Joint Optimization of Wireless Fronthaul and Access Links in CRAN with a Massive MIMO Central Unit

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***Abstract*—We propose a new downlink cloud radio access network (CRAN) architecture with wireless fronthaul and access links and a central unit (CU) that is equipped with a very large antenna array to serve multiple multi-antenna remote radio heads (RRHs) that in turn serve a number of user equipments (UEs). The use of a very large antenna array at the CU allows to improve the fronthaul capacity leading to improved capacity of the whole network. We propose to minimize the total transmit power at the CU and RRHs subject to maximum powers allowed at the CU and RRHs and rate constraints of UEs via jointly optimizing the power allocation at the CU, the precoders at the RRHs, and the quantization noise covariance matrices. Both independent and joint compression schemes are considered. An iterative algorithm is proposed via reformulating the non-convex optimization problem as a semidefinite relaxation (SDR) problem of which the solution is proved to be also optimal for the original problem. Simulation results show that the performance of the proposed system can significantly decrease the total transmit power compared to two benchmark schemes.**

***Index Terms*—CRAN, massive MIMO, wireless fronthauling, decompress-and-forward.**

I. Introduction

Cloud radio access network (CRAN) is a promising technology for the next generation of wireless communication networks [1]. With the centralization of the baseband signal processing, the central unit (CU) undertakes most of computations from the conventional distributed base stations (BSs), which are referred to as remote radio heads (RRHs) in CRAN. One of the major advantages of CRAN systems is to alleviate the burden of the conventional BSs by reducing their computational and hardware complexities and power consumption. The CU is responsible for most of the processing and also provides large-area cooperation and interference management.

However, a full centralized structure, which is the most promising structure, requires high capacity fronthaul links [1]. Large amounts of data need to be transmitted uplink/downlink through high-speed fronthaul links. In most of the existing works, the fronthaul links were considered as wired links [2] or finite-capacity links [3], [4]. Although wired links can provide high throughput, the lack of flexibility and high cost of deployment and maintenance are non-negligible. For this reason, some works considered wireless fronthaul links and jointly designed the fronthaul and access links [2], [5], [6]. In particular, [2] compared the wired and wireless fronthaul links in CRAN in terms of the economical spectral efficiency (SE). Two forwarding schemes, namely decode-and-forward (DF) and decompress-and-forward (DCF), in CRAN with wireless fronthauling were compared in [5]. [6] aimed to maximize the weighted sum-rate by considering both the user association problem and the design of the beamformers for the fronthaul and access links. However, the above works considered very few antennas at the CU where the capacity of fronthaul links is highly restrained.

In this paper, we propose to use massive MIMO in the wireless fronthaul link in a CRAN architecture. Massive MIMO has been recently studied in CRANs [7], [8]. In [7], the authors studied secrecy and energy efficiency for downlink data transmission in heterogeneous CRAN with BSs equipped with large antenna arrays. However, the fronthaul links were considered as optical fiber links with limited capacity. [8] considered massive MIMO in the fronthaul links in CRAN to maximize the minimum rate. However, single antenna unmanned aerial vehicles (UAVs) were deployed as flying RRHs. Unlike the aforementioned works, in this work, by jointly optimizing both fronthaul and access links and assuming the DCF scheme, we propose to minimize the total transmit power subject to the maximum available power at the CU and RRHs and guaranteed minimum rate for each UE. The corresponding problem is non-convex and hence difficult to solve optimally. To solve it, we first convexify the non-convex constraints through linearizing relevant terms in the constraints and then reformulate it as a semidefinite relaxation (SDR) problem for which ready–to–use solvers exist. The proposed algorithm is proved to be convergent and the solutions are optimal to the original problem. The superior reduction of the total power consumption has been verified and compared with two benchmark schemes in the numerical results.

The rest of this paper is organized as follows. In Section II, we introduce the proposed CRAN system model. In Section III, the power minimization problem is formulated and the proposed algorithm to solve it is presented. Section IV provides numerical results and discusses the proposed system performance. Finally, we conclude the paper in Section V.

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II. System Model

We consider a downlink transmission of a CRAN consisting of a CU, N_{R} RRHs and N_{U} multi-antenna UEs. The CU is equipped with a massive uniform linear array of size $M \gg N_{\text{R}}$. Each UE is equipped with K antennas. The antenna spacing is equal to half of the wavelength for all nodes. The CU communicates with the UEs through the RRHs, which are serving as relays. The fronthaul links (CU-RRHs links) and access links (RRHs-UEs links) are assumed to be wireless and separated in the time domain to avoid interference between them. The RRHs are equipped with one antenna for reception and N antennas for transmission. This is because in the fronthaul link we have a massive MIMO channel and hence the assumption of using single antenna receivers (here RRHs) is of practical interest.

Let $M_k \in \{1, 2, \ldots, 2^{nR_k}\}$ denote the message to be transmitted by the CU to the kth UE, where n and R_k are the coding block length (assumed to be large) and the rate of message M_k, respectively. We adopt the DCF relaying strategy, used in the standard CRAN architecture [9], where the CU precodes the signals intended for the UEs, quantizes and compresses the precoded signals then transmits the resulting compressed signal to the RRHs through the fronthaul links. Subsequently, each RRH decompresses the received signal before forwarding it to the UEs through the access links. The CU encodes the messages M_k as $\mathbf{s}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$, $k = 1, \ldots, N_{\text{U}}$. Then, it precodes the encoded signals $\mathbf{s}_k$ as $\bar{\mathbf{x}}_{R,i} = \sum_{k=1}^{N_{\text{U}}} \mathbf{U}_{k,i}\mathbf{s}_k$, where $\mathbf{U}_{k,i}$ denotes the precoding matrix for signal $\mathbf{s}_k$. To account for the limited capacity of the wireless fronthaul links, the CU quantizes and compresses the precoded signals $\bar{\mathbf{x}}_{R,i}$. We adopt the Gaussian quantization test channel to model the quantization process [9], and hence, the resulting quantized signal, $\mathbf{x}_{R,i}$, can be written as

$$\mathbf{x}_{R,i} = \bar{\mathbf{x}}_{R,i} + \mathbf{q}_i = \sum\nolimits_{k=1}^{N_{\text{U}}} \mathbf{U}_{k,i}\mathbf{s}_k + \mathbf{q}_i, \tag{1}$$

where $\mathbf{q}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ii})$ represents the Gaussian quantization noise, which is independent of $\bar{\mathbf{x}}_{R,i}$. $\mathbf{Q}_{ii}$ denotes the covariance of the compression noise. Intuitively, the smaller the quantization noise level the more accurate the signal, and vice versa. The CU then compresses the quantized signal $\mathbf{x}_{R,i}$ to generate the compression index $V_i \in \{1, 2, \ldots, 2^{nC_i}\}$, where C_i is the rate of message V_i that can be interpreted as the capacity of the fronthaul link between the CU and RRH i. Note that the signals $\mathbf{x}_{R,i}$, $i \in \{1, 2, \ldots, N_{\text{R}}\}$, could be either compressed independently or jointly [10]. In particular, if the signals are independently compressed the quantization noises for different RRHs will be uncorrelated, i.e., $E[\mathbf{q}_i\mathbf{q}_j^H] = \mathbf{0}$, for $i \neq j$. However, if the signals are jointly compressed the quantization noises for different RRHs will be correlated, i.e., $E[\mathbf{q}_i\mathbf{q}_j^H] \neq \mathbf{0}$, for $\forall i, j \in \{1, 2, \ldots, N_{\text{R}}\}$. Using joint compression allows to improve the achievable rates compared to independent compression. Since independent compression can be seen as a special case of joint compression, we adopt joint compression for the model and their performance will be compared in the numerical results. The compressed signal V_i is then encoded as a baseband signal $u_i \sim \mathcal{CN}(0, 1)$. The signal u_i is then beamformed towards RRH i, using the beamforming vector $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$, to produce the following transmit signal

$$\mathbf{x}_C = \sum\nolimits_{i=1}^{N_{\text{R}}} \sqrt{p_i}\mathbf{w}_i u_i, \tag{2}$$

where p_i is the CU transmit power corresponding to signal u_i intended for RRH i. In this work, without loss of generality, we consider two conventional low-complexity linear precoders, namely, the maximum-ratio transmission (MRT) and zero-forcing (ZF) precoding methods [11]. Hence, $\mathbf{w}_i$, $i = 1, 2, \ldots, N_{\text{R}}$, is expressed as

$$\mathbf{w}_i = \frac{\mathbf{b}_i}{\|\mathbf{b}_i\|}. \tag{3}$$

where $\mathbf{b}_i$ is the ith column of $\mathbf{B}$. For MRT, $\mathbf{B} = \mathbf{H}^*$ and for ZF, $\mathbf{B} = \mathbf{H}^*(\mathbf{H}^T\mathbf{H}^*)^{-1}$, where $\mathbf{H} = [\mathbf{h}_1\mathbf{h}_2 \ldots \mathbf{h}_{N_{\text{R}}}] \in \mathbb{C}^{M\times N_{\text{R}}}$ denotes the channel matrix between the CU and N_{R} RRHs, with $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ denoting the channel vector between the CU and the ith RRH. The CU and each RRH have a clear LoS path between them, which can be achieved by appropriately placing the CU and RRHs. Hence, we assume that all the CU-RRHs channels are LoS channels, i.e.,

$$\mathbf{h}_i = \sqrt{\beta_i^{\text{LoS}}}\left[1 \; e^{j\frac{2\pi}{\lambda}d\sin(\phi_i)} \; \ldots \; e^{j\frac{2\pi}{\lambda}(M-1)d\sin(\phi_i)}\right]^T, \tag{4}$$

where β_i^{LoS}, λ, d, and $\phi_i \in [0, 2\pi)$ representing the large-scale fading coefficient, the wavelength, the antenna spacing, and the angle of arrival to RRH i from the CU, respectively. β_i^{LoS} is assumed to be constant over frames and known at the CU as both CU and RRHs are fixed. We assume $\phi_i \neq \phi_j$ for $i \neq j$ such that $\lim\limits_{M\to\infty} |\bar{\mathbf{h}}_i^H\bar{\mathbf{h}}_j| \to 0$ if $i \neq j$ [11]. Assuming that the CU has a maximum average transmit power P_{C}, from (2) and (3), the average transmit power at the CU is constrained as $E[\|\mathbf{x}_C\|^2] = \sum_{i=1}^{N_{\text{R}}} p_i \leq P_{\text{C}}$. The received signal at the ith RRH is given by

$$\mathbf{y}_{R,i} = \mathbf{h}_i^T\mathbf{x}_C + \mathbf{n}_{R,i}, \tag{5}$$

where $\mathbf{n}_{R,i} \sim \mathcal{CN}(\mathbf{0}, \sigma_R^2\mathbf{I}) \in \mathbb{C}^{M\times 1}$ is the additive white Gaussian noise vector with zero mean and covariance matrix $\sigma_R^2\mathbf{I}$. From (2) and (5), the received signal at RRH i can be recast as

$$\mathbf{y}_{R,i} = \sqrt{p_i}\mathbf{h}_i^T\mathbf{w}_i u_i + \sum\nolimits_{j\neq i}^{N_{\text{R}}} \sqrt{p_j}\mathbf{h}_i^T\mathbf{w}_j u_j + \mathbf{n}_{R,i}. \tag{6}$$

Next, RRH i decodes u_i based on $\mathbf{y}_{R,i}$ and consequently recovers the message V_i. The achievable rate of message V_i, C_i, is

$$C_i = C_{fr,i}(\mathbf{p}) \triangleq \log_2 \frac{\sum_{j=1}^{N_{\text{R}}} p_j\mathbf{h}_i^T\mathbf{w}_j\mathbf{w}_j^H\mathbf{h}_i^* + \sigma_R^2}{\sum_{j\neq i}^{N_{\text{R}}} p_j\mathbf{h}_i^T\mathbf{w}_j\mathbf{w}_j^H\mathbf{h}_i^* + \sigma_R^2}, \tag{7}$$

where $\mathbf{p} = [p_1, p_2, \ldots, p_{N_{\text{R}}}]^T$. Based on the decoded message V_i, RRH i can determine the quantized signal $\mathbf{x}_{R,i}$. With joint compression at the CU, in order for RRH i to recover $\mathbf{x}_{R,i}$, according to the rate-distortion theory, the condition

$$\varphi_m(\{\mathbf{U}_{j,i}\},\mathbf{Q}) \triangleq \sum\nolimits_{i\in\mathcal{S}_m} \log\det\left(\sum\nolimits_{j=1}^{N_{\text{U}}} \mathbf{U}_{j,i}\mathbf{U}_{j,i}^H + \mathbf{Q}_{ii}\right) - \log\det\left(\mathbf{\Gamma}_{\mathcal{S}_m}^H\mathbf{Q}\mathbf{\Gamma}_{\mathcal{S}_m}\right) \leq \sum\nolimits_{i\in\mathcal{S}_m} C_{fr,i}(\mathbf{p}), \tag{8}$$

must be satisfied for all subsets $\mathcal{S}_m \subseteq \{1, \ldots, N_R\}$, where

$m = 1, \ldots, 2^{N_\mathrm{R}} - 1$ is subset index [9]. Matrix $\mathbf{\Gamma}_{\mathcal{S}_m}$ denotes a matrix that is obtained by stacking matrices $\mathbf{\Gamma}_i$, $i \in \mathcal{S}_m$, horizontally, e.g., if $\mathcal{S}_m = \{1, 2, 4\}$ then $\mathbf{\Gamma}_{\mathcal{S}_m} = [\mathbf{\Gamma}_1 \mathbf{\Gamma}_2 \mathbf{\Gamma}_4]$. $\mathbf{\Gamma}_i \in \mathbb{C}^{N N_\mathrm{R} \times N}$ is an all zero matrix except the submatrix from row $\sum (i-1)N + 1$ to iN, which is an identity matrix of size N. $\mathbf{Q} = E[\mathbf{q}\mathbf{q}^H]$, where $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_1^T, \ldots, \mathbf{q}_{N_\mathrm{R}}^T]^T$. Note that in order to decompress the received signal, each RRH needs to be informed by the CU about the used compression codebooks. The decoded signal $\mathbf{x}_{R,i}$ is then transmitted by RRH i to all UEs through the access links. Assuming that RRH i has a maximum average transmit power $P_{R,i}$, from (1), the average transmit power at RRH i must be constrained as

$$E[\|\mathbf{x}_{R,i}\|^2] = \sum_{j=1}^{N_\mathrm{U}} \mathrm{tr}\left(\mathbf{U}_{j,i}\mathbf{U}_{j,i}^H\right) + \mathrm{tr}\left(\mathbf{Q}_{ii}\right) \leq P_{R,i}. \quad (9)$$

The received signal by the kth UE is given by

$$\mathbf{y}_{U,k} = \sum_{i=1}^{N_\mathrm{R}} \mathbf{G}_{k,i}\mathbf{x}_{R,i} + \mathbf{n}_{U,k}, \quad (10)$$

where $\mathbf{G}_{k,i} \in \mathbb{C}^{K \times N}$ denotes the channel matrix between RRH i and UE k, and $\mathbf{n}_{U,k} \in \mathbb{C}^{K \times 1}$ is Gaussian noise at the kth UE. We assume that the access channels, $\mathbf{G}_{k,i}$, exhibit block fading, i.e., they remain constant over a time block and change independently from one block to another, and that they are Rayleigh flat-fading channels, i.e., $\mathbf{G}_{k,i} = \sqrt{\alpha_{k,i}}\ \tilde{\mathbf{G}}_{k,i}$, where $\alpha_{k,i}$ is the large-scale fading coefficient of the channel between RRH i and UE k. $\tilde{\mathbf{G}}_{k,i} \in \mathbb{C}^{K \times N}$ is a matrix of independent Rayleigh coefficients with elements modeled as $\mathcal{CN}(0, 1)$. By substituting (1) into (10), the received signal at UE k can be rewritten as

$$\mathbf{y}_{U,k} = \mathbf{G}_k\mathbf{U}_k\mathbf{s}_k + \sum_{j \neq k}^{N_\mathrm{U}} \mathbf{G}_k\mathbf{U}_j\mathbf{s}_j + \sum_{i=1}^{N_\mathrm{R}} \mathbf{G}_{k,i}\mathbf{q}_i + \mathbf{n}_{U,k}, \quad (11)$$

where $\mathbf{G}_k = [\mathbf{G}_{k,1}, \mathbf{G}_{k,2}, \ldots, \mathbf{G}_{k,N_\mathrm{R}}]$ and $\mathbf{U}_k = [\mathbf{U}_{k,1}^T, \mathbf{U}_{k,2}^T, \ldots, \mathbf{U}_{k,N_\mathrm{R}}^T]^T$, $k = 1, \ldots, N_\mathrm{U}$. The first, second, and third terms in (9) represent the desired signal to be decoded, the interference from other UEs, and the quantization noise contribution, respectively. Based on the received signal in (11), UE k decodes the message M_k and its achievable rate is given by

$$R_k = C_{\mathrm{ac},k}(\{\mathbf{U}_j\}, \mathbf{Q}) \triangleq \log\det\left(\mathbf{G}_k\left(\sum_{j=1}^{N_\mathrm{U}} \mathbf{U}_j\mathbf{U}_j^H + \mathbf{Q}\right)\mathbf{G}_k^H + \sigma_U^2\mathbf{I}\right) - \log\det\left(\mathbf{G}_k\left(\sum_{j \neq k}^{N_\mathrm{U}} \mathbf{U}_j\mathbf{U}_j^H + \mathbf{Q}\right)\mathbf{G}_k^H + \sigma_U^2\mathbf{I}\right). \quad (12)$$

In the next section, we present the design of the power allocation coefficients p_i, precoders $\mathbf{U}_j$, and quantization noise covariance matrix $\mathbf{Q}$, according to the minimization of the total transmit power.

III. Total Transmit Power Minimization

In some communication applications, we are required to ensure a given QoS for each UE. In this section, we are interested in achieving the required QoS with the smallest transmit power possible. The QoS metric here is each UE rate, where each UE k has a desired rate $R_k \geq \gamma_k$. Hence, we jointly optimize the fronthaul and access links by minimizing the total transmit power at the CU and RRHs subject to UEs' rates, and CU and RRHs transmit power constraints. Let $\bar{\mathbf{U}}_j = \mathbf{U}_j\mathbf{U}_j^H \succeq \mathbf{0}$. The corresponding optimization problem can be formulated as

$$\underset{\mathbf{p} \geq 0, \{\mathbf{U}_j\}, \mathbf{Q} \succeq \mathbf{0}}{\mathrm{minimize}} \quad \sum\nolimits_{j=1}^{N_\mathrm{U}} \mathrm{tr}\left(\bar{\mathbf{U}}_j\right) + \mathrm{tr}\left(\mathbf{Q}\right) + \sum\nolimits_{i=1}^{N_\mathrm{R}} p_i \quad (13a)$$

$$\text{s.t. } \varphi_m(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \leq \sum\nolimits_{i \in \mathcal{S}_m} C_{fr,i}(\mathbf{p}),\ m \in \mathcal{N}_\mathcal{S}, \quad (13b)$$

$$C_{\mathrm{ac},k}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \geq \gamma_k,\ k \in \mathcal{N}_\mathrm{U} \quad (13c)$$

$$\sum\nolimits_{i=1}^{N_\mathrm{R}} p_i \leq P_\mathrm{C}, \quad (13d)$$

$$\sum\nolimits_{j=1}^{N_\mathrm{U}} \mathrm{tr}\left(\mathbf{\Gamma}_i^H\bar{\mathbf{U}}_j\mathbf{\Gamma}_i\right) + \mathrm{tr}\left(\mathbf{Q}_{ii}\right) \leq P_{R,i}, i \in \mathcal{N}_\mathrm{R}, \quad (13e)$$

$$\mathrm{rank}\left(\bar{\mathbf{U}}_j\right) \leq \min(K, NN_\mathrm{R}),\ j \in \mathcal{N}_\mathrm{U}, \quad (13f)$$

where (13d) and (13e) denote power constraints at the CU and RRHs, respectively. Constraint (13f) ensures that the data streams can be decoded properly at UEs. It is obvious that optimization problem (13) is nonconvex due the non-convexity of constraints (13b), (13c) and (13f). Now, to transform problem (13) into a convex problem, we use the majorization minimization (MM) method. In particular, we express each function of (13b) and (13c) in the nonconvex constraint as a difference of two functions and then linearize the part of the function causing the nonconvexity of the constraint. For constraint (13b) to be convex, $\varphi_m(\{\mathbf{U}_j\}, \mathbf{Q})$ and $C_{fr,i}(\mathbf{p})$ need to be convex and concave, respectively. (7) can be rewritten as $C_{fr,i}(\mathbf{p}) = \log_2\left(\mathbf{f}_i\mathbf{p} + \sigma_R^2\right) - \log_2\left(\bar{\mathbf{f}}_i\mathbf{p} + \sigma_R^2\right)$ where $\mathbf{f}_i = [f_{i,1}, f_{i,2}, \ldots, f_{i,N_\mathrm{R}}]$ with its jth element $f_{i,j} = \mathbf{h}_i^T\mathbf{w}_j\mathbf{w}_j^H\mathbf{h}_i^*$, and $\bar{\mathbf{f}}_i = [f_{i,1}, \ldots, f_{i,i-1}, 0, f_{i,i+1}, ..., f_{i,N_\mathrm{R}}]$. In order to convert function $C_{fr,i}(\mathbf{p})$ into a concave function, we need to linearize the second term $\log_2\left(\bar{\mathbf{f}}_i\mathbf{p} + \sigma_R^2\right)$. To achieve this, we use the first order Taylor expansion at a given feasible $\tilde{\mathbf{p}}$. Since, the second term is convex, it is upper bounded by its first order Taylor expansion and hence $C_{fr,i}(\mathbf{p})$ is lower bounded as

$$C_{fr,i}(\mathbf{p}) \geq C_{fr,i}^{\mathrm{lb}}(\mathbf{p}) \triangleq \log_2\left(\mathbf{f}_i\mathbf{p} + \sigma_R^2\right) - \log_2\left(\bar{\mathbf{f}}_i\tilde{\mathbf{p}} + \sigma_R^2\right) - \frac{1}{\ln(2)}\frac{\bar{\mathbf{f}}_i}{\bar{\mathbf{f}}_i\tilde{\mathbf{p}} + \sigma_R^2}(\mathbf{p} - \tilde{\mathbf{p}}), \quad (14)$$

where $C_{fr,i}^{\mathrm{lb}}(\mathbf{p})$ is a convex function. Similarly, to convert function $\varphi_m(\{\bar{\mathbf{U}}_j\}, \mathbf{Q})$ into a convex one, we can linearize its concave term, which is the first term in the right hand side of the equality in the first line of (8), by using the first order Taylor expansion at $(\tilde{\bar{\mathbf{U}}}_j, \tilde{\mathbf{Q}})$. Hence, the nonconvex function $\varphi_m(\{\bar{\mathbf{U}}_j\}, \mathbf{Q})$ can be upper bounded by the convex function $\varphi_m^{\mathrm{ub}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q})$ as

$$\varphi_m(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \leq \varphi_m^{\mathrm{ub}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \triangleq -\log\det\left(\mathbf{\Gamma}_{\mathcal{S}_m}^H\mathbf{Q}\mathbf{\Gamma}_{\mathcal{S}_m}\right) + \frac{1}{\ln(2)}\mathrm{tr}\left(\mathbf{\Theta}_i^{-1} \times \left(\mathbf{\Gamma}_i^H\sum_{j=1}^{N_\mathrm{U}}\bar{\mathbf{U}}_j\mathbf{\Gamma}_i + \mathbf{Q}_{ii} - \mathbf{\Theta}_i\right)\right) + \sum_{i \in \mathcal{S}_m}\log\det\mathbf{\Theta}_i, \quad (15)$$

where $\boldsymbol{\Theta}_i \triangleq \boldsymbol{\Gamma}_i^H \sum_{j=1}^{N_{\mathrm{U}}} \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii}$. Hence, the nonconvex constraint (13b) can be transformed into the following convex constraint

$$\varphi_m^{\mathrm{ub}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \le \sum_{i \in \mathcal{S}_m} C_{\mathrm{fr},i}^{\mathrm{lb}}(\mathbf{p}). \tag{16}$$

Next, for the left-hand side of the constraint (13c), the first term of $C_{\mathrm{ac},k}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q})$ is a concave function. In order to convert constraint (13c) into a convex constraint, we need to linearize the second term using the first order Taylor expansion at a point $(\tilde{\bar{\mathbf{U}}}_j, \tilde{\mathbf{Q}})$. Since, the second term is convex, it is upper bounded by its first order Taylor expansion and hence $C_{\mathrm{ac},k}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q})$ is lower bounded as

$$\begin{aligned}
&C_{\mathrm{ac},k}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \ge C_{\mathrm{ac},k}^{\mathrm{lb}}(\{\mathbf{U}_j\}, \mathbf{Q}) \\
&\triangleq \log\det\left(\mathbf{G}_k\left(\sum\nolimits_{j=1}^{N_{\mathrm{U}}} \bar{\mathbf{U}}_j + \mathbf{Q}\right)\mathbf{G}_k^H + \sigma_U^2 \mathbf{I}\right) - \log\det(\boldsymbol{\Xi}_k) \\
&- \frac{1}{\ln(2)} \mathrm{tr}\left(\boldsymbol{\Xi}_k^{-1} \times \left(\mathbf{G}_k\left(\sum\nolimits_{j\neq k}^{N_{\mathrm{U}}} \bar{\mathbf{U}}_j + \mathbf{Q}\right)\mathbf{G}_k^H - \boldsymbol{\Xi}_k\right)\right),
\end{aligned} \tag{17}$$

where $\boldsymbol{\Xi}_k \triangleq \mathbf{G}_k\left(\sum_{j\neq k}^{N_{\mathrm{U}}} \tilde{\bar{\mathbf{U}}}_j + \tilde{\mathbf{Q}}\right)\mathbf{G}_k^H + \sigma_U^2 \mathbf{I}$. Hence, we have the convexified constraint

$$C_{\mathrm{ac},k}^{\mathrm{lb}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \ge \gamma_k. \tag{18}$$

Furthermore, in order to solve problem (13), we relax the rank constraint (13f) and the resulting problem is the following semidefinite relaxation (SDR) problem by applying the convex constraints (16) and (18)

$$\begin{aligned}
\underset{\mathbf{p}\ge 0, \{\bar{\mathbf{U}}_j\}\succeq \mathbf{0}, \mathbf{Q}\succeq\mathbf{0}}{\text{minimize}} \quad & \sum\nolimits_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}\left(\bar{\mathbf{U}}_j\right) + \mathrm{tr}\left(\mathbf{Q}\right) + \sum\nolimits_{i=1}^{N_{\mathrm{R}}} p_i && \text{(19a)}\\
\text{s.t.} \quad & \varphi_m^{\mathrm{ub}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \le \sum\nolimits_{i\in\mathcal{S}_m} C_{\mathrm{fr},i}^{\mathrm{lb}}(\mathbf{p}),\ m \in \mathcal{N}_{\mathcal{S}}, && \text{(19b)}\\
& C_{\mathrm{ac},k}^{\mathrm{lb}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) \ge \gamma_k,\ \ k \in \mathcal{N}_{\mathrm{U}}, && \text{(19c)}\\
& \text{(13d)},\ \text{(13e)},
\end{aligned}$$

which can optimally be solved using interior point algorithms such as the CVX optimization toolbox [12]. As the constraint on the rank of $\bar{\mathbf{U}}_j$ is relaxed, it is not guaranteed that the obtained solution $\bar{\mathbf{U}}_j^\star$ satisfies the rank constraint. However, we can prove that the rank of the obtained optimal solution $\bar{\mathbf{U}}_j^\star$ of the relaxed problem always satisfies the rank constraint (13f) and hence $\bar{\mathbf{U}}_j^\star$ is also an optimal solution to problem (13). We have the following theorem.

Theorem 1: The solution to problem (19), $\bar{\mathbf{U}}_j^\star$, always satisfies $\mathrm{rank}(\bar{\mathbf{U}}_j^\star) \le \min(K, NN_{\mathrm{R}})$, $j \in \mathcal{N}_{\mathrm{U}}$.

Proof 1: Please see the Appendix.

From the optimal solution $\bar{\mathbf{U}}_j^\star$, we can obtain the optimal solution $\mathbf{U}_j^\star$ by using the eigenvalue decomposition. In particular, we have $\bar{\mathbf{U}}_j^\star = \mathbf{V}_j \boldsymbol{\Lambda}_j \mathbf{V}_j^H$. Hence, $\mathbf{U}_j^\star = \bar{\mathbf{V}}_j \bar{\boldsymbol{\Lambda}}_j^{\frac{1}{2}}$, where $\bar{\boldsymbol{\Lambda}}_j$ is a diagonal matrix whose diagonal elements are the non-zero diagonal elements of the eigenvalue matrix $\boldsymbol{\Lambda}_j$ and the columns of $\bar{\mathbf{V}}_j$ are the corresponding eigenvectors. By iteratively solving the convex SDR problem (19) as shown in Algorithm 1, due to the convexity of the problem, the total transmit power at each iteration will be non-increasing and hence convergence is guaranteed.

Algorithm 1 Block Coordinate Descent Algorithm

1: Set $r := 0$. Initialize $\mathbf{p}^{(0)} \ge 0$, $\{\bar{\mathbf{U}}_j\}^{(0)} \succeq \mathbf{0}$ and $\mathbf{Q}^{(0)} \succeq \mathbf{0}$.
2: **repeat**
3: Solve the SDR problem (19) for the given $\left\{\mathbf{p}^{(r)}, \{\bar{\mathbf{U}}_j\}^{(r)}, \mathbf{Q}^{(r)}\right\}$, and denote the optimal solution as $\left\{\mathbf{p}^{(r+1)}, \{\bar{\mathbf{U}}_j\}^{(r+1)}, \mathbf{Q}^{(r+1)}\right\}$.
4: Update $r := r + 1$.
5: **until** convergence

IV. Numerical Results

In this section, we evaluate the performance of the proposed system by simulating a 2 GHz downlink communication channel. The parameters in our simulation, unless otherwise stated, are set as follows: $N_{\mathrm{R}} = 5$, $N_{\mathrm{U}} = 10$, $M = 200$, $N = 2$, $K = 1$. We assume the UEs are uniformly distributed inside a square area of 1 km^2. The heights of CU, RRHs and UEs are assumed as 25 m, 10 m and 1.5 m from the ground, respectively. The minimum distance among all RRHs and between CU and RRHs is set as 100 m. The large scale fading of the founthaul links β^{LoS} and the access links α^{NLoS} are in line with the urban macrocell and urban microcell Street Canyon model in Table 7.4.1-1 of [13], respectively. The noise level σ_{R}^2, σ_{U}^2 are modelled as thermal noise with the room temperature 290 K and 5 dB noise figure over 20 MHz bandwidth channel.

Fig. 1 plots the feasibility of problem (19) against the minimum required rate per user for various values of P_{C} and P_{R} with both joint and independent compression schemes. It can be observed that the feasibility increases with increasing power budgets P_{C} and P_{R}. Also, joint compression can achieve higher feasibility of problem (19) compared to independent compression under all conditions, which demonstrates the superiority of the joint compression scheme.

Fig. 2 shows the convergence of the proposed algorithm. The minimum required rate per user is set as $\gamma_k = 0.5$ bps/Hz. We compared the proposed algorithm with two benchmarks: (1) Benchmark 1 applies ZF as the precoding method at the access links where $\mathbf{U} \triangleq \boldsymbol{\Lambda}_U \mathbf{G}^*(\mathbf{G}^T\mathbf{G}^*)^{-1}$ and satisfies

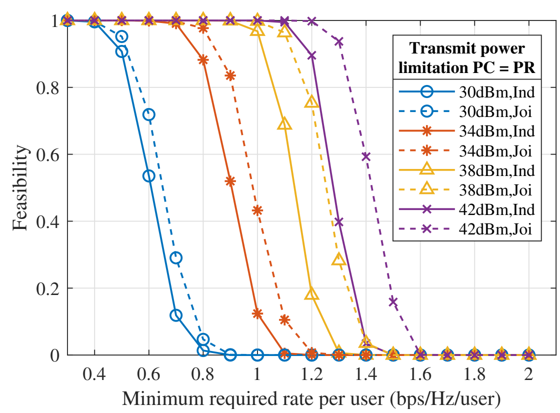

Fig. 1. Feasibility of the proposed scheme versus minimum required rate per user for independent and joint compression schemes, and for different power constraints P_{C} and P_{R} with $M = 200$.

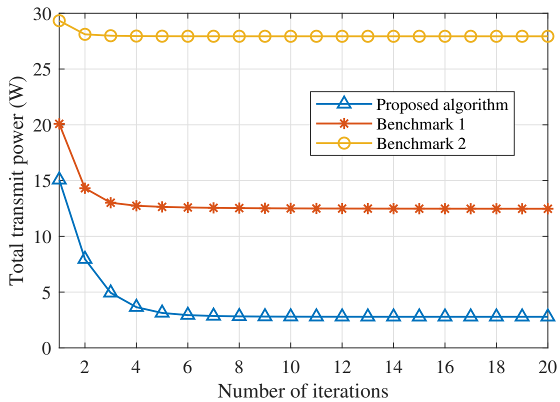

Fig. 2. Convergence rate of the proposed algorithm and two benchmarks for both independent and joint compression schemes with $M = 200$, $\gamma_k = 0.5$ bps/Hz for $K = 10$ UEs and $P_C = P_R = 40$ dBm.

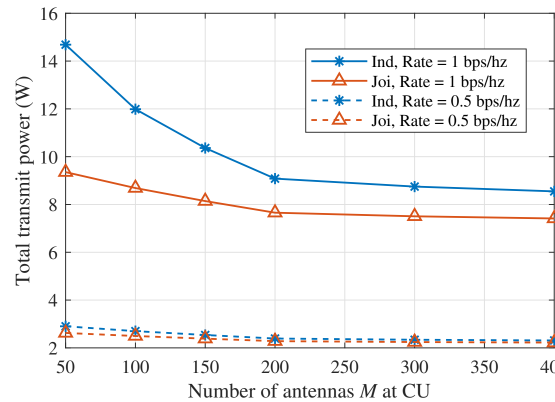

Fig. 3. Total power consumption of the proposed algorithm versus the number of antennas M at CU for both independent and joint compression schemes with $\gamma_k = 1$ bps/Hz and $\gamma_k = 0.5$ bps/Hz for $K = 10$ UEs.

constraint (13e). $\boldsymbol{\Lambda}_U$ is a real-valued diagonal matrix, where $\boldsymbol{\Gamma}_i^H \boldsymbol{\Lambda}_U \boldsymbol{\Gamma}_i$ corresponds to the transmit power matrix for the ith RRH, $i \in \{1, \ldots, N_R\}$. Instead of $\mathbf{U}$, $\boldsymbol{\Lambda}_U$ is considered as an optimization variable in problem (19); (2) Benchmark 2 considers a non-optimized compression noise, where $\mathbf{q}_i$ are generated randomly following the normal distribution. It can be observed that the proposed algorithm converges quickly and can significantly decrease the power consumption and outperforms both benchmark schemes. Specifically, the proposed algorithm reduces the total transmit power by 77.66% and 90.03% compared to benchmark schemes 1 and 2, respectively.

Fig. 3 investigates the total power consumption versus the number of antennas M under different minimum required rates. The optimized power consumption achieved by the proposed algorithm notably decreases with increasing M while the gain is relatively flat when $M \geq 200$. Furthermore, it is shown that a higher minimum rate requirement needs more transmit power and the proposed algorithm with joint compression saves more power than independent compression.

V. Conclusion

In this paper, we proposed a joint design of wireless fronthaul and access links in a CRAN with a massive MIMO CU. In particular, an optimization problem was formulated aiming to minimize the total transmit power subject to individual minimum rate per UE and maximum available power at the CU and RRHs. An iterative algorithm, with a guaranteed convergence, was proposed to slove the convexified problem. The resulting solution is proved to be optimal to the original problem. Numerical results showed that superior decrease in power consumption can be achieved by the proposed system compared to two benchmark schemes. Also, the use of joint compression and adequate number of antennas at the CU can further reduce the power consumption.

Appendix

In this appendix, we prove Theorem 1. By following the framework used in [14, Chapter 9], the Lagrangian of problem (19) is given by

$$\begin{aligned}\mathcal{L} = &\sum\nolimits_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}(\bar{\mathbf{U}}_j) + \mathrm{tr}(\mathbf{Q}) + \sum\nolimits_{i=1}^{N_{\mathrm{R}}} p_i - \sum\nolimits_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}(\mathbf{F}_j \bar{\mathbf{U}}_j) - \mathrm{tr}(\mathbf{T}\mathbf{Q}) \\ &+ \sum\nolimits_{m \in \mathcal{N}_{\mathcal{S}}} \eta_m \left(\varphi_m^{\mathrm{ub}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) - \sum\nolimits_{i \in \mathcal{S}_m} C_{\mathrm{fr},i}^{\mathrm{lb}}(\mathbf{p}) \right) \\ &- \sum\nolimits_{k=1}^{N_U} \lambda_k \left(C_{ac,k}^{\mathrm{lb}}(\{\bar{\mathbf{U}}_j\}, \mathbf{Q}) - \gamma_k \right) + \theta \left(\sum\nolimits_{i=1}^{N_{\mathrm{R}}} p_i - P_C \right) \\ &+ \sum\nolimits_{i=1}^{N_{\mathrm{R}}} \rho_i \left(\sum\nolimits_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}(\boldsymbol{\Gamma}_i^H \bar{\mathbf{U}}_j \boldsymbol{\Gamma}_i) + \mathrm{tr}(\mathbf{Q}_{ii}) - P_{R,i} \right), \end{aligned} \quad (20)$$

where $\{\eta_m\}$, $\{\lambda_k\}$, θ, and $\{\rho_i\}$, $\{\mathbf{F}_j\}$, and $\mathbf{T}$ are the Lagrange multipliers corresponding to constraints of problem (19), $\{\bar{\mathbf{U}}_j\} \succeq \mathbf{0}$, and $\mathbf{Q} \succeq \mathbf{0}$, respectively. Since we are interested in the rank of $\bar{\mathbf{U}}_j$, we can reexpress the Lagrangian as a function of $\bar{\mathbf{U}}_j$ and relevant Lagrange multipliers as follows

$$\begin{aligned}\mathcal{L} = &\sum_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}\left(\bar{\mathbf{U}}_j\right) - \sum_{j=1}^{N_{\mathrm{U}}} \mathrm{tr}\left(\mathbf{F}_j \bar{\mathbf{U}}_j\right) + \sum_{i=1}^{N_{\mathrm{R}}} \sum_{j=1}^{N_{\mathrm{U}}} \rho_i \mathrm{tr}\left(\boldsymbol{\Gamma}_i^H \bar{\mathbf{U}}_j \boldsymbol{\Gamma}_i\right) + \Psi \\ &+ \sum_{m \in \mathcal{N}_{\mathcal{S}}} \sum_{i \in \mathcal{S}_m} \frac{\eta_m}{\ln(2)} \times \mathrm{tr}\left(\left(\sum_{j=1}^{N_{\mathrm{U}}} \boldsymbol{\Gamma}_i^H \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii} \right)^{-1} \sum_{j=1}^{N_{\mathrm{U}}} \boldsymbol{\Gamma}_i^H \bar{\mathbf{U}}_j \boldsymbol{\Gamma}_i \right) \\ &+ \frac{1}{\ln(2)} \mathrm{tr}\left(\left(\mathbf{G}_k \left(\sum_{j \neq k} \tilde{\bar{\mathbf{U}}}_j + \tilde{\mathbf{Q}} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right)^{-1} \times \sum_{j \neq k} \mathbf{G}_k \bar{\mathbf{U}}_j \mathbf{G}_k^H \right) \\ &- \sum_{k=1}^{N_{\mathrm{U}}} \lambda_k \left(\log\det\left(\mathbf{G}_k \left(\sum_{j=1}^{N_{\mathrm{U}}} \bar{\mathbf{U}}_j + \mathbf{Q} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right), \right. \end{aligned} \quad (21)$$

where Ψ contains all the remaining terms of the Lagrangian in (20) that are independent of $\bar{\mathbf{U}}_j$. Since problem (19) is convex and satisfies Slater's condition, Karush-Kuhn-Tucker (KKT) conditions provide necessary and sufficient conditions for optimality. The KKT conditions relevant to our derivation are

$$\frac{\partial \mathcal{L}}{\partial \bar{\mathbf{U}}_\ell^\star} = \mathbf{0}, \ \ell \in N_{\mathrm{U}}, \quad (22)$$

$$\mathbf{F}_\ell^\star \bar{\mathbf{U}}_\ell = \mathbf{0}, \ \ell \in N_{\mathrm{U}}. \quad (23)$$

From (22), we have

$$\frac{\partial \mathcal{L}}{\partial \bar{\mathbf{U}}_\ell} = \mathbf{I} + \sum\nolimits_{i=1}^{N_\mathrm{R}} \rho_i \boldsymbol{\Gamma}_i^* \boldsymbol{\Gamma}_i^T - \mathbf{F}_\ell^T$$
$$+ \sum\nolimits_{m\in\mathcal{N}_\mathcal{S}} \sum\nolimits_{i\in\mathcal{S}_m} \frac{\eta_m}{\ln(2)} \boldsymbol{\Gamma}_i^* \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \boldsymbol{\Gamma}_i^H \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii} \right)^{-T} \boldsymbol{\Gamma}_i^T$$
$$- \sum\nolimits_{k=1}^{N_\mathrm{U}} \frac{\lambda_k}{\ln(2)} \mathbf{G}_k^T \left(\mathbf{G}_k \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \bar{\mathbf{U}}_j + \mathbf{Q} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right)^{-T} \mathbf{G}_k^*$$
$$+ \sum\nolimits_{k\neq\ell}^{N_\mathrm{U}} \frac{\lambda_k}{\ln(2)} \mathbf{G}_k^T \left(\mathbf{G}_k \left(\sum\nolimits_{j\neq k}^{N_\mathrm{U}} \tilde{\bar{\mathbf{U}}}_j + \tilde{\mathbf{Q}} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right)^{-T} \mathbf{G}_k^*$$
$$= 0. \tag{24}$$

Hence

$$\mathbf{F}_\ell^\star = \mathbf{I} + \sum\nolimits_{i=1}^{N_\mathrm{R}} \rho_i^\star \boldsymbol{\Gamma}_i \boldsymbol{\Gamma}_i^H$$
$$+ \sum\nolimits_{m\in\mathcal{N}_\mathcal{S}} \sum\nolimits_{i\in\mathcal{S}_m} \frac{\eta_m^\star}{\ln(2)} \boldsymbol{\Gamma}_i \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \boldsymbol{\Gamma}_i^H \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii} \right)^{-1} \boldsymbol{\Gamma}_i^H$$
$$+ \sum\nolimits_{k\neq\ell}^{N_\mathrm{U}} \frac{\lambda_k^\star}{\ln(2)} \mathbf{G}_k^H \left(\mathbf{G}_k \left(\sum\nolimits_{j\neq k}^{N_\mathrm{U}} \tilde{\bar{\mathbf{U}}}_j + \tilde{\mathbf{Q}} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right)^{-1} \mathbf{G}_k$$
$$- \sum\nolimits_{k=1}^{N_\mathrm{U}} \frac{\lambda_k^\star}{\ln(2)} \mathbf{G}_k^H \left(\mathbf{G}_k \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \bar{\mathbf{U}}_j^\star + \mathbf{Q}^\star \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \right)^{-1} \mathbf{G}_k . \tag{25}$$

Let

$$\boldsymbol{\Phi}_k = \mathbf{G}_k \left(\sum\nolimits_{j\neq k}^{N_\mathrm{U}} \tilde{\bar{\mathbf{U}}}_j + \tilde{\mathbf{Q}} \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I} \tag{26}$$
$$\boldsymbol{\Omega}_k = \mathbf{G}_k \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \bar{\mathbf{U}}_j^\star + \mathbf{Q}^\star \right) \mathbf{G}_k^H + \sigma_U^2 \mathbf{I}. \tag{27}$$

Hence, we have

$$\mathbf{F}_\ell = \mathbf{I} + \sum\nolimits_{i=1}^{N_\mathrm{R}} \rho_i^\star \boldsymbol{\Gamma}_i \boldsymbol{\Gamma}_i^H + \sum\nolimits_{k\neq\ell}^{N_\mathrm{U}} \frac{\lambda_k^\star}{\ln(2)} \mathbf{G}_k^H \left(\boldsymbol{\Phi}_k^{-1} - \boldsymbol{\Omega}_k^{-1} \right) \mathbf{G}_k$$
$$+ \sum\nolimits_{m\in\mathcal{N}_\mathcal{S}} \sum\nolimits_{i\in\mathcal{S}_m} \frac{\eta_m^\star}{\ln(2)} \boldsymbol{\Gamma}_i \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \boldsymbol{\Gamma}_i^H \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii} \right)^{-1} \boldsymbol{\Gamma}_i^H$$
$$- \frac{\lambda_\ell^\star}{\ln(2)} \mathbf{G}_\ell^H \boldsymbol{\Omega}_\ell^{-1} \mathbf{G}_\ell. \tag{28}$$

As shown in [15, Theorem 1], the MM algorithm is guaranteed to converge to a stationary point of the original non-convex problem. Therefore, after convergence, we have: $\tilde{\bar{\mathbf{U}}}_j = \bar{\mathbf{U}}_j^\star$ and $\tilde{\mathbf{Q}} = \mathbf{Q}^\star$. Hence, from (26) and (27), we obtain $\boldsymbol{\Omega}_k = \boldsymbol{\Phi}_k + \mathbf{G}_k \bar{\mathbf{U}}_k \mathbf{G}_k^H$. Since $\bar{\mathbf{U}}_k \succeq \mathbf{0}$, then $\mathbf{G}_k \bar{\mathbf{U}}_k \mathbf{G}_k^H \succeq \mathbf{0}$ leading to $\boldsymbol{\Omega}_k \succeq \boldsymbol{\Phi}_k$; Consequently, $\boldsymbol{\Phi}_k^{-1} \succeq \boldsymbol{\Omega}_k^{-1}$. Since $\lambda_k \geq 0$, this results in $\sum\nolimits_{k\neq\ell}^{N_\mathrm{U}} \frac{\lambda_k^\star}{\ln(2)} \mathbf{G}_k^H \left(\boldsymbol{\Phi}_k^{-1} - \boldsymbol{\Omega}_k^{-1} \right) \mathbf{G}_k \succeq \mathbf{0}$. Let

$$\boldsymbol{\Upsilon} = \mathbf{I} + \sum\nolimits_{i=1}^{N_\mathrm{R}} \rho_i^\star \boldsymbol{\Gamma}_i \boldsymbol{\Gamma}_i^H + \sum\nolimits_{k\neq\ell}^{N_\mathrm{U}} \frac{\lambda_k^\star}{\ln(2)} \mathbf{G}_k^H \left(\boldsymbol{\Phi}_k^{-1} - \boldsymbol{\Omega}_k^{-1} \right) \mathbf{G}_k$$
$$+ \sum\nolimits_{m\in\mathcal{N}_\mathcal{S}} \sum\nolimits_{i\in\mathcal{S}_m} \frac{\eta_m^\star}{\ln(2)} \boldsymbol{\Gamma}_i \left(\sum\nolimits_{j=1}^{N_\mathrm{U}} \boldsymbol{\Gamma}_i^H \tilde{\bar{\mathbf{U}}}_j \boldsymbol{\Gamma}_i + \tilde{\mathbf{Q}}_{ii} \right)^{-1} \boldsymbol{\Gamma}_i^H . \tag{29}$$

Clearly, $\mathrm{rank}(\boldsymbol{\Upsilon}) = NN_\mathrm{R}$ since $\boldsymbol{\Upsilon}$ is the sum of an identity matrix of size $NN_\mathrm{R} \times NN_\mathrm{R}$ and positive semidefinite matrices. On the other hand, since $\mathrm{rank}(\boldsymbol{P}_\ell) = K$ (full rank) and $\mathrm{rank}(\mathbf{G}_\ell) = \min(K, NN_\mathrm{R})$, then $\mathrm{rank}\left(\frac{\lambda_\ell^\star}{\ln(2)} \mathbf{G}_\ell^H \boldsymbol{\Omega}_\ell^{-1} \mathbf{G}_\ell \right) = \mathrm{rank}(\mathbf{G}_\ell^H \boldsymbol{\Omega}_\ell^{-\frac{1}{2}} \boldsymbol{\Omega}_\ell^{-\frac{1}{2}} \mathbf{G}_\ell) = \mathrm{rank}(\mathbf{G}_\ell) = \min(K, NN_\mathrm{R})$. Thus, we have

$$\mathrm{rank}(\mathbf{F}_\ell) = \mathrm{rank}(\boldsymbol{\Upsilon} - \frac{\lambda_\ell^\star}{\ln(2)} \mathbf{G}_\ell^H \boldsymbol{\Omega}_\ell^{-1} \mathbf{G}_\ell)$$
$$\geq \mathrm{rank}(\boldsymbol{\Upsilon}) - \mathrm{rank}(\frac{\lambda_\ell^\star}{\ln(2)} \mathbf{G}_\ell^H \boldsymbol{\Omega}_\ell^{-1} \mathbf{G}_\ell)$$
$$= NN_\mathrm{R} - \min(K, NN_\mathrm{R}). \tag{30}$$

where made use of $\mathrm{rank}(\mathbf{A} - \mathbf{B}) \geq \mathrm{rank}(\mathbf{A}) - \mathrm{rank}(\mathbf{B})$ [16]. From (23), we have

$$\mathrm{rank}(\bar{\mathbf{U}}_\ell^\star) \leq \mathrm{Nullity}(\mathbf{F}_\ell^\star) = NN_\mathrm{R} - \mathrm{rank}(\mathbf{F}_\ell^\star)$$
$$\leq \min(K, NN_\mathrm{R}), \tag{31}$$

where $\mathrm{Nullity}(\mathbf{A})$ denotes the dimension of the null space of $\mathbf{A}$. For the special case of $K = 1$ and from (31), we have $\mathrm{rank}(\bar{\mathbf{U}}_\ell^\star) \leq 1$. Also, since $\bar{\mathbf{U}}_\ell^\star \neq \mathbf{0}$, i.e., $\mathrm{rank}(\bar{\mathbf{U}}_\ell^\star) \geq 1$, this implies that $\mathrm{rank}(\bar{\mathbf{U}}_\ell^\star) = 1$ and thus concludes the proof.

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