# Energy Efficiency Optimization for Integrated Sensing and Communications Systems

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Abstract—In this paper, we consider an energy efficient waveform design in integrated sensing and communication (ISAC) systems. The transmitted waveform simultaneously serves multiple communication users and estimates the parameters of a moving target. In order to improve its energy efficiency (EE) while guaranteeing target estimation performance, we maximize the EE of the emitted dual-use waveform, under a Cramér-Rao bound (CRB) constraint. However, the considered optimization problem is a fractional function that is highly non-convex. Thus, we firstly adopt fractional programming based on Dinkelbach' method and then, solve the sub-problem by leveraging semi-definite relaxation (SDR). Numerical results demonstrate superior performance than the benchmark and show the trade-off between EE and CRB.

Index Terms—Integrated sensing and communication, energy efficiency maximization, non-convex optimization

# I. INTRODUCTION

Integrated sensing and communication (ISAC) is considered as an essential technology for future wireless networks, as its high capability of combining communication and sensing systems to efficiently exploit wireless resources, and even to pursue mutual benefits. The intention of ISAC is that a radio emission might transfer communication data while also simultaneously extracting environmental data from dispersed echoes [1]. Here we focus on the joint waveform design problem. In [2], the dual-functional beamformer matrix are formulated where the multi-user interference is minimized as well as formulating an appropriate radar beampattern. Also, the trade-off between communication and radar is studied. Recent work in [3] considers Cramér-Rao bound (CRB) minimization with guaranteed SINRs for multiple communication users and solves the non-convex problems.

On the other hand, an increasing demand of data traffic and communication services brings about a rapid growth of energy consumption in wireless networks in recent years, which has led to higher cost in the devices and infrastructure, also causing environment and health problems in the daily life. A consensus has emerged that the energy efficiency (EE) should be increased in data transmission rather than simply raising the transmit power [4].

In order to increase the EE performance in wireless networks, various wireless transmission algorithms and signal processing techniques have been proposed. Generally, EE maximization can be considered as balancing the trade-off between the total power consumption and the sum rate of

users, which is widely investigated in various scenarios [5]. It is proved that EE optimization is an efficient solution to the surging demand of communication services in various scenarios, but the EE optimization of ISAC waveform [6]–[8] is not fully investigated.

Although the existing joint waveform designs achieve favorable performance trade-offs between sensing and communications, the EE in ISAC is not fully-investigated in the existing works. Different from the EE maximization in traditional communication systems whose performance is generally guaranteed by the SINR or transmission rate constraints, designs in ISAC system should take both communication performance and radar sensing performance into account. As a consequence, the performance constraint of target estimation should also be especially considered in the EE problem design. While the EE optimization in the scenario of radar-communication spectrum sharing is studied in [9], where a MIMO communication system is considered with the interference caused by a surveillance radar, it only considers the EE under spectrum interference, but does not consider the waveform design of dual-functional transmitter. In dual-functional transmitters, the waveform design should guarantee the communication and sensing performance at the same time, which is different from the design of communication waveforms.

In this paper, we propose an energy-efficient waveform in the case of joint design in ISAC architecture. Particularly, an communication-centric EE maximization optimization problem is formulated and constrained by the power budget and CRB limitations, in order to guarantee the target estimation performance of the radar. However, the optimization problem is non-convex and hard to track, due to the fractional objective function and the CRB constraints. To tackle this problem, we adopt nonlinear fractional programming based on Dinkelbach's method. Then, we relax the CRB constraint by leveraging Schur complement condition and semi-definite relaxation (SDR). Simulations under different system parameter settings demonstrate its effectiveness performance on EE compared with sum power minimization method. Also, the trade-off between CRB and communication-centric EE is shown in our experiments.

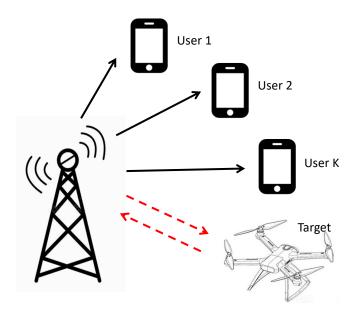


Fig. 1. An example of an ISAC system.

#### II. SYSTEM MODEL

We consider a dual-functional MIMO base station (BS) with M element antenna array, broadcasting M downlink narrowband communication signals to K single-antenna users, while simultaneously tracking a point-like target from its reflected echoes, as shown in Fig 1.

#### A. Communication Model

Let  $\mathcal{K} \triangleq \{1, \dots, K\}$  denotes the communication user set. Then, we can denote the user k's received signals at time slot n as

$$y_k[n] = \mathbf{h}_k^T \mathbf{w}_k s_k[n] + \sum_{\substack{k \in \mathcal{K} \\ j \neq k}} \mathbf{h}_k^T \mathbf{w}_j s_j[n] + z_c[n], \quad (1)$$

where  $s_k[n]$  represent the data symbol intended for user k, with unit power  $|s_k[n]|^2=1, \forall k\in\mathcal{K}$ . Vectors  $\mathbf{h}_k\in\mathbb{C}^{M\times 1}$ ,  $\mathbf{w}_k\in\mathbb{C}^{M\times 1}$  represent the communication channel vector and the beamforming vector, respectively. The white noise  $z_c[n]$  obeys complex Gaussian distribution with variance  $\sigma_c^2$ . After beamforming, the transmitted waveform for user k can be given as a M dimensional vector,

$$\mathbf{x}_k[n] = \mathbf{w}_k s_k[n]. \tag{2}$$

We assume that the channel matrix is perfectly estimated, which is known by both the users and BS. By defining  $M \times K$  dimensional beamforming matrix  $\mathbf{W} \triangleq [\mathbf{w}_1, \cdots, \mathbf{w}_k]$  at BS side, the SINR at user k, denoted as  $\gamma_k$  is,

$$\gamma_k(\mathbf{W}) = \frac{\left|\mathbf{h}_k \mathbf{w}_k\right|^2}{\sigma_c^2 + \sum_{\substack{k \in \mathcal{K} \\ j \neq k}} \left|\mathbf{h}_k \mathbf{w}_j\right|^2},$$
 (3)

On the basis of SINR, we give the data rate of user k as,

$$R_k(\mathbf{W}) = \log(1 + \gamma_k(\mathbf{W})). \tag{4}$$

### B. Sensing Model

We assume that the BS is collecting target echos in N time slots for sensing usage. When the transmitted waveform is formed towards a target at direction  $\theta$ , the corresponding steering vector is denoted as  $\mathbf{a}(\theta) \in \mathbb{C}^{M \times 1}$ . Considering the BS working under monostatic sensing mode, the corresponding array response matrix is represented as

$$\mathbf{A}(\theta) = \mathbf{a}(\theta)\mathbf{a}^{T}(\theta),\tag{5}$$

where  $(\cdot)^T$  denotes transpose operation. Note that  $\theta$  denotes the azimuth angle of the target relative to BS, which depends the elements of  $\mathbf{A}(\theta)$ . Then, we can represent the target echoes at BS receiver by,

$$\mathbf{y}_T[n] = \alpha \mathbf{A}(\theta) \mathbf{x}[n] + \mathbf{z}_T[n], \quad n = 1, ..., N, \tag{6}$$

where  $\alpha$  is the roundtrip path-loss of the received sensing waveform, and  $\mathbf{z}_T[n]$  denotes the additive Gaussian write noise with variance  $\sigma_T^2$ . For simplicity, the receive signals can be recast as a matrix form,

$$\mathbf{Y}_T = \alpha \mathbf{A}(\theta) \mathbf{X} + \mathbf{Z}_T \tag{7}$$

where  $\mathbf{Y}_T = [\mathbf{y}_T[1], \mathbf{y}_T[2], \cdots, \mathbf{y}_T[n]] \in \mathbb{C}^{M \times N},$   $\mathbf{X} = [\mathbf{x}[1], \mathbf{x}[2], \cdots, \mathbf{x}[n]] \in \mathbb{C}^{M \times N}, \quad \mathbf{Z} = [\mathbf{z}_T[1], \mathbf{z}_T[2], \cdots, \mathbf{z}_T[n]] \in \mathbb{C}^{M \times N}$  denote the target response matrix, the transmit sensing waveform matrix and noise matrix, respectively.

Before going further, we provide the coherent matrix of transmitted sensing waveform as,

$$\mathbf{R}_X = \frac{1}{N} \mathbf{X} \mathbf{X}^H = \frac{1}{N} \mathbf{W} \mathbf{S} \mathbf{S}^H \mathbf{W}^H = \mathbf{W} \mathbf{W}^H = \sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H$$
(8)

where  $(\cdot)^H$  stands for Hermitian transpose and  $\mathbf{S} \in \mathbb{C}^{K \times N}$  whose (k,n) entry is  $s_k[n]$ . For target estimation that we consider in this model, CRB offers the theoretical lower bound on the variance of unbiased estimators, which is widely adopted as the sensing metric of the parameter estimation performance. Since the CRBs of  $\theta$  and  $\alpha$  have the similar forms [3], in this paper we only consider  $\mathrm{CRB}(\theta)$  which is derived in [10], as shown in Equation (9), where  $\dot{\mathbf{a}}(\theta)$  denotes the derivative of  $\mathbf{a}(\theta)$ .

# C. Energy Model

Traditionally, EE tend to be defined as a ratio of transmission sumrate  $\sum_k R_k(\mathbf{W})$  to the total power consumption P,

$$EE = \frac{\sum_{k} R_k(\mathbf{W})}{P}.$$
 (10)

For the total power consumption, the linear power model is adopted in this paper where P is represented as,

$$P = \frac{1}{\epsilon} P_d + P_0, \tag{11}$$

where the power amplifier efficiency  $\epsilon \in [0,1]$  and the circuit power  $P_0$  is consumed by circuit blocks, power supply, cooling system, etc. When the transmitted signal power is

$$CRB(\theta) = \frac{\sigma_T^2}{2N \left|\alpha\right|^2 \left(M\dot{\mathbf{a}}^H(\theta)\mathbf{R}_{\mathbf{x}}^T\dot{\mathbf{a}}(\theta) + \mathbf{a}^H(\theta)\mathbf{R}_{\mathbf{x}}^T\mathbf{a}(\theta) \left\|\dot{\mathbf{a}}(\theta)\right\|^2 - \frac{M|\mathbf{a}^H(\theta)\mathbf{R}_{\mathbf{x}}^T\dot{\mathbf{a}}(\theta)|^2}{\mathbf{a}^H(\theta)\mathbf{R}_{\mathbf{x}}^T\mathbf{a}(\theta)}\right)},\tag{9}$$

unit, the transmit power consumption can be given by  $P_d =$  $\sum_{k} \|\mathbf{w}_{k}\|_{2}^{2}$ . Hence, we obtain the EE of the to-be-designed ISAC waveform as our objective function,

$$EE = \frac{\sum_{k} R_k(\mathbf{W})}{\frac{1}{\epsilon} \sum_{k} \|\mathbf{w}_k\|_2^2 + P_0}.$$
 (12)

#### III. PROBLEM FORMULATION

In this section, we formulate ISAC waveform design problem based on the EE maximization objective function. The objective of this paper is to maximize the EE defined in (12). Under a given power limitation and the CRB constraint, the problem is formulated as follows,

$$\max_{\{\mathbf{w}_k\}_{k=1}^K} EE = \frac{\sum_k R_k(\mathbf{W})}{\frac{1}{\epsilon} \sum_k \|\mathbf{w}_k\|_2^2 + P_0}$$

$$s.t. \sum_k \|\mathbf{w}_k\|_2^2 \le P_{\text{max}},$$
(13a)

s.t. 
$$\sum_{k} \|\mathbf{w}_k\|_2^2 \le P_{\text{max}},$$
 (13b)

$$CRB(\theta) \le \rho,$$
 (13c)

$$\mathbf{R}_{\mathbf{X}} = \sum_{k=1}^{K} \mathbf{w}_k \mathbf{w}_k^H, \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_K],$$
(13d)

where  $P_{\text{max}}$  denotes the maximum transmission power and (13b) is the power constraints. To guarantee the performance of target estimation, we set a pre-define parameter  $\rho$  as the upper limit of CRB and formulate the CRB constraints in (13c).

To tackle with problem (13), it can be noted that both the optimization objective and the constraint (13c) are non-convex owing to their fractional structure.

#### IV. PROBLEM SOLUTION

To tackle with the non-convexity of problem (13), we leverage non-linear fractional programming that has been proved as a successful approach in the field of wireless communications [11], [12]. In this section, we first solve the non-convexity of the fractional structure in objective function by fractional programming; and then we focus on relaxing the sub-problem to a convex program by exploiting SDR technique.

# A. Nonlinear Fractional Programming

Here we firstly deals with the objective function in (13) by ignoring its constraints. For simplicity, we denote  $g_1(\mathbf{W}) \triangleq$  $\sum_{k} R_k(\mathbf{W}), g_2(\mathbf{W}) \triangleq \frac{1}{\epsilon} \sum_{k} \|\mathbf{w}_k\|_2^2 + P_0$ ; then, the objective function can be formulate as

$$\max_{\{\mathbf{w}_k\}_{k=1}^K} \frac{g_1(\mathbf{W})}{g_2(\mathbf{W})},\tag{14}$$

where  $g_1, g_2: S \to \mathbb{R}$ . Problem (14) is indeed a concaveconvex fractional program [13], where both function  $g_1$  and  $g_2$ are concave. The optimizing variable  $\{\mathbf{w}_k\}_{k=1}^K \in \mathbb{R}$  is indeed in a convex set with respect to W1. By introducing a new variable  $\lambda$ , we recast problem (14) as the following equivalent

$$\max_{\{\mathbf{w}_k\}_{k=1}^K, \lambda \in \mathbb{R}} \lambda \tag{15a}$$

s.t. 
$$\frac{g_1(\mathbf{W})}{g_2(\mathbf{W})} - \lambda \ge 0,$$
 (15b)  
(13b), (13c), (13d).

Then, the problem can be given by

$$\max_{\{\mathbf{w}_k\}_{k=1}^K, \lambda \in \mathbb{R}} \lambda \tag{16a}$$

$$s.t. \ g_1(\mathbf{W}) - \lambda g_2(\mathbf{W}) \ge 0, \tag{16b}$$

s.t. 
$$g_1(\mathbf{W}) - \lambda g_2(\mathbf{W}) \ge 0,$$
 (16b)  
(13b), (13c), (13d).

We further observe that, for a given value of  $\lambda$ , if  $g_1(\mathbf{W})$  –  $\lambda g_2(\mathbf{W}) \geq 0$ , the optimization problem is feasible over **W**. In particular,  $\mathbf{W}^*$  is optimal for (14) if and only if it is optimal for (16) with optimal  $\lambda^*$ . Let  $G(\lambda) = g_1(\mathbf{W}) - \lambda g_2(\mathbf{W})$ . It's obvious that  $G(\lambda)$  is convex, continuous and strictly decreasing in  $\lambda$  [11] and that  $\lambda^*$  is the unique zero of  $G(\lambda) = \max_{\{\mathbf{w}_k\}_{k=1}^K}$ . Then, solving problem (16) is equal to find the unique zero of  $G(\lambda)$ , denoted as  $G(\lambda^*) = g_1(\mathbf{W}) - \lambda^* g_2(\mathbf{W}) = 0$ . Note that the initial value of  $\lambda$ , denoted as  $\lambda_0$ , should satisfy  $G(\lambda_0) \geq 0$ . Thus, the sub-optimization problem is recast as,

$$\max_{\{\mathbf{w}_k\}_{k=1}^K} \sum_{k} R_k(\mathbf{W}) - \lambda \left(\frac{1}{\epsilon} \sum_{k} \|\mathbf{w}_k\|_2^2 + P_0\right)$$
 (17)   
s.t. (13b), (13c), (13d).

In Algorithm 1, we give the details of the iterative process to solve problem (18).

Algorithm 1: The Dinkelbach's Algorithm for EE Optimization

set 
$$j=0, \, \delta>0, \mathbf{W}_0\in\mathcal{S}$$
  
Initialize  $\lambda_0$  satisfying  $g_1(\mathbf{W}_0)-\lambda g_2(\mathbf{W}_0)\geq 0$   
**repeat**

$$j\leftarrow j+1\;;$$

$$\lambda_j\leftarrow \frac{g_1(\mathbf{W}_{j-1})}{g_2(\mathbf{W}_{j-1})}\;;$$
Solve sub-problem (17) to obtain  $\mathbf{W}_j\;;$ 
**until**  $g_1(\mathbf{W}_j)-\lambda_jg_2(\mathbf{W}_j)$  is less than  $\delta.$ 

<sup>&</sup>lt;sup>1</sup>Here,  $g_1 \ge 0$ . Otherwise  $g_2$  should be affine.

# B. Solution of Sub-Problem

In this subsection, we relax (17) based on the well-known semi-definite relaxation. Recall  $h_k$  is perfectly known by the transmitter and let  $\mathbf{Q}_k = \mathbf{h}_k \mathbf{h}_k^H$ , it is derived in [14] that  $\gamma_k(\mathbf{W}) = \mathbf{w}_k^H \mathbf{Q}_k \mathbf{w}_k$ . Therefore, problem (17) can be rewritten as,

$$\max_{\{\mathbf{w}_k\}_{k=1}^K} \quad \sum_{k} \log(1 + \mathbf{w}_k^H \mathbf{Q}_k \mathbf{w}_k) - \lambda \sum_{k} \|\mathbf{w}_k\|_2^2 \quad (18)$$

$$s.t. \quad \sum_{k} \|\mathbf{w}_k\|_2^2 \le P_{\text{max}},$$

$$(13c), \mathbf{R}_X = \sum_{k}^K \mathbf{w}_k \mathbf{w}_k^H,$$

By defining  $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ , the objective function can be reformulated as

$$\max_{\mathbf{W}_{k}} \sum_{k} \log(1 + \operatorname{tr}(\mathbf{Q}_{k} \mathbf{W}_{k})) - \lambda \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_{k})$$

$$s.t. \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_{k}) \leq P_{\max},$$

$$(13c), \mathbf{R}_{X} = \sum_{k=1}^{K} \mathbf{w}_{k} \mathbf{w}_{k}^{H}.$$

Note that here the rank constraint of  $\operatorname{rank}(\mathbf{W}_k) = 1$  is dropped to obtain a relaxed convex problem. Then, we concentrate on solving the non-convexity of constraint (13c). First, we rewritten (13c) as

$$\mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) - \frac{M \left| \mathbf{a}^{H}(\theta) \mathbf{R}_{\mathbf{x}}^{T} \dot{\mathbf{a}}(\theta) \right|^{2}}{\mathbf{a}^{H}(\theta) \mathbf{R}^{T} \mathbf{a}(\theta)} - \Gamma \ge 0, \qquad (20)$$

where

$$\mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) \triangleq M\dot{\mathbf{a}}^{H}(\theta)\mathbf{R}_{\mathbf{x}}^{T}\dot{\mathbf{a}}(\theta) + \mathbf{a}^{H}(\theta)\mathbf{R}_{\mathbf{x}}^{T}\mathbf{a}(\theta) \|\dot{\mathbf{a}}(\theta)\|^{2},$$
(21)

and,

$$\Gamma = \frac{2N\rho \left|\alpha\right|^2}{\sigma_z^2}.$$
 (22)

By leveraging Schur complement condition, (20) is equivalent to the following:

$$\begin{bmatrix} \mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) - \Gamma & M\mathbf{a}^{H}(\theta)\mathbf{R}_{\mathbf{x}}^{T}\dot{\mathbf{a}}(\theta) \\ M\dot{\mathbf{a}}^{H}(\theta)\mathbf{R}_{\mathbf{x}}^{T}\mathbf{a}(\theta) & \mathbf{a}^{H}(\theta)\mathbf{R}_{\mathbf{x}}^{T}\mathbf{a}(\theta) \end{bmatrix} \succeq 0. \tag{23}$$

Therefore, the sub-problem (13) can be recast as

$$\max_{\mathbf{W}_{k}} \sum_{k} \log(1 + \operatorname{tr}(\mathbf{Q}_{k}\mathbf{W}_{k})) - \lambda \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_{k})$$

$$s.t. \sum_{k=1}^{K} \operatorname{tr}(\mathbf{W}_{k}) \leq P_{\max},$$

$$\left[ \mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) - \Gamma - M\mathbf{a}^{H}(\theta) \sum_{k=1}^{K} \mathbf{W}_{k}^{T} \dot{\mathbf{a}}(\theta) \right]$$

$$(24)$$

$$\begin{bmatrix} \mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) - \Gamma & M\mathbf{a}^H(\theta) \sum\limits_{k=1}^K \mathbf{W}_k^T \dot{\mathbf{a}}(\theta) \\ M\dot{\mathbf{a}}^H(\theta) \sum\limits_{k=1}^K \mathbf{W}_k^T \mathbf{a}(\theta) & \mathbf{a}^H(\theta) \sum\limits_{k=1}^K \mathbf{W}_k^T \mathbf{a}(\theta) \end{bmatrix} \succeq 0,$$

where  $\mathcal{F}(\mathbf{R_X}, \mathbf{a}(\theta))$  is rewritten with  $\mathbf{W}_k$  as Equation (25). Problem (24) is convex and can be solved by using the well-known CVX toolbox [15]. It's worth noting that the optimal point of the relaxed problem may not be necessarily identical with the original optimization problem. However, one can find the rank 1 solution by employing eigenvalue decomposition and then, choosing the eigenvector corresponding to the maximum eigenvalue as the kth beamformer.

#### V. SIMULATION RESULTS

For numerical simulations, we consider a dual-functional BS equipped with M=16 antennas, with the frame length N set to 30. The maximum transmit power is  $P_T=30$  dBm. The power amplifier efficiency is  $\epsilon=0.35$  and circuit power consumption is  $P_0=33$  dBm. For the target estimation of radar, the target angle is  $\theta=0^\circ$ . We adopt sum power minimization algorithm as the benchmark whose objective function is the transmission power and the constraints are the same with the proposed algorithm.

Fig. 2 demonstrates the algorithm iteration of the proposed EE optimization with random beamforming initialization. We evaluate the proposed algorithm as illustrated in Algorithm. 1 by iteratively solving the sub-problem (24) for each updated  $\lambda$ . Over 100 random beamforming initializations are tested and we choose the highest value of EE as the optimal result. By the curve of example initializations, we can see that the proposed algorithm tends to converge after 10-th iteration. It is also proved in [11] that  $\lambda_j$  is able to converge to  $\lambda^*$  at a super-linear convergence rate. Despite the fact that the achievable EE performance and the convergence speed may be slightly influenced by different initialization status, the proposed algorithm converges to over 95% of the optimal performance in most cases, indicating the near-optimal EE performance.

In Fig. 3, EE of our proposed algorithm is compared with the transmission power minimization algorithm. EE of the proposed algorithm is significantly higher than that of the benchmark. Also, When the transmit power increases, EE of the proposed algorithm tend to be saturated. Besides,

$$\mathcal{F}(\mathbf{R}_{\mathbf{X}}, \mathbf{a}(\theta)) = M\dot{\mathbf{a}}^{H}(\theta) \sum_{k=1}^{K} \mathbf{W}_{k}^{T} \dot{\mathbf{a}}(\theta) + \mathbf{a}^{H}(\theta) \sum_{k=1}^{K} \mathbf{W}_{k}^{T} \mathbf{a}(\theta) \|\dot{\mathbf{a}}(\theta)\|^{2}.$$
(25)

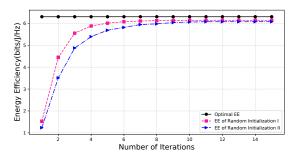


Fig. 2. Convergence behavior with different values of random beamforming initialization.

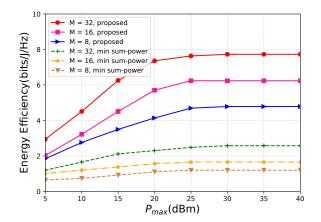


Fig. 3. EE comparison versus  $P_{\text{max}}$  for different M.

their EE performance with different numbers of antennas is also illustrated here. Furthermore, the EE values tend to increase with the increasing number of BS antennas, which demonstrates the competitiveness in massive-antenna systems.

The average EE versus different  $P_0$  for different optimization designs is studied in Fig. 4. It's observed that the proposed algorithm offers significant gains over the sum power minimization criteria on EE, especially when  $P_0$  is small. When the circuit power is relatively low, the algorithm tends to use lower transmit power to increase EE because the transmission data power,  $P_d$ , accounts for the majority of the total power consumption P. In such cases, the results are similar to the sum power minimization algorithm which encourages the BS to transmit at a low power level. As  $P_0$  increase, the gap between between two methods reduces since the circuit power consumption has a significant impact on EE.

In Fig. 5, we study the trade-off between EE and the CRB constraint. The EE of both algorithms increase with the increasing CRB thresholds. As for sum power minimization method, despite the increase of CRB thresholds, EE remains at a lower level.

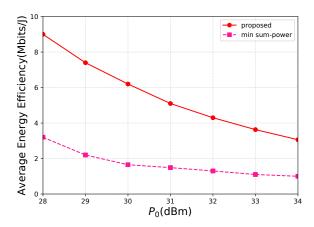


Fig. 4. The EE comparison of the proposed algorithm and sum power minimization algorithm versus the different circuit power  $P_0$ .

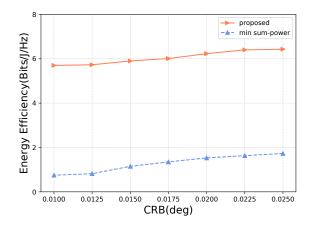


Fig. 5. Trade-off between the CRB and the EE, compared with the sum power minimization algorithm.

# VI. CONCLUSION

In this paper, we study the ISAC waveform design in the scenario of the downlink multi-user communication and downlink sensing. We aim to maximize EE at the communication side while guarantees target CRB at the sensing side. However, the problem is obviously non-convex and hard to solve. To deal with this issue, we firstly employ the fractional programming technology based on Dinkelbach's algorithm. The subsequent sub-problem is relaxed by semi-definite relaxation technology. Numerical results demonstrate the advantages of proposed algorithm than the sum power minimization algorithm under different settings.

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