Stochastic Geometry Analysis of Spectrum Sharing Among Seller and Buyer Mobile Operators

Elaheh Ataeebojd[†], Mehdi Rasti[†], Hossein Pedram[†], and Pedro H. J. Nardelli[§]
[†]Department of Computer Engineering, Amirkabir University of Technology, Tehran, Iran
[§] Lappeenranta-Lahti University of Technology, Lappeenranta, Finland

2022 IEEE Wireless Communications and Networking Conference (WCNC) | 978-1-6654-4266-4/22/$31.00 ©2022 IEEE | DOI: 10.1109/WCNC51071.2022.9771800

***Abstract*—Sharing the licensed frequency spectrum among mobile network operators (MNOs) is a promising approach to improve licensed spectrum utilization. In this paper, we model and analyze a non-orthogonal spectrum sharing system consisting of seller and buyer MNOs where buyer MNOs can lease several licensed sub-bands from different seller MNOs. All base stations (BSs) owned by a buyer MNO can also utilize various licensed sub-bands simultaneously, which are also used by other buyer MNOs. To reduce the interference that a buyer MNO imposes on one seller MNO sharing its licensed sub-band, this buyer MNO has a limitation on the maximum interference caused to the corresponding seller MNO's users. We assume each MNO owns its BSs and users whose locations are modeled as two independent homogeneous Poisson point processes. Applying stochastic geometry, we derive expressions for the downlink signal-to-interference-plus-noise ratio coverage probability and the average rate of both seller and buyer networks. The numerical results validate our analysis with simulation and illustrate the effect of the maximum interference threshold on the total sum rate of the network.**

***Index Terms*—average rate, coverage probability, mobile network operator, non-orthogonal spectrum sharing, stochastic geometry**

I. Introduction

Due to the scarcity of frequency spectrum and increasing demand for various mobile services [1], mobile network operators (MNOs) encounter many challenges in providing efficient ways to use resources more efficiently to serve their users in 5G and beyond. In recent studies, spectrum sharing is considered as a robust solution in 5G and beyond to utilize the existing resources more efficiently [2]. In spectrum sharing, different MNOs, named buyer MNOs, are allowed to use (or lease) the frequency sub-bands of other MNOs, called seller MNOs, to afford mobile data services to their users [3].

In [4] – [7], various aspects and performance of spectrum sharing among MNOs are studied in a cellular setting. These studies consider a deterministic model in which the location of base stations (BSs) and user equipment (UE) are known, resulting in unwieldy problem formulation when the number of BSs and UEs is large. Hence, a stochastic geometry model is adopted as a tractable approach to model and analyze various cellular wireless systems owned by different MNOs as the case in [8] – [10] instead of a deterministic model. For instance, in [10], the performance of a system with a single buyer and a single seller MNO is investigated, and performance metrics such as signal-to-interference-plus-noise ratio (SINR) coverage probability and per-user average rate are derived. Besides, in [10], a power control strategy is adopted based on the distance between each buyer BS and the nearest seller UE to it. In [9], a stochastic geometry framework is presented to analyze a non-orthogonal spectrum sharing system in an indoor small cell network considering a single buyer MNO and several seller MNOs. Spectrum sharing among several MNOs is also studied in [8], where each MNO owns its BSs and UEs distributed based on independent Poisson Point Process (PPP).

A significant limitation of prior work [4] – [9] is that in sharing the frequency spectrum between different MNOs, there is no power control over the transmit power of the BSs, especially the BSs of buyer MNOs. Consequently, the interference imposed on UEs is unpredictable, leading to a reduction in the overall performance of the networks. The other limitation is the assumption of a single buyer MNO in [9] – [10]. To study the impact of sharing sub-bands of one seller MNO with several buyer MNOs and the effect of simultaneous leasing sub-bands from several seller MNOs by each buyer MNO (opposing with [8]), a more general framework containing several seller and buyer MNOs is needed.

In this paper, to address the issues mentioned above, we consider a non-orthogonal spectrum sharing system where buyer MNOs lease several licensed sub-bands from seller MNOs. In this scenario, all BSs of each buyer MNO can utilize several sub-bands. Also, each seller MNO allows buyer MNOs to utilize each of its sub-band simultaneously. Moreover, inspired by [11], we introduce a distribution function for the downlink transmit power of buyer MNOs so that the interference imposed on seller MNOs' UEs does not violate a tolerable interference threshold. Using a stochastic geometry approach, we analyze the SINR coverage probability and the average rate considering a large-scale cellular network. Our contributions in this paper are summarized as follows:

- A more general framework is proposed containing several seller and buyer MNOs to study the impact of sharing sub-bands between seller and buyer MNOs.
- Employing stochastic geometry, we analytically obtain the SINR coverage probability and the average rate as performance metrics under random parameters such as channel gains, distance of UEs from BSs, as well as the transmit power of buyer BSs. To do this, we introduce a power control strategy to justify the transmit power of

This work is supported by the Academy of Finland: (a) ee-IoT n.319009, (b) EnergyNet n.321265/n.328869, and (c) FIREMAN n.326270/CHISTERA-17-BDSI-003; and by JAES Foundation via STREAM project.

buyer MNOs' BSs on the licensed sub-bands of the seller MNOs.
- Numerical results validate our analytical results with simulation and show the performance of our adopted power control strategy.

The rest of this paper is organized as follows: The system model and our assumptions are explained in Section II. Employing stochastic geometry, we analyze the probability of SINR coverage and the average rate for both seller and buyer MNOs in Section III. In Section IV, the numerical results are presented. Finally, the conclusion is summarized in Section V.

II. System Model and Assumptions

A. System Model

Consider a downlink cellular wireless network with a set of seller MNOs $\mathcal{S}$ lease their frequency sub-bands to a set of buyer MNOs $\mathcal{B}$ (see Fig. 1). The set of all MNOs is denoted by $\mathcal{O} = \mathcal{S} \cup \mathcal{B}$. We assume that seller MNO $s \in \mathcal{S}$ owns a license for an orthogonal spectrum divided into L_s licensed sub-bands. Moreover, the set of licensed sub-bands for seller MNO $s \in \mathcal{S}$ is represented by $\mathcal{L}_s$ ($|\mathcal{L}_s| = L_s$). Additionally, there is no interference among the licensed sub-bands of different seller MNOs. Each MNO owns an independent network consisting of its own BSs and UEs. The location of BSs and UEs for MNO $k \in \mathcal{O}$ are modeled by two independent PPPs Φ_k and Ψ_k with intensity λ_k and μ_k, respectively. Also, the set of UEs and BSs owned by MNO $k \in \mathcal{O}$ are given by $\mathcal{U}_k$ and $\mathcal{F}_k$, respectively. Each BS of seller MNOs transmits with a fixed power denoted by P^{S} on their sub-bands. Nonetheless, each BS of buyer MNOs is limited to transmit at a certain power $P^{\mathrm{B}}_{l_s}$ on sub-band $l_s \in \mathcal{L}_s$ of seller MNO s to ensure that the interference imposed on each UE of seller MNO s is below the maximum interference threshold ζ_s. Furthermore, it is assumed that a UE associated with a BS of its MNO, providing the maximum average received power. We perform the analysis based on a typical seller UE (SUE) of a seller MNO and a typical buyer UE (BUE) of a buyer MNO for downlink cellular communication. We use a general power-law path loss model with path loss exponent $\alpha > 2$. Besides, we assume all channel gains are modeled by Rayleigh fading, which means that all fading variables are exponentially distributed with unit mean.

B. License Sharing Group

Seller MNO s can share each of its licensed sub-bands, e.g., $l_s \in \mathcal{L}_s$, with different buyer MNOs at the same time (see Fig. 1). Let us denote all MNOs sharing licensed sub-band $l_s \in \mathcal{L}_s$ by Q_{l_s}, where seller MNO s is surely belong to Q_{l_s}. Accordingly, in each sub-band $l_s \in \mathcal{L}_s$, a typical BUE of buyer MNO b experiences interference from the BSs of its MNO, the BSs of seller MNO s, and the BSs of the other buyer MNOs sharing sub-band $l_s \in \mathcal{L}_s$ simultaneously. Similarly, in each sub-band $l_s \in \mathcal{L}_s$, a typical SUE of seller MNO s will experience interference from the BSs of its MNO and the BSs of buyer MNOs sharing sub-band $l_s \in \mathcal{L}_s$ at the same time.

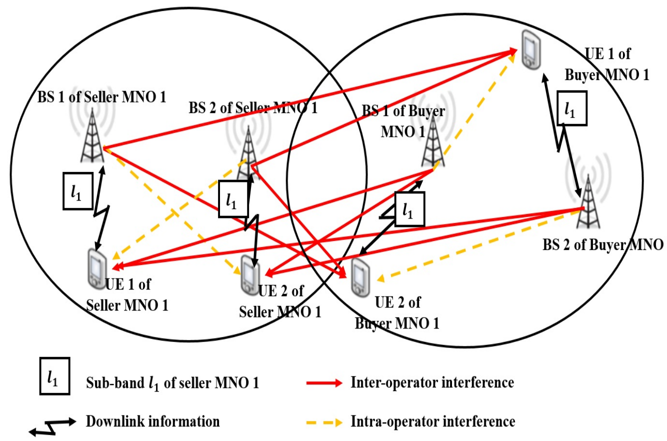

Fig. 1. A cellular network with one seller and one buyer MNO.

In both cases, the set of interfering BSs is owned by MNOs, members of the sharing group Q_{l_s}.

C. SINR Model

We assume that a typical SUE of seller MNO s is located at the origin. Thus, the downlink received SINR at the typical SUE of seller MNO $s \in \mathcal{S}$ on sub-band $l_s \in \mathcal{L}_s$ is determined by

$$\gamma^{\mathrm{S}}_{s,l_s} = \frac{P^{\mathrm{S}} h^{\mathrm{S}}_{0,l_s} {x^{\mathrm{S}}_0}^{-\alpha}}{I^{\mathrm{S}}_{s,l_s} + \sigma^2}, \tag{1}$$

where x^{S}_0 is the distance between this typical SUE and its associated BS, h^{S}_{0,l_s} represents the channel gain between this typical SUE and its associated BS on sub-band $l_s \in \mathcal{L}_s$, σ^2 denotes the noise power, and I^{S}_{s,l_s} is the interference experienced by this typical SUE of seller MNO s on sub-band $l_s \in \mathcal{L}_s$ from all interfering BSs. It is calculated as $I^{\mathrm{S}}_{s,l_s} = \sum_{f \in \mathcal{F}_s \setminus \{0\}} P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} {x^{\mathrm{S}}_f}^{-\alpha} + \sum_{b \in Q_{l_s} \setminus \{s\}} \sum_{f \in \mathcal{F}_b} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha}$, where h^{S}_{f,l_s} is the channel gain between this typical SUE and BS f of the corresponding seller MNO on sub-band $l_s \in \mathcal{L}_s$, h^{B}_{f,l_s} is the channel gain between this typical SUE and BS f of any buyer MNO on sub-band $l_s \in \mathcal{L}_s$, x^{S}_f is the distance between this typical SUE and BS f of the corresponding seller MNO, and x^{B}_f is the distance between this typical SUE and BS f of any buyer MNO.

Likewise, we assume that a typical BUE of buyer MNO b is located at the origin. Therefore, the downlink received SINR at the typical BUE of buyer MNO b on shared sub-band $l_s \in \mathcal{L}_s$ is expressed as

$$\gamma^{\mathrm{B}}_{b,l_s} = \frac{P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{0,l_s} {x^{\mathrm{B}}_0}^{-\alpha}}{I^{\mathrm{B}}_{b,l_s} + \sigma^2}, \tag{2}$$

where x^{B}_0 represents the distance between this typical BUE and its associated BS, h^{B}_{0,l_s} denotes the channel gain between this typical BUE and its associated BS on sub-band $l_s \in \mathcal{L}_s$, and I^{B}_{b,l_s} is the interference experienced by this typical BUE of buyer MNO b on sub-band $l_s \in \mathcal{L}_s$ from all interfering BSs.

It is calculated as $I^{\mathrm{B}}_{b,l_s} = \sum_{f\in\mathcal{F}_b\setminus\{0\}} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha} + \sum_{f\in\mathcal{F}_s} P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} {x^{\mathrm{S}}_f}^{-\alpha} + \sum_{k\in Q_{l_s}\setminus\{s,b\}} \sum_{f\in\mathcal{F}_k} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha}$.

D. Power Control Strategy

Inspired by the approach in [11], each buyer BS is limited to transmit at a certain power $P^{\mathrm{B}}_{l_s}$ on sub-band $l_s \in \mathcal{L}_s$ to ensure that the interference imposed on each UE of seller MNO s is below the maximum interference threshold ζ_s. That is, given the maximum interference threshold ζ_s, for any SUE $i \in \mathcal{U}_s$ and an arbitrary buyer BS which are located at $\mathrm{y}^{\mathrm{S}}_i$ and x^{B}, respectively, we have $P^{\mathrm{B}}_{l_s} h_{i,0,l_s} ||\mathrm{x}^{\mathrm{B}} - \mathrm{y}^{\mathrm{S}}_i||^{-\alpha} \le \zeta_s$, where $h_{i,0,l_s}$ is the channel gain between UE i and this arbitrary BS on sub-band $l_s \in \mathcal{L}_s$. Accordingly, given the maximum interference threshold ζ_s, it is sufficient to satisfy $\max_{\substack{\mathrm{y}^{\mathrm{S}}_i\in\Psi_s \\ \forall i\in\mathcal{U}_s}} P^{\mathrm{B}}_{l_s} h_{i,0,l_s} ||\mathrm{x}^{\mathrm{B}} - \mathrm{y}^{\mathrm{S}}_i||^{-\alpha} \le \zeta_s$. By doing this, the interference constraint is surely satisfied for all other UEs of seller MNO s. Let us denote the largest interference channel gain on sub-band l_s associated with each buyer BS as $H^{\mathrm{B}}_{l_s}$. It is defined as $H^{\mathrm{B}}_{l_s} = \max_{\substack{\mathrm{y}^{\mathrm{S}}_i\in\Psi_s \\ \forall i\in\mathcal{U}_s}} h_{i,0,l_s} ||\mathrm{x}^{\mathrm{B}} - \mathrm{y}^{\mathrm{S}}_i||^{-\alpha}$. If each buyer BS transmits with the maximum allowable power on licensed sub-band $l_s \in \mathcal{L}_s$, $P^{\mathrm{B}}_{l_s}$ is calculated by

$$P^{\mathrm{B}}_{l_s} = \zeta_s / H^{\mathrm{B}}_{l_s}. \tag{3}$$

III. Analysis of Performance Metrics

In this section, different performance metrics are analyzed. In subsections III-A and III-B, the coverage probability and average rate, as performance metrics, for UEs are obtained, respectively. According to (1) and (2), SINR is dependent on $P^{\mathrm{B}}_{l_s}$ which is a random variable, therefore the distribution of $P^{\mathrm{B}}_{l_s}$ is needed to drive the coverage probability and average rate of both BUEs and SUEs. Since $P^{\mathrm{B}}_{l_s}$ is dependent on $H^{\mathrm{B}}_{l_s}$ based on (3), we first introduce the distribution of $H^{\mathrm{B}}_{l_s}$ and then the distribution of $P^{\mathrm{B}}_{l_s}$ in the following Lemmas [11].

Lemma 1. *The cumulative distribution function (CDF) and the probability density function (PDF) of $H^{B}_{l_s}$ are expressed respectively as*

$$F_{H^{\mathrm{B}}_{l_s}}(z) = \exp\left(\frac{-\pi\mu_s}{z^{2/\alpha}}\Gamma(1+\frac{2}{\alpha})\right), \tag{4}$$

and

$$f_{H^{\mathrm{B}}_{l_s}}(z) = \frac{2\pi\mu_s\Gamma(1+\frac{2}{\alpha})}{\alpha z^{1+\frac{2}{\alpha}}} \exp\left(\frac{-\pi\mu_s}{z^{2/\alpha}}\Gamma(1+\frac{2}{\alpha})\right), \tag{5}$$

where $\Gamma(\kappa)$ is the complete Gamma function calculated as $\Gamma(\kappa) = \int_0^{+\infty} t^{\kappa-1}e^{-t}\mathrm{d}t$.

Lemma 2. *The CDF and PDF of $P^{B}_{l_s}$ are expressed respectively as*

$$F_{P^{\mathrm{B}}_{l_s}}(z) = 1 - \exp\left(\frac{-\pi\mu_s z^{2/\alpha}}{\zeta_s^{2/\alpha}}\Gamma(1+\frac{2}{\alpha})\right), \tag{6}$$

and

$$f_{P^{\mathrm{B}}_{l_s}}(z) = \frac{2\pi\mu_s z^{2/\alpha-1}\Gamma(1+\frac{2}{\alpha})}{\alpha\zeta_s^{\frac{2}{\alpha}}} \exp\left(\frac{-\pi\mu_s z^{2/\alpha}}{\zeta_s^{2/\alpha}}\Gamma(1+\frac{2}{\alpha})\right). \tag{7}$$

Now, having the distribution of $P^{\mathrm{B}}_{l_s}$, we aim to analyze the coverage probability and average rate for both BUEs and SUEs. To do this, without loss of generality, we analyze the coverage probability and average rate for a typical SUE of a seller MNO and a typical BUE of a buyer MNO considering a network in which seller MNOs lease their sub-bands to buyer MNOs. Similarly, the coverage probability and average rate for other BUEs and SUEs are analyzed.

A. Coverage probability for buyer and seller MNOs

For a typical UE of MNO $k \in \mathcal{O}$ and a threshold β, the SINR coverage probability on sub-band $l_s \in \mathcal{L}_s$ is defined as the probability that SINR at this typical UE on sub-band $l_s \in \mathcal{L}_s$ is above β, that is

$$\mathcal{C}^{\mathcal{X}}_{k,l_s} = \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[\mathbb{P}\left[\gamma^{\mathcal{X}}_{k,l_s} > \beta\right]\right], \tag{8}$$

where (8) holds for any arbitrary UE owned by MNO $\mathcal{X} \in \{\mathrm{S},\mathrm{B}\}$, where S and B represent the seller and buyer MNOs, respectively. In the following Theorems, the coverage probability for a typical BUE and a typical SUE is computed, respectively.

Theorem 1. *The coverage probability for a typical BUE of buyer MNO b on sub-band $l_s \in \mathcal{L}_s$ is*

$$\begin{aligned}\mathcal{C}^{\mathrm{B}}_{b,l_s} &= \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[\mathbb{P}\left[\gamma^{\mathrm{B}}_{b,l_s} > \beta\right]\right] \\ &= \int_0^{+\infty} \pi\lambda_b \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right] \exp\left(-\beta\sigma^2 z^{\alpha/2}\right) \\ &\times \exp\left(-\pi\lambda_b \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right] z\right) \mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}\left(\beta z^{\alpha/2}\right)\mathrm{d}z,\end{aligned} \tag{9}$$

where $\rho(\alpha,\beta) = \int_{\beta^{-2/\alpha}}^{+\infty} \frac{1}{1+\nu^{\alpha/2}}\mathrm{d}\nu$, $\rho(\alpha,\infty) = \int_0^{+\infty} \frac{1}{1+\nu^{\alpha/2}}\mathrm{d}\nu$, and $\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}$ is the Laplace transform of the interference I^{B}_{b,l_s}. $\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}(\kappa)$ is defined as

$$\begin{aligned}\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}(\kappa) &= \exp\left(\pi\lambda_b \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\kappa^{2/\alpha}\rho(\alpha,\beta)\right) \\ &\times \exp\left(\pi\lambda_s {P^{\mathrm{S}}}^{2/\alpha}\kappa^{2/\alpha}\rho(\alpha,\infty)\right) \\ &\times \exp\left(\pi\lambda_{b,l_s} \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\kappa^{2/\alpha}\rho(\alpha,\infty)\right),\end{aligned} \tag{10}$$

where $\lambda_{b,l_s} = \sum_{k\in Q_{l_s}\setminus\{s,b\}} \lambda_k$.

Proof. The coverage probability for a typical BUE of buyer MNO b on sub-band $l_s \in \mathcal{L}_s$ is given by

$$\begin{aligned}
\mathcal{C}^{\mathrm{B}}_{b,l_s} &= \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[\mathbb{P}\left[\gamma^{\mathrm{B}}_{b,l_s} > \beta\right]\right] \\
&= \mathbb{E}_{P^{\mathrm{B}}_{l_s}}[\mathbb{P}(\frac{P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{0,l_s} {x^{\mathrm{B}}_0}^{-\alpha}}{I^{\mathrm{B}}_{b,l_s}+\sigma^2} > \beta)] \\
&\overset{(a)}{=} \mathbb{E}_{P^{\mathrm{B}}_{l_s}}[\int_0^\infty \mathbb{P}(\frac{P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{0,l_s} {x^{\mathrm{B}}_0}^{-\alpha}}{I^{\mathrm{B}}_{b,l_s}+\sigma^2} > \beta) f_b(x)\mathrm{d}x] \\
&\overset{(b)}{=} \mathbb{E}_{P^{\mathrm{B}}_{l_s}}[\int_0^\infty (P^{\mathrm{B}}_{l_s})^{2/\alpha} \mathbb{P}(\frac{h^{\mathrm{B}}_{0,l_s} u^{-\alpha}}{I^{\mathrm{B}}_{b,l_s}+\sigma^2} > \beta) f_b(u)\mathrm{d}u] \\
&\overset{(c)}{=} \int_0^\infty K(\lambda_n)\mathbb{E}[\exp\{-u^\alpha \beta(\sigma^2 + I^{\mathrm{B}}_{b,l_s})\} f_b(u)\mathrm{d}u] \\
&\overset{(d)}{=} \int_0^\infty K(\lambda_n)\exp(-u^\alpha\beta\sigma^2)\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}(u^\alpha\beta) f_b(u)\mathrm{d}u.
\end{aligned} \tag{11}$$

Where, (a) is from the fact that the typical user of buyer MNO b is associated to the BS of its MNO network providing the maximum average received power, in which the PDF of distance x between the typical user and its nearest BS is computed by

$$f_b(x) = 2\pi\lambda_b x\exp(-\pi\lambda_b x^2), \tag{12}$$

(b) is obtained using the transformation $x = (P^{\mathrm{B}}_{l_s})^{1/\alpha} u$, (c) is due to (3) and the definition of $K(\lambda_n)$, and (d) is from the moment generating function of h^{B}_{0,l_s}.

$\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}(\kappa)$, where $\kappa = u^\alpha\beta$ is the Laplace transform of I^{B}_{b,l_s} and is given by

$$\begin{aligned}
&\mathcal{L}_{I^{\mathrm{B}}_{b,l_s}}(\kappa) = \mathbb{E}[\exp(-\kappa(\sum_{f\in\mathcal{F}_b\setminus\{0\}} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha} + \\
&\sum_{f\in\mathcal{F}_s} P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} {x^{\mathrm{S}}_f}^{-\alpha} + \sum_{k\in Q_{l_s}\setminus\{s,b\}}\sum_{f\in\mathcal{F}_k} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha}))] \\
&\overset{(a)}{=} \mathbb{E}[\exp(-\kappa\sum_{f\in\mathcal{F}_b\setminus\{0\}} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha})] \times \mathbb{E}[\exp(-\kappa\sum_{f\in\mathcal{F}_s} \\
&P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} {x^{\mathrm{S}}_f}^{-\alpha})] \times \mathbb{E}[\exp(-\kappa\sum_{k\in Q_{l_s}\setminus\{s,b\}}\sum_{f\in\mathcal{F}_k} P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} {x^{\mathrm{B}}_f}^{-\alpha})] \\
&\overset{(b)}{=} \exp\big(-2\pi\lambda_b\mathbb{E}_{P^{\mathrm{B}}_{l_s}, h^{\mathrm{B}}_{f,l_s}}\big[\int_0^\infty (1-\exp(-\kappa P^{\mathrm{B}}_{l_s} h^{\mathrm{B}}_{f,l_s} x^{-\alpha})) \\
&x\mathrm{d}x]\big) \times \exp\big(-2\pi\lambda_s\mathbb{E}_{h^{\mathrm{S}}_{f,l_s}}\big[\int_0^\infty (1-\exp(-\kappa P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} x^{-\alpha})) \\
&x\mathrm{d}x]\big) \times \exp\big(-2\pi\lambda_{b,l_s}\mathbb{E}_{P^{\mathrm{B}}_{l_s}, h^{\mathrm{B}}_{f,l_s}}\big[\int_0^\infty (1-\exp(-\kappa P^{\mathrm{B}}_{l_s} h^{\mathrm{S}}_{f,l_s} \\
&x^{-\alpha}))x\mathrm{d}x]\big) \\
&\overset{(c)}{=} \exp\big(-2\pi\lambda_b\mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\mathbb{E}_{h^{\mathrm{B}}_{f,l_s}}\big[\int_0^\infty (1-\exp(-\kappa h^{\mathrm{B}}_{f,l_s} \\
&u^{-\alpha}))u\mathrm{d}u]\big) \times \exp\big(-2\pi\lambda_s\mathbb{E}_{h^{\mathrm{S}}_{f,l_s}}\big[\int_0^\infty (1-\exp(-\kappa P^{\mathrm{S}} h^{\mathrm{S}}_{f,l_s} \\
&x^{-\alpha}))x\mathrm{d}x]\big) \times \exp\big(-2\pi\lambda_{b,l_s}\mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\mathbb{E}_{h^{\mathrm{B}}_{f,l_s}}\big[\int_0^\infty (1- \\
&\exp(-\kappa h^{\mathrm{S}}_{f,l_s} u^{-\alpha}))u\mathrm{d}u]\big) \\
&\overset{(d)}{=} \exp\left(\pi\lambda_b\mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\kappa^{2/\alpha}\rho(\alpha,\beta)\right) \\
&\times\exp\left(\pi\lambda_s P^{\mathrm{S}\,2/\alpha}\kappa^{2/\alpha}\rho(\alpha,\infty)\right) \\
&\times\exp\left(\pi\lambda_{b,l_s}\mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\kappa^{2/\alpha}\rho(\alpha,\infty)\right).
\end{aligned} \tag{13}$$

Where, (a) is from independence of MNOs, (b) is due to PGFL of a PPP, (c) is obtained by using the transformation $x = {P^{\mathrm{B}}_{l_s}}^{1/2} u$ in the first and third terms, and (d) is due to using the MGF of h^{B}_{f,l_s} and h^{S}_{f,l_s}.

Substituting (12) and (13) in (11), and using the transformation $u^2 = z$, the proof of Theorem is completed. ■

In the following Theorem, the coverage probability for a typical SUE is given.

Theorem 2. *The coverage probability for a typical SUE of seller MNO s on sub-band $l_s \in \mathcal{L}_s$ is*

$$\begin{aligned}
&\mathcal{C}^{\mathrm{S}}_{s,l_s} = \mathbb{E}_{P^{B}_{l_s}}\left[\mathbb{P}\left[\gamma^{\mathrm{S}}_{s,l_s} > \beta\right]\right] = \int_0^{+\infty} \pi\lambda_s P^{\mathrm{S}\,2/\alpha} \\
&\times\exp\left(-\beta\sigma^2 z^{\alpha/2}\right)\exp\left(-\pi\lambda_s P^{\mathrm{S}\,2/\alpha} z\right)\mathcal{L}_{I^{\mathrm{S}}_{s,l_s}}\left(\beta z^{\alpha/2}\right)\mathrm{d}z,
\end{aligned} \tag{14}$$

where $\mathcal{L}_{I^{\mathrm{S}}_{s,l_s}}(t)$ is the Laplace transform of the interference I^{S}_{s,l_s}, that is

$$\begin{aligned}
\mathcal{L}_{I^{\mathrm{S}}_{s,l_s}}(\kappa) &= \exp\left(\pi\lambda_s P^{\mathrm{S}\,2/\alpha}\kappa^{2/\alpha}\rho(\alpha,\beta)\right) \\
&\times\exp\left(\pi\lambda_{s,l_s}\mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[{P^{\mathrm{B}}_{l_s}}^{2/\alpha}\right]\kappa^{2/\alpha}\rho(\alpha,\infty)\right),
\end{aligned} \tag{15}$$

where $\lambda_{s,l_s} = \sum_{b\in Q_{l_s}\setminus\{s\}}\lambda_b$.

Proof. The proof is similar to Theorem 1. The only difference is that the computations for the buyer and seller MNOs are interchanged and the downlink transmit power of seller MNOs is deterministic. ■

B. Average rate for buyer and seller MNOs

The average rate of a typical BUE of MNO b on all shared sub-bands is given by

$$\mathcal{R}^{\mathrm{B}}_b = \sum_{l_s:\, b\in Q_{l_s}} \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[\mathbb{E}_{\gamma^{\mathrm{B}}_{b,l_s}}\left[\ln\left(1+\gamma^{\mathrm{B}}_{b,l_s}\right)\right]\right], \tag{16}$$

where the sum is over all licensed sub-bands which are leased by buyer MNO b. Now, $\mathcal{R}^{\mathrm{B}}_b$ is presented in the following Theorem.

Theorem 3. *The average rate for a typical BUE of buyer MNO b is*

$$\begin{aligned}
\mathcal{R}^{\mathrm{B}}_b &= \sum_{l_s: b\in Q_{l_s}} \mathbb{E}_{P^{B}_{l_s}}\left[\mathbb{E}_{\gamma^{\mathrm{B}}_{b,l_s}}\left[\ln\left(1+\gamma^{\mathrm{B}}_{b,l_s}\right)\right]\right] \\
&= \sum_{l_s: b\in Q_{l_s}} \mathbb{E}_{P^{\mathrm{B}}_{l_s}}\left[\int_0^\infty \mathbb{P}\left[\ln\left(1+\gamma^{\mathrm{B}}_{b,l_s}\right) > t\right]\mathrm{d}t\right]
\end{aligned}$$

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
$\lambda_s, s \in \mathcal{S}$	$8/(\pi \times 500^2)$	$\lambda_b, b \in \mathcal{B}$	$[8, 16]/(\pi \times 500^2)$
$\mu_s, s \in \mathcal{S}$	$[50, 70]/(\pi \times 500^2)$	$\mu_b, b \in \mathcal{B}$	$50/(\pi \times 500^2)$
$P^{\rm S}$	10 dBm	$\zeta_s, s \in \mathcal{S}$	-100 dBm
α	5	σ^2	-120 dBm

$$= \sum_{l_s:b\in Q_{l_s}} \int_0^{\infty} \int_0^{+\infty} \pi\lambda_b \mathbb{E}_{P_{l_s}^{\rm B}} \left[{P_{l_s}^{\rm B}}^{2/\alpha} \right]$$
$$\times \exp\left(-(e^t-1)\sigma^2 z^{\alpha/2} \right) \exp\left(-\pi\lambda_b \mathbb{E}_{P_{l_s}^{\rm B}} \left[{P_{l_s}^{\rm B}}^{2/\alpha} \right] z \right)$$
$$\times \mathcal{L}_{I_{b,l_s}^{\rm B}} \left((e^t-1) z^{\alpha/2} \right) {\rm d}z \, {\rm d}t. \tag{17}$$

The average rate of a typical SUE of MNO s on all its sub-bands is expressed as

$$\mathcal{R}_s^{\rm S} = \sum_{l_s \in \mathcal{L}_s} \mathbb{E}_{P_{l_s}^{\rm B}} \left[\mathbb{E}_{\gamma_{s,l_s}^{\rm S}} \left[\ln\left(1+\gamma_{s,l_s}^{\rm S}\right) \right] \right], \tag{18}$$

that is provided in the following Theorem.

Theorem 4. *The average rate for a typical SUE of seller MNO s is*

$$\mathcal{R}_s^{\rm S} = \sum_{l_s \in \mathcal{L}_s} \mathbb{E}_{P_{l_s}^{\rm B}} \left[\mathbb{E}_{\gamma_{s,l_s}^{\rm S}} \left[\ln\left(1+\gamma_{s,l_s}^{\rm S}\right) \right] \right]$$
$$= \sum_{l_s \in \mathcal{L}_s} \mathbb{E}_{P_{l_s}^{\rm B}} \left[\int_0^{\infty} \mathbb{P}\left[\ln\left(1+\gamma_{s,l_s}^{\rm S}\right) > t \right] {\rm d}t \right]$$
$$= \sum_{l_s \in \mathcal{L}_s} \int_0^{\infty} \int_0^{+\infty} \pi\lambda_s {P^{\rm S}}^{2/\alpha} \exp\left(-(e^t-1)\sigma^2 z^{\alpha/2} \right)$$
$$\times \exp\left(-\pi\lambda_s {P^{\rm S}}^{2/\alpha} z \right) \mathcal{L}_{I_{s,l_s}^{\rm S}} \left((e^t-1) z^{\alpha/2} \right) {\rm d}z \, {\rm d}t. \tag{19}$$

IV. NUMERICAL RESULTS

In this section, we validate the analytical results and evaluate the performance of our adopted power control strategy. To this end, we consider a cellular network with four MNOs: two seller MNOs and two buyer MNOs. The BSs and UEs of MNOs are spatially distributed according to independent PPPs inside a circular area of 500 meter radius (as in [8] – [11]). All simulations have been carried out using MATLAB on a laptop with Intel Core i7 at 2.80 GHz and 8 GB of RAM. In addition, simulation results are obtained by averaging over 10000 simulation iterations. The summary of other simulation parameters is given in Table I.

Fig. 2 illustrates the coverage probability for seller and buyer MNOs versus SINR threshold (β) for different values of λ_b ($b \in \mathcal{B}$). For this purpose, we consider a scenario in which the first and second seller MNOs share a licensed sub-band with the first and second buyer MNOs, respectively. This scenario is called Scenario 1. It is worth noting that seller (buyer) MNOs have the same curves because their simulation parameters are the same. So, for simplicity, the curve of one of them is drawn. From Fig. 2, we can observe that our analytical expressions provided in Theorem 1 and 2 and simulations are perfectly matched. From this figure, it can be seen that increasing the value of buyer MNOs' BS intensity ($\lambda_b, b \in \mathcal{B}$) results in improving the coverage probability of the buyer MNOs, while bringing about a negligible impact on the seller MNOs' coverage probability. It is worth noting that the seller MNOs and buyer MNOs can gain significant coverage probability for their UEs by selecting a suitable value of BS intensity.

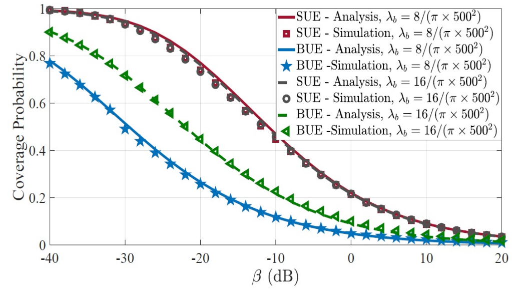

Fig. 2. Coverage probability of seller and buyer MNOs versus SINR threshold (β) for different values of λ_b ($b \in \mathcal{B}$).

Fig. 3 demonstrates the coverage probability for seller and buyer MNOs versus SINR threshold (β) for different values of μ_s ($s \in \mathcal{S}$) in Scenario 1. Similar to Fig. 2, the curve of one of buyer and seller MNOs is drawn. From Fig. 3, it is shown that with increasing the value of seller MNOs' UE intensity, the coverage probability of the seller MNOs remains fixed. Nonetheless, the coverage probability of the buyer MNOs decreases. The reason is that with increasing the UE intensity of the seller MNOs, $P_{l_s}^{\rm B}$ may decrease based on its definition in Lemma 2 leading to a lower coverage probability for the buyer MNOs. This claim has been confirmed in both simulation and analytical results in Fig. 3.

The per-user average rate of both the seller and buyer MNOs in different license sharing groups versus the maximum interference threshold (ζ_s) is observed in Fig. 4. For this purpose, we consider three scenarios. Scenario 1 is described in Fig, 2. In Scenario 2, the first seller MNO shares one more licensed sub-band with the first buyer MNO compared with Scenario 1, while this licensed sub-band is shared with both buyer MNOs in Scenario 3. For simplicity, the curves of the first seller MNO as Seller MNO-1 and the first buyer MNO as Buyer MNO-1 are plotted in Fig. 4. From this figure, we realize that increasing the maximum interference threshold (ζ_s) leads to a decrease in the per-user average rate for the seller MNO and an increase in the per-user average rate of the buyer MNO. The reason is that when the maximum interference threshold increases, the transmit power of buyer MNO's BSs ($P_{l_s}^{\rm B}$) based on (3) can increase that causes to a higher per-user average rate. Since the transmit power of the seller's BSs is constant, when the maximum interference threshold (ζ_s) increases, the seller MNO's per-user average rate decreases according to Theorem 4. Besides, the per-user average rate of the first buyer MNO is improved in Scenario 2 compared with

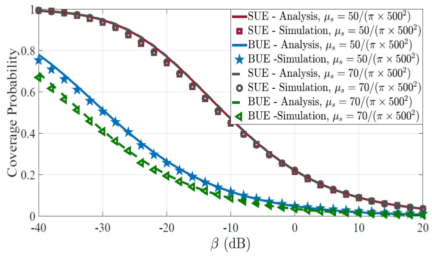

Fig. 3. Coverage probability of seller and buyer MNOs versus SINR threshold (β) for different values of μ_s ($s \in \mathcal{S}$).

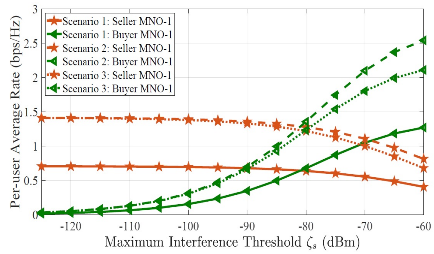

Fig. 4. The per-user average rate of the seller and buyer MNOs in different license sharing groups versus maximum interference threshold (ζ_s).

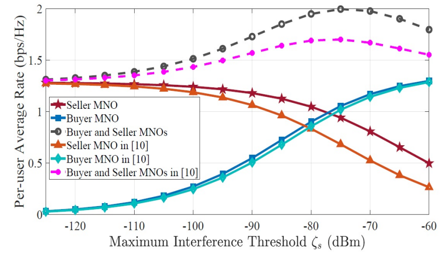

Fig. 5. The per-user average rate of the seller and buyer MNOs, as well as the total sum-rate versus maximum interference threshold (ζ_s).

Scenario 1 because the buyer MNO has two sub-bands, but it has one sub-band in Scenario 1. On the other hand, the per-user average rate of the first buyer MNO in Scenario 3 is less than Scenario 2 since the sub-bands of the first seller MNO are shared with both buyer MNOs, simultaneously, which leads to an increase in interference imposed on the buyer users.

The per-user average rate of both the seller and buyer MNOs, as well as their total sum-rate versus the maximum interference threshold (ζ_s) is observed in Fig. 5. From this figure, we realize that increasing the maximum interference threshold (ζ_s) leads to a decrease in the per-user average rate for the seller MNO and an increase in the per-user average rate of the buyer MNO. The reason is that when the maximum interference threshold increases, the transmit power of buyer MNO's BSs ($P_{l_s}^{\mathrm{B}}$) based on (3) can increase that causes to a higher per-user average rate. Since the transmit power of the seller's BSs is constant, when the maximum interference threshold (ζ_s) increases, the seller MNO's per-user average rate decreases according to Theorem 4. For a particular network setting, increasing the maximum interference threshold (ζ_s) contributes to an increasing total sum-rate; nonetheless, after some points of ζ_s, the total sum-rate decreases because the interference on SUEs is intensified. Another interesting observation is that there is a measurable value for the maximum interference threshold (ζ_s) that maximizes the total sum-rate of MNOs. Besides, we compare our adopted power control strategy provided in subsection II-D with the power control strategy introduced in [10]. In [10], a system consisting of one seller and one buyer MNO is considered where the buyer MNO adjusts the transmit power of its BSs according to the distance of the nearest SUE to each of its BSs. From Fig. 5, we see that our adopted power control strategy outperforms the another strategy proposed in [10]. In fact, our adopted power control strategy adjusts the transmit power of buyer MNO's BSs in such a way that the interference imposed on all corresponding SUEs is below ζ_s. Thus, the per-user average rate of seller MNO along with the total sum-rate improves significantly.

V. Conclusion

In this paper, we modeled a non-orthogonal spectrum sharing among seller and buyer MNOs employing stochastic geometry, where each seller MNO may lease several licensed sub-bands to buyer MNOs. Moreover, each buyer MNO could lease several licensed sub-bands from different seller MNOs. We assumed each MNO owns its BSs and UEs distributed based on two independent homogeneous PPPs. To reduce the interference on one seller MNO's UEs on one licensed sub-band, buyer MNOs had a limitation on the maximum interference imposed on the corresponding SUEs. After that, we analyzed the downlink coverage probability and the average rate of both seller and buyer networks. Finally, we validated our analysis with simulation and illustrated the performance of our adopted power control strategy via numerical results.

References

[1] Cisco, "Cisco visual networking index: Global mobile data traffic forecast update 2017-2022," White Paper, Feb. 2019.

[2] G. Gui, M. Liu, F. Tang, N. Kato, and F. Adachi, "6G: Opening new horizons for integration of comfort, security, and intelligence," *IEEE Wireless Communications*, vol. 27, no. 5, pp. 126-132, Oct. 2020.

[3] R. Umar, A. U. H. Sheikh, M. Deriche, M. Shoaib, and M. Hadi, "Multi-operator spectrum sharing in next generation wireless communications networks: A short review and roadmap to future," in *2017 International Symposium on Wireless Systems and Networks (ISWSN)*, pp. 1-5, 2017.

[4] S. K. Joshi, K. B. S. Manosha, M. Codreanu, and M. Latva-aho, "Dynamic inter-operator spectrum sharing via lyapunov optimization," *IEEE Transactions on Wireless Communications*, vol. 16, no. 10, Oct. 2017.

[5] Y. Xiao, M. Hirzallah, and M. Krunz, "Distributed resource allocation for network slicing over licensed and unlicensed bands," *IEEE Journal on Selected Areas in Communications*, vol. 36, no. 10, pp. 2260-2274, Oct. 2018.

[6] P. Luoto, M. Bennis, P. Pirinen, S. Samarakoon, and M. Latva-aho, "Enhanced co-primary spectrum sharing method for multi-operator networks," *IEEE Transactions on Mobile Computing*, vol. 16, no. 12, pp. 3347-3360, Dec. 2017.

[7] M. Asaduzzaman, R. Abozariba, and M. Patwary, "Dynamic spectrum sharing optimization and post-optimization analysis with multiple operators in cellular networks," *IEEE Transactions on Wireless Communications*, vol. 17, no. 3, pp. 1589-1603, March 2018.

[8] A. K. Gupta, J. G. Andrews, and R. W. Heath, "On the feasibility of sharing spectrum licenses in mmWave cellular Systems," *IEEE Transactions on Communications*, vol. 64, no. 9, pp. 3981-3995, Sept. 2016.

[9] T. Sanguanpuak, S. Guruacharya, E. Hossain, N. Rajatheva, and M. Latva-aho, "On spectrum sharing among micro-operators in 5G," in *2017 European Conference on Networks and Communications (EuCNC)*, pp. 1-6, 2017.

[10] A. K. Gupta, A. Alkhateeb, J. G. Andrews, and R. W. Heath, "Gains of restricted secondary licensing in millimeter wave cellular systems," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 11, pp. 2935-2950, Nov. 2016.

[11] Z. Yan, X. Zhang, H. Liu, and Y. Liang, "An efficient transmit power control strategy for underlay spectrum sharing networks with spatially random primary users," *IEEE Transactions on Wireless Communications*, vol. 17, no. 7, pp. 4341-4351, July 2018.