A Backbone-Listener Relative Localization Scheme for Distributed Multi-agent Systems

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***Abstract*—Reliable and accurate localization awareness is of great importance for the distributed multi-agent system (D-MAS). Instead of global information, measurements only between neighbors pose locatability and accuracy challenges for distributed systems, which leads to the development and application of relative localization. In this paper, we put forward a backbone-listener localization scheme for the D-MAS. Agents switch backbone-listener modes through a node selection strategy. Position and orientation angle of agents are jointly estimated by range and angle information fusion. A distributed multidimensional scaling method is proposed for backbone agents to maintain the topology estimation. And listener agents ensure the localization capacity through a least square range and angle fusion algorithm. Extensive simulation and real-world experiments validate that our method achieves decimeter-level accuracy relative localization.**

***Index Terms*—cooperative localization, distributed multi-agent system, hardware implementation.**

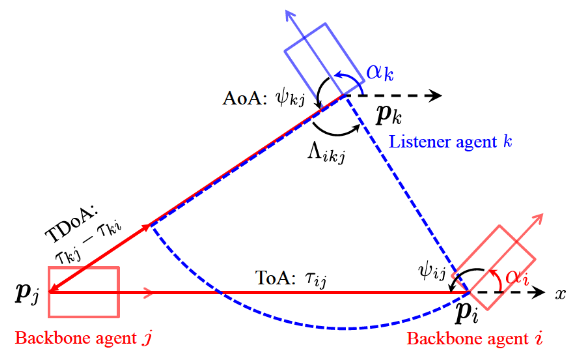

Fig. 1. Illustration of the proposed backbone-listener scheme, where two red backbone agents i and j are transmitting signals to obtain ToA and AoA measurements, while the blue listener agent k only receives signals to obtain TDoA and AoA measurements.

I. Introduction

During the past decades, the distributed multi-agent system (D-MAS) has experienced tremendous development in both academic and industrial applications, due to its potential applications in 6G wireless systems. With multiple cooperative agents, the D-MAS can achieve better performance than a single agent system. In a D-MAS, localization awareness has become a prerequisite and an important guarantee [1], and numerous relative localization methods have been proposed to provide high-precision and low-latency localization capability.

The most common-used localization method is the global navigation satellite system (GNSS). But there are a lot of GNSS-denied scenarios for confidential tasks in harsh and sheltered environments [2]. Generally, the D-MAS can achieve location awareness through radio communications with the aid of cooperative localization techniques [3]. Some common positional methods fuse inter-user measurements, e.g., ranges and angles among agents, to perform spatial cooperation with other nodes [4]. .Nonetheless, one main challenge is the high-density deployment which is too cost-prohibitive to implement in real systems. Therefore, high accuracy localization without infrastructure requirements has received considerable attention in recent years.

For infrastructure-free scenarios, optimization methods based on multidimensional scaling (MDS) and least square (LS) are capable of giving an overall relative estimation of positional relationships. However,previous studies show that MDS-based and LS-based algorithms are accomplished in a centralized manner. For a D-MAS, agents can only get measurements among neighborhoods, which leads to some refinements of distributed MDS estimation. In [5], a distributed weighted MDS algorithm called dwMDS(E) is proposed, which adaptively emphasizes the lowest relative error within the sensor networks. However, a large number of communication links and high-complexity calculations limit the use of MDS algorithms in large-scale systems. Some LS optimization methods have been proposed to complete the information fusion of range and angle measurements. Y.T. Chan [6] proposed an approach for localizing a source from a set of hyperbolic curves defined by time-difference-of-arrival (TDoA) measurements. In [7], the LS optimization is extended to ranging and angle fusion with a linearization of measurement noises. Nevertheless, most mentioned-above localization methods are with position-known anchor references, and are not capable of giving the orientation estimation. There still needs improvements in terms of the link reduction and distributed execution for relative localization.

In this paper, we propose a backbone-listener scheme for distributive localization in large-scale MAS. Agents in large-scale networks complete the position and orientation estimation simultaneously, and adaptively switch backbone-listener modes to maintain the localization capacity and accuracy. Numerical results of both simulations and experiments demonstrate that the proposed scheme achieves decimeter-level relative localization accuracy.

II. Problem Formulation

A. Network Setting

Considering a two-dimensional MAS that consists of N agents whose set is denoted as $\mathcal{N}$. The global position of agent i is denoted as $\boldsymbol{p}_i = [x_i,\ y_i]^{\mathrm{T}}$, and the global position parameter vector is denoted as $\boldsymbol{p} = [\boldsymbol{p}_1^{\mathrm{T}}, \ldots, \boldsymbol{p}_N^{\mathrm{T}}]^{\mathrm{T}}$, which can be also regarded as a *formation*. In the local coordinate system of agent i, the relative position of any other agent j is denoted as $\boldsymbol{p}_{i\leftarrow j} = [x_j - x_i,\ y_j - y_i]^{\mathrm{T}}$. The relative position parameter of the formation is denoted as $\boldsymbol{P}_{i\leftarrow\tilde{i}} = [\boldsymbol{p}_{i\leftarrow 1}, \ldots, \boldsymbol{p}_{i\leftarrow N}]$, where $\tilde{i}$ refers to all the index values that are not equal to i. Besides, the orientation angle of the agent i itself is denoted as α_i.

All the N agents are capable of transmitting and receiving signals, and are divided into *backbone* agents and *listener* agents according to whether they turn on the signal transmitting mode. Limited by resources, the MAS can only afford the maximum of N_{a} backbone agents' communication cost. The remaining agents complete the self-localization only by listening measurements in the network. The backbone-listener scheme is shown in Fig. 1. The red triangles represent the chosen N_{a} backbone agents that transmit wireless signals actively, whose set is defined as $\mathcal{N}_{\mathrm{a}} = \{1, 2, \ldots, N_{\mathrm{a}}\}$. The blue ones represent the rest of agents referred to as the listener agents that only receive wireless signals sent by the backbone agents, and the set of N_{b} listener agents is defined as $\mathcal{N}_{\mathrm{b}} = \{N_{\mathrm{a}}+1, N_{\mathrm{a}}+2, \ldots, N_{\mathrm{a}}+N_{\mathrm{b}}\}$. For either a backbone or a listener agent $i(i \in \mathcal{N}, \mathcal{N} = \mathcal{N}_{\mathrm{a}} \cup \mathcal{N}_{\mathrm{b}}, N = N_{\mathrm{a}} + N_{\mathrm{b}})$, the set of neighbor agents of agent i is denoted as $\mathcal{N}_i$, and the number of elements in the set $\mathcal{N}_i$ is denoted as N_{i}.

B. Measurement Model

The backbone agents i and j transmit and receive wideband signals actively to obtain the ToA τ_{ij} and the AoA ψ_{ij} between each other, using the double-sided two-way ranging method [8] and the maximum likelihood method [9], respectively. For the listener agent k, it obtains the time difference of arrival $\Delta\tau_{ij}^k = \tau_{ki} - \tau_{kj}$ between the backbone agents i and j, and the angle of arrival ψ_{ki}, ψ_{kj} from i and j, respectively. The corresponding ToA, TDoA and AoA measurements are modeled as

$$\begin{aligned} \mathsf{\tau}_{ij} &= \tau_{ij} + \mathsf{n}_{\tau,ij} \\ \Delta\mathsf{\tau}_{ij}^k &= \tau_{ki} + \mathsf{n}_{\tau,ki} - \tau_{kj} - \mathsf{n}_{\tau,kj} \\ \mathsf{\psi}_{ij} &= \psi_{ij} + \mathsf{n}_{\psi,ij} \end{aligned} \tag{1}$$

where $\mathsf{n}_{\tau,ij} \sim \mathcal{N}\big(0, \sigma_{\tau,ij}^2\big)$ and $\mathsf{n}_{\psi,ij} \sim \mathcal{N}\big(0, \sigma_{\psi,ij}^2\big)$ are independent additive white Gaussian measurement noises with the variance of $\sigma_{\tau,ij}^2, \sigma_{\psi,ij}^2$.

Note that the orientation angle α and the AoA measurement ψ are different parameters as shown in Fig. 1. For backbone agents, double sides AoA are available, so the relationship between orientation angle and AoA is denoted as

$$\alpha_i + \psi_{ij} = \mathbf{mod}(\alpha_j + \psi_{ji} + \pi, 2\pi) \tag{2}$$

where $\mathbf{mod}(x, y)$ is a function that returns the remainder after the division of x divided by y.

For listener agents, even the AoA ψ_{kj} is available when the listener agent k receives the signal from the backbone agent j, the measurement of ψ_{jk} is still missing since agent k does not transmit signals actively. Hence, the estimation of the orientation angle shall be executed based on the a priori position $\boldsymbol{p}_j$ of agent j and its own position estimation $\hat{\boldsymbol{p}}_k = [\hat{x}_k, \hat{y}_k]^{\mathrm{T}}$ as follows,

$$\alpha_k + \psi_{kj} = \mathbf{atan2}\left(y_j - \hat{y}_k, x_j - \hat{x}_k\right), \tag{3}$$

where $\mathbf{atan2}\,(y, x)$ is a function that returns the angle between the ray to the point (x, y) and the positive x axis.

III. Backbone-Listener Localization Scheme

A. Node Selection

The relative squared position error bound (rSPEB) of the network measures how each agent contributes differently to the information gain of the network, which is denoted as $\mathrm{tr}\{\boldsymbol{J}_{\mathrm{e}}^{\dagger}(\boldsymbol{p})\}$ in [10]. The purpose of node selection is to specify those backbone agents that contribute more of the error reduction, which can be managed by solving the following optimization problem

$$\begin{aligned} \text{minimize} \quad & \mathrm{tr}\big\{\boldsymbol{J}_{\mathrm{e}}^{\dagger}(\boldsymbol{p})\big\} \\ \text{subject to} \quad & \mathcal{N}_{\mathrm{a}} \cup \mathcal{N}_{\mathrm{b}} = \mathcal{N} \\ & \mathcal{N}_{\mathrm{a}} \cap \mathcal{N}_{\mathrm{b}} = \emptyset \\ & |\mathcal{N}_{\mathrm{a}}| \leqslant N_{\mathrm{a}} \end{aligned}. \tag{4}$$

It has been proven that (4) is a non-convex problem. Consequently, we do not optimize the rSPEB directly, on the other hand, we draw some inspirations by some convex relaxation methods of (4). Denote the inverse of the square of the distance $1/d_{ij}^2$ between two given agents i and j as the potential field. A heuristic greedy node selection (GNS) method is proposed as follows.

1. Randomly pick an agent i and get the distance measurements of i: $\mathsf{d}_{ij} = c \cdot \mathsf{\tau}_{ij}, j \in \mathcal{N}_i$, and find $j_0 = \arg\min_j 1/\mathsf{d}_{ij}^2$ with the lowest potential field from i as the auxiliary agent.

2. Get the distance measurements of j_0: $\mathsf{d}_{j_0 j} = c \cdot \mathsf{\tau}_{j_0 j}, j \in \mathcal{N}_{j_0}$, and select $j_1 = \arg\min_j 1/\mathsf{d}_{j_0 j}^2$ with the lowest potential field from j as the first agent in $\mathcal{N}_{\mathrm{a}}$. Select agent j_1 into $\mathcal{N}_{\mathrm{a}}$, and set the index number $n = 2$.

3. select the agent $j_n \in \mathcal{N}\backslash\mathcal{N}_{\mathrm{a}}$ iteratively such that the sum of the potential field function to other agents already in set $\mathcal{N}_{\mathrm{a}}$ is minimized $j_n = \underset{j \in \mathcal{N}\backslash\mathcal{N}_{\mathrm{a}}}{\arg\min} \sum_{i \in \mathcal{N}_{\mathrm{a}}} 1/\mathsf{d}_{ij}^2$, until the number of backbone agents is satisfied.

Algorithm 1 Backbone Distributed Multidimensional Scaling (BD-MDS)

Input:

ToA measurements, $\mathsf{\tau}_{ij}, i, j \in \mathcal{N}_\mathrm{a}$;

AoA measurements, $\mathsf{\psi}_{ij}, i, j \in \mathcal{N}_\mathrm{a}$;

Output:

Position and orientation estimation of backbone agents, $\hat{\mathbf{P}}_{\mathcal{N}_\mathrm{a}}, \hat{\mathsf{\alpha}}_i, i \in \mathcal{N}_\mathrm{a}$;

1: Get the index of the backbone agents from Algorithm 1;
2: Find $j_0 = \arg\min_j \mathsf{d}_{ij}$, set $\hat{\mathsf{\alpha}}_i = -\mathsf{\psi}_{ij_0}, j_0 \in \mathcal{N}_i$;
3: Calculate $\hat{\mathsf{\alpha}}_j = \mathbf{mod}(\hat{\mathsf{\alpha}}_i + \mathsf{\psi}_{ij} + \pi, 2\pi) - \mathsf{\psi}_{ji}$;
4: Calculate $\mathbf{D}_{i\leftarrow\tilde{i}} = \mathbf{diag}(\mathbf{d}_{i\leftarrow\tilde{i}})$;
5: Calculate $\mathbf{\Lambda}_i$ according to (5), and get the singular value decomposition of $\mathbf{U}_i$ and $\sqrt{\mathbf{\Sigma}_i}$ according to (8);
6: Calculate $\hat{\mathbf{P}}_{i\leftarrow\tilde{i}}$ according to (10);
7: Set agent i as the origin and obtain $\hat{\mathbf{P}}_{\mathcal{N}_\mathrm{a}}$ according to (11).

B. Distributed MDS for Backbone Agents

Once the set of the backbone agents is determined, the selected agents will transmit signals to obtain ToA and AoA measurements. Based on the measurement model, we propose a distributed multidimensional scaling algorithm for backbone agents. For each certain agent i, we collect the ToA and AoA information from neighbours as $\boldsymbol{\tau}_{i\leftarrow\tilde{i}} = [\cdots, \tau_{ij}, \cdots]^\mathrm{T}, \boldsymbol{\psi}_{i\leftarrow\tilde{i}} = [\cdots, \psi_{ij}, \cdots]^\mathrm{T}, j \in \mathcal{N}_i$. The distance matrix is denoted as $\boldsymbol{D}_{i\leftarrow\tilde{i}} = \mathbf{diag}(c \cdot \boldsymbol{\tau}_{i\leftarrow\tilde{i}})$, where $\mathbf{diag}(\boldsymbol{x})$ denotes the operator that generates diagonal matrices using elements in vector $\boldsymbol{x}$. The orientation angle of agent i is obtained according to (2). Denote the angle between the link from agent j_1 towards the reference agent i and the link from agent j_2 towards the reference agent i as $\Lambda_{j_1 i j_2} = \psi_{ij_1} - \psi_{ij_2}$ as shown in Fig. 1, where $j_1, j_2 \in \mathcal{N}_i$, i, j_1, j_2 are unequal. Accordingly, the cosine angle matrix is denoted as

$$\boldsymbol{\Lambda}_i = \begin{bmatrix} 1 & \cos\Lambda_{1i2} & \cdots & \cos\Lambda_{1iN_\mathrm{i}} \\ \cos\Lambda_{2i1} & 1 & \cdots & \cos\Lambda_{2iN_\mathrm{i}} \\ \vdots & \vdots & \ddots & \vdots \\ \cos\Lambda_{N_\mathrm{i}i1} & \cos\Lambda_{N_\mathrm{i}i2} & \cdots & 1 \end{bmatrix}_{N_\mathrm{i}\times N_\mathrm{i}}. \tag{5}$$

Denote the position parameter of the backbone agents as $\boldsymbol{P}_{\mathcal{N}_\mathrm{a}} = [\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_{N_\mathrm{a}}]$. We then derive a distributed MDS-based estimation of $\boldsymbol{P}_{\mathcal{N}_\mathrm{a}}$ by constructing a kernel function which is given by

$$\begin{aligned} \boldsymbol{K}_i &= \boldsymbol{P}_{i\leftarrow\tilde{i}}^\mathrm{T} \cdot \boldsymbol{P}_{i\leftarrow\tilde{i}} = \left(c^2 \cdot \boldsymbol{\tau}_{i\leftarrow\tilde{i}} \cdot \boldsymbol{\tau}_{i\leftarrow\tilde{i}}^\mathrm{T}\right) \odot \boldsymbol{\Lambda}_i \\ &= \boldsymbol{D}_{i\leftarrow\tilde{i}} \cdot \boldsymbol{\Lambda}_i \cdot \boldsymbol{D}_{i\leftarrow\tilde{i}} \end{aligned} \tag{6}$$

where the element in the mth row and nth column ($m \neq n$) of matrix $\boldsymbol{K}_i$ is

$$\begin{aligned} [\boldsymbol{K}_i]_{mn} &= \langle \boldsymbol{p}_{i\leftarrow m}; \boldsymbol{p}_{i\leftarrow n} \rangle \\ &= \|\boldsymbol{p}_{i\leftarrow m}\| \cdot \|\boldsymbol{p}_{i\leftarrow n}\| \cos\left([\boldsymbol{\Lambda}_i]_{mn}\right). \end{aligned} \tag{7}$$

Notice that $\boldsymbol{\Lambda}_i$ is a symmetric and semi-definite matrix with rank 2.[1] We decompose it by singular value decomposition

$$\boldsymbol{\Lambda}_i = \boldsymbol{U}_i \boldsymbol{\Sigma}_i \boldsymbol{U}_i^\mathrm{T} = \left(\boldsymbol{U}_i\sqrt{\boldsymbol{\Sigma}_i}\right) \cdot \left(\boldsymbol{U}_i\sqrt{\boldsymbol{\Sigma}_i}\right)^\mathrm{T}. \tag{8}$$

Replacing $\boldsymbol{\Lambda}_i$ with (8) in (6), we get

$$\boldsymbol{K}_i = \boldsymbol{D}_{i\leftarrow\tilde{i}} \cdot \left(\boldsymbol{U}_i\sqrt{\boldsymbol{\Sigma}_i}\right) \cdot \left(\boldsymbol{U}_i\sqrt{\boldsymbol{\Sigma}_i}\right)^\mathrm{T} \cdot \boldsymbol{D}_{i\leftarrow\tilde{i}}^\mathrm{T}. \tag{9}$$

Comparing (6) with (9), we choose

$$\hat{\boldsymbol{P}}_{i\leftarrow\tilde{i}} = \left[\left(\boldsymbol{U}_i\sqrt{\boldsymbol{\Sigma}_i}\right)^\mathrm{T} \boldsymbol{D}_{i\leftarrow\tilde{i}}^\mathrm{T}\right]_{1:2,1:N_\mathrm{a}-1} \tag{10}$$

as one feasible estimation of other agents' relative positions towards agent i. If we choose agent i as the origin, it follows that the estimation of the position $\hat{\boldsymbol{P}}_{\mathcal{N}_\mathrm{a}}$ can be obtained by

$$\hat{\boldsymbol{P}}_{\mathcal{N}_\mathrm{a}} = \left[\mathbf{0}_2 \quad \hat{\boldsymbol{P}}_{i\leftarrow\tilde{i}}\right]. \tag{11}$$

We summarize the overall derivation in Algorithm 2 as Backbone Distributed Multidimensional Scaling (BD-MDS). In the above derivation, we assume noisy-free measurements to simplify the process of explaining the principle, but the derived algorithm works even with noisy measurements, which is displayed in the algorithm description as bold letters. A distributed TDoA and AoA information fusion method will be introduced for listener agents in the next subsection.

C. Distributed TDoA/AoA fusion for Listener Agents

For agents that can obtain TDoA and AoA measurements, an information fusion algorithm is proposed in [7]. But our setting is different from [7]: First, the AoA ψ_{ik} from the listener agent $k, k \in \mathcal{N}_\mathrm{b}$ to the backbone agent i is unavailable since the listener agents do not transmit signals. Therefore, the orientation α_k cannot be estimated directly by (2). Second, agent k receives signals from N_a backbone agents, but multi-angle fusion has not been discussed in the literature.

To solve these problems, we propose a 2-step iterative method called Listener Distributed Least Square (LD-LS) algorithm to complete a high accuracy position as well as orientation estimation for listener agents. The proposed LD-LS algorithm has two steps, summarized in Algorithm 3:

Step 1: We first estimate the position $\boldsymbol{p}_k$ as $\hat{\mathbf{p}}_k^0$ using only TDoA information by *Chan algorithm* [6], where a maximum likelihood estimator can be formulated:

$$\boldsymbol{q}_k^0 = \arg\min_{\boldsymbol{q}_k} (\boldsymbol{h}^0 - \boldsymbol{G}_a^0 \boldsymbol{q}_k)^\mathrm{T} (\boldsymbol{\Psi}^0)^{-1} (\boldsymbol{h} - \boldsymbol{G}_a^0 \boldsymbol{q}_k) \tag{12}$$

$\boldsymbol{q}_k = [\boldsymbol{p}_k^\mathrm{T}, r_1]^\mathrm{T}$ is an augmented vector composed of the position $\boldsymbol{p}_k$ of listener agent k, and r_1 is the distance from agent k to reference agent 1. Other parameters and variables are borrowed from [6] which are omitted here. Solving (12) gives us an initial estimated position $\hat{\mathbf{p}}_k^0 = [\boldsymbol{q}_k^0]_{1:2}$, which can be used as a prior to calculate the orientation angle $\hat{\mathsf{\alpha}}_k^0$ according to (3).

Step 2: Based on the initial estimated $\hat{\mathsf{\alpha}}_k^0$ from step 1, AoA information can be used to obtain refined orientation

[1] The proof of the semi-definite property is simple but cumbersome, which is omitted due to the limited space.

Algorithm 2 Listener Distributed Least Sqaure (LD-LS)

Input:

Estimated backbone network topology, $\hat{\mathbf{P}}_{\mathcal{N}_\mathrm{a}}$, $\hat{\alpha}_i, i \in \mathcal{N}_\mathrm{a}$
TDoA measurements, $\Delta\tau^k_{i_1,i_2}, i_1, i_2 \in \mathcal{N}_\mathrm{a}, k \in \mathcal{N}_\mathrm{b}$;
AoA measurements, $\psi_{ki}, i \in \mathcal{N}_\mathrm{a}, k \in \mathcal{N}_\mathrm{b}$;

Output:

Position and orientation estimation of the listener agents, $\hat{\mathbf{p}}_k, \hat{\alpha}_k, k \in \mathcal{N}_\mathrm{b}$;

1: Get the initial estimated position $\hat{\mathbf{p}}^0_k$ by the *Chan algorithm*;
2: Calculate the 1-step orientation $\hat{\alpha}^0_k$ according to (3);
3: Calculate $\boldsymbol{G}^*_a$ and $\boldsymbol{h}^*$ according to (13) using AoA measurements;
4: Update $\boldsymbol{\Psi}^*$ with (14);
5: Calculate the 2-step $\hat{\mathbf{p}}_k$ according to (15);
6: Use the estimation $\hat{\mathbf{p}}_k$ to update $\hat{\alpha}_k$ according to (3).

estimation according to (2). Notice the geometric relationship of $d_{k1} \sin \mathrm{n}_{\psi,k1} = y_{1\leftarrow k} \cos \psi_{k1} - x_{1\leftarrow k} \sin \psi_{k1}$, which can be approximated as $0 \approx -d_{k1}\mathrm{n}_{\psi,k1} + y_k \cos \psi_{k1} - x_k \sin \psi_{k1}$, since $\sin \mathrm{n}_{\psi,k1} \approx \mathrm{n}_{\psi,k1}$ when $\mathrm{n}_{\psi,k1}$ is small. Therefore, $\boldsymbol{G}^0_a$ and $\boldsymbol{h}^0$ can be updated to $\boldsymbol{G}^*_a$ and $\boldsymbol{h}^*$ by the known TDoA and AoA measurements, in a form of an augmented matrix as follows:

$$
\begin{aligned}
\boldsymbol{G}^*_a &= \begin{bmatrix} & \boldsymbol{G}^0_a & \\ -\sin\psi_{k1} & \cos\psi_{k1} & 0 \\ \cdots & \cdots & \cdots \\ -\sin\psi_{kN_\mathrm{a}} & \cos\psi_{kN_\mathrm{a}} & 0 \end{bmatrix} \\
\boldsymbol{h}^* &= \frac{1}{2}\begin{bmatrix} 2\boldsymbol{h}^0 \\ \mathbf{0}_{N_\mathrm{a}} \end{bmatrix}.
\end{aligned} \tag{13}
$$

The covariance matrix $\boldsymbol{\Psi}^*$ can be updated by

$$
\boldsymbol{\Psi}^* = \begin{bmatrix} \boldsymbol{\Psi}^0 & \\ & \mathbf{diag}\left([\sigma^2_{\psi,k1}d_{k1}, \cdots, \sigma^2_{\psi,kN_\mathrm{a}}d_{kN_\mathrm{a}}]^\mathrm{T}\right) \end{bmatrix}. \tag{14}
$$

Then the estimated positions of the listener agents can be obtained by

$$
\begin{aligned}
\boldsymbol{q}^\mathrm{r}_k &= \arg\min_{\boldsymbol{q}_k} \left(\boldsymbol{h}^* - \boldsymbol{G}^*_a\boldsymbol{q}_k\right)^\mathrm{T} \left(\boldsymbol{\Psi}^*\right)^{-1} \left(\boldsymbol{h}^* - \boldsymbol{G}^*_a\boldsymbol{q}_k\right) \\
\hat{\mathbf{p}}_k &= [\boldsymbol{q}^\mathrm{r}_k]_{1:2}
\end{aligned}. \tag{15}
$$

Finally, the orientation estimation can be updated by (3) using the latest $\hat{\mathbf{p}}_k$. The proposed LD-LS method jointly fuses TDoA and AoA measurements without priori orientation angles.

IV. Numerical Results

A. Experiment Setup

We validate the proposed backbone-listener localization scheme by both simulation and hardware implementation.

In the simulation part, parameters are set as follows: the positions of agents follow a uniform distribution of 10 m by 10 m, i.e., $\boldsymbol{p} \sim \mathcal{U}(10, 10)$, and the orientation angles follow a uniform distribution in $(0, 2\pi)$, i.e., $\alpha \sim \mathcal{U}(0, 2\pi)$. The frequency of the TDoA and AoA measurements is set to be 10 Hz. The measurement noise of ranges and angles are set as $\sigma_{\tau,ij} = 0.2/c$ m, $\sigma_{\psi,ij} = \pi\ /90$ rad, respectively, where c is the propagation speed of the signal.

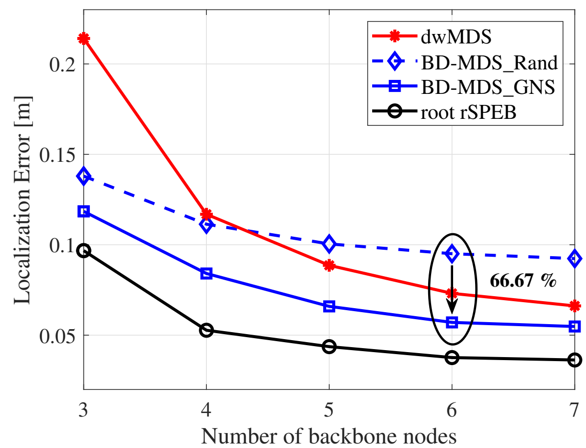

Fig. 2. Localization performance comparison of backbone agents.

B. Simulation Results

We first validate the effectiveness and efficiency of the proposed algorithms through numerical simulations. For each round of the simulation, 10000 different topologies are generated randomly to compute the root mean squared error (RMSE). The performance gain of node selection strategy and BD-MDS algorithm is shown in Fig. 2: The dwMDS proposed in [5] (marked as red asterisks and is used as a benchmark of our method) updates the position estimate by minimizing a local cost function which emphasizes the lowest relative error within the network; the root rSPEB (marked as black circles) is the lower bound of the localization error; the proposed BD-MDS algorithm is executed in two different situations, one is to select backbone agents randomly (marked as blue diamonds), another (marked as blue squares) is by the GNS algorithm described in Section III-A. As can be seen, the proposed BD-MDS method significantly outperforms the benchmark when the number of the backbone agents is small. And the GNS algorithm can provide about a 66.67% performance gain.

Second, we evaluate the performance of listener agents. Fig. 3 presents the cumulative distribution function of different methods. The root rSPEB (marked as black circles) is the lower bound of the localization error, and the TDoA/AoA information fusion method proposed in [7] (marked as red asterisks) is a benchmark of the proposed LD-LS method, which is also carried out randomly (marked as blue diamonds) and by the GNS algorithm (marked as blue squares). The simulation results still verify the advantages of the proposed LD-LS algorithm as well as the effectiveness of the GNS algorithm.

C. Experimental Results

Next, we validate the performance of the proposed methods using real measurements on the self-developed hardware platform. Each agent is equipped with a four-antenna UWB

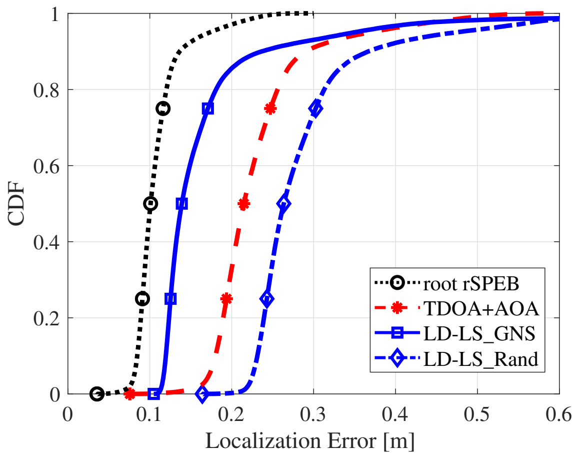

Fig. 3. The CDF of the listener agents' localization error.

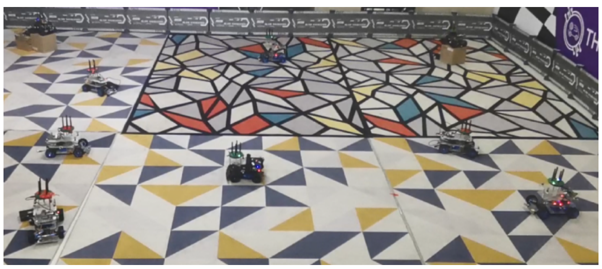

Fig. 4. Photo of the experiment in an actual scene.

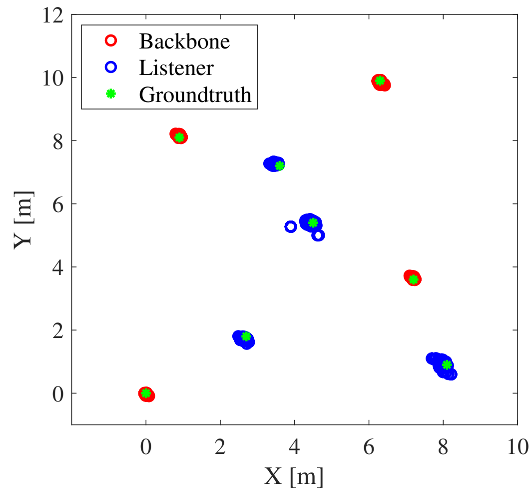

Fig. 5. The experimental result of the the backbone-listener localization scheme. The RMSE is 8.65 cm.

array connected to a DecaWave DW1000 UWB transceiver. All the transceivers are driven by the same clock reference with ARM STM32F103RB. All algorithms are implemented with Raspberry Pi 4B using ARM Cortex-A72 CPU. Fig. 4 represents a top-down panorama of an actual scene. Eight agents are deployed at marked locations in a 10 m $\times$ 10 m indoor venue. Then we use the backbone-listener localization scheme to obtain the relative localization estimation. We have conducted several experiments with agents in different positions, and the experimental results verify that the localization error is less than 10 cm. Due to space limitations, Only one experimental result is shown in Fig. 5, where the red and blue circles are the location estimation of the backbone and listener agents, respectively, while the blue circles are the estimation of and the green asterisks represent the groundtruth we marked in advance. More demos and results can be found at https://sgroupresearch.github.io/gc2022/.

V. Conclusion

This paper introduces a backbone-listener localization scheme for distributed multi-agent systems. The scheme composes of the node selection strategy, the topology estimation of backbone agents and the localization of listener agents. Agents in the backbone mode are selected by the GNS strategy, and the BD-MDS algorithm is used to maintain the estimation of the network topology. The remaining listener jointly estimate the positions and orientation angles of agents using the LD-LS method. The proposed scheme enables agents in the D-MAS to accomplish distributed localization. Simulation and experimental results demonstrate that the relative localization accuracy of 10 cm is achieved.

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