Spatial and Spectral Resource Allocation for Energy-Efficient Massive MIMO 5G Networks

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***Abstract*—To meet the targets of net-zero green house gas (GHG) emissions, future wireless networks must operate highly energy efficient. To this end, various aspects of energy efficiency (EE) maximization have been addressed. On the one hand, careful selection of active number of antennas in massive multiple-input multiple-output (MIMO) systems has shown significant gains. Whereas, switching off physical resource blocks (PRBs) and carrier shutdown saves energy in low load scenarios. However, the joint optimization of both dimensions, the spectral PRB allocation with carrier aggregation (CA) and spatial layering, has not been accounted for. In this paper, we propose a power consumption model that captures the joint effect of CA and spatial layering on the total power consumption of a 5G network. We characterize the optimal resource allocation in spatial and spectral dimensions under practical constraints. Our results show that only in very low load scenarios, a single spatial layer achieves the lowest energy consumption and in most cases with high rate requirements and more users, spatial layering is required with carefully optimized number of active antennas and active PRBs. The gains compared to activating all available antennas and using all available PRB resources are tremendous. Finally, we study the point where switching on another frequency band results in better EE depending on the attenuation model.**

***Index Terms*—Energy efficiency, wireless cellular communications, resource allocation, power control, 5G, massive MIMO**

I. Introduction

The 5G networks, due to their larger capacity and improved hardware, can provide an improved EE in bits per Joule compared to previous generations. However, the predicted exponential increase in data traffic under full load conditions, the deployment of more sites for providing the same coverage at higher frequencies, and the processing required to address wider bandwidths and more antennas can lead to high energy consumption. As a result, the carbon footprint of the 5G networks may substantially increase [1]. To meet the ambitious targets of net-zero GHG emissions, it is, thus, of utmost importance to design the future wireless networks to be much more energy efficient.

It has been reported that 57% of the total network energy is consumed by the base stations (BSs) [2], where the power amplifier, transceivers and cables consume about 65% of the total BS energy [3]. Therefore, significant attention has been directed towards reducing the energy consumed by the BS. It has been identified that a simple linear or affine model as a function of the transmit power is inadequate, because it leads to an unbounded EE in a massive MIMO network as the number of antennas grow large [4]. In fact, in massive MIMO systems, it is important to consider the power consumed by the different BS components. Accounting for the circuit power consumption, in [5] and [6], the EE has been shown to be a quasi-concave function of the number of antennas and user equipments (UEs), as well as the transmit power, where the EE is optimized by increasing the transmit power with the number of antennas. In [7], it is advised to turn off a fraction of the antennas to reduce the total power consumption, late at night, when the traffic demand is low. Following a similar thinking, in [8], the downlink EE is maximized by adapting the number of antennas to temporal load variations over the day, whereas, in [9], the authors adjust not only the number of antennas but also the transmit data rate to maximize the EE, while achieving an uplink energy-efficient resource allocation in very large multi-user MIMO systems. Considering network densification, in [10], the authors instead aimed at finding the optimal dense network configuration, and concluded that reducing the cell size leads to a higher EE. However, they also showed how the EE saturates when the circuit power dominates over the transmit power.

While various aspects for the EE maximization have been considered in the above mentioned works, the interplay between spatial layering and CA on the EE has not been addressed in any of them. The 3rd Generation Partnership Project (3GPP) New Radio (NR) supports massive MIMO and CA, with up to 16 carrier components (CCs), which can be efficiently used to enhance the capacity and the data rates via wider bandwidths and load balancing. Moreover, CA allows to reduce the transmit power needed to achieve a specific rate requirement by using multiple CCs. However, when a large number of spatial layers and CCs are used, and if the CCs are inappropriately utilized, the hardware energy consumption of the BS can significantly increase [11]. Therefore, this work aims at investigating optimal joint resource allocation in spatial and spectral dimensions under practical constraints. More specifically, given a 5G network and quality of service (QoS) requirement for the UEs, we aim at finding the most energy efficient operating point for a multi-cell downlink massive MIMO system, seeking answers to questions such as, *i)* how many CCs and PRBs should be used, in addition to *ii)* what number of antennas per BS should be de/activated?

The rest of the paper is organized as follows. Section II-A describes the system model under consideration. In Sections II-B and II-C, the power consumption model is defined and the EE optimization problem is formulated, respectively. Section III and IV present the analysis and results, as well as the related discussions, respectively. The conclusion and future work are presented in Section V.

II. System Model and Preliminaries

A. Macro-cell Scenario

We consider a massive MIMO system where a macro cell operates three different frequency bands, e.g. the 4.9 and 2.6 GHz bands with 100 MHz bandwidth each and the 700 MHz band with 40 MHz bandwidth. Each frequency band is comprised of a set of PRBs - $N_{\mathfrak{h}}$, $N_{\mathfrak{m}}$, $N_{\mathfrak{l}}$ - corresponding to the 4.9 GHz, 2.6 GHz, and 700 MHz bands, respectively, to which the UE served by the macro cell can be assigned following different multi-band architectures. Such a scenario is depicted in Figure 1.

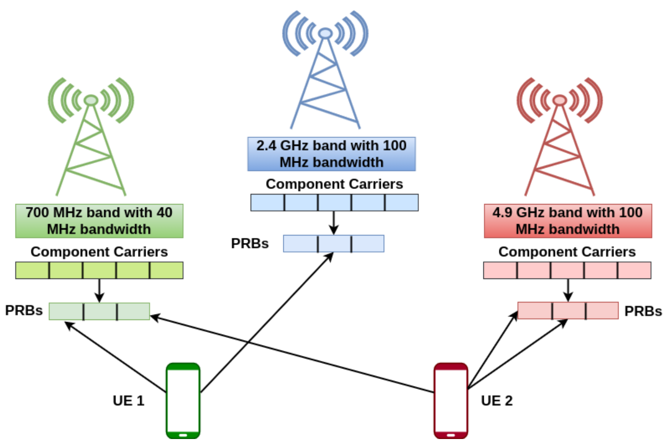


Fig. 1: A macro cell operating different frequency bands showing PRBs within CCs in each band and the assignment of PRBs to UEs.

In the defined system model, $\mathcal{B} = \{\mathfrak{l}, \mathfrak{m}, \mathfrak{h}\}$ and $\mathcal{M} = \{1, ..., M\}$ denote the set of available frequency bands and the set of macro cells, respectively, $\mathcal{T} = \{1, \ldots, T\}$ denotes the set of transceivers used at each macro cell, $CC(m)$ returns the set of CCs[1] belonging to macro cell m, and $\mathcal{N}_{m,c} = \{1, ..., \alpha_{m,c}\}$ is the set of available PRBs at the m^{th} macro cell on the c^{th} CC. The number of active PRBs used by the m^{th} macro cell on the c^{th} CC is denoted by $\alpha_{m,c}$, with a maximum of $\bar{A}_{m,c}$ PRBs, i.e., $0 \leq \alpha_{m,c} \leq \bar{A}_{m,c}$. The number of active transmit antennas connected to the t^{th} transceiver at the m^{th} macro cell on the c^{th} CC is denoted by $a_{m,t}$, with a maximum of A_m available active transmit antennas.

The assignment of the k^{th} UE to the available PRBs in the m^{th} macro cell is realized by a matching function μ as $\mu(k) = [\pi_1^k, ..., \pi_{N(k)}^k]$ that contains the list of all assigned PRBs to the k^{th} UE, where $N(k) = \pi_l^k = |\mu(k)|$ is the number of PRBs assigned to such k^{th} UE. The functions $m(\pi_\ell^k)$, $c(\pi_\ell^k)$, and $n(\pi_\ell^k)$ return the macro cell index, the CC index, and the PRB index of PRB π_ℓ^k, respectively. Vice versa, the matching function μ is overloaded, and also returns the set of UEs assigned to PRB π, described by the parameters m, c, and n, or directly by π, i.e., $\mu(m, c, n) = \mu(m(\pi), c(\pi), n(\pi))$ with

$$\mu(\pi) = \begin{cases} \mathcal{K}_{m,c,n} & \text{if UEs assigned} \\ \emptyset & \text{if unassigned or not used.} \end{cases} \tag{1}$$

In the downlink, the transmit signal is generated at the base station by precoding and scaling the data symbols [12]. Let $p_{m,k,\pi}$ be the normalized transmit power applied by the m^{th} macro cell for the k^{th} UE in PRB π. Then, the data rate achieved over different PRBs from the same band and CC is calculated using their average signal to interference and noise ratio (SINR), as all these PRBs are coded together. Instead, the data rate achieved over different PRBs of different bands or from the same band but different CCs are simply added, because they are coded separately. Therefore, the data rate of the k^{th} UE can be defined as

$$R_k = \sum_{c \in CC(m)} (\bar{b} \cdot |\mu_c(k)|) \log (1 + \overline{\sigma}_k), \tag{2}$$

where $\mu_c(k) = \{\pi \in \mu(k) : c(\pi) = c\}$ is the set of PRBs allocated to the k^{th} UE on the c^{th} CC, $\bar{b}$ is the PRB bandwidth[2], and the average SINR ($\overline{\sigma}$) for the k^{th} UE with a zero-forcing precoder is computed as[3]

$$\overline{\sigma}_k = \frac{1}{|\mu_c(k)|} \left(\sum_{\substack{\pi \in \mu_c(k) \\ c(\pi) = c}} \frac{(a_{m(\pi),t} - |\mu(\pi)|) p_{m,k,\pi} \beta^c_{k,m(\pi)}}{1 + \sum\limits_{\substack{m(\pi') \neq m(\pi) \\ n(\pi') = n(\pi) \\ c(\pi') = c(\pi)}} \sum\limits_{k'} p_{m',k',\pi'} \beta^c_{k,m(\pi')}} \right), \tag{3}$$

where $a_{m(\pi),t}$ is the number of active transmit antennas connected to the t^{th} transceiver at the m^{th} macro cell, $|\mu(\pi)|$ is the number of UEs spatially multiplexed on PRB π, and $\beta^c_{k,m} \in \beta^{\mathfrak{l}}_{k,m}, \beta^{\mathfrak{m}}_{k,m}, \beta^{\mathfrak{h}}_{k,m}$ are the distance-dependent large scale fadings, which depend on the assignment of the k^{th} UE to the available PRBs in the respective CC of the respective frequency band in the respective macro cell.

B. Power Consumption Model

Since a linear or affine power consumption model is insufficient for a massive MIMO system, the model must include the power consumed by the power amplifiers and different base station components, such as transceivers, analog filters, etc. Inspired from [5] and [13], we consider the following power consumption model:

[1]In this work, we use carrier and CC interchangeably, where the latest terminology is borrowed from CA.

[2]180KHz in LTE and NR with a sub-carrier spacing of 15KHz.

[3]It should be noted that the SINR expression has been normalized by the noise power, and thus it has been included as part of the large scale fading.

$$P_{tot} = \sum_{m=1}^{M} \frac{1}{\eta_{PA}} \sum_{c\in CC(m)} \sum_{k\in K(c)} \sum_{\pi\in\mu_c(k)} p_{m,k,\pi} \tag{4}$$
$$+ \sum_{m=1}^{M} I_{\mu_m} \left(\frac{|CC(m)|}{\lambda_{m,0}} P_{m,FIX} + \frac{|CC(m)|}{\lambda_{m,1}} P_{m,SYNC} \right)$$
$$+ \sum_{m=1}^{M} D_{m,0} \left(\sum_{t=1}^{T_m} I_{\alpha_m, c_m(t)} a_t \right) + C$$
$$+ \sum_{m=1}^{M} \left(D_{m,1} \sum_{t=1}^{T_m} I_{\alpha_m, c_m(t)} a_t \sum_{c\in CC(m)} \sum_{k\in K(c)} |\mu_c(k)| \right),$$

where η_{PA} is the power amplifier's efficiency, I_{μ_m} is an indicator function indicating if any PRB in any CC of the m^{th} macro cell is being used[4], $|CC(m)|$ is the total number of CCs in use in the m^{th} macro cell, $P_{m,FIX}$ is the load-independent power consumption in the m^{th} macro cell required for site-cooling, control signaling, backhaul infrastructure, and base-band processors, $P_{m,SYNC}$ is the load-independent power consumed by the local oscillator, $D_{m,0}$ is the power consumed by the transceiver attached to an antenna, including converters, mixers, filters, etc., T_m is the number of transceivers in the m^{th} macro cell, $c_m(t)$ is the set of CCs served by the t^{th} transceiver, $I_{\alpha_m,c_m(t)}$ is an indicator function that indicates whether any PRB in any CCs served by t^{th} transceiver are being used, a_t is the number of active transceiver chains per transceiver[5], $D_{m,1}$ is the power consumed by the signal processing of a MIMO layer across a PRB, through $\lambda_{m,0}$ and $\lambda_{m,1}$ the power consumption model can capture a linear, sub-linear, or an independent relationship between P_{FIX}, P_{SYN} and $D_{m,0}$ and number of CCs used, indicating the level of hardware sharing among such CCs in the m^{th} macro cell [13], and C is the fixed power consumed by the coding at a massive MIMO BS and backhaul support.

C. Problem Formulation

In the problem setup, we assume that the matching function $\mu(k)$ is already fixed. Firstly, there exist algorithms to find a proper matching. Secondly, the following analysis can be used to design the optimal matching. Thus, for a given individual rate constraint R_k the power consumption minimization problem is formulated as

$$\min_{\mathbf{p},\mathbf{a},\boldsymbol{\alpha}} \quad P_{tot} \tag{5}$$
$$\text{s.t.} \quad R_k \geq \underline{R}, \tag{6}$$
$$p_{m,k,\pi} \geq 0, \tag{7}$$
$$\sum_{c\in CC(m)} \sum_{k\in K(c)} \sum_{\pi\in\mu_c(k)} p_{m,k,\pi} \leq P_m \; \forall m, \tag{8}$$
$$0 \leq \alpha_m \leq \pi_{max}, \tag{9}$$
$$\max |\mu(\pi)| < a_t, \tag{10}$$

with the minimum rate constraints per UE in (6), the non-negativeness power constraints per PRB in (7), the sum power constraints per macro cell and CC in (8), the maximum number of available PRBs constraints in (9), and the minimum number of required antennas constraints in (10).

With this problem definition, we aim at finding the optimal values of the number of active PRBs $\alpha_{m,c}$, the number of active transmit antennas $a_{m,t}$, and the transmit power $p_{m,k,\pi}$.

III. Analysis and Results

The power consumption model in (4) is non-convex, and represents a mixed-integer programming problem including the assignment of UEs to PRBs, which cannot be efficiently solved by standard methods. Therefore, we first consider a single macro cell to understand the behavior of the objective function P_{tot} as the number of antennas, the number of PRBs, the number of UEs and the QoS constraints change.

A. Single UE Scenario

The rate constraint for a single UE, given the minimum QoS requirement $\underline{R}$, can be computed from (2) as

$$\bar{b} \cdot \alpha \log\left(1 + \frac{1}{\alpha} \sum_{n=1}^{\alpha} (a-1)\beta p\right) \geq \underline{R}, \tag{11}$$

where α is the number of PRBs, a is the number of active transmit antennas, β is the large scale fading, and p is the transmit power. Assuming a uniform power allocation for each PRB assigned to the UE, the total power consumption of the macro cell can be expressed as

$$P_{tot} = \frac{\alpha}{\eta_{PA}} p + D_0 a + D_1 \alpha a, \tag{12}$$

where the constant C[6] is omitted and p is obtained from (11) as

$$p \geq \frac{2^{\frac{\underline{R}}{\bar{b}\alpha}} - 1}{(a-1)\beta}. \tag{13}$$

To understand how the number of PRBs and antennas affect the behavior of the objective function, (12) with p from (13) is analyzed by minimizing it with respect to them separately. The optimum number of PRBs α^* and the optimum number of antennas a^* are computed by equating the derivative of (12) with respect to α and a to zero, yielding

$$\alpha^* = \left[\frac{\underline{R}\log(2)}{\bar{b}} \frac{1}{W\left(\frac{(D_1\eta_{PA}a(a-1)\beta)-1}{e}\right)+1} \right]_0^{\pi_{max}}, \tag{14}$$
$$a^* = \left[\sqrt{\frac{\alpha(2^{\frac{\underline{R}}{\bar{b}\alpha}}-1)}{\eta_{PA}\beta(D_0+D_1\alpha)}} + 1 \right]_{\max(|\mu(\pi)|)}, \tag{15}$$

where $W(\cdot)$ in (14) is the Lambert W function[7], and e is the Euler's number. The two optimality criteria in (14) and (15) implicitly characterize the optimal tuple (α^*, a^*), which can be found numerically via a fixed point algorithm.

[4]Note that, with this indicator function we realise an ideal carrier shutdown in which the BS does not consume anything when no PRB is used.

[5]The chains are connected to ports though which antennas can be turned on/off.

[6]C along with the power consumption terms P_{FIX} and P_{SYNC} in (4) have been omitted because there is no dependency on the optimization variables.

[7]To obtain a positive real-valued α^*, it should be noted that $D_1\eta_{PA}\beta a(a-1)-1 \geq 0$, such that the optimum number of PRBs lie on the principal branch (W_0) of the Lambert $W(\cdot)$ function.

B. Multi UE Scenario

We consider the following matching of the UEs to PRBs $\mu_c(k)$ (implicitly $\mu(\pi)$):

1) Single band and single spatial layer (NSM): $|\mu(\pi)| = 1, \forall\pi$ s.t. $\sum_{k=1}^{K}\alpha_k \leq \pi_{max}$
2) Single band and multiple spatial layers (SM): $|\mu(\pi)| = K, \forall\pi$ s.t. $\max\{\alpha_1,\ldots,\alpha_k\} \leq \pi_{max}$
3) Multi-Band with equal split among $\mathcal{B}$ bands such that NSM: $|\mu(\pi)| = 1, \forall\pi$ SM: $|\mu(\pi)| = \frac{K}{|\mathcal{B}|}, \forall\pi$.

The total power consumption per band, then, for a multi-UE scenario with K UEs can be approximated as

$$P_{tot}^{NSM}(\boldsymbol{\alpha}) = \frac{1}{\eta_{PA}}\sum_{k=1}^{K}\alpha_k p_k + D_0 a + D_1 a K \sum_{k=1}^{K}\alpha_k, \quad (16)$$

$$P_{tot}^{SM}(\boldsymbol{\alpha}) = \frac{1}{\eta_{PA}}\sum_{k=1}^{K}\alpha_k p_k + D_0 a + D_1 a K \max\{\alpha_1,\ldots,\alpha_k\}, \quad (17)$$

where $\boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_K]$ is the PRB allocation vector. In the following, we restrict our analysis to a symmetric and low load scenario, and derive the following lemma and propositions:

Lemma 1. *For a symmetric scenario with K UEs, where $\beta_1 = \ldots = \beta_K = \beta$ and $R_1 = \cdots = R_K = R$. Then, the minimum power consumption is achieved for $\alpha_1 = \ldots = \alpha_K = \frac{\pi}{K}$, if π is the total number of PRBs used at the macrocell*[8].

Proof. The proof is given in Appendix A. □

Proposition 1. *For a symmetric scenario, if the rate requirement $\underline{R}$ approaches small values, i.e. $\underline{R} \to 0$, the total power consumption for UEs with a single spatial layer is lower than for UEs with multiple spatial layers. Then, the optimal number of PRBs per UE is one, and the minimum number of antennas should be used*[9].

Proof. The proof is given in Appendix B. □

The next result specifies this point for two UEs.

Proposition 2. *For two UEs, there exists a specific number of PRBs π^* for which $P_{tot}^{NSM} = P_{tot}^{SM}$. For all $\pi < \pi^*$, $P_{tot}^{NSM} < P_{tot}^{SM}$, whereas, for all $\pi > \pi^*$, $P_{tot}^{NSM} > P_{tot}^{SM}$*[10].

Proof. The proof is given in Appendix C. □

Corollary 1. *The asymptotic per-antenna power consumption as $a \to \infty$ when UEs are spatially and not spatially multiplexed is equal.*

[8]This simplifies the optimization problem to a scalar π, which can be efficiently found by bisection method.

[9]The asymptotic characterization in Proposition 1 indicates that there might exist a range of operating points with low rate requirements for which NSM outperforms SM.

[10]In a low load scenario, a small number of PRBs are assigned to UEs. For all $\pi < \pi^*$, the total power consumption for NSM is lower than SM.

IV. Numerical Analysis

A. Single UE Scenario

Table I shows the number of antennas and PRBs required to achieve the minimum of the total power consumption computed via (12) for various values of the propagation channel (β). For a large β, it suffices to use 3 antennas and 3 PRBs to achieve the minimum. However, as the channel conditions worsen, the required number of antennas and PRBs increase, where for the smallest β all available spectral and spatial resources should be utilized.

β	2059.20	33.340	1.473	0.0651	0.00288	0.000047
α^*	3	12	45	55	62	192
a^*	3	3	3	10	40	45

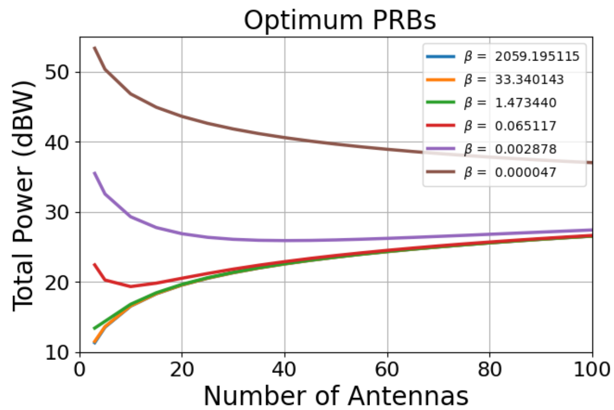
TABLE I: Optimal number of antennas and PRBs required to achieve the minimum total power consumption for each β, with $\eta_{PA} = 0.48$, $D_0 = 4.49$ W, $D_1 = 0.0062$ W, $\underline{R} = 5000$ Kbps and $\bar{b} = 180$ KHz.

Figure 2a and 2b show how the total power consumption changes as a function of the total number of antennas with the optimal number of PRBs α^* and as a function of the total number of PRBs with the optimal number of antennas a^*, respectively. Using α^*, the total power consumption increases for large β as the number of antennas increases. Whereas, it first decreases for small β before it increases again. Furthermore, the total power consumption first decreases and then slightly increases as the number of PRBs increases using a^*. This indicates that an optimal number of PRBs exists, i.e., (14), to achieve a given rate requirement, while consuming the least total power. Moreover, depending on the parameters η_{PA}, D_0, D_1, β, and $\underline{R}$, an optimal number of antennas also exists, i.e., (15).

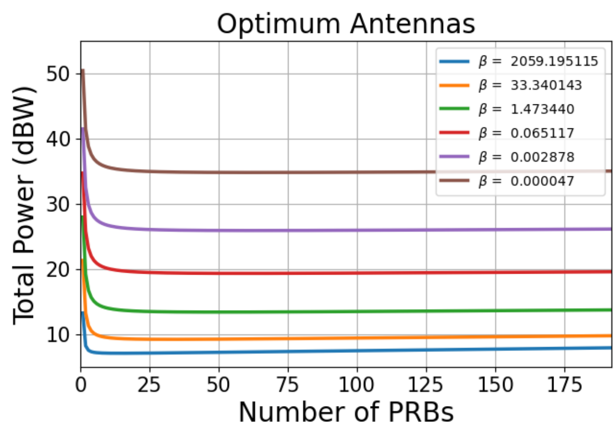
B. Multi UE Scenario

1) Single Band: Depicted in Figure 3 is the minimum of each total power consumption curve in a single band for 2-10 UEs with SM and NSM for $\beta = 0.042$ and various data rates. Firstly, for the lowest data rates, i.e., 3000 Kbps, NSM consumes a lower power than SM for all UEs. For higher data rates, a switching behavior is observed such that, from 4 to 9 UEs and 4000 Kbps and for more than two UEs and 5000 Kbps, SM achieves a lower power consumption. Finally, for the highest rate, i.e., 6000 Kbps, SM consumes a lower power for all UEs. The observed behavior is in accordance with Proposition 1, which suggests to spatially multiplex UEs to achieve a lower power consumption for large data rates and as the number of UEs increases.

2) Multi-Band: Figure 4 compares the minimum power consumption for 4 UEs with NSM and SM for $\beta = 0.0651$. The channels are further degraded by a factor of 2, 4, and 8, corresponding to a 3dB, 6dB, and 9dB degradation, respectively. Firstly, the power consumption increases as the channel conditions worsen. However, the power consumption is the lowest for SM in a single band. Secondly, as the channel conditions degrade, NSM in single band performs



(a) Total power consumption over antennas for optimum number of PRBs.



(b) Total power consumption over PRBs for optimum number of antennas.

Fig. 2: Total power consumption for a single UE with $\eta_{PA} = 0.48$, $D_0 = 4.49$ W, $D_1 = 0.0062$ W, $\underline{R} = 5000$ Kbps, and $\bar{b} = 180$KHz.

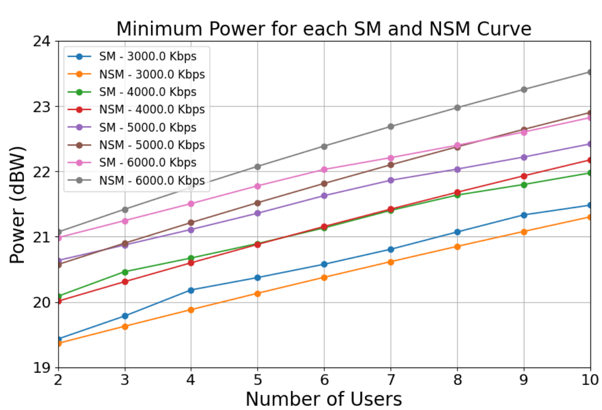


Fig. 3: Minimum of each total power consumption curve showing the effects of number of UEs and data rates in a single band for SM and NSM with poor channels, with $\eta_{PA} = 0.48$, $D_0 = 4.49$ W, $D_1 = 0.0062$ W, and $\bar{b} = 180$ KHz.

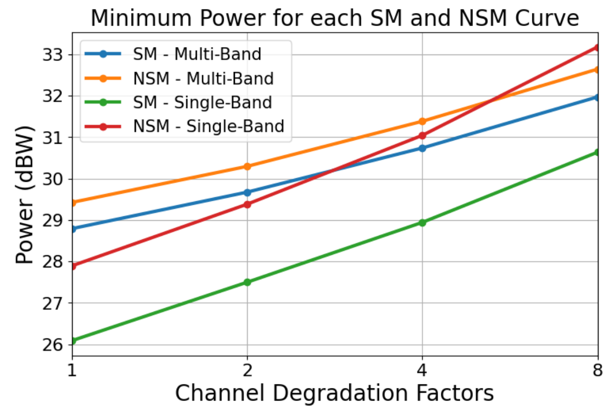


Fig. 4: Minimum of each total power consumption curve highlighting the effects of channel conditions for 4 spatially and non-spatially multiplexed UEs with $\eta_{PA} = 0.48$, $D_0 = 4.49$ W, $D_1 = 0.0062$ W, $\underline{R} = 50000$ Kbps, and $\bar{b} = 180$ KHz.

worse than NSM and SM in multi-band. Therefore, to consume less power, all UEs should be assigned to a single band and spatially multiplexed. While this holds true for the matching described in Section IV-B, further analysis is required using the optimal per UE PRB allocation.

V. Conclusion and Future Work

In this work, we have introduced a power consumption model capturing the interplay between —and addressing the joint optimization of— the spectral PRB allocation with CA and the massive MIMO spatial layering to analyze and minimize the power consumption of a macro-cell 5G network. Our analysis leads to the following key observations: *i)* a single spatial layer per PRB achieves the lowest energy consumption in low load scenarios, *ii)* spatial layering is required for higher rate requirements and when having more UEs, *iii)* as the channel conditions worsen, i.e., weak channel gain, SM lead to a lower power consumption, and *iv)* switching on additional frequency bands at higher frequencies managed by different hardware leads to a higher power consumption for both SM and NSM.

While the analysis in this paper pertains to a single macro-cell scenario at low load, in the future work, we not only aim at analysing the system and power model for high load scenarios, but also at incorporating the effects of multiple macro-cells on the EE of 5G networks.

Appendix A
Proof of Lemma 1.

For a PRB allocation vector $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K]$ satisfying $\sum_{k=1}^{K} \alpha_k = \pi$, majorization can be applied to show that if $\boldsymbol{\alpha} \geq \boldsymbol{\delta}$, i.e., $\sum_{k=1}^{M} \alpha_{[k]} \geq \sum_{k=1}^{M} \delta_{[k]}$ for $1 \leq M \leq K$ and $\sum_{k=1}^{K} \alpha_k \geq \sum_{k=1}^{K} \delta_k$, then $P_{tot}(\boldsymbol{\alpha}) \geq P_{tot}(\boldsymbol{\delta})$, where $P_{tot}(\boldsymbol{\alpha})$ is a Schur-convex function because it is the sum of convex functions in α_k and it also scales linearly with α_k. Furthermore, $\sum_{k=1}^{K} \alpha_k$ in (16) and $\max\{\alpha_1, \ldots, \alpha_k\}$ in (17) are also convex in $\boldsymbol{\alpha}$.

Appendix B
Proof of Proposition 1.

For a symmetric scenario with $\alpha_1 = \ldots = \alpha_k = \frac{\pi}{K}$, the total power consumption for NSM and SM is computed as

$$P_{tot}^{NSM} = \frac{\pi}{\eta_{PA}} \frac{2^{\frac{K\underline{R}}{\overline{b}\pi}} - 1}{(a-1)\beta} + D_0 a + D_1 K \pi a, \tag{18}$$

$$P_{tot}^{SM} = \frac{K\pi}{\eta_{PA}} \frac{2^{\frac{\underline{R}}{\overline{b}\pi}} - 1}{(a-K)\beta} + D_0 a + D_1 K \pi a, \tag{19}$$

respectively. (18) and (19) show that at least $a > 1$ and $a \geq K+1$ antennas are required for NSM and SM, respectively. Furthermore, as $\underline{R} \to 0$, P_{tot}^{NSM} and P_{tot}^{SM} result in

$$\lim_{\underline{R} \to 0} P_{tot}^{NSM} = D_0 a + D_1 K \pi a, \tag{20}$$

$$\lim_{\underline{R} \to 0} P_{tot}^{SM} = D_0 a + D_1 K \pi a, \tag{21}$$

respectively. However, (20) and (21) provide no insights about the minimum number of PRBs and antennas required to serve K UEs. Therefore, an auxiliary variable t[11] is introduced and number of PRBs and antennas, and the data rate are defined as a function of t as follows:

$$a(t) = \begin{cases} (K+1)(1+t) & \text{if spatial multiplexing,} \\ 2(1+t) & \text{if no spatial multiplexing} \end{cases} \tag{22a}$$

$$\pi(t) = 1 + t \tag{22b}$$

$$\underline{R}(t) = t. \tag{22c}$$

Then, computing P_{tot}^{NSM} and P_{tot}^{SM} using (22a), (22b) and (22c) is equivalent to computing P_{tot}^{NSM} and P_{tot}^{SM} as $\underline{R} \to 0$, yielding

$$\lim_{t \to 0} P_{tot}^{NSM} = 2D_0 + 2D_1 K \tag{23}$$

$$\lim_{t \to 0} P_{tot}^{SM} = D_0(K+1) + D_1 K(K+1) \tag{24}$$

and from (23) and (24) it is evident that $P_{tot}^{NSM} < P_{tot}^{SM}$ as $t \to 0$.

Appendix C
Proof of Proposition 2.

To find a fixed number of PRBs where NSM equals SM, (18) and (19) can be equated and solved for π as

$$2^{\frac{K\underline{R}}{\overline{b}\pi}}(a-K) - 2^{\frac{\underline{R}}{\overline{b}\pi}}(K(a-1)) - a = 0. \tag{25}$$

The solution to (25) for two UEs is found by substituting $K = 2$, $x = 2^{\frac{\underline{R}}{\overline{b}\pi}}$ and solving the quadratic equation, yielding

$$\pi^* = \frac{\underline{R}}{\overline{b}} \frac{1}{\log_2(a) - \log_2(a-2)}. \tag{26}$$

(26) shows that at least 3 antennas are required. Furthermore, for any a, (26) shows the point where P_{tot}^{NSM} and P_{tot}^{SM} are equal. However, it does not inform about which multiplexing technique performs better. Therefore, (18) and (19) as a function of π are analyzed in the vicinity of π^*. Thus, an auxiliary variable $\epsilon > 0$ is multiplied with π, yielding

$$\frac{\epsilon\pi}{\eta_{PA}} \frac{2^{\frac{2\underline{R}}{\overline{b}\epsilon\pi}} - 1}{(a-1)\beta} = \frac{2\epsilon\pi}{\eta_{PA}} \frac{2^{\frac{\underline{R}}{\overline{b}\epsilon\pi}} - 1}{(a-2)\beta}. \tag{27}$$

The common terms in (27) cancel out. Then, let

$$f(\epsilon) = \frac{2^{\frac{2\underline{R}}{\overline{b}\epsilon\pi}} - 1}{(a-1)} - 2\frac{2^{\frac{\underline{R}}{\overline{b}\epsilon\pi}} - 1}{(a-2)}. \tag{28}$$

The goal is to compute $f'(\epsilon)$ with respect to ϵ at $\epsilon = 1$ and show that it is less than zero. $f'(\epsilon)$ can be computed as

$$f'(\epsilon) = \frac{2\kappa \log(2)}{\epsilon^2} \left(\frac{-2^{\frac{2\kappa}{\epsilon}}}{a-1} + \frac{2^{\frac{\kappa}{\epsilon}}}{a-2} \right), \tag{29}$$

where $\kappa = \log_2(1 + \frac{2}{a-2})$. Upon further simplifications $f'(\epsilon = 1) = 2a \log\left(\frac{a-2}{a}\right)$, where $a > 0$ and $\log\left(\frac{a-2}{a}\right) < 0$ shows that approaching $\epsilon > 1$, $P_{tot}^{NSM} < P_{tot}^{SM}$, whereas, approaching $\epsilon < 1$, $P_{tot}^{NSM} > P_{tot}^{SM}$.

[11] Under the assumption that as $\underline{R} \to 0$ the number of PRBs and antennas decrease somewhat linearly.

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