. It should be mentioned that the work [11] has assumed a constant transmit power without any power control behaviour. To the best of our knowledge, the **optimal** joint design for power control and continuous trajectory design in UAV-assisted network is still missing in the literature.

Therefore, in this work, we are motivated to extend the novel trajectory design approach in [11] to a UAV-assisted energy constrained communication scenario. Applying UAV as a mobile base station, we consider a single-user network with limited energy budget for the wireless communication. Given starting point and ending point for UAV trajectory, we aim at optimally addressing the throughput maximization problem

This work was supported in part by NSFC Grant with No.62101389, in part by the Federal Ministry of Education and Research (BMBF, Germany) as part of the 6G Research and Innovation Cluster 6G-RIC under Grant 16KISK020K and in part by DFG SCHM2643/16.

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with power control and UAV trajectory to be jointly optimized. To keep the continuity of UAV trajectory, we first analyse the joint optimization problem via a dual approach. Based on characterized optimal power control scheme and mechanical equivalence approach, we propose an efficient algorithm for optimally solving the dual problem. Finally, the proved strong duality ensures the optimality of the constructed solution from dual problem, which is further verified via numerical results. Our main contributions are:

- **Optimal joint design in UAV-assisted networks**: For the first time, we have obtained an optimal joint solution for resource allocation and trajectory design in UAV-assisted networks. The analysis process and mechanical equivalence based design will also enable the construction of the optimal joint design with other types of resource allocation in UAV-assisted network.
- **Efficient closed-form solution**: We have also for the first time constructed the closed-form solution for the joint optimization of power control and trajectory design. With closed-form expression, an extremely lower complexity can be expected from the proposed algorithm for optimal solution. The whole approach for the optimal closed-form solution can also be directly extended to the scenarios with a general model, like probabilistic LoS model.
- **Numerical results**: Via simulations, we also validate the proposed algorithm and verify the optimality of constructed closed-form solution.

The remaining of this paper is organized as follows: In Section II, we state the focused problem, which is characterized via a dual approach in Section III. Based on the results, we introduce a mechanical equivalence in solving the dual problem and propose an efficient algorithm in Section IV. Finally, the solution is validated via simulations in Section V and the work is concluded in Section VI.

II. Problem Statement

We consider a wireless communication network assisted by a UAV, which is deployed at a fixed altitude H and acts as a mobile base station. Aiming at attaining fundamental insights for the system design in UAV-assisted network, in the considered network, we assume a single user distributed on the ground and served by the mobile UAV. Denote by (w_x, w_y) the ground position of the user. The whole communication period between the UAV and user is scheduled as $\mathcal{T} \triangleq [0, T]$, which is possibly decided by the scheduling centre. At each time point $t \in \mathcal{T}$, we represent by $(x(t), y(t))$ the instantaneous horizontal position of UAV, which constitutes the continuous UAV trajectory $\{x(t), y(t)\}$. In particular, the speed of UAV is constrained by a maximum V, i.e., we have $||\dot{x}(t), \dot{y}(t)||_2 \leq V$, where $\dot{x}(t)$ and $\dot{y}(t)$ are respectively the first-order derivatives of $x(t)$ and $y(t)$ in t. Moreover, in addition to the operation period $\mathcal{T}$, both the starting and ending points for UAV's operation are very likely to be predetermined by the scheduling centre, especially when multiple tasks are assigned to UAV. Thus, in this work, we denote the horizontal positions of the starting and ending points for UAV trajectory, respectively by (x_1, y_1) and (x_2, y_2).

Furthermore, for the wireless communication between the UAV and user, we assume the transmit power at each time point $t \in \mathcal{T}$ is given by $P(t)$. For the uplink communication, the limited energy storage at the user may constrict the available energy for data uploading, while the energy budget at UAV may also be pre-scheduled in the case of downlink communication. Thus, we assume the total available energy for the transmissions is restricted by E, i.e., we have $\int_0^T P(t)dt \leq E$. As for the channel modelling, since the air-to-ground links in UAV-assisted networks are generally LoS dominant, we adopt a free-space path loss model without loss of generality, as frequently applied in [8]–[10]. More specifically, at each time point $t \in \mathcal{T}$, the channel gain between UAV and the single user can be expressed by

$$h(t) = \frac{\beta_0}{d(t)^2 + H^2} = \frac{\beta_0}{(x(t) - w_x)^2 + (y(t) - w_y)^2 + H^2}, \quad (1)$$

where $d(t)$ denotes the horizontal distance between UAV and user at time t and β_0 is the reference channel gain at unit distance. As a result, the signal-to-noise ratio (SNR) at time t can be obtained as

$$\gamma(t) = \frac{h(t)P(t)}{\sigma^2}, \quad (2)$$

where σ^2 is the noise power level. Accordingly, the maximum achievable transmission rate at time t can be formulated as

$$R(t) = \log_2(1 + \gamma(t)), \quad (3)$$

which leads to an overall throughput for the whole operation period $\mathcal{T}$ expressed as

$$U(\{x(t), y(t)), P(t)\}) = \int_0^T R(t)dt. \quad (4)$$

To sum up, in order to enhance the communication between the UAV and user, we target at maximizing the overall throughput with constrained energy via jointly optimizing the power control scheme and UAV trajectory. Note that in this work we consider a pre-design for the whole operation period and focus on the maximal achievable throughput. The possible performance decay in practical deployment has been ignored to gain a fundamental insight of the joint optimization. The formulated original problem is given by

$$\begin{aligned}
\text{(OP)}: \max_{\{x(t), y(t), P(t)\}} \quad & U(\{x(t), y(t), P(t)\}) && (5a)\\
s.t. \quad & ||(\dot{x}(t), \dot{y}(t))||_2 \leq V, \ \forall t \in \mathcal{T}, && (5b)\\
& (x(0), y(0)) = (x_1, y_1), && (5c)\\
& (x(T), y(T)) = (x_2, y_2), && (5d)\\
& \int_0^T P(t)dt \leq E, && (5e)\\
& 0 \leq P(t) \leq P_{\max}, \ \forall t \in \mathcal{T}, && (5f)
\end{aligned}$$

where $P_{\max}$ is the maximum transmit power which is determined by the hardware. In problem (OP), both the transmit power $P(t)$ and UAV trajectory $\{x(t), y(t)\}$ have contained infinite variables with respect to the time scale and accordingly make the problem analysis rather complicated. Besides, the continuity of UAV trajectory also makes the problem intractable. Different from most of the existing works which are based on time discretization and can only guarantee a suboptimal result, in this work, we keep the continuity of UAV trajectory and aim at constructing the globally optimal joint solution for (OP). In the next section, we start with the characterization of the optimal design.

III. Optimal Joint Solution Characterization

Hereby, we perform the optimal joint design characterization via a dual approach. Focusing on the energy constraint (5e) and given a Lagrange multiplier $\lambda \geq 0$, we build the partial Lagrangian for problem (OP) as

$$L(\{x(t), y(t), P(t)\}, \lambda) = -\int_0^T R(t)dt + \lambda(\int_0^T P(t)dt - E), \quad (6)$$

such that the corresponding dual function $L_D(\lambda)$ is

$$L_D(\lambda) = \min_{\{x(t), y(t), P(t)\}} L(\{x(t), y(t), P(t)\}, \lambda) \quad (7)$$
$$s.t. \quad \text{(5b), (5c), (5d), and (5f)},$$

where all the constraints are inherited from (OP) except the energy constraint (5e). Accordingly, the dual problem for (OP) can be formulated as

$$\text{(DP)}: \quad \max_{\lambda} \quad L_D(\lambda) \quad (8a)$$
$$s.t. \quad \lambda \geq 0. \quad (8b)$$

A. Optimal Power Control Scheme Characterization

To address problem (DP), we first analyse the dual function $L_D(\lambda)$, which is obtained from optimizing the problem in (7). By observing (7), we find that with any given $\lambda \geq 0$ and UAV trajectory $\{x(t), y(t)\}$, the optimal power control scheme $P(t)$ can be attained by solving the following simplified problem

$$\min_{\{P(t)\}} \quad \int_0^T \big(- R(t) + \lambda P(t) \big) dt \quad (9a)$$
$$s.t. \quad 0 \leq P(t) \leq P_{\max}, \ \forall t \in \mathcal{T}. \quad (9b)$$

According to (3), given UAV trajectory $\{x(t), y(t)\}$, the maximum transmission rate $R(t)$ can be considered as a function of $P(t)$. Therefore, we can drop the time variable t in solving (9), such that for any time t, the optimal transmit power $P(t)$ is optimized from problem

$$\min_{P(t)} \quad -\log_2(1 + \frac{h(t)P(t)}{\sigma^2}) + \lambda P(t) \quad (10a)$$
$$s.t. \quad 0 \leq P(t) \leq P_{\max}. \quad (10b)$$

Namely, for any given λ and UAV trajectory $\{x(t), y(t)\}$, we have the optimal transmit power for (7) defined as

$$P_\lambda(t) = \min\Big\{P_{\max}, \big(\frac{1}{\lambda \ln 2} - \frac{\sigma^2}{h(t)}\big)^+\Big\}, \quad (11)$$

where the operator $(\cdot)^+$ is defined as $(x)^+ \triangleq \max\{0, x\}$. Since $h(t)$ is a function in $x(t)$ and $y(t)$ according to (1), the optimal power $P_\lambda(t)$ is actually a function in λ and $(x(t), y(t))$. By inserting (11) into (7), we can equivalently reform the dual function $L_D(\lambda)$ as

$$L_D(\lambda) = \min_{\{x(t), y(t)\}} L(\{x(t), y(t), P_\lambda(t)\}, \lambda) \quad (12)$$
$$s.t. \quad \text{(5b), (5c), and (5d)},$$

which becomes a pure UAV trajectory design problem. Clearly, dual problem (DP), which is always convex, can be optimally addressed if we can optimally solve the pure trajectory design problem (12) for any given λ. However, in the dual approach, solving the dual problem (8) can only provide an upper bound for problem (OP), i.e., we have $-d^* \geq p^*$ where d^* and p^* are respectively the optimal of problems (DP) and (OP). Hence, we are motivated to investigate the corresponding duality.

B. Strong Duality

To prove the strong duality, i.e., $-d^* = p^*$, we analyse both problems (OP) and (DP) and respectively obtain the following two lemmas, i.e., Lemma 1 and Lemma 2.

Lemma 1. *For the optimal solution of problem (OP) $\{x^*(t), y^*(t), P^*(t)\}$, it must holds that $\int_0^T P^*(t)dt = \min\{E, P_{\max}T\}$.*

Proof. When $E \geq P_{\max}T$, the energy budget E is sufficient to support the communication with maximum transmit power $P_{\max}$ during the whole operation period $\mathcal{T}$. As more power will definitely result in a higher maximum transmission rate and correspondingly larger throughput, the optimal power control scheme will be $P^*(t) = P_{\max}$, $\forall t \in \mathcal{T}$. When $E < P_{\max}T$, $P^*(t) = P_{\max}$ does not hold for all $t \in \mathcal{T}$. If $\int_0^T P^*(t)dt < E$, the spared energy $E - \int_0^T P^*(t)dt$ can be allocated to any time slot where $P^*(t) < P_{\max}$ and further improve the throughput such that a better solution than the optimal can be obtained. Therefore, it must hold that $\int_0^T P^*(t)dt = E$. To sum up, we have $\int_0^T P^*(t)dt = \min\{E, P_{\max}T\}$. □

Lemma 2. *For the dual function $L_D(\lambda)$, we denote by $P_\lambda^*(t)$ the optimized $P_\lambda(t)$ after the optimization of (12). Then, we have $\int_0^T P_\lambda^*(t)dt$ monotonically decreasing in $\lambda \geq 0$.*

In particular, there exists $\lambda^* \geq 0$, such that when $\lambda = \lambda^*$, it holds that $\int_0^T P_\lambda^*(t)dt = \min\{E, P_{\max}T\}$.

Proof. For the optimization problem (12), the objective can be equivalently considered as a weighted superposition of two terms, $-\int_0^T R(t)dt$ and $\int_0^T P_\lambda(t)dt$, respectively with weights of 1 and λ. Then, increasing λ will lead to a larger weight for $\int_0^T P_\lambda(t)dt$, such that the optimized $\int_0^T P_\lambda^*(t)dt$ will be suppressed during the objective minimization, i.e., monotonically decreasing in $\lambda \geq 0$.

In particular, according to (11), when λ approaches 0, it holds $P_\lambda(t) = P_{\max}$, $\forall t \in \mathcal{T}$ and $\int_0^T P_\lambda^*(t)dt = P_{\max}T$. When λ approaches infinity, we have $P_\lambda(t) = 0$, $\forall t \in \mathcal{T}$ and $\int_0^T P_\lambda^*(t)dt = 0$. Thus, recalling the monotonic property of $\int_0^T P_\lambda^*(t)dt$, we can find such $\lambda^* \geq 0$ that $\int_0^T P_\lambda^*(t)dt = \min\{E, P_{\max}T\}$ when $\lambda = \lambda^*$. $\square$

Combining two lemmas, we can conclude that when $\lambda = \lambda^*$, the problem in (12) will result in the same optimal UAV trajectory as (OP). This is due to the facts that the problem (12) is equivalent to (7) and the optimal solutions in both (7) and (OP) are feasible for each other when $\lambda = \lambda^*$. In addition, when $E \geq P_{\max}T$, there may be multiple applicable λ^*. In this case, we simply define $\lambda^* = 0$ when $E \geq P_{\max}T$. As a result, we will always have $d^* \geq L_D(\lambda^*) = -p^*$. As $-d^* \geq p^*$ always holds, it will hold that $-d^* = p^*$, namely the strong duality between (OP) and (DP) holds. The corresponding λ^* will also be the optimal solution for (DP).

After all, due to the strong duality, (OP) can eventually be optimally addressed via solving dual problem (DP). More simply, based on the previous discussions, as long as the optimized λ^* can be found, we will be able to formulate the optimal power control scheme according to (11) and construct the optimal UAV trajectory via solving the pure trajectory design problem in (12). Therefore, the remaining tasks for optimally solving problem (OP) will remain to be optimally solving (12) for any $\lambda \geq 0$ and searching for the optimal λ^*, which inspires us to propose an algorithm for the optimal solution construction in the next section.

IV. Optimal Joint Solution Construction

A. Optimal UAV Trajectory Design

To assist the construction of the optimal joint solution, we first focus on the pure UAV trajectory design problem in (12). Since $-\lambda E$ is constant for any given λ, we can simply drop the term $-\lambda E$ during the optimization of (12). According to our previous work [11], the pure trajectory design problem (12) in single-user scenario can be completely equivalent to a mechanical problem (MP), in which the UAV trajectory with a speed limit is represented by an extremely soft rope with a line density constraint[1]. More specifically, the equivalent mechanical problem can be formulated as

$$\begin{aligned}
\text{(MP)}: \min_{\{\hat{x}(s),\hat{y}(s)\},\rho(s),S} \quad & \int_0^S \hat{R}(\hat{x}(s),\hat{y}(s))\rho(s)\mathrm{d}s && \text{(13a)}\\
s.t. \quad & \rho(s) \geq \rho_{\min},\ \forall s \in [0,S], && \text{(13b)}\\
& \int_0^S \rho(s)\mathrm{d}s = m, && \text{(13c)}\\
& (\hat{x}(0),\hat{y}(0)) = (x_1,y_1), && \text{(13d)}\\
& (\hat{x}(S),\hat{y}(S)) = (x_2,y_2), && \text{(13e)}
\end{aligned}$$

where $\rho(s)$ is the rope line density with $\frac{1}{\rho(s)}$ denoting UAV speed, the variable s denotes the passed path length from the starting point, S is the total length of the rope and $\{\hat{x}(s),\hat{y}(s)\}$ represents the equivalent rope shape which has the same shape as the UAV trajectory path. Moreover, (13b) represents the equivalent constraint of the UAV speed constraint in trajectory design while $\rho_{\min} = \frac{1}{V}$. The equivalent rope has a total mass of $m = T$, as shown in (13c), and the same ending points as UAV trajectory, as indicated in (13d), (13e). In particular, to ensure the complete problem equivalence, the artificial potential field (APF) $\hat{R}(x,y)$ defined as

$$\hat{R}(x,y) = -\log_2\Big(1 + \frac{\beta_0 \hat{P}_\lambda(d(x,y))}{\sigma^2(d(x,y)^2 + H^2)}\Big) + \lambda \hat{P}_\lambda(d(x,y)), \quad (14)$$

where $d(x,y)$ is the distance from point (x,y) to (w_x,w_y), similar to the horizontal distance $d(t)$ from UAV to user in trajectory design, and $\hat{P}_\lambda(d(x,y))$ is obtained based on (11) via replacing $d(t)$ with $d(x,y)$. As a result, the APF $\hat{R}(x,y)$ is purely a function in $d(x,y)$ and differs in λ. The mechanical problem (MP) will be a rope shape design problem for minimizing the overall artificial potential energy on the rope.

Since the problem equivalence from (12) to (MP) is a complete equivalence according to [11], we can thus equivalently solve (MP) in (13) and accordingly reconstruct the optimal UAV trajectory based on the optimized rope solution. In the potential energy minimization problem (MP), according to the minimum total potential energy principle, the optimal rope solution must stay in a state of equilibrium, which has largely facilitated the construction of the optimal rope solution. In addition, we have Lemma 3 for the APF in (14).

Lemma 3. *The APF $\hat{R}(x,y)$ is a monotonically increasing function in $d(x,y) \geq 0$.*

Proof. When $\hat{P}_\lambda(d(x,y)) = 0$ or $\hat{P}_\lambda(d(x,y)) = P_{\max}$, $\hat{R}(x,y)$ is clearly monotonically increasing in $d(x,y)$. When $0 < \hat{P}_\lambda(d(x,y)) < P_{\max}$, from (11), we have

$$\hat{P}_\lambda(d(x,y)) = \frac{1}{\lambda \ln 2} - \frac{\sigma^2}{\beta_0}(d(x,y)^2 + H^2) > 0, \quad (15)$$

such that the APF can be simplified as

$$\begin{aligned}
\hat{R}(x,y) = &-\log_2 \frac{\beta_0}{\sigma^2 \lambda \ln 2} + \frac{1}{\ln 2} + \log_2(d(x,y)^2 + H^2)\\
&- \frac{\lambda\sigma^2}{\beta_0}(d(x,y)^2 + H^2). \quad (16)
\end{aligned}$$

Subsequently, we can obtain that

$$\frac{\partial \hat{R}(x,y)}{\partial(d(x,y)^2 + H^2)} = \frac{\lambda \hat{P}_\lambda(d(x,y))}{d(x,y)^2 + H^2} > 0. \quad (17)$$

Namely, APF $\hat{R}(x,y)$ is monotonically increasing in $d(x,y)^2 + H^2$, i.e., increasing in $d(x,y)$, when $0 < \hat{P}_\lambda(d(x,y)) < P_{\max}$. In addition, we can easily prove the continuity of $\hat{R}(x,y)$ in $d(x,y)$. The monotonic increasing property for $\hat{R}(x,y)$ can thus be proved. $\square$

According to [11], if the APF has a single centre and is monotonically increasing with respect to the distance to the centre, the optimal rope solution in the defined APF can be expressed in a closed form. Letting d_1 and d_2 respectively denote the distances $d(x_1,y_1)$ and $d(x_2,y_2)$, we will have

- when $T = m \geq \frac{d_1+d_2}{V}$, the optimal rope solution will contain two straight line segments with minimum line density $\rho_{\min}$

[1] More details about the mechanical equivalence and mechanical problem formulation can be found in Section III of work [11].

respectively from (x_1, y_1) and (x_2, y_2) to (w_x, w_y), and a mass point at (w_x, w_y), namely the optimal UAV trajectory will be a direct flight from (x_1, y_1) to (w_x, w_y) then back to (x_2, y_2) along straight lines with maximum speed V, while the remaining operation time will become a hovering behaviour above (w_x, w_y);

- when $T = m < \frac{d_1+d_2}{V}$, the optimal rope solution will have a curve shape $\{\hat{x}^*(s), \hat{y}^*(s)\}$ with line density always equal to $\rho_{\min}$, such that the optimal trajectory can be reconstructed as

$$(x^*(t), y^*(t)) = (\hat{x}^*(Vt), \hat{y}^*(Vt)),\ \forall t \in \mathcal{T}. \qquad (18)$$

In particular, when $T = m < \frac{d_1+d_2}{V}$, the optimal rope shape $\{\hat{x}^*(s), \hat{y}^*(s)\}$ is constructed based on the force balance rule in equilibrium, such that the whole rope shape can be generated from an optimal initial tension at the ending point (x_1, y_1)[2]. According to [11], the proved monotonic property in Lemma 3 will enable an efficient algorithm for obtaining the optimal initial tension and correspondingly the optimal rope shape. More specifically, for the optimal initial tension which includes a tension angle α^* and a tension value Q_0^*, based on Lemma 3, we can first find for any angle α the unique tension value $Q_0^*(\alpha)$ letting the rope exactly pass through point (x_2, y_2) and then again based on the monotonic property find the unique α^* which make the total rope length S exactly equal to VT. After all, the resulted angle α^* and corresponding $Q_0^*(\alpha^*)$ will comprise the optimal initial tension and accordingly enable the construction of the optimal UAV trajectory.

B. Optimal Lagrange Multiplier Design

So far, we have provided an efficient approach for optimally solving the problem (12) for any given $\lambda \geq 0$. The final task will be looking for the corresponding optimal λ^*. Specially, when $E \geq P_{\max}T$, we have $\lambda^* = 0$. When $E < P_{\max}T$, according to Lemma 2, the optimized $\int_0^T P_\lambda^*(t)dt$ from solving (12) is monotonically decreasing in $\lambda \geq 0$. Since the optimal λ^* will lead to $\int_0^T P_\lambda^*(t)dt = E$, we can start with a feasible interval for λ, repeatedly solve (12) with λ being the midpoint and shorten the interval according to the value of $\int_0^T P_\lambda^*(t)dt$. Eventually, the optimal λ^*, letting $\int_0^T P_\lambda^*(t)dt = E$, will be obtained. Afterwards, with $\lambda = \lambda^*$, the global optimal UAV trajectory can be thus attained from addressing (12) and the corresponding optimal power control scheme can also be constructed from (11) based on the optimized UAV trajectory. The whole algorithm flow for the optimal solution is shown in Algorithm 1. Due to the closed-form expression, the algorithm complexity is expected to be $\mathcal{O}(1)$.

V. Simulation Results

In this section, via numerical results, we evaluate our proposed construction algorithm for the optimal joint design and subsequently observe the diverse optimal behaviour of the joint design under different system setups. The default simulation setup is defined as follows: $P_{\max} = 10$W, $E = 200$J, $\beta_0 = -30$dB, noise power $\sigma^2 = -50$dBW, $H = 20$m, $T = 35$s and maximum UAV speed $V = 2$m/s. In particular, we assume the single user is deployed at position $(0, 0)$m, while for the operation period the UAV starts from position $(50, 20)$m and ends at $(10, 50)$m.

[2]See (27)-(34) in [11] for more details. In this work, the optimal rope solution with $T = m < \frac{d_1+d_2}{V}$ can be obtained by directly replacing the APF in [11] with our defined APF (14).

Algorithm 1 : Optimal Joint Solution Construction.

If $E \geq P_{\max}T$
a) $\lambda^* = 0$.
Else
b) $(\lambda_{\min}, \lambda_{\max}) = (0, \frac{\beta_0}{\sigma^2 H^2 \ln 2})$.
c) $\lambda = (\lambda_{\min} + \lambda_{\max})/2$.
d) Optimally solve (12).
If $\int_0^T P_\lambda^*(t)dt < E$
e) $\lambda_{\max} = \lambda$, back to **c)**.
Elseif $\int_0^T P_\lambda^*(t)dt > E$
f) $\lambda_{\min} = \lambda$, back to **c)**.
End
g) $\lambda^* = \lambda$.
End
h) Obtain $\{x^*(t), y^*(t)\}$ via solving (12) with $\lambda = \lambda^*$.
i) Construct optimal $P^*(t)$ according to (11).

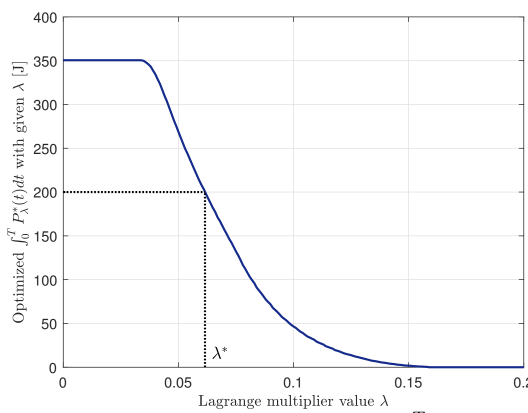

Fig. 1. Monotonic behaviour of optimized $\int_0^T P_\lambda^*(t)dt$ in λ.

First of all, we solve the pure trajectory design problem (12) with different λ and show the relation between optimized $\int_0^T P_\lambda^*(t)dt$ and λ in Fig. 1. As shown in Fig. 1, the optimized $\int_0^T P_\lambda^*(t)dt$ is monotonically decreasing in λ, which confirms the statement in Lemma 2 and validates the proposed algorithm for obtaining the optimal λ^*. In particular, with the default setup $E = 200$J, the corresponding optimal λ^* has been illustrated in Fig. 1. Clearly, when λ approaches to 0, $\int_0^T P_\lambda^*(t)dt$ will be equal to $P_{\max}T$, i.e., 350J.

Afterwards, we turn to evaluate the communication performance of the optimal joint design under different energy budgets E in Fig. 2. The achieved capacity, defined as $\frac{U}{T}$, where U denotes the optimized throughput, is considered as the indicator for the network performance. As clarified in Fig. 2, more energy resources will result in a higher capacity, while this benefit disappears when the energy budget E exceeds $P_{\max}T$, which accords with the discussions in Lemma 1. Moreover, we can also observe that more time resources (larger T) and higher UAV mobility (larger V) will significantly enhance the performance. Increasing T will also enlarge the effective energy budget range for improving the overall capacity. In addition, we also provide in Fig.2 the solution based on the

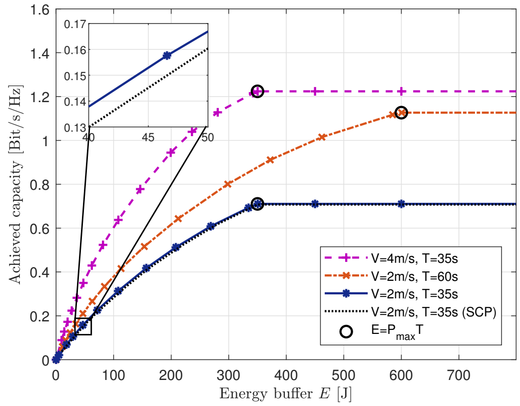

Fig. 2. Performance observation with different energy budget E. (Optimal solution has more than 6% performance gain than the SCP-based benchmark.)

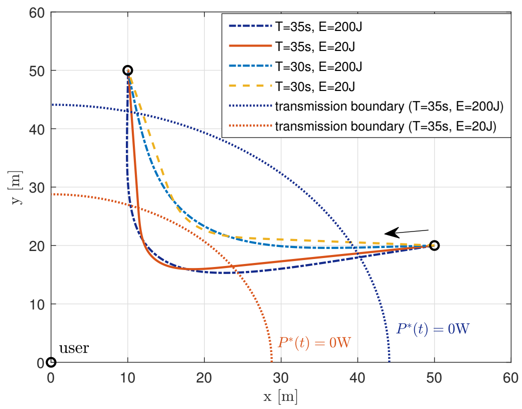

Fig. 3. Optimal trajectory observation.

popular time discretization strategy. After discretization, the joint design can also be addressed via alternately optimizing the power control scheme and UAV trajectory. However, due to the approximation approach, the obtained solution always shows to be worse than the optimal one, which confirms the optimality of our solution.

At last, we finalize the evaluation by displaying in Fig. 3 the optimal continuous UAV trajectories with different setups. We find that the optimal UAV trajectory tends to be closer to the user with more operation time T, which verifies the performance advantage of larger T in Fig. 2. In particular, we also depict in Fig. 3 the transmission boundary respectively for the cases $E = 200$J and $E = 20$J. Namely, the transmit power will be 0W, i.e., with no transmissions, when UAV is out of the boundary. As indicated, the optimal UAV trajectory out of the boundary will become purely straight lines, since without transmission intention the UAV will prefer to spend as shorter time as possible to reach the transmission area. Moreover, with different energy budgets E, the optimal trajectory differs obviously. When more energy is available, most part of the optimal UAV trajectory tends to be a curve to optimally exploit both UAV mobility and the sufficient energy budget for the maximum throughput. By contrast, with few energy resources, UAV in the optimal joint design will tend to fly closer to the user without communication performance, i.e., $P(t) = 0$, and then the limited energy is only implemented when the distance between UAV and user is relatively shorter.

VI. Conclusion

In this paper, we have investigated a UAV-assisted energy constrained communication scenario. Aiming at maximizing the throughput between the user and UAV, we have analytically constructed the optimal joint solution for transmit power control and continuous UAV trajectory design. In particular, via a dual approach, the optimal power control scheme is characterized as a function in UAV position, which enables us to optimally address the dual problem via solving pure trajectory design problems. Afterwards, keeping the trajectory continuity, we have established the optimal solution for pure trajectory design problems based on mechanical equivalence and subsequently completed the construction of the analytical joint optimal solution with a proposed algorithm. At last, the simulation results confirm the validation of our proposed algorithm. It should be stressed that as a main contribution, the whole approach has the potentials to be extended for more analytical joint solutions in UAV-assisted network and under various power control capability, e.g., with discrete power control choices.

References

[1] Y. Zeng, R. Zhang and T. J. Lim, “Wireless communications with unmanned aerial vehicles: opportunities and challenges,” *IEEE Commun. Mag.*, vol. 54, no. 5, pp. 36-42, May 2016.

[2] Y. Zeng, Q. Wu and R. Zhang, “Accessing From the Sky: A Tutorial on UAV Communications for 5G and Beyond," *Proceedings of the IEEE*, vol. 107, no. 12, pp. 2327-2375, Dec. 2019.

[3] S. Shakoor, Z. Kaleem, D.-T. Do, O. A. Dobre and A. Jamalipour, “Joint Optimization of UAV 3-D Placement and Path-Loss Factor for Energy-Efficient Maximal Coverage,” *IEEE Int. Things J.*, vol. 8, no. 12, pp. 9776-9786, 15 June15, 2021.

[4] R. Fan, J. Cui, S. Jin, K. Yang, and J. An, “Optimal Node Placement and Resource Allocation for UAV Relaying network,” *IEEE Commun. Lett.*, vol. 22, no. 4, pp. 808–811, Apr. 2018.

[5] P. Li and J. Xu, “Fundamental Rate Limits of UAV-Enabled Multiple Access Channel with Trajectory Optimization,” *IEEE Trans. Wireless Commun.*, vol. 19, no. 1, pp. 458-474, Jan. 2020.

[6] Y. Zeng, J. Xu, and R. Zhang, “Energy Minimization for Wireless Communication with Rotary-Wing UAV,” *IEEE Trans. Wireless Commun.*, vol. 18, no. 4, pp. 2329-2345, Apr. 2019.

[7] X. Yuan, T. Yang, Y. Hu, J. Xu and A. Schmeink, “Trajectory Design for UAV-Enabled Multiuser Wireless Power Transfer With Nonlinear Energy Harvesting,” *IEEE Trans. Wireless Commun.*, vol. 20, no. 2, pp. 1105-1121, Feb. 2021.

[8] N. Zhao et al., “Joint Trajectory and Precoding Optimization for UAV-Assisted NOMA Networks,” *IEEE Trans. Commun.*, vol. 67, no. 5, pp. 3723-3735, May 2019.

[9] X. Yuan, Y. Hu and A. Schmeink, “Joint Design of UAV Trajectory and Directional Antenna Orientation in UAV-Enabled Wireless Power Transfer Networks,” *IEEE J. Sel. Areas Commun.*, vol. 39, no. 10, pp. 3081-3096, Oct. 2021.

[10] X. Zhou, Q. Wu, S. Yan, F. Shu, and J. Li, “UAV-Enabled Secure Communications: Joint Trajectory and Transmit Power Optimization,” *IEEE Trans. Veh. Technol.*, vol. 68, no. 4, pp. 4069–4073, Apr. 2019.

[11] X. Yuan, Y. Hu, D. Li and A. Schmeink, “Novel Optimal Trajectory Design in UAV-Assisted Networks: A Mechanical Equivalence-Based Strategy,” *IEEE J. Sel. Areas Commun.*, early access.