Joint Subchannel Allocation and Beamforming for Multicast in Ultra-Dense LEO Backbone Network

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***Abstract*—Nowadays, the ultra-dense low earth orbit (LEO) satellite network has become a promising paradigm in the next generation mobile communication network. With the development of content centric communication, multicast technology also attracts much attention. In this paper, we consider the downlink multicast transmission in the ultra-dense LEO satellite network. Multiple LEO satellites provide multicast service for multiple ground user (GU) groups under their coverage, where each GU group requests the same content. To improve the multicast performance, we propose an optimal subchannel allocation and beamforming scheme to maximize the system max-min fair (MMF) capacity of GUs. By leveraging the many-to-many matching model, we obtain the optimal subchannel allocation solution, and we propose a successive convex approximation (SCA) based algorithm for the downlink beamforming in the matching process. The many-to-many matching algorithm is convergent to a stable solution after finite iterations. Simulation results show the superiority and the effectiveness of the proposed subchannel allocation and beamforming method compared with other baseline schemes.**

***Index Terms*—ultra-dense LEO satellite network, multicast, beamforming, subchannel allocation, matching theory.**

I. Introduction

With recent significant advances in low earth orbit (LEO) satellites, the ultra-dense LEO satellite network has been envisioned as a supplement to the terrestrial communication system to provide high data rate, high reliability, and ubiquitous connectivity [1]. It can support some communication services which can not be accomplished by terrestrial communication system [2], [3], e.g., remote multicast service.

Wireless multicast transmission can be effectively used in the content centric communication to make full use of wireless communication resources, such as music streaming and video streaming, where multiple ground users (GUs) may simultaneously request the same content [4]–[6]. Work [7] studied the multicast beamforming scenario with multiple groups, where solving the beamforming problem is considered in two NP-hard subproblems, i.e., the total transmit power minimization with the quality of service (QoS) constraint and the max-min fair (MMF) problem with the power constraint. However, sometimes GUs can not be well served only by terrestrial base stations (TBSs). For instance, in remote areas, the GUs requesting the same contents may distribute far away from each other and can not be simultaneously covered by TBSs. In this scenario, the LEO satellite network is considered a predictable and achievable way to provide multicast service for GUs. In [8], Li et al. optimized the robust multigroup multicast beamforming in the multi-beam satellite communication system by maximizing the worst-case outage signal to interference plus noise ratio (SINR) under the per-beam power constraints, where full spectrum reuse is considered. Zhu et al. [9] investigated the cooperative multigroup multicast transmission in the integrated terrestrial-satellite network, where TBSs and a single satellite cooperatively provide multicast services for GUs with reusing the entire bandwidth.

Since each GU will suffer interference from the undesired signals in the same frequency band, we consider subchannel partition in the multicast transmission to achieve performance improvement [10]. Meanwhile, in an ultra-dense LEO network, multiple LEOs can cooperatively multicast for GUs distributed in a large geographic area to improve transmission efficiency. Then, the downlink beamforming of multiple LEOs should be well designed in a cooperative manner. Facing these challenges, in this paper, we investigate the joint subchannel allocation and downlink beamforming to maximize the system MMF capacity in the ultra-dense LEO network. The novelty and contributions of this paper are highlighted as follows:

- We investigate a general downlink cooperative multigroup multicast transmission in the ultra-dense LEO satellite network, where multiple LEO satellites provide the multicast service for GUs within their coverage.
- We propose the many-to-many matching algorithm for subchannel allocation to maximize the worst GU's rate, and obtain a stable solution in finite iterations.
- In each step of the matching process, we adopt an effective successive convex approximation (SCA) based iterative algorithm to acquire the optimal downlink beamforming vectors. Numerous simulation results validate the performance of the proposed algorithm.

The rest part is organized as follows. Section II describes the system model and problem formulation. Section III and Section IV provide the problem solution and simulation results, respectively, while Section V concludes the paper.

II. System Model

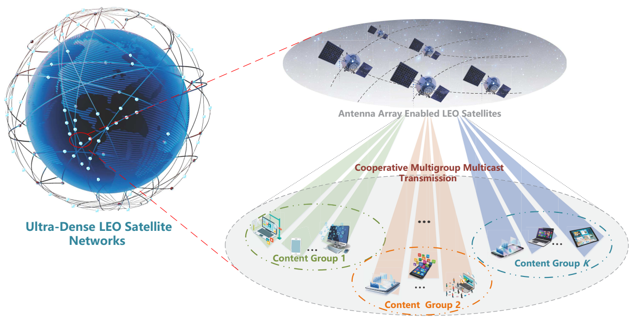


Fig. 1. Downlink multicast in the ultra-dense LEO network.

We consider a downlink multigroup multicast transmission scenario in the ultra-dense LEO satellite network as shown in Fig. 1. Although LEO satellites admit high mobility, they can keep serving GUs in an appointed area during a time period. Due to the large scale of the ultra-dense LEO satellite network, we suppose that there are M LEO satellites simultaneously providing multicast service for N GUs within their coverage during a given time period, and each LEO is equipped with a single antenna. Each GU has one single receiving antenna, and GUs are partitioned into K groups according to different contents. We denote GUs requesting the same content k as $\mathcal{G}_k = \{\mathcal{U}_{1,k}, \cdots, \mathcal{U}_{G_k,k}\}$, where $\mathcal{U}_{i,k}$ is the GU i requesting content k, and G_k is the number of GU group k. Each GU is assumed to request only one content at one time slot.

Each GU will receive the signal including the desired content together with the interference contents from other groups. All GUs share the total bandwidth B, which can be divided into H subchannels. Each group of GUs occupies the same subchannels. We introduce a binary matrix $\boldsymbol{X} \in \mathbb{R}^{K\times H}$ to depict the subchannel allocation, where $x_{k,h} = 1$ implies group k occupies subchannel h, and $x_{k,h} = 0$ otherwise. Then the transmit signal of LEO satellites over subchannel h is $\boldsymbol{x}_h = \sum_{k=1}^{K} \boldsymbol{w}_{k,h} x_{k,h} s_k$, where $\boldsymbol{w}_{k,h} = [w_{k,h}^1, \cdots, w_{k,h}^M] \in \mathbb{C}^M$ denotes the aggregation beamforming vector of group k in subchannel h with $w_{k,h}^l \in \mathbb{C}$ representing the beamforming of LEO l, and $s_k \in \mathbb{C}$ is the transmit signal of content k for group k with $\mathbb{E}[|s_k|^2] = 1$. Provided that the waveforms are mutually uncorrelated, the total transmit power of LEO l is $\sum_{h=1}^{H}\sum_{k=1}^{K} \|\boldsymbol{e}_l \boldsymbol{w}_{k,h}\|^2$, where $\boldsymbol{e}_l \in \mathbb{R}^{1\times M}$ is a vector with the l-th element being 1 and others being 0.

Accordingly, the received signal of GU i in group j sent by LEO satellites over the subchannel h is formulated as

$$y_{i,j,h} = \boldsymbol{g}_{i,j,h}^{\mathrm{H}} \boldsymbol{x}_h + n_{i,j,h} \tag{1}$$

$$= \boldsymbol{g}_{i,j,h}^{\mathrm{H}} \boldsymbol{w}_{j,h} x_{j,h} s_j + \boldsymbol{g}_{i,j,h}^{\mathrm{H}} \sum_{k=1,k\neq j}^{K} \boldsymbol{w}_{k,h} x_{k,h} s_k + n_{i,j,h}, \tag{2}$$

where $\boldsymbol{g}_{i,j,h} \in \mathbb{C}^M$ is the channel vector from LEO satellites to GU i in group j over subchannel h generated by the composite multi-beam satellite channel model in [11], and $n_{i,j,h}$ is the additive white Gaussian noise (AWGN) with noise variances $\sigma_{i,j,h}^2$. Accordingly, the received SINR of GU i in group j on subchannel h is derived as

$$\gamma_{i,j,h} = \frac{x_{j,h} |\boldsymbol{g}_{i,j,h}^{\mathrm{H}} \boldsymbol{w}_{j,h}|^2}{\sum\limits_{k=1,k\neq j}^{K} x_{k,h} |\boldsymbol{g}_{i,j,h}^{\mathrm{H}} \boldsymbol{w}_{k,h}|^2 + \sigma_{i,j,h}^2}. \tag{3}$$

Then, the achievable rate of GU i in group j is obtained by $R_{i,j} = \sum_{h=1}^{H} B_h \log(1 + \gamma_{i,j,h})$.

Analogous to [5], [7], we aim at maximizing the worst achievable rate of all GUs. Meanwhile, we consider a predetermined target achievable rate $R_{i,j}^0$ for GU $\mathcal{U}_{i,j}$ to describe different service levels. Then, we formulate the weighted MMF subchannel allocation and downlink beamforming as

$$(\mathcal{F}) \quad \max_{\{\boldsymbol{w}_{j,h}\},\boldsymbol{X}} \ \min_{j\in\{1,\cdots,K\}} \ \min_{i\in[1,G_j]} \ \frac{1}{R_{i,j}^0} R_{i,j} \tag{4}$$

$$\text{s.t.} \ \sum_{h=1}^{H}\sum_{j=1}^{K} x_{j,h} \|\boldsymbol{e}_l \boldsymbol{w}_{j,h}\|^2 \leqslant P_{\max}, \ l = 1, \cdots, M, \tag{4a}$$

$$x_{j,h} \in \{0,1\}, \ j = 1, \cdots, K, \ h = 1, \cdots, H, \tag{4b}$$

where $P_{\max}$ is the maximum transmit power of LEO satellite.

Notice that, problem $(\mathcal{F})$ with a special case $H = 1$ and $\boldsymbol{X} = [1, \cdots, 1]'$ is shown to be NP-hard in [7]. Therefore, it is an immediate result that problem $(\mathcal{F})$ is NP-hard.

III. Algorithm Design for Weighted MMF Problem

Since the beamforming and the subchannel allocation are coupled, we first discuss how to solve $\boldsymbol{w}$ with fixed $\boldsymbol{X}$ and then determine the optimal $\boldsymbol{X}$.

A. Beamforming Method

For a given $\boldsymbol{X}$, we first introduce an auxiliary variable $t > 0$, and then problem $(\mathcal{F})$ is reduced to

$$(\mathcal{F}_w) \quad \max_{\{\boldsymbol{w}_{j,h}\},t} \ t \tag{5}$$

$$\text{s.t.} \ \frac{1}{R_{i,j}^0} R_{i,j} \geqslant t, \ \forall\ i,\ j, \tag{5a}$$

$$\sum_{h=1}^{H}\sum_{j=1}^{K} x_{j,h} \|\boldsymbol{e}_l \boldsymbol{w}_{j,h}\|^2 \leqslant P_{\max}. \tag{5b}$$

Obviously, problem $(\mathcal{F}_w)$ is non-convex and NP-hard. Hence, we apply some approximations on $R_{i,j}$ to transform problem $(\mathcal{F}_w)$ to a convex one. First, we equivalently convert vector variables in problem $(\mathcal{F}_w)$ to matrix variables, and set

$$\begin{aligned} \boldsymbol{w}_h^{\mathrm{H}} &= \left(\boldsymbol{w}_{1,h}^{\mathrm{H}}, \cdots, \boldsymbol{w}_{K,h}^{\mathrm{H}}\right) \in \mathbb{C}^{1\times KM}, \\ \boldsymbol{W}_h &= \boldsymbol{w}_h \boldsymbol{w}_h^{\mathrm{H}} \in \mathbb{C}^{KM\times KM}, \\ \tilde{\boldsymbol{Q}}_{i,j,h} &= \boldsymbol{g}_{i,j,h} \boldsymbol{g}_{i,j,h}^{\mathrm{H}} \in \mathbb{C}^{M\times M}, \\ \boldsymbol{Q}_{k;i,j,h} &= D\{\boldsymbol{0}, \cdots, \boldsymbol{0}, \tilde{\boldsymbol{Q}}_{i,j,h}, \boldsymbol{0}, \cdots, \boldsymbol{0}\} \in \mathbb{C}^{KM\times KM}, \end{aligned} \tag{6}$$

where $D\{\cdot\}$ means block diagonalization, while $\boldsymbol{Q}_{k;i,j,h}$ is a matrix with the k-th block being the M-dimensional matrix $\tilde{\boldsymbol{Q}}_{i,j,h}$ and other blocks being M-dimensional zero matrices.

With these notations, problem $(\mathcal{F}_w)$ is equivalent to

$$(\mathcal{F}_W)\ \max_{\{\boldsymbol{W}_h\},t} \quad t \tag{7}$$

$$\text{s.t.}\ \frac{1}{R^0_{i,j}}\bar{R}_{i,j} \geqslant t,\ \forall\ i,\ j, \tag{7a}$$

$$\sum_{h=1}^{H} \text{tr}(\boldsymbol{A}_{h,l}\boldsymbol{W}_h) \leqslant P_{\max},\ \forall\ l, \tag{7b}$$

$$\text{rank}(\boldsymbol{W}_h) = 1, \boldsymbol{W}_h \geqslant \boldsymbol{0},\ \forall\ h, \tag{7c}$$

where $\boldsymbol{A}_{h,l} = D\{x_{1,h}\boldsymbol{e}_l^{\rm T}\boldsymbol{e}_l, \cdots, x_{K,h}\boldsymbol{e}_l^{\rm T}\boldsymbol{e}_l\} \in \mathbb{C}^{KM\times KM}$, and $\bar{R}_{i,j}$ in constraint (7a) is given by

$$\begin{aligned}\bar{R}_{i,j} =& \sum_{h=1}^{H} B_h \log\left(\text{tr}\Big[\sum_{k=1}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\boldsymbol{W}_h\Big] + \sigma^2_{i,j,h}\right)\\ &- \sum_{h=1}^{H} B_h \log\left(\text{tr}\Big[\sum_{k=1,k\neq j}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\boldsymbol{W}_h\Big] + \sigma^2_{i,j,h}\right).\end{aligned} \tag{8}$$

Denote $\bar{R}_{i,j} = f_1(\boldsymbol{W}_h) - f_2(\boldsymbol{W}_h)$. Then both $f_1(\boldsymbol{W}_h)$ and $f_2(\boldsymbol{W}_h)$ are concave. Based on the SCA of Proposition B.3 in [12], we give the following lemma.

Lemma 1: Given a matrix $\bar{\boldsymbol{W}}_h$, we have

$$f_2(\boldsymbol{W}_h) \leqslant f_2(\bar{\boldsymbol{W}}_h) + \text{tr}[\nabla f_2(\bar{\boldsymbol{W}}_h)^{\rm T}(\boldsymbol{W}_h - \bar{\boldsymbol{W}}_h)], \tag{9}$$

in which $\nabla f_2(\bar{\boldsymbol{W}}_h) = \dfrac{(\log e)\sum_{k=1,k\neq j}^{K} x_{k,h}\boldsymbol{Q}^{\rm T}_{k;i,j,h}}{\text{tr}\Big[\sum_{k=1,k\neq j}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\bar{\boldsymbol{W}}_h\Big] + \sigma^2_{i,j,h}}$.

According to the above lemma, $R_{i,j}$ has a concave lower bound, which is shown in the following theorem.

Theorem 1: $R_{i,j}$ is lower bounded by

$$\begin{aligned}\tilde{R}_{i,j} =& \sum_{h=1}^{H} B_h \log\left(\text{tr}\Big[\sum_{k=1}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\boldsymbol{W}_h\Big] + \sigma^2_{i,j,h}\right)\\ &- \sum_{h=1}^{H} B_h \log\left(\text{tr}\Big[\sum_{k=1,k\neq j}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\bar{\boldsymbol{W}}_h\Big] + \sigma^2_{i,j,h}\right)\\ &- \sum_{h=1}^{H} B_h \frac{(\log e)\sum_{k=1,k\neq j}^{K} x_{k,h}\text{tr}[\boldsymbol{Q}_{k;i,j,h}(\boldsymbol{W}_h - \bar{\boldsymbol{W}}_h)]}{\text{tr}\Big[\sum_{k=1,k\neq j}^{K} x_{k,h}\boldsymbol{Q}_{k;i,j,h}\bar{\boldsymbol{W}}_h\Big] + \sigma^2_{i,j,h}},\end{aligned} \tag{10}$$

where $\bar{\boldsymbol{W}}_h$ is an arbitrarily given matrix. Furthermore, $\tilde{R}_{i,j}$ is concave with respect to $\boldsymbol{W}_h$, and $\tilde{R}_{i,j} = R_{i,j}$ if $\bar{\boldsymbol{W}}_h = \boldsymbol{W}_h$.

Replacing $R_{i,j}$ by $\tilde{R}_{i,j}$ in Eq. (10) and removing the rank-1 constraint, we can approximately convert problem $(\mathcal{F}_W)$ to

$$(\mathcal{F}_r)\ \max_{\{\boldsymbol{W}_h\},t} \quad t \tag{11}$$

$$\text{s.t.}\ \frac{1}{R^0_{i,j}}\tilde{R}_{i,j} \geqslant t,\ \forall\ i,\ j, \tag{11a}$$

$$\sum_{h=1}^{H} \text{tr}(\boldsymbol{A}_h\boldsymbol{W}_h) \leqslant P_{\max}, \tag{11b}$$

$$\boldsymbol{W}_h \geqslant \boldsymbol{0},\ \forall\ h. \tag{11c}$$

Note that, problem $(\mathcal{F}_r)$ is convex, which can be solved by convex solver CVX. Moreover, constraint set (11a) is a subset of (5a), so the optimal objective value of problem $(\mathcal{F}_W)$ is lower bounded by that of problem $(\mathcal{F}_w)$. We can solve problem $(\mathcal{F}_r)$ iteratively for multiple times to enhance the quality of the solution. According to [13], the iterative procedure can converge to a local minimum or a saddle point.

After getting the optimal solution $\boldsymbol{W}_h$, if $\text{rank}(\boldsymbol{W}_h) = 1$, we can acquire the optimal beamforming vector by using the eigenvalue decomposition. When the obtained solution $\boldsymbol{W}_h$ does not satisfy the rank-1 constraint, the Gaussian randomization method [5] is adopted to generate candidate beamforming vectors. Specifically, with the eigenvalue decomposition given by $\boldsymbol{W}_h = \boldsymbol{U}_h\boldsymbol{\Sigma}_h\boldsymbol{U}_h^{\rm H}$, we can calculate $\boldsymbol{w}_h$ as $\boldsymbol{w}_h = \boldsymbol{U}_h\boldsymbol{\Sigma}_h^{\frac{1}{2}}\boldsymbol{\varphi}_h$, with $\boldsymbol{\varphi}_h \sim CN(\boldsymbol{0}, \boldsymbol{I})$ being a Gaussian random vector. Then, it holds $\mathbb{E}[\boldsymbol{w}_h\boldsymbol{w}_h^{\rm H}] = \boldsymbol{W}_h$. Note that, the power constraint is not considered when generating the candidate beamforming vectors. Hence, according to the beamforming vectors obtained, we rewrite the MMF problem as the following power scaling problem and then get the optimal power scaling factors p_h

$$(\mathcal{F}_p)\ \max_{\{p_h\},t} \quad t \tag{12}$$

$$\begin{aligned}\text{s.t.}\ & \sum_{h=1}^{H} B_h \log\Big(1 + \frac{x_{j,h}p_h|\boldsymbol{g}^{\rm H}_{i,j,h}\boldsymbol{w}_{j,h}|^2}{\sum_{k=1,k\neq j}^{K} x_{k,h}p_h|\boldsymbol{g}^{\rm H}_{i,j,h}\boldsymbol{w}_{k,h}|^2 + \sigma^2_{i,j,h}}\Big)\\ &\quad \geqslant tR^0_{i,j},\ \forall\ i,\ j,\\ &\sum_{h=1}^{H} p_h \sum_{j=1}^{K} x_{j,h}\|\boldsymbol{e}_l\boldsymbol{w}_{j,h}\|^2 \leqslant P_{\max},\ \forall\ l,\\ &p_h \geqslant 0,\ h = 1, \cdots, H.\end{aligned}$$

Problem $(\mathcal{F}_p)$ is a convex problem, which is easily solved by CVX. For each set of the candidate beamforming vectors generated according to $\boldsymbol{W}_h$, we can obtain the optimal power scaling factors p_h, and then choose the set of scaled beamforming vectors $\sqrt{p_h}\boldsymbol{w}_h$ with the best performance as the final solution. We summarize the overall algorithm as Algorithm 1.

Algorithm 1 SCA Based Multicast Beamforming

Input: $\boldsymbol{R}^0$, total transmit power of LEO satellites $P_{\max}$, prescribed threshold ϵ.

Output: Optimal beamforming vectors $\boldsymbol{w}_{j,h}$, $j = 1, \cdots, K$, $h = 1, \cdots, H$.

1: Solve problem $(\mathcal{F}_r)$ to obtain $\boldsymbol{W}_h$.
2: Generate candidate beamforming vectors $\tilde{w}_{j,h}$ using Gaussian randomization method for all rank($\boldsymbol{W}_h$)> 1.
3: Solve problem $(\mathcal{F}_p)$ to obtain $\{p_h\}$.
4: Compute $w_{j,h} = \sqrt{p_h}\tilde{w}_{j,h}$ as the final solution.

The computational complexity of Algorithm 1 mainly depends on solving convex problems $(\mathcal{F}_r)$ and $(\mathcal{F}_p)$, with corresponding computation complexities are $O((K^2M^2 +$

$1)^3\log(\epsilon^{-1}))$ and $O((H+1)^3\log(\epsilon^{-1}))$, where ϵ is the accepted duality gap. Therefore, the total computational complexity of Algorithm 1 is formulated as

$$O\left(((K^2M^2+1)^3+(H+1)^3)\log(\epsilon^{-1})\right). \tag{13}$$

B. Subchannel Allocation Algorithm

In this subsection, we will leverage the matching game to determine the optimal $\boldsymbol{X}$. Since the elements of $\boldsymbol{X}$ are binary variables, the subchannel allocation can be formulated as a matching problem. In our setup, each GU group can be matched with multiple subchannels, and each subchannel can be matched with multiple groups of GUs. Hence, the subchannel allocation problem can be regarded as a many-to-many matching game [14]. For integrity and clarity, we first give the following definition [14].

Definition 1: A mapping μ from set $\mathcal{G}$ to set $\mathcal{C}$ is a many-to-many matching if and only if every $g \in \mathcal{G}$ and $c \in \mathcal{C}$ satisfy: 1) $\mu(g) \subseteq \mathcal{C},\ \forall\ g \in \mathcal{G}$; 2) $\mu(c) \subseteq \mathcal{G},\ \forall\ c \in \mathcal{C}$; 3) $c \in \mu(g)$ if and only if $g \in \mu(c)$; where $\mu(g)$ is the set of partners on c, and $\mu(c)$ is the set of partners for g under μ.

In our scenario, there are K groups of GUs $\mathcal{G} = \{\mathcal{G}_1, \cdots, \mathcal{G}_K\}$, and H subchannels $\mathcal{C} = \{C_1, \cdots, C_H\}$. Considering $\mathcal{G}$ and $\mathcal{C}$ as two sets of players in the many-to-many matching relation, the binary matrix $\boldsymbol{X}$ represents the matching result. Each subchannel can be allocated to multiple GU groups in set $\mathcal{G}$, and each GU group can occupy a subset of subchannels in set $\mathcal{C}$. For each GU group, the preference list refers to the received power from each subchannel. In particular, with the gain of subchannel h for GU i in group j given by $\boldsymbol{g}_{i,j,h}$, we can obtain the received power of GU group j from h as $\min\limits_{i\in[1,G_j]} \|\boldsymbol{g}_{i,j,h}\|^2$.

Use the notation $\Omega' \succ \Omega''$ to indicate that group $g \in \mathcal{G}$ prefers the subchannels in subset Ω' than subset Ω'', where $\Omega' \subseteq \mathcal{C}$ and $\Omega'' \subseteq \mathcal{C}$. We can make an analogy for any subchannel c in set $\mathcal{C}$. The preference of subchannel c is referred to the overall system benefit, i.e., the minimum achievable rate among all GUs, which is defined as

$$thr = \min_{j\in\{1,\cdots,K\}} \min_{i\in[1,G_j]} \frac{1}{R_{i,j}^0} R_{i,j}. \tag{14}$$

If GU group g chooses subchannel c, then c accepts the choice if and only if the system benefit will be improved.

For the subchannel-GU group mapping problem, we aim to seek a stable solution, where there are no unmatched players and they all prefer to be partners. Because the choice of subchannel players is based on the system performance when choosing partners from set $\mathcal{G}$, we can envision the stable solution as the optimal solution for the mapping problem. Considering a large number of players ($\mathcal{G} \cup \mathcal{C}$) in the scenario, it is intractable to identify optimal partners for each player. Hence, we solve the many-to-many matching problem through identifying partners one by one from the opposite set. For the matching model, this manner of choosing partners can lead to pair-wise stability, whose definition is shown as follows. Before that, we first introduce two notations. With a set $\hat{\mathcal{C}} \subseteq \mathcal{C}$ of possible partners, we denote $\mathcal{C}_g(\hat{\mathcal{C}})$ as the subset of set $\hat{\mathcal{C}}$ that a player $g \in \mathcal{G}$ prefers to match with. Similarly, for a set $\hat{\mathcal{G}} \subseteq \mathcal{G}$ of possible partners, denote $\mathcal{G}_c(\hat{\mathcal{G}})$ as the subset of set $\hat{\mathcal{G}}$ that a subchannel $c \in \mathcal{C}$ wishes to match with.

Definition 2: The matching μ is pair-wise stable if and only if there is no a pair of (g, c) with $g \notin \mu(c)$ and $c \notin \mu(g)$ such that for $g' \in \mathcal{G}_c(\mu(c) \cup \{g\})$ and $c' \in \mathcal{C}_g(\mu(g) \cup \{c\})$, both $\{g'\} \succ \mu(c)$ and $\{c'\} \succ \mu(g)$ are satisfied.

Note that the pair-wise stability is important in the many-to-many matching model. We tend to let each player (i.e., GU group or subchannel) choose the partner one-by-one instead of a subset. Algorithm 2 is proposed to achieve the stable many-to-many matching solution.

Algorithm 2 Matching Based Subchannel Allocation

Input: Group set $\mathcal{G}$, subchannel set $\mathcal{C}$.
Output: Optimal subchannel allocation scheme $\boldsymbol{X}^*$.

```
Initialization: ∀g ∈ G, Ω_g = ∅; ∀c ∈ C, G_c = ∅.
For group G_j ∈ G produces its preference list.
for ite = 1, ··· , Ite_max do
   for j = 1, ··· , K do
      c ← Best unallocated subchannel in preference list
      of group G_j (denoted as g).
      Strategy set T = ∅, throughput set T_o = ∅.
      Construct addition strategy t: Add group g to set G_c,
      and insert the strategy t to set T.
      Construct replacement strategy s: Replace g' ∈ G_c
      by g, and insert the strategy t to set T.
      for t ∈ T do
         Update the subchannel set C' for the strategy t.
         Adjust power allocation of g' ∈ G_{c'}, ∀c' ∈ C'.
         thr ← Benefit defined in (14) of all subchannels
         in C' due to strategy t.
         if thr is larger than the benefit of C' before
         applying strategy t then
            Insert strategy t to CT and thr to T_0.
         end if
      end for
      t_Best ← arg max_{t∈CT} T_0
      if t_Best is the addition strategy then
         Ω_g ← Ω_g ∪ {c}, and G_c ← G_c ∪ {g}.
         Adjust power allocation of subchannels in Ω_g,
         update the preference list of group g.
      else
         Ω_g ← Ω_g ∪ {c}, Ω_{g'} ← Ω_{g'}/{c}, and G_c ←
         (G_c/{g'}) ∪ {g}.
         Adjust power allocation of subchannels in Ω_g and
         Ω_{g'}, update preference lists of g and g'.
      end if
      Terminate this loop when t_Best is empty.
   end for
   Stop the iteration until thr no longer changes.
end for
```

Based on [14], we have the following result of Algorithm 2.

Theorem 2: Algorithm 2 can be guaranteed to achieve a pair-wise stable matching solution in finite iterations.

According to [14], the many-to-many matching algorithm admits a polyhedral time complexity, which is much lower than the brute-force searching operation.

IV. Numerical Simulations

In the simulation part, numerous numerical simulation results are given to validate the performance of the proposed subchannel allocation and beamforming scheme.

With the downlink multicast scenario in the ultra-dense LEO satellite network, we assume that there are $M = 6$ LEO satellites with overall $M = 6$ antennas simultaneously serving $K = 3$ GU groups in a given time period. The corresponding parameters of LEO satellites are the same as in [11], where the total bandwidth is set to $B = 500$ MHz. The bandwidth is evenly partitioned over $H = 5$ subchannels. Then, the bandwidth in each subchannel is derived by $B_h = B/H$. Hence, the AWGN power is $\sigma^2_{i,j,h} = B_h N_0$ with $N_0 = -174$ dBm/Hz being the AWGN power spectral density. Suppose that the maximum transmit power of each LEO satellite is $P_{\max} = 50$ dBm. Moreover, the number of GUs in each group is assumed to be the same and $G_k = G = 3$. The target achievable rate of each GU is set to be $R^0_{i,j} = 1$.

To measure the system performance, we give the definitions

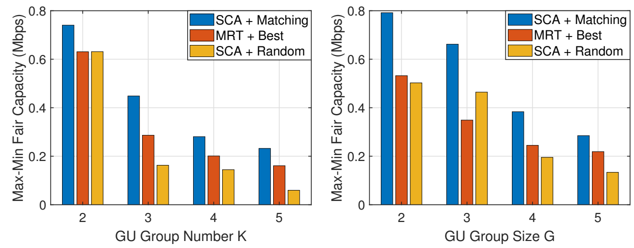
$$C_j = \min_{i \in [1, G_j]} \frac{1}{R^0_{i,j}} R^*_{i,j} \quad \text{and} \quad C = \min_{j \in \{1, \cdots, K\}} C_j$$

as the MMF capacity of group j and the system MMF capacity respectively, where $R^*_{i,j}$ is the achievable rate with corresponding beamforming and subchannel selection. Obviously, the higher the MMF capacity is, the better performance the method admits. To validate the superiority of the proposed algorithm, we make comparisons of the proposed method (called "SCA+Matching") with two benchmark methods (called "SCA+Random" and "MRT+Best", respectively). For the SCA+Random method, the subchannel allocation matrix $\boldsymbol{X}$ is a randomly generated binary matrix, while the beamforming vectors are derived by Algorithm 1. Meanwhile, we randomly generate the subchannel allocation matrix $\boldsymbol{X}$ for 10 times to compute the average. For the MRT+Best method, each group selects the subchannel with the best channel condition, and each beamforming vector is generated by the maximum ratio transmission (MRT) [15], which is given by

$$\boldsymbol{w}_{j,h} = \sqrt{P^{\text{ave}}_{h,j}} \frac{\boldsymbol{e}_l \boldsymbol{v}_{j,h}}{\|\boldsymbol{e}_l \boldsymbol{v}_{j,h}\|},$$

where $\boldsymbol{v}_{j,h} = \boldsymbol{g}_{i_0,j,h}$ with $i_0 = \arg\max_i \|g_{i,j,h}\|^2$, and $P^{\text{ave}}_{h,j} = P_{\max}/(H_0 N^h_j)$ with H_0 being the number of used subchannels and N^h_j is the number of groups served over subchannel h.

For different group number $K = \{2, 3, 4, 5\}$, we set other parameters as default. As observed in Fig. 2(a), the performance of the proposed SCA+Matching method is better than those of the MRT+Best method and the SCA+Random method. Meanwhile, the system MMF capacities of the three methods are in decreasing trends with the increase of GU group number. The decreasing trend can be accounted for more inter-group interference from more groups, which deteriorates the system performance. Similarly, with other parameters

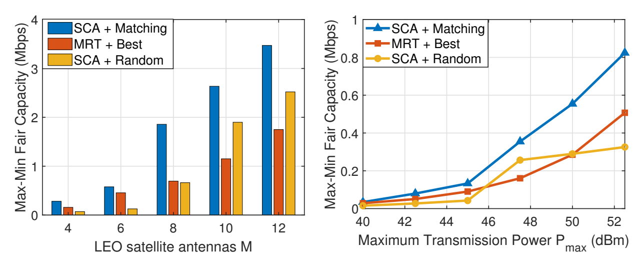


(a) Different group numbers. (b) Different group sizes.

Fig. 2. Comparison of three methods with different K and G.

given by fault, we consider different numbers of GUs in each group as $G = \{2, 3, 4, 5\}$. It is easily observed from Fig. 2(b) that the SCA+Matching method is superior to the other two methods for all cases. Furthermore, all the methods have a same trend in terms of the system MMF capacity. By combining Fig. 2(a) and Fig. 2(b), it illustrates that the system MMF capacity decreases fast when the GU scale exceeds the number of antennas.

For $M = \{2, 4, 6, 8, 10\}$, we display the system MMF capacities of the SCA+Matching method, the SCA+Random method and the MRT+Best method in Fig. 3(a). For the five cases, the system performance of the SCA+Matching method exceeds those of the other two methods. It is easy to see that, with the increase of antennas M, the system MMF capacity also becomes larger for the three methods. This is because larger LEO satellite antenna number M will lead to less inter-group interference to GUs. Fig. 3(b) displays



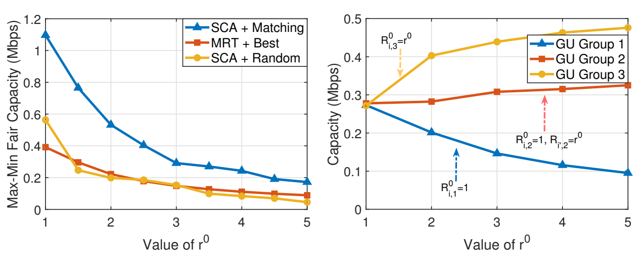
(a) Different LEO satellite antennas. (b) Different maximum power.

Fig. 3. Comparison of three methods with different M and $P_{\max}$.

the comparison of the SCA+Matching method, MRT+Best method, and SCA+Random method with 7 different cases of $P_{\max} = \{40, 42.5, 45, 47.5, 50, 52.5\}$ dBm. First, the system MMF capacity of the SCA+Matching method is better than the other two methods. Meanwhile, when $P_{\max}$ is increasing, the system MMF capacities of the three methods also increase. Since larger $P_{\max}$ leads to a larger achievable rate for each GU, the MMF capacity is naturally larger.

To show the impact of different target achievable rate R_0

on the system performance, we set $M = 6$, $H = 5$, $K = 3$, and $G = 4$. With other parameters fixed, the optimal MMF capacity only depends on the relative value of $R^0_{i,j}$ of GUs. The basic value is set to $R^0_{i,j} = 1$. For a given r^0, we set the target achievable rates of the three groups as follows: (1) In group 1, we set $R^0_{i,1} = 1$ for all GUs; (2) In group 2, we set $R^0_{i,2} = 1$ for half of GUs, and $R^0_{i',2} = r^0$ for the rest of GUs; (3) In group 3, we set $R^0_{i,3} = r^0$ for all GUs. In order to investigate the system MMF capacity with different target achievable rates, we consider 9 cases of r^0 as $r^0 = \{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$. Fig. 4(a) exhibits that, the system performance of the SCA+Matching method is much larger than the other two methods for all cases. Meanwhile, the system MMF capacities of the three methods are decreasing with the increase of r^0. Since it is the GU with the worst achievable rate that determines the capacity of each group, the target achievable rate of each GU in the group will contribute to the actual received achievable rate cumulatively. A larger r^0 will lead to more resources allocated to GUs with higher target achievable rate, and then the achievable rate of the worst GU will decrease, and thus the system capacity becomes lower. Moreover, we depict the capacity of each group in



(a) System Performance comparison. (b) Group performance comparison.

Fig. 4. Comparison of different r^0.

Fig. 4(b) for cases of $r^0 = \{1, 2, 3, 4, 5\}$. Note that group 3 and group 1 have the highest and lowest target achievable rates, respectively. As can be seen from Fig. 4(b), the capacity of group 3 is the largest, while that of group 1 is the worst. Moreover, with the increase of r^0, the capacities of group 2 and group 3 are increasing, whereas the capacity of group 1 is decreasing. Since the system resources are mainly allocated to GUs in group 2 and 3 with higher target achievable rates, which leads to the capacity of group 1 decreasing.

V. Conclusion and Future Work

In this paper, we have addressed the downlink cooperative multigroup multicast transmission for GUs requiring different contents in the ultra-dense LEO satellite network. Specifically, by leveraging the SCA and matching game methodologies, we have proposed an effective iterative algorithm to maximize the system MMF capacity by jointly optimizing the beamforming vector and the subchannel allocation matrix. Numerical results have demonstrated the effectiveness and the superiority of the proposed method. The work could provide useful guidelines for the downlink cooperative multigroup multicast transmission in ultra-dense LEO satellite networks. In future work, we can further investigate the multigroup multicast transmission together with subchannel allocation in the space-air-ground integrated network to provide efficient service for GUs.

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